

# **CSC2/455 Software Analysis and Improvement**

## **Type Inference - II**

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URCS

Type Systems for Realistic Languages

Type Checking ChocoPy

Postscript

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# Statements and Expressions

- The type system we studied in the last class was for a functional language
  - Programs consist of expressions
  - All well-formed expressions have a type
- Some languages, notably imperative languages (Python, C, etc.), differentiate between *statements* and *expressions*
  - Expressions have types
  - Statements do not have types
  - “Procedures” do not have return values

## Additional judgements for imperative languages

- $\Gamma \vdash C$ 
  - $C$  is a well-formed command (or statement) in  $\Gamma$

In general, block-structured languages may have  $C$  be a block as well.

# Subtyping

- Most languages have a subtype relation
  - $C$  has *unsigned short* is a subtype of *unsigned int*
  - Subtypes can be used wherever the "supertype" is used
- Usually denoted by  $A \leq B$ 
  - $A$  is a subtype of  $B$
  - $A \leq A$ , obviously
- Type hierarchy, a tree of types
  - Usually object as the root
  - A edge from  $B$  to  $A$  if  $A < B$

# Outline

Type Systems for Realistic Languages

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- ChocoPy is simpler version of Python3
  - Developed as a “toy” language for compiler courses
- Fully-specified
  - Syntax (BNF)
  - Type System
  - Semantics (Operational Semantics)
- We will only look at its Type System



# ChocoPy Types

- Basic types: `object`, `int`, `bool`, `str`
- List type: `[T]`, where `T` is another type
  - e.g. `[int]`
  - `T` be a list as well, this is an example of a *recursive type*
  - `[[int]]`
- Special types
  - `<None>`
  - `<Empty>`, indicating an empty list `[]`

# ChocoPy Type Hierarchy

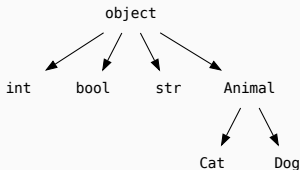
- $A \leq A$ , for all types  $A$
- $C \leq A$  if  $C$  is a subclass of  $A$ 
  - `int`, `bool`, `str` are all subclasses of `object`
- $[T] \leq \text{object}$ 
  - but no relationship between two different list types
- $\langle \text{None} \rangle \leq \text{object}$
- $\langle \text{Empty} \rangle \leq \text{object}$

# ChocoPy Assignment Compatibility

A type  $T_1$  is assignment compatible ( $\leq_a$ ) to  $T_2$  iff values of type  $T_1$  can be used wherever values of  $T_2$  are expected.

- $T_1 \leq T_2$  (note: this is subtyping  $\leq$ )
- $T_1$  cannot be `<None>` if  $T_2$  is `int`, `bool` or `str`
- $T_1$  can be `<Empty>` only if  $T_2$  is `[T]`
- $T_1$  can be `[<None>]` if  $T_2$  is `[T]`, where `<None>  $\leq_a$  T`
  - Note that means T can only be object (\*)

# Join of types



The join  $\sqcup$  of two types  $A$  and  $B$  denoted as  $A \sqcup B$  is defined as:

- $B$  if  $A \leq_a B$ 
  - This is commutative, so  $B \sqcup A$  is also  $B$  in this case
- $C$  where  $C$  is the least common ancestor of  $A$  and  $B$  in the type hierarchy defined by  $\leq$ 
  - So  $\text{int} \sqcup \text{str}$  is  $\text{object}$
  - Note this is the *least upper bound* or LUB (analogous to GLB in meet lattices)

# Type Environment

- Judgements of the form:
  - $O, M, C, R \vdash \dots$
- Where the type environment consists of:
  - $O$ , “local” environment
  - $M$ , method/attribute environment, i.e. for  $A.x$ -style code
  - Both are maps
- And:
  - $C$ , the name of the current class, can be  $\perp$  if outside class
  - $R$ , the return type of the current function, can be  $\perp$  if outside function

## Function Type Notation in ChocoPy

```
def contains(items:[int], x:int) -> bool:
    i:int = 0
    while i < len(items):
        if items[i] == x:
            return True
        i = i + 1
    return False
```

The general type for a function  $f$  is  $O(f) = \{T_1 \times T_2 \times \dots \times T_n \rightarrow T_0; x_1, x_2, \dots, x_n; v_1 : T'_1, v_2 : T'_2, \dots, v_m : T'_m\}$

- $T_1 \times T_2 \times \dots \times T_n \rightarrow T_0$  is the type of arguments and the return type
  - Here,  $[int] \times int \rightarrow bool$
- $x_1, x_2, \dots, x_n$  are names of the arguments: `items, x`
- $v_1 : T'_1, \dots, v_m : T'_m$  are types of variables/functions declared inside the function
  - Here `i : int`

# Method/Attribute Type Notation in ChocoPy

```
class Point1D:  
    x: int = 0  
  
    # the quotes are used since Point1D is being defined ...  
    def distance(self: "Point1D", o: "Point1D") -> int:  
        ...
```

- Attributes are simply  $M(C, a) = T$ , where  $C$  is the class,  $a$  is the attribute, and  $T$  is its type
  - So,  $M(\text{Point1D}, x) = \text{int}$
- And methods  $M(C, m)$  are just like functions in the previous slide
  - So,  
 $M(\text{Point1D}, \text{distance}) = \{\text{Point1D} \times \text{Point1D} \rightarrow \text{int}; \dots\}$

# Type Judgements in ChocoPy

- $O, M, C, R \vdash e : T$ 
  - Expression  $e$  has type  $T$  in the environment  $O, M, C, R$
- $O, M, C, R \vdash b$ 
  - $b$  is a well-formed (block) statement in  $O, M, C, R$
  - can also be a single statement, obviously
- If reading the ChocoPy reference, make sure to distinguish the colon in type judgements  $e : T$  from the colon in Python syntax
  - I'll use  $:$  to indicate the latter, and  $\vdash$  to indicate the former



# Expressions

$$\frac{}{O, M, C, R \vdash \text{True} : \text{bool}} \text{(BOOL-TRUE)}$$

$$\frac{}{O, M, C, R \vdash \text{False} : \text{bool}} \text{(BOOL-FALSE)}$$

$$\frac{i \text{ is an integer literal}}{O, M, C, R \vdash i : \text{int}} \text{(INT)}$$

$$\frac{s \text{ is a string literal}}{O, M, C, R \vdash s : \text{str}} \text{(STR)}$$

$$\frac{O(id) = T \quad T \text{ is not a function type}}{O, M, C, R \vdash id : T} \text{(VAR-READ)}$$

## Variable Definitions

$$\frac{O(id) = T \quad O, M, C, R \vdash e_1 : T_1 \quad T_1 \leq_a T}{O, M, C, R \vdash id : T = e_1} \text{(VAR-INIT)}$$

# Statements

$$O, M, C, R \vdash s_1$$
$$O, M, C, R \vdash s_2$$
$$\vdots$$
$$O, M, C, R \vdash s_n$$

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$$O, M, C, R \vdash s_1 \text{ NEWLINE } s_2 \dots s_n \text{ NEWLINE} \quad (\text{STMT-DEF-LIST})$$

# Operators

$$\frac{\begin{array}{l} O, M, C, R \vdash e_1 : int \\ O, M, C, R \vdash e_2 : int \\ op \in \{+, -, *, //, \%\} \end{array}}{O, M, C, R \vdash e_1 \text{ op } e_2 : int} \text{(ARITH)}$$

$$\frac{\begin{array}{l} O, M, C, R \vdash e_1 : int \\ O, M, C, R \vdash e_2 : int \\ \bowtie \in \{<, <=, >, >=, ==, !=\} \end{array}}{O, M, C, R \vdash e_1 \bowtie e_2 : bool} \text{(COMPARE-INT)}$$

$$\frac{\begin{array}{l} O, M, C, R \vdash e_0 : bool \\ O, M, C, R \vdash e_1 : T_1 \\ O, M, C, R \vdash e_2 : T_2 \end{array}}{O, M, C, R \vdash e_1 \text{ if } e_0 \text{ else } e_2 : T_1 \sqcup T_2} \text{(IF-EXPR)}$$

## Return and Class Definitions

$$\frac{O, M, C, R \vdash e : T \quad T \leq_a R}{O, M, C, R \vdash \text{return } e} \text{(RETURN-E)}$$

$$\frac{O, M, C, R \vdash b}{O, M, \perp, R \vdash \text{class } C(S) : b} \text{(CLASS-DEF)}$$

# Function Definitions

```
x: str = "hello, world"

def somefn(x: int, y: str):
    # what is the type of x here?
```

- Function argument names:
  - shadow names outside the function (like `x` here)
  - or are introduced in the definition (like `y`)
- The notation for shadowing or introducing a variable is:
  - $O[T/x]$  which is defined as  $O[T/x](x) = T$  and  $O[T/x](y) = O(y)$  where  $y \neq x$
  - i.e.,  $O[T/x]$  is a new map with  $x$  mapped to  $T$  and the other mappings unchanged
  - repeated applications are possible:  $O[T_1/x][T_2/y]$

## Function Definition - II

$$T = \begin{cases} T_0, & \text{if } \rightarrow \text{ is present,} \\ \langle \text{None} \rangle, & \text{otherwise.} \end{cases}$$

$$O(f) = \{T_1 \times \dots \times T_n \rightarrow T; x_1, \dots, x_n; v_1 : T'_1, \dots, v_m : T'_m\}$$

$$n \geq 0 \quad m \geq 0$$

$$O[T_1/x_1] \dots [T_n/x_n][T'_1/v_1] \dots [T'_m/v_m], M, C, T \vdash b$$

$$\frac{O[T_1/x_1] \dots [T_n/x_n][T'_1/v_1] \dots [T'_m/v_m], M, C, T \vdash b}{O, M, C, R \vdash \mathbf{def} f(x_1:T_1, \dots, x_n:T_n) [\rightarrow T_0]^? : b}$$

[FUNC-DEF]

## Method Definition

$$T = \begin{cases} T_0, & \text{if } \rightarrow \text{ is present,} \\ \langle \text{None} \rangle, & \text{otherwise.} \end{cases}$$

$$M(C, f) = \{T_1 \times \dots \times T_n \rightarrow T; x_1, \dots, x_n; v_1 : T'_1, \dots, v_m : T'_m\}$$

$$n \geq 1 \quad m \geq 0$$

$$C = T_1$$

$$O[T_1/x_1] \dots [T_n/x_n][T'_1/v_1] \dots [T'_m/v_m], M, C, T \vdash b$$

$$\frac{}{O, M, C, R \vdash \text{def } f(x_1:T_1, \dots, x_n:T_n) \llbracket \rightarrow T_0 \rrbracket^? : b}$$

[METHOD-DEF]



# Outline

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## References

- A self-contained introduction to type systems
  - Luca Cardelli, Type Systems, Handbook of Computer Science and Engineering, 2nd Ed
- An updated version (available only through the library)
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- Leandro TC Melo, Rodrigo G Riberio, Breno CF Guimaraes, Fernando Magno Quintao, Type Inference for C: Applications to the Static Analysis of Incomplete Programs, ACM TOPLAS
  - The psychec tool