Outline

Review

Live Variable Analysis

Live Variables using Iterative Data flow Analysis

Postscript
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What we know how to do so far

- Basic blocks to Control Flow Graphs
  - Section 5.4.3 in Cooper and Turczon
- Local optimizations (Value Numbering)
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Postscript
void example(int x) {
    int a = 0;
    int b = 0;

    if(x > 1) {
        a = b;
    } else {
        a = x;
    }

    return a;
}

Must the compiler emit code to initialize a to 0?
Motivation

```c
void example(int x) {
    int a = 0;
    int b = 0;

    if(x > 1) {
        a = b;
    } else {
        a = x;
    }

    return a;
}
```

A is *defined* (i.e. written) thrice in the code above. Which definitions are “alive” at `return a`?
A variable $x$ is live at a definition (i.e. a write) if there is a subsequent read of the variable along a program path from that definition.

Can we identify live variables just by analyzing basic blocks (i.e. local analysis)?
void test() {
    int i;

    // begin delay
    for(i = 0; i < 10000; i++);
    // end delay
}

GCC without optimization

test():
    push rbp
    mov rbp, rsp
    mov DWORD PTR [rbp-4], 0
    .L3:
    cmp DWORD PTR [rbp-4], 9999
    jg .L4
    add DWORD PTR [rbp-4], 1
    jmp .L3
    .L4:
    nop
    pop rbp
    ret

Code courtesy Compiler Explorer at godbolt.org (this is more convenient than gcc -Wa if you have the Internet)
GCC with -O3

test():
    ret

Code courtesy Compiler Explorer at godbolt.org
Example 1

```c
x = 0;
if(y > 0)
    x = 2;
else
    x = 3;
printf("%d\n", x);
```
Example 2

```c
x = 0
i = 0
while (i++ < 10) {
    if (i % 3 == 0) {
        x = 0;
    }
    x = x + 1;
}
```

▶ Which definition(s) of `x` are live at entry of loop?
▶ Which definition(s) of `x` are live in the loop?
▶ Which definition(s) of `x` are live at exit of loop?
i = 1;
L1:
    if(!(i < 10000)) goto L2;
i++;
goto L1;
L2:
    return

Where in the CFG must i be “dead” to eliminate it?
Some Definitions

For a basic block $s$:

- $\text{DEF}(s)$ (for “defined”) is the set of variables written by $s$
- $\text{USE}(s)$ is the set of variables read by $s$

For the blocks below, what are the $\text{DEF}$ and $\text{USE}$ sets?

$\text{BB}_\text{eg}_0$:
\[
i = i + 1
\]

$\text{BB}_\text{eg}_1$:
\[
a = x; \\
x = 3;
\]

$\text{BB2}_\text{eg}_2$:
\[
t = a;
\]
Effects of definitions and reads

BB_eg_3:
  a = x;
  x = y;

▶ What happens to a definition of a in a basic block that is before BB_eg_3 on some path through the CFG?
▶ What can we say about definitions of y in other basic blocks that are before BB_eg_3 on some path through the CFG?
More definitions

- Upwardly exposed $\text{UE}(s)$ are those variables that are read before they are written to in basic block $s$.
- A variable is alive in a basic block if it is in the set: $\ ?$
- A variable is killed in a basic block if it is in the set: $\ ?$
Finding out if a definition is live

- Start from the definition, and see if it “reaches” a use
  - i.e., it is not killed on some path (in the CFG) to a use
- Start from a use, and walk backwards to see if you can reach the definition
  - Repeat for all uses

When does enumerating paths work in practice?
A More Practical Way: Iterative Data flow Analysis

- Setup data flow equations
- Solve these data flow equations
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Equations for Live Variables

- Remember that each basic block has “incoming” edges (from predecessors) and “outgoing” edges (to successors).
- When is an “incoming” variable live “into” a block?
  - it is in UE for that block
- When is a variable live out of a block?
  - it is live into a successor block
  - first try: \( \text{LIVEOUT}(n) = \bigcup_m \text{UE}(m) \) (where \( m \in \text{succ}(n) \))
  - what are we missing?
LIVEOUT(n) = \bigcup_{m \in \text{succ}(n)} \text{UE}(m) \cup (\text{LIVEOUT}(m) \cap \overline{\text{DEF}(m)})

The variables live out of block n must:

- be upwardly exposed in a successor m
- OR be live out of m and NOT be killed by m (i.e. \overline{\text{DEF}(m)})
  - note this is equivalent to (\text{LIVEOUT}(m) - \text{DEF}(m))

How would you begin solving this equation?
Iterative Data flow Analysis Algorithm for Live Variables

- Live variables is a *backwards dataflow analysis*
  - ‘Facts” flow from a node’s successors to it
- Initialization:
  - Compute DEF(s) for all blocks s
  - Compute UE(s) for all blocks s
  - Set LIVEOUT(s) = ∅ for all s
- Repeat (iterate):
  - Compute LIVEOUT(s) for all blocks s
  - Until no set LIVEOUT(s) changes – i.e. until a fixpoint is reached.
Termination and correctness

- **Termination**
  - Note \( \text{LIVEOUT}(s) \) only increases in size or remains the same in every iteration
  - What is the maximum size of \( \text{LIVEOUT}(s) \)?

- **Correctness**
  - When the algorithm terminates, all computed \( \text{LIVEOUT}(s) \) sets satisfy the equation
  - and hence meet the definition of “liveness”.
Quality of solution?

How does the iterative solution differ from the path-based solution?
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References

▶ Chapter 8 of Cooper and Turczon
  ▶ Section 8.6.1
▶ Also recommended:
  ▶ Aho, Lam, Sethi and Ullman, Chapter 9, Section 9.2.5