advice

Λ 124 $\begin{array}{c|c} \|A\| & 124 \\ \triangle & 124 \end{array}$ \_ Ø 123  $\stackrel{\scriptstyle p}{\equiv}{}^{p}_{k\text{-}tt} \quad 98,99 \\ \geq_{\text{lex}} \quad 82$  $\geq lex \in \mathbb{R}$   $\cap 124$ 124 $\oplus$ |a|12363 85, 105–107, 115, 117  $\begin{array}{c} 85,105{-}107\\ \leq_{poly-T}^{e} 115\\ \leq_{pos}^{poly-T} 8,81,8\\ \leq_{ptt}^{p} 8,85,86\\ \leq_{r} 8,61,85,\\ \leq_{T}^{e} 115\\ \leq_{T}^{p} 2,8,27,\\ \leq_{T}^{sn} 42,44,1\\ \leq_{T}^{sn} 42,44,1\\ \leq_{t}^{p} 8,61,62\\ - 124 \end{array}$ 8, 81, 85, 86, 88, 115 8, 85, 86, 116 8, 61, 85, 94, 117, 118, 122 2, 8, 27, 42, 44, 67, 68, 76, 99, 11542, 44, 116 8, 61, 62, 65-70, 94, 108, 116 124 $\neg 124$  $\vee$ 124 $\prec$ 124U 124 $\sim 26$ Abadi 40 advice 19, 20, 24, 27, 28, 30-36, 39, 52, 56, 109 $- \sim$  for nondeterministically selective sets 32  $- \sim \text{for P-selective sets} = 17, 20$  $- \sim$  interpreter see interpreter, analog

- class of  $\sim$  interpreters 31 improvement below linear  $\sim$  for recognition of P-selective sets 31– length-bounded  $\sim$  19 - linear  $\sim$  24, 26, 30, 31, 39 - logarithmic  $\sim$  39 - lying  $\sim$  33 – polynomial  $\sim$  19, 39 - quadratic  $\sim 30$ - recursive  $\sim$  39 - small  $\sim$  3 – strong  $\sim$  35–37 Agrawal 77, 113 algorithm 16, 17, 24, 26, 30, 47, 52, 64, 70, 72, 86, 90, 92-94  $-\sim$  finding the king of a tournament - deterministic polynomial-time  $\sim v$ , 1, 2, 5, 29, 30, 64-66, 69, 72, 89- intelligent  $\sim$  vi – linear-exponential-time  $\sim$  27 nondeterministic polynomial-time  $\sim$ 14,90,93 - nondeterministic recursive  $\sim$  92 – output of a polynomial-time  $\sim$  – v - P-time decoding  $\sim$  for advice 23 – polynomial-time membership  $\sim 2$ – recursive  $\sim$  v – semi-membership  $\sim s - 2$  $- \text{ smart} \sim - \text{vi}$ - standard brute-force conversion of a nondeterministic  $\sim$  to a deterministic – theory of semi-feasible  $\sim s$  iii Allender vii, viii, 58, 59 ambiguity – polynomial  $\sim$  119 Amir 59 - complexity-theoretic  $\sim v, 1$ 

- complexity-theoretic  $\sim$  of the low and high hierarchies from recursive function theory 58 – nondeterministic  $\sim$  of P – 9 – nondeterministic  $\sim$  to standard deterministic results on self-reducibility 75Appel 16 approximability 105 - bounded  $\sim$  113 APT - almost polynomial time 2,16 argument 1, 4, 10, 17, 23, 32, 33, 63, 69, 76, 84, 93, 106, 108 - counting  $\sim 22$ - divide and conquer  $\sim 54, 57$ Arvind 77, 113 assignment - collection of  $\sim s = 72, 73$ – nondeterministic selection of one  $\sim$ 38- satisfying  $\sim$  72 associativity 105, 111-113 assumption 2, 29, 30, 32, 35, 37, 46, 67, 74, 94, 101 - complexity-theoretic  $\sim s = 2$ Babai 59 Balcázar vii, 58, 59 bAPP 113 Barrington 16.39 Beigel 59, 76, 77, 103, 113 bit 17, 19, 23, 24, 27, 29, 31, 33, 66, 70-73, 82, 86, 87, 91, 97-99, 116  $- \sim s \text{ of advice} \quad 19, 20$ - input  $\sim$ s 18 nondeterministic guess  $\sim 62, 69$ bitflip 99 bitstring 29, 71, 97, 98, 109, 116 Book 16, 58, 59 bound 24, 32, 46, 48, 51 - absolute lower  $\sim$  on lowness 46 - absolute upper  $\sim s$  46 - adjacent upper and lower  $\sim s$  46 - length  $\sim$  24 - lower  $\sim$  20, 30–32, 39, 46, 48, 58, 59 - nontrivial lower  $\sim$  on lowness 48 – polynomial  $\sim$  47 - query  $\sim$  43 - relativized lower  $\sim$ s on lowness 58 - time  $\sim$  16, 26, 96 - upper  $\sim$  17, 20, 21, 25, 28, 32, 34, 46, 49, 51, 53, 55, 57-59

upper  $\sim$ s on the amount of advice for P-selective sets 21– upper and lower  $\sim s$  49 - upper and lower  $\sim$ s on lowness 46 Bovet vii BPP 122 Brauer viii Buhrman vii, viii, 76, 77, 103 Burtschick 39,76 Cai viii, 39, 40, 59, 76, 77, 113 certificate 33  $-\sim$  certifying the guessed answer 12 – advice  $\sim$  34 – membership  $\sim$  33, 34 succinct  $\sim 12$ Chakaravarthy 40 Chang 59 characterization - complete  $\sim$  of the P-selective sets 6 – complete  $\sim$  of the semi-recursive sets 6 Chari 59 circuit 18, 19 –  $\sim$  for a P-selective set 18  $- \sim$  for an arbitrary language 18  $- \sim \text{for } \Sigma^*$ 18  $-\sim$  recognizing a sparse set 17 - collection of  $\sim s$  18 encoding of a  $\sim$  18 – exponential-size  $\sim s = 18$ – families of  $\sim s$  17 polynomial-size  $\sim s$  19 polynomial-sized family of  $\sim$ s for P-selective sets 18 – size-bounded  $\sim s$  19  $- \text{ small } \sim s \quad v, 3, 18, 107$ claim 4, 13, 24, 25, 45, 48, 52, 63, 68, 74, 96, 103 relatively typical optimal lowness  $\sim$ 48- relativizable  $\sim$  13 class 11, 12, 41, 43, 48, 105, 106, 113 –  $\sim$  closed under composition with logspace functions 20  $-\sim$  containing Turing self-reducible complete sets 75  $- \sim \text{known as MA}_{\text{EXP}}$  76  $-\sim$  of advice interpreters 31  $-\sim$  of functions 121

 $- \sim$  of functions computable via  $\mathcal{O}(\log n)$  Turing queries to NP 68

59

117

81

 $-\sim$  of functions computable via – incomparable  $\sim$ es: P-sel and polynomial-time truth-table access weakly-P-rankable 110 to NP 68  $- L_{\Delta_k^p} \sim es \quad 58$  $- \sim \text{ of languages } 120, 121, 123$ – length-based advice  $\sim$  117  $-\sim \text{of oracles}$  121 - low  $\sim$ es 43  $- \sim \text{ of P-selective sets} \quad 81$ - lowness  $\sim es$  43  $- \sim$  of sets having interactive proofs  $- \mathcal{L}_{\Sigma_k^p} \sim \text{es} \quad 43, 58$ 59 $-L_{\Theta_{L}^{p}} \sim es 58$  $- \sim$  of sets of simple organization 43 – membership in nondeterministic  $\sim$ es  $- \sim \text{ of sets reducible to P-sel}$ 2680  $-\sim$  of the form C/poly 34 – membership in the complexity  $\sim P$  $-\sim$ es in the extended low hierarchy 1 45– nondeterministic function  $\sim es$  9  $-\sim$ es of nondeterministically selective nondeterministic selectivity  $\sim es - 9$ , sets 49 81  $\sim$ es that are not subsets of P/poly optimal lower bounds for most 76extended-lowness  $\sim es$  $\sim$ es that lack hard P-selective sets reduction  $\sim$  97 61 reduction and equivalence  $\sim es - 94$ , - advice  $\sim$  19, 34 95- advice upper bounds for reductions – refined advice  $\sim$  20 to selectivity  $\sim es = 22$ - refinement of multivalued nondeteradvice upper bounds for selectivity ministic function  $\sim es$  40  $\sim es = 21$ - relationships between nonde-- arbitrary  $\sim$  of selector functions 9 terministic selectivity  $\sim es = 12$ , - classic low  $\sim es$  41 13- collapse of  $\sim es = 12$ , see collapse – relativizable  $\sim$  43 – complement of a  $\sim$  11 - relativized  $\Sigma_k^p \sim -43$ complexity  $\sim 3, 8, 16, 19, 41, 61, 62$ , - selectivity  $\sim es = 10$ 79, 116, 117, 119 semi-recursive sets as a  $\sim$  from complexity  $\sim$ es near polynomial time recursive function theory v separation of reduction  $\sim$ es from - degree of organizational simplicity of equivalence  $\sim es$  96 selectivity  $\sim es$  43 - set of equivalence  $\sim es -7, 124$  $-\operatorname{EL}_{\Delta_k^p} \sim \operatorname{es} -58$  set-wise complements of a complexity  $- \operatorname{EL}_{\Sigma^p_{\iota}} \sim \operatorname{es} 58$  $\sim 116$  $- \operatorname{EL}_{\Theta_{h}^{p}} \sim \operatorname{es} 58$ structure of polynomial-time equalities and inequalities of recomplexity  $\sim es$  48 duction and equivalence  $\sim es$  of  $\Theta^p_{k} \sim \text{es} \quad 43$ P-selective sets – time-bounded  $\sim$  20 103- equivalence  $\sim 8, 97, 100, 124$ – token-based advice  $\sim$ exponential-time complexity  $\sim es$ 7. classification 16–  $\sim$  of sets in NP using the low hierarchy 42 first two levels of the high hierarchy are well-known  $\sim es$  42 clique - function  $\sim es$  19, 119, see function,  $-q-\sim$ 63 class of closure - function analog of  $\sim es$  68  $-\sim$  properties of P-sel 6 – hardness for complexity  $\sim es$ 61  $-\sim$  under 2-ary connectives of P-sel, - high  $\sim$  42 NPSV<sub>t</sub>-sel, and NPMV<sub>t</sub>-sel 84- inclusion properties of nondetermin-–  $\sim$  under bounded-truth-table istic advice  $\sim es$  39 reductions of a class

 $\mathbf{2}$ 

135

| _  | $\sim$ under combined self-reducibility<br>and 1-truth-table reductions of P-sel<br>103                                       | $- \sim$ of the polynomial hierarchy being<br>a consequence of unique solutions for<br>SAT 38   |
|----|---|---|
| -  | ~ under complement of NPMV-sel $15.82$  | - surprising $\sim$ of complexity classes 61  |
| _  | ~ under complement of NPMV <sub>t</sub> -sel 11, 15, 34   | – unexpected $\sim$ of complexity classes 3   |
| -  | $\sim$ under complement of NPSV-sel<br>and NPMV <sub>t</sub> -sel 81  | collection $- \sim \text{ of strings}  53, 56, 57$  |
| _  | $\sim$ under complement of P-sel 24,<br>79, 81, 83  | commutativity 111, 112<br>comparability   |
| _  | $\sim$ under conjunctive reductions of NP 79  | - membership $\sim 105, 113$<br>- $\mathcal{O}(\log n)$ membership $\sim 108$   |
| _  | $\sim$ under connectives of P-sel 102<br>$\sim$ under connectives of selectivity  | complement<br>$- \sim \text{ of a P-selective set}  24, 26, 79$   |
| _  | classes $83, 84$<br>~ under disjunctive reductions of NP  | $- \sim \text{ of a set}  124$<br>$- \sim \text{ of an NPSV-selective set}  12$   |
| _  | 79<br>$\sim$ under intersection of P-sel 82   | complementation $- \sim$ and connectives 84   |
| _  | ~ under <i>k</i> -ary connectives of P-sel,<br>NPSV <sub>4</sub> -sel and NPMV <sub>4</sub> -sel 84                           | $\begin{array}{ll} - \text{ closure under } \sim \text{ of NPMV-sel} & 82 \\ - \text{ closure under } \sim \text{ of NPMV}_t\text{-sel} & 11, \end{array}$                    |
| -  | $\sim$ under many-one reductions of NP 79   | 15 - closure under ~ of NPSV <sub>t</sub> -sel and  |
| _  | $\sim$ under many-one reductions of the levels of the low hierarchy 46  | NPMV <sub>t</sub> -sel 81<br>– closure under ~ of P-sel 5, 24, 79,  |
| -  | $\sim$ under nonpositive reductions of P-sel 79   | 81, 83<br>completeness 61, 117  |
| -  | $\sim$ under NXOR and XOR of nondeterministic selectivity classes   | $-\sim$ for NP v<br>$-\sim$ for NP under many-one reductions  |
| _  | $^{83}$ $\sim$ under positive reductions of P-sel   | $^{2}$ - $\sim$ for NP under Turing reductions  |
| _  | 103 $\sim$ under positive Turing reductions   | $- \leq_{m}^{p} \sim \text{for UP}  66$   |
| _  | of P-sel 79, 80 $\sim$ under reductions of P-sel 80, 85   | $- \leq_T \sim \text{ for NP}  42$ $- \mathcal{C} \sim  61,117$   |
| -  | $\sim$ under Turing reductions of EXP 79  | $\begin{array}{ccc} -\mathcal{C}-\underline{\leq_{m}}-\infty & 117\\ -\mathcal{C}-\underline{\leq_{r}}-\infty & 61,117\\ \text{high hierarchy of a hierarchy of} \end{array}$ |
| -  | boolean $\sim$ of a complexity class 79, 81   | $\begin{array}{c} $   |
| _  | downward $\sim$ of P-selective sets 79<br>downward $\sim$ under 1-truth-table   | complexity 8  |
| _  | reductions 88 extension of $\sim$ to NPSV <sub>t</sub> -selectivity   | - advice $\sim 111$   |
| _  | 103 reduction $\sim 85.88$  | - arbitrary $\sim -8, 17, 59$<br>- capture of $\sim - v$  |
| -  | relativized world $\sim$ , optimal for<br>self-reducible P-selective sets 103   | - computational $\sim$ of a gappy left cut<br>31  |
| cc | ollapse   | - computational $\sim$ of a P-selective set   |
| -  | $\sim$ of the boolean hierarchy 76  | 18  |
| _  | $\sim$ of the polynomial hierarchy 48<br>$\sim$ of the polynomial hierarchy vi, 2,<br>10, 38, 40, 42–44, 54, 55, 57, 108, 121 | $\begin{array}{llllllllllllllllllllllllllllllllllll$  |

– left cuts capture the  $\sim$  of real numbers v - nonuniform  $\sim$  17  $\operatorname{cut}$ – semi-membership  $\sim$  1, 3 - types of  $\sim v$ computability  $- \sim$  in deterministic polynomial time reals v 119 $-\sim \text{in FP}_{tt}^{\text{NP}}$  70  $-\sim$  in polynomial time 120 31– deterministic polynomial-time  $\sim$ 119- easy  $\sim$  100 – partial polynomial-time  $\sim$  7 degree - polynomial-time  $\sim v, 6, 62, 88$ - recursive  $\sim$  31 computation 19, 20, 33, 97, 116, 121 $-\sim$  of the value of a circuit 18  $-\leq^p_m -\sim 8$ - accepting  $\sim$  93  $\leq^p_{pos}$ -~ – feasible  $\sim$  2  $-\leq_{r}\sim 8$ – nondeterministic guess of a  $\sim$  –14  $\begin{array}{c} \underline{\leq}_{T}^{p} \\ \underline{\leq}_{T}^{p} \\ \underline{\leq}_{tt}^{p} \\ \end{array} \\ \sim$ 8 – polynomial-time  $\sim 2$ 8 – semi-feasible  $\sim$  v–vii, 1, 2, 16 – world of  $\sim -v$ computers - NP  $\sim$  -8– making  $\sim$  smarter vi computing – intuitive  $\sim$  vi coNE 8 connection - structural  $\sim$  v connective  $-\sim$ s and complementation 84 – almost-completely degenerate  $\sim$ s Díaz vii 80,83 domain - boolean  $\sim s$  79,80 - complementation as a  $\sim$  84 - completely degenerate  $\sim s = 80, 83, 84$ – degenerate  $\sim s = 84$ 121– identity  $\sim$  84 Du vii - nondegenerate  $\sim s = 83, 84$ – under which  $\sim$ s are P-selective sets  $\mathbf{E}_{1\text{-}T}^{p}$ - ~(P-sel) closed 79 coNP 12-16, 20-22, 24-26, 32-34, 37, 38, 40, 42, 44, 54–56, 68, 74–76, 78, 82,  $\mathbf{E}_{1-tt}^{p}$ 88, 90, 116, 117, 120, 122  $-\sim$ (P-sel) - are nondeterministically selective EATCS i  $\mathbf{E}_{btt}^{p}$ - ~(P-sel) sets hard for  $\sim$  73 coNP/poly 21, 33, 34  $\mathbf{E}_{k-T}^{p}$ - ~(P-sel) containment – nonuniform  $\sim s$  67 coR 123  $\mathbf{E}_{k-tt}^{p}$  $\sim$  (P-sel) count

– transition  $\sim$ 97, 98Crescenzi vii - computational complexity of a gappy left  $\sim 31$ - gappy left  $\sim 31, 96, 99$ – left  $\sim$ s capture the complexity of nonempty parts of a gappy left  $\sim$ standard left  $\sim$  of real numbers 3 - time-bounded left  $\sim s$  16 decidability 8  $-\sim in NP = 8$ –  $\sim$  of organizational simplicity of selectivity classes 43 8 - maximum out- $\sim$  63 - node with maximum out- $\sim$  76 – recursively enumerable  $\sim s = 15$ - reducibility  $\sim 8$ – sets in an NP  $\sim$ 8 - sets in an NP ~  $\Delta_0^{p,A}$  117  $\Delta_2^p$  61, 66, 67, 122  $\Delta_k^p$  43, 117, 118  $\Delta_{k}^{p,A}$  45, 117, 118 diagonalization 96, 97, 99-102  $\sim$  of an NPSV function 39 DSPACE 118, 121 DTIME 2, 8, 31, 68, 71, 82, 118, 119,  ${\rm E} \quad 8, 22, 26, 27, 109, 115, 118 \\$ 9595, 9797,100 97, 100, 103 97, 99, 100, 103

 $\operatorname{EL}_{\Delta_k^p}$ 45, 46, 48, 58, 118  $\operatorname{EL}_{\Delta_{L}^{p}}^{W}$ 48, 118 ELH<sup>-</sup> 45, 54, 55, 118  $EL_{\Sigma_{h}^{p}}$  45–50, 54, 55, 58, 59, 107, 118 45, 46, 49, 55, 58, 107, 118  $EL_{\Theta_{i}^{p}}$ van Emde Boas vii, 103 enumerability see P-enumerability  $\epsilon = 123$ equality 43,68 - complete  $\sim$  versus weak  $\sim$  16 – notion of  $\sim$  for partial functions 16 equivalence 81,99 – Turing  $\sim$  between tally sets and P-selective sets 17, 30  $E_T^p$  $-\sim$  (P-sel) 97, 99, 100  $\mathbf{E}_{tt}^p$  $-\sim$  (P-sel) 97,100 example 2, 3, 5, 6, 8, 9, 13–15, 17, 18, 21, 41, 43, 46, 48, 58, 61, 71, 74--76, 79,81, 82, 85, 90, 97, 105, 108, 109 - classic  $\sim$  of P-selectivity 3 - counter $\sim$  50 EXP 27, 37, 38, 66, 67, 76, 79, 119  $\mathcal{F}$ -sel 9, 119, 120 fair-S(k) 106 fair-S(n)106, 107feasibility – semi- $\sim$  vii, 1 Feigenbaum 40 FewP 68, 69, 119 FEXP 82,109 FEXP-sel 82 flier - taking a  $\sim$  vi formula - boolean  $\sim s$  35, 36 - satisfiable  $\sim$ s vi, 5, 38, 73 Fortnow viii, 59, 76 FP 9, 10, 12–15, 32, 68, 70, 71, 77, 88, 96, 111, 112, 115, 119, 121 - relativized ~ 14  $\operatorname{FP}^{\operatorname{NP}[\mathcal{O}(\log n)]}$  68, 70, 71, 77 FPP 109, 113  $\begin{array}{c} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \end{array}$ 68, 70, 71, 77 $FP^X$  96 fraction - dyadic rational  $\sim 3$ function

 $-\sim$  computable by a deterministic polynomial-time Turing machine 119-(A,k)-sort  $\sim$  113 - advice  $\sim$  19,23 – almost completely degenerate  $\sim$  – 80 - associative  $\sim$  111 - boolean  $\sim$  79,83 - characteristic  $\sim$  81,84 - class of  $\sim$ s 9, 19, 117, 119 – class of  $\sim$ s computable via  $\mathcal{O}(\log n)$ Turing queries to NP 68 class of  $\sim$ s computable via truth-table access to NP 68 - collection of  $\sim s = 20, 117$ - completely degenerate  $\sim$  80 - complexity-theoretic study of one-way  $\sim s = 16$ - computable  $\sim$  62, 64, 65 - degenerate  $\sim 80$ deterministic polynomial-time  $\begin{array}{rl} \text{computable} \sim & 119 \\ - & \text{FPP-selector} \sim & 109 \end{array}$ – general classes of  $\sim s$ 16- logspace  $\sim$ s 20 – multivalued  $\sim$  122 - multivalued nondeterministic polynomial-time  $\sim s = 120$ – multivalued symmetric  $\sim$ 10 – nondegenerate boolean  $\sim$  84 - nondeterministic selector  $\sim 32$ - notion of equality for partial  $\sim s$ 16- NPMV  $\sim$  10 - NPMV<sub>t</sub>  $\sim$  10 - NPSV  $\sim$  10 – NPSV-selector  $\sim$  32 - NPSV<sub>t</sub>  $\sim -10$ - P-selector  $\sim 1, 3-7, 21, 23, 25, 26,$ 28, 31, 32, 35, 49, 63-65, 85, 88, 89, 92, 94-96, 98, 99, 102, 111-113, 124 - partial  $\sim 16$ - partial multivalued  $\sim 9, 10, 122$ polynomial-time computable  $\sim$  7, 65, 107- probabilistic selector  $\sim 105, 109$ - ranking  $\sim$  110 selector  $\sim$  v, 7, 10, 32, 53, 57, 81, 90, 92, 96, 98, 105-107, 109, 121 - selector ~ for NPSV<sub>t</sub>-sel sets 32– selector  $\sim$ s sensitive to the order of the arguments 4 - single-valued  $\sim$  10, 56, 120

- single-valued deterministic polynomial-time computable  $\sim$ 9 - single-valued nondeterministic polynomial-time  $\sim 120$ - symmetric selector  $\sim 4, 22, 28, 34,$ hierarchy 96,101 - total  $\sim$  117, 119 - total multivalued nondeterministic polynomial-time  $\sim 120$ - total recursive  $\sim$  96 – total selector  $\sim$  32 - total single-valued  $\sim$  119 - total single-valued nondeterministic polynomial-time  $\sim 120$ – uncomputable  $\sim$  18 Furst 39 - high  $\sim$  42 - low  $\sim$  41 Gabarró vii – lowness  $\sim$ Gasarch vii, 59, 113 gate 18 – and  $\sim$ 18 - exponentially many  $\sim s = 18$ 123- not  $\sim$  18 - or  $\sim$  18 – polynomial number of  $\sim s = 18$ Hofmann viii Gavaldà 59 generalization 15, 105-110, 121 Homan vii  $-\sim$ s of P-selectivity 105Homer vii Gill 113 Glaßer 76 Goldsmith 15,113 de Graaf vii immunity guess  $-C-\sim 110$ - nondeterministic  $\sim$  bits 62 - nondeterministic  $\sim$  of a computation - P-sr-∼ 111 path 53 Gundermann 76 hardness 61, 117  $-\leq_{tt}^{p}\sim -66$  $-\operatorname{coNP-}{\leq_m^p}{\sim}$ 73inequality - NP- $\sim$  61 interpreter - NP- $\leq_{1-tt}^{p}$ -~ 74 $- \operatorname{NP-}_{\gamma}^{-1} \sim 74$  $- \operatorname{NP-}_{tt}^{p} \sim 68$ – truth-table  $\sim$  for NP 67- Turing  $\sim$  for NP 67 28Hartmanis 59,76 hashing – half  $\sim$ 40Hemachandra 15, 58, 59, 76, 103, see intuition vi Hemaspaandra

Hemaspaandra iii, iv, vii, viii, 15, 16, 39, 40, 58, 59, 76–78, 102, 103, 112, 113, see Hemachandra Hempel vii, 113 HH 42, 44, 119 - arithmetical  $\sim 8,122$ - close connection of the extended low  $\sim$  to the low  $\sim$  45 collapse of the boolean  $\sim$ 76collapse of the low  $\sim 48$ collapse of the polynomial  $\sim$  vi, 2, 10, 40, 42-44, 54, 55, 57, 108, 121decomposition of NP via the low  ${\sim}{\rm s}$ and high  $\sim s$  44 extended low  $\sim$ 118– Kleene  $\sim$  122 43 - multiselectivity  $\sim 107$ polynomial  $\sim$  and small circuits 35 relativized polynomial  $\sim 117, 122,$  $- S(k) \sim 106$ Hoene vii, 16, 39, 40, 76-78, 103, 113 Holzwarth viii  $H_{\Sigma_{k}^{p}}$  42, 44, 119 - P-∼ 110, 111 – weakly-P-rankable- $\sim$  111 incomparability  $\begin{array}{l} & -\sim \text{ of } \mathbf{E}_{T}^{p}(\text{P-sel}) \text{ and } \mathbf{R}_{btt}^{p}(\text{P-sel}) \\ & -\sim \text{ of } \mathbf{E}_{T}^{p}(\text{P-sel}) \text{ and } \mathbf{R}_{tt}^{p}(\text{P-sel}) \\ & -\sim \text{ of } \mathbf{E}_{tt}^{p}(\text{P-sel}) \text{ and } \mathbf{R}_{btt}^{p}(\text{P-sel}) \end{array}$ 10010010029, 42- advice  $\sim$  20, 24, 31, 34, 35 - advice  $\sim$  for SAT 35-37 – class of advice  $\sim s$  31 nondeterministic advice  $\sim$ 24, 26,NP advice  $\sim 25, 26, 30$ – P advice  $\sim$  30 – recursive advice  $\sim$ - 39 – skating on  $\sim$  vi

Jain 103 Jenner 77 Jiang vii, 58, 102, 112 Jockusch 15, 16, 103 Joseph 15, 16, 113 k-walk 97-99 - self-avoiding  $\sim$  97 Kadin 76 Kämper 40 Karloff 59 Karp 20, 35, 39, 40 Karp-Lipton  $-\sim$  Theorem 35 – relativized version of the  $\sim$  Theorem 38Kilian 40 king  $-\sim$  of a tournament 24 Kleene 16  $-\sim$  hierarchy 122 Ko vii, 16, 39, 58, 59, 75, 112 Köbler 16, 40, 58, 59 Kummer 77, 103, 113 Landau 39 language *see* set - tally  $\sim$  see set, tally  $L_{\Delta_k^p}$  43–45, 48, 58  $L^{\overline{W}}_{\Delta^p_k}$ 48Lemma - Toda Ordering  $\sim$  76 – Toda's  $\sim$  – 76 length - advice  $\sim$  34 - bit- $\sim$  23 - linear  $\sim$  19–22, 26, 28, 30, 39, 105, 112 polynomial ~ 19-24, 27, 32-40, 49,54, 55, 67, 68, 76, 78, 105, 107–109, 120, 121 quadratic ~ 19, 20, 24, 39, 112LH 41, 42, 44, 46, 48, 57, 58, 119 Lindner 39,76 linear 19 Lipton 20, 35, 39, 40 list- query  $\sim$  116  $\mathbf{L}_{k}^{W}$  119  $L_{NP}$  41 logarithm – implicit base of  $\sim$ 23Long 16, 58, 59

loop - self  $\sim$ s 63 Low  $-\sim(\mathcal{C})$  119  $-\sim \stackrel{\sim}{(\Sigma_0^p)} \\ -\sim \stackrel{\sim}{(\Sigma_1^p)}$ 41 41 $-\sim (\Sigma_2^{\dagger})$ 41 $-\sim (\Sigma_3^{\tilde{p}})$ 41lowness 41-43, 45, 46, 55, 58, 59 –  $\sim$  of all four types of nondeterministically selective sets 58  $\sim$  of nondeterministically selective sets 49,58  $-\sim of NP \cap P-sel 48$  $-\sim$  of P-selective sets 46,49 - analysis in terms of  $\sim$  58 - best currently known upper bounds for  $\sim$  of selective sets 51 - best currently known upper bounds for extended  $\sim$  of selective sets 50 - definition of  $\sim$  41 - extended  $\sim$  43, 46–49, 59 - extended  $\sim$  of all four types of nondeterministically selective sets 58- extended- $\sim$  bounds 50 – extended- $\sim$  structure of P-selective 48, 49sets extended- $\sim$  upper and lower bounds 46 extended- $\sim$  upper bounds 55 generalization of  $\sim 45$ - more general  $\sim$  result 40 - nontrivial lower bound on  $\sim$  46,48 - refined  $\sim$  43 - upper and lower bounds on  $\sim$  46, 49- upper and lower bounds on extended  $\sim 49$ L<sub>P</sub> 41  $L_{\Sigma_{t}^{p}}$  41–45, 47–49, 51, 54, 55, 57–59, 107, 119 $L_{\Theta_{k}^{p}}$  43-45, 50, 55, 58 Lund 59 machine 9, 24, 27, 28, 32, 47, 52, 56, 65, 67, 69, 81, 92, 95, 99, 100, 102, 103, 116, 118- bottleneck  $\sim s$  39 deterministic f(n)-space Turing ~ 118– deterministic f(n)-time Turing ~ 118

- deterministic polynomial-time Turing nonclosure  $\sim 2, 47, 88, 106, 118, 119, 123$  $- \sim \text{ of } \operatorname{EL}_{\Sigma_{r}^{p}}$  under many-one – enumeration of partial recursive  $\sim s$ reductions 46 31  $-\sim$  of the extended low hierarchy - exponential-time  $\sim$  27  $- \sim$  under a function of P-sel 84 - FewP  $\sim$  68 –  $\sim$  under intersection for all selectivity - FP  $\sim$  12 classes 82  $- \mathrm{FP}_{tt}^A \sim -71$  $-\sim$  under k-ary connectives of - function-computing Turing  $\sim 10$ , NPSV-sel and NPMV-sel 84 120 $- \sim$  under nondegenerate connectives nondeterministic polynomial-time of NPSV-sel and NPMV-sel 84 function-computing Turing  $\sim 10$ –  $\sim$  under reductions of P-sel  $\,$  85  $\,$ nondeterministic polynomial-time  $- \sim$  under union of selective sets 82 Turing  $\sim 10, 11, 14, 29, 38, 47, 50,$ – simultaneous capture of  $\sim$  under 55, 62, 65, 69, 90, 109, 115, 120, 121 intersection for all versions of - NP  $\sim$  53, 56, 67 selectivity 82 - oracle  $\sim$  47,86 - P<sup>A \oplus SAT</sup>  $\sim$  47 nondeterminism vi, 39 - linear amount of  $\sim$  39 – polynomial-time oracle  $\sim$ 29 – understanding of  $\sim$  9 – polynomial-time Turing  $\sim$  -27, 69notation 123 probabilistic polynomial-time Turing notion  $\sim 123$ - query-clocked  $\sim$  100  $- \sim$  of being "easily k-countable" 113 - simulating  $\sim$  27  $- \sim$  of equality for partial functions - Turing  $\sim 5,116$ 16 – Turing reduction  $\sim$  94 - ~s closely related to P-selectivity unambiguous polynomial-time Turing 109- ~s related to membership compara- $\sim 67,123$ – unambiguous Turing  $\sim$ 67 bility 108 Magklis viii - advice  $\sim$  35 Mayer viii – complexity-theoretic  $\sim$  vii McLaughlin 16refinements of the  $\sim$  of membership measure comparability 108  $-\sim$  of resource 19 NP v, 2, 3, 5, 8, 9, 11-16, 20-22, 25, 26, - complexity  $\sim s = 107$ 29, 30, 32-59, 61, 66-72, 74-79, 81, 82, measurement 88, 90, 91, 94, 103, 105, 107, 108, 111, - fine-grained  $\sim$  of advice 20112, 116, 117, 119-122 Meyer 16 complete for  $\sim v$ MinimumPath 62, 65-67, 69, 70 – completeness for  $\sim$  under many-one reductions 2  $\begin{array}{cc} \mathbb{N} & 124 \\ \mathbb{N}^+ & 124 \end{array}$ – completeness for  $\sim$  under Turing reductions 2 Naik vii, 16, 39, 40, 59, 76-78, 103 - relativized  $\sim$  42, 43, 47, 48, 119, 120 Nasipak vii, 39 NP/poly 21, 33, 34 nature NPMV 10, 11, 15, 16, 34, 39, 54, 74, 91, – nondeterministic  $\sim$  of gamma 92, 120 reductions 90 NPMV-sel 10, 11, 13, 21, 34, 41, 50, 51, NE 8,119 54, 55, 57, 58, 74, 75, 82-84, 120 NEXP 119 Nickelsen vii, viii, 113 NPMV<sub>t</sub> 10-12, 15, 16, 34, 54, 74, 75,81, 90, 91, 120 Nisan 59 NNT 15 NPMV<sub>t</sub>-sel 11-13, 21, 34, 41, 50, 51,54, 55, 57–59, 74, 75, 81–84, 90, 120 - implicitly membership-testable sets  $NP^{NP}$  55, 57, 121 2,119

141

46

 $\begin{array}{l} - & \mathbf{E}_{1-tt}^{p}(\sim) & 95,97 \\ - & \mathbf{E}_{btt}^{p}(\sim) & 97,100 \\ - & \mathbf{E}_{k-T}^{p}(\sim) & 97,100,103 \\ - & \mathbf{E}_{k-tt}^{p}(\sim) & 97,99,100,1 \end{array}$ NPSV 9, 10, 12, 15, 16, 32, 34, 38, 39, 49, 50, 52, 55, 56, 74, 120 NPSV-sel 10-13, 21, 32-34, 41, 49-51, 54, 55, 57 - 59, 68, 74, 83, 84, 120 $\begin{array}{l} - \mathbf{E}_{k-tt}^{p}(\sim) & 97, 99, 100, 103 \\ - \mathbf{E}_{T}^{p}(\sim) & 97, 99, 100 \end{array}$ NPSV<sub>t</sub> 10, 12–16, 74, 81, 88, 90, 103,  $\begin{array}{l} - \mathrm{E}_{T}^{*}(\sim) & 97, 99, 10\\ - \mathrm{E}_{tt}^{*}(\sim) & 97, 100\\ - \mathrm{R}_{1-T}^{*}(\sim) & 95\\ - \mathrm{R}_{1-tt}^{*}(\sim) & 95, 96\\ - \mathrm{R}_{2-tt}^{*}(\sim) & 96\\ - \mathrm{R}_{btt}^{*}(\sim) & 97, 100\\ \end{array}$ 120NPSV<sub>t</sub>-sel 11-14, 21, 32, 41, 50, 51, 54,57, 58, 68, 74, 81, 83, 84, 120 NT near-testable sets 2, 121 NTIME 119, 121 - relativized  $\mathbf{R}_{k-tt}^{p}(\sim)$ 103 $\begin{array}{l} - \mathbf{R}_{k-T}^{p}(\sim) & 95, 97, 99 \\ - \mathbf{R}_{k-tt}^{p}(\sim) & 95, 97, 99, 103 \\ - \mathbf{R}_{n\mathcal{O}(1)-T}^{e}(\sim) & 28 \\ - \mathbf{R}_{\mathcal{O}(\log n)-T}^{p}(\sim) & 95 \end{array}$ Ogihara vii, viii, 16, 39, 40, 59, 77, 78, 103, 113, see Ogiwara Ogiwara 76, 103, see Ogihara optimality  $- \operatorname{R}^{e}_{O(n^k)-T}(\sim) \quad 27$ - relativized  $\sim$  48  $- R_T^p(\sim) 35,97,100,102$ oracle 12, 14, 27-29, 41, 43, 47, 52-54,  $- R_{tt}^{p}(\sim) = 95, 97, 99, 100, 102, 108$ 56, 57, 59, 80, 81, 88, 98–103, 115, 118, P-sr 110, 111, 113 120, 121, 123 - the polynomial-time semi-rankable  $-\sim$  query 5 sets 110 - low sets as  $\sim s$  41 P/poly 21–24, 35–37, 39, 78, 105, 107, - NP  $\sim$  71 108 - open  $\sim$  questions 59 Papadimitriou vii ordering Parkins vii, 39 - lexicographical  $\sim$  71, 108, 110 Pasanen 16 - linear  $\sim$  7,124 - linear  $\sim$  of  $\{1\}^*$ Paterson 16 6 path 11, 12, 14, 53, 65, 81, 92, 93 – linear ~ of  $\Sigma^*$ 6 accepting  $\sim 11, 29, 56, 62, 66-70,$ – linear ~ of  $\Sigma^*$ 6 90, 94, 120, 121, 123 - partial  $\sim$  7 accepting  $\sim$  of a FewP machine 68 – partial  $\sim$  7 accepting  $\sim$  of a function-computing output machine 10, 120  $-\sim$  of a polynomial-time algorithm v - computation  $\sim$  14, 33, 56, 62, 66, – linearly bounded  $\sim$  of an advice 69, 70, 87, 109, 121 function 19 directed  $\sim 63, 64, 76$ - quadratically bounded  $\sim$  of an advice - directed  $\sim$  in a tournament 64 function 19 – guess bits of an accepting  $\sim$ Owings 113 70– guessed  $\sim$  14 v, 1-9, 12-16, 19-24, 26, 28-30, 32, guessed computation  $\sim 11$ Ρ 33, 35-39, 41-49, 54, 55, 58, 61, 62, minimum accepting  $\sim$  of a nondeter-64-68, 76-79, 82, 83, 85, 88, 94, 95, ministic Turing machine 62, 65–67, 103, 105-112, 115, 117, 118, 120-123 69,70 P-close 2, 49, 121 - nondeterministic  $\sim$  53, 54, 57 P-enumerability - nondeterministic guess of a P-mc 108 computation  $\sim 11, 53$  $- \sim (\text{const})$ 108– nondownward  $\sim$  20  $\mathbf{R}_{btt}^{p}(\sim(\text{const}))$  108 rejecting  $\sim$  of a function-computing P-sel 1-5, 8, 9, 11-13, 18, 20-27, 30, 31, machine 10, 120 35, 39, 41, 43, 46-51, 66-68, 70, 72, 76, - rejecting  $\sim s$  66 - short  $\sim$ s in a tournament 63 77, 79, 81-85, 88, 94-96, 102, 103, 105, 106, 110--112, 121– simulated  $\sim$  53  $- E_{1-T}^{p}(\sim) - 95$  $\mathbf{PH}$ 

- polynomial hierarchy 15, 35, 37–40, 42, 48, 58, 67, 68, 74–76, 78, 108, 113, 121 $\Pi_{k}^{p}$  37, 38 poly 19 Popeye 41 70 $-\sim$  the Sailor Man 41 - cotton candy is low for  $\sim$  41 - spinach is not low for  $\sim$  41 power 41, 49, 94  $-\sim s \text{ of two} 20$ - distinguishing the  $\sim$  of reductions – relative  $\sim -2$ - separating the  $\sim$  of reducibilities 2, 15 PP 28-30, 46-49, 55, 75, 109, 121 PP/poly 109 R 94 predecessor – lexicographical  $\sim 2$ 123preorder 7  $\mathbf{R}_{1-T_{i}}^{p}$ procedure – nondeterministic polynomial-time  $\sim$  $\mathbf{R}_{1-tt}^{p}$ 91program – Selman's  $\sim$  – 16 – Selman's structural  $\sim$  – v pronouncement 77 proof - nonrelativizable  $\sim$  59 - relativizable  $\sim$  46 property - closure  $\sim$  84 - closure  $\sim$ s of P-sel 6,79 rational - closure  $\sim$ s of P-sel, NPSV<sub>t</sub>-sel, and  $NPMV_t$ -sel 84  $\mathbf{R}_{btt}^{p}$ PSPACE 37, 58, 59, 66, 67, 75, 88, 105, 109, 121  $\mathbf{P}^W$ -sel 48 qP - the quasipolynomial time sets 2, 16, 121quadratic 19 quantification 88 - universal  $\sim$  33 quantifier - alternating  $\sim s$  43 – number of  $\sim$ s needed to remove a set's ability to provide useful information 43 103– polynomially bounded  $\sim$  9 - unbounded  $\sim$  9

query 12, 14, 26-28, 47, 52-54, 56, 57, 65, 66, 68-71, 73, 81, 86, 89-102, 107, 108, 115, 116, 118, 121, 123 - answers to  $\sim$ s 69,70 – answers to  $\sim$ s on the MinimumPath – linear limit to number of  $\sim$ s 27 linear number of  $\sim$ s by a Turing reduction 26 membership  $\sim$ 70– nonadaptive  $\sim$ 102- oracle  $\sim$  5 – polynomial number of  $\sim$ s to a P-selective set 76 - possible answers to  $\sim s$  69 - set of answers to  $\sim$  71 - truth-table  $\sim$  70 - random polynomial time 68, 122,  $\sim$ (P-sel) 95 $\sim$ (P-sel) 95,96  $\begin{array}{c} \mathbf{R}_{2\text{-}tt}^{p} \\ - \sim (\text{P-sel}) \begin{array}{c} 96 \\ \mathbf{J}_{\text{rop}} \end{array}$ Ramachandran 59range 101  $-\sim$  of natural senses v Ranjan 59 rank 110 rankability - P-semi- $\sim$  110 – dyadic  $\sim$  3  $-\sim$  (P-mc(const)) 108  $- \sim$  (P-sel) 97,100 realization  $-\sim$  of an FP<sup>NP</sup><sub>tt</sub> function 77 recursiveness - semi- $\sim$  2, 6, 82, 83, 85 reducibility 8,117 – 1-truth-table  $\sim$  to a P-selective set - 2-disjunctive self- $\sim$  5,118  $- \sim \text{degree} \quad 8$ - disjunctive self- $\sim$  5, 38, 118 - disjunctive self- $\sim$  of SAT 35 - self- $\sim$  vi, 16, 74, 75, 79, 80, 88, 90, Turing  $\sim 26$ - Turing self- $\sim$  5, 74, 75, 88–92, 123

- Turing self- $\sim$ , classes containing - standard set  $\sim$ s 123 complete sets with that property 75– structural  $\sim$ – transitive  $\sim$  7 reduction relativization 13, 41, 43, 49, 55, 88, 90, – advice upper bounds for  $\sim$ s to selectivity classes 22 119 $-\sim$  of a proof 32 - comparison of polynomial-time  $\sim$ s  $\mathbf{2}$  $-\sim$  on a per set basis 38  $-\sim$  remains a useful approach 59 completeness for NP under many-one - positive  $\sim$  59  $\sim s = 2$ completeness for NP under Turing survey of open  $\sim$  questions 59 requirement 20  $\sim s$ 2 research v-vii, 2, 3, 5 - conjunctive  $\sim s = 81$ – unification of semi-feasibility  $\sim$  vi - disjunctive  $\sim s = 81$ result 5, 7, 12–16, 20, 23, 34, 37, 39, 40, – exponential-time Turing  $\sim$ 11545-49, 55, 61, 66-68, 73-80, 85, 88, 90, - gamma  $\sim$  90, 115 – linear Turing  $\sim$  -2691, 100, 102, 105, 106, 108, 110–113  $\sim$ s distinguishing reducibility notions - locally positive Turing  $\sim$  103, 115 2- many-one  $\sim s$  81 –  $\sim$ s on topics beyond selectivity 3 - parity  $\sim s = 81$ advice  $\sim s = 54$ - polynomial-time Turing  $\sim 95,99$ classic  $\sim$  of Hartmanis and Stearns - positive Turing  $\sim 29, 79, 81, 85, 86,$ 76115- classic  $\sim s = 3$ - positive-truth-table  $\sim$  87 - complexity  $\sim 107$ strong nondeterministic Turing  $\sim$ – extended-lowness  $\sim$ 47, 48, 54 116– hardness  $\sim$  62 – truth-table  $\sim s$  116 immediate corollaries of lowness  ${\sim}{\rm s}$ - Turing  $\sim 26, 29, 94, 100, 115$ 46- variants of positive  $\sim s = 80$ links between oracle  $\sim$ s and  $\sim$ s in refinement 16, 39, 40, 110 the real world 59  $- \sim \text{ of P-selectivity} \quad 110, 111$ - lowness  $\sim$ s 43, 46, 48, 55  $-\sim$  of the amount of advice 20 - meaning and weight of relativization  $-\sim$  of the P-selective sets 105 $\sim s$ -59 $-\sim$  of the semi-feasible sets vnonrelativized  $\sim s$  76 - NPSV  $\sim$ 39- relativizable  $\sim$ s 58 Regan vii - relativized  $\sim s$  76 relation 7,29 - relativizing  $\sim s$  90  $-\sim$  between nondeterministic value of relativized  $\sim s$  49 selectivity classes 12  $\mathbf{R}_{k-T}^{p}$  $\sim$  between the multiselectivity -~(P-sel) 95, 97, 99hierarchy and the extended low  $\begin{array}{c} \mathbf{R}_{k-tt}^{p} \\ - \sim (\text{P-sel}) \end{array}$ hierarchy 107 95, 97, 99, 103  $\sim$  between truth-table equivalence  $\begin{array}{c} \mathbf{R}_{k-tt}^{p,X} \\ - \sim (\text{P-sel}) \end{array}$ classes and Turing equivalence classes 103100 $\mathbf{R}_m^p$ – close  $\sim$  between P-selective sets and  $\sim$ (S(k)) 106standard left cuts 39  $\mathbf{R}^{e}_{n^{\mathcal{O}(1)}\text{-}T}$  $\begin{array}{ll} - \mbox{ equivalence } \sim & 7,105,124 \\ - \mbox{ equivalence } \sim & 7 \end{array}$  $\sim$ (P-sel) 28 $\mathbf{R}^{p}_{\mathcal{O}(\log n) - T}$ – equivalence ~ on  $\Sigma^*$ 7  $-\sim$  (P-sel) – preorder  $\sim$  7 95– reflexive  $\sim$  7  $\mathbf{R}^{e}_{O(n^k)-T}$ – reflexive and transitive  $\sim$  7  $-\sim$ (P-sel) 27- standard arithmetic  $\sim s$  123 Rogers 40

Rohatgi 59 Rothe vii, 16, 58, 112 Royer 40 Rozenberg viii  $R_r(\mathcal{C}) = 122$  $\mathbf{R}_T^p$  $-\sim$  (P-sel) 35, 97, 100, 102  $-\sim$ (SPARSE) 35  $\mathbf{R}_{tt}^p$  $- \sim$  (P-sel) 95, 97, 99, 100, 102, 108  $- \sim$ (TALLY) 35  $S(\sqrt{n}) = 106$ S(2) = 107 $\begin{array}{c} \mathbf{S}(k) & 106 \\ - \mathbf{R}_m^p(\sim) & 106 \end{array}$  $S(\log n)$  106 S(n) 106  $\begin{array}{c} S_2 \\ S_2^{\rm NP\cap coNP} \\ S_2^{\rm NP\cap coNP} \\ \end{array} \begin{array}{c} 40, 76, 78, 113, 122 \\ 40, 76, 78, 122 \end{array}$ SAT 5, 9, 14, 15, 35–39, 45, 47, 54, 68, 71-73, 75, 76, 82, 108, 118- initial  $\sim$  of a linear ordering 6 - initial  $\sim$  of a polynomial-time computable linear ordering 6 - initial  $\sim$  of a recursive linear ordering 6  $- \sim$  and self-reducible sets 88  $-\sim$  via general functions 119 - associative  $\sim$  111–112 – commutative  $\sim$  111–112 - four types of nondeterministic  $\sim$  15 – generalizations of  $\sim$  105 - multi $\sim$  105, 106, 112, 113 - nondeterministic  $\sim 9,80$ - NP-∼ 38 - NP2V-∼ 39 - NPMV- $\sim$  11, 15, 16, 34, 74, 91 - NPMV<sub>t</sub>- $\sim$  11, 12, 15, 16, 74, 75, 90,

- NPSV- $\sim$  12, 15, 32, 38 - NPSV<sub>t</sub>-~ 12, 14–16, 88, 90, 103 - other types of  $\sim$  than P- $\sim$  102  $- P-\sim 1, 3-7, 9, 12, 14-16, 31, 49, 62,$ 64, 68, 74, 82, 88, 102, 103, 105, 109, 110– P-∼ 1 – probabilistic  $\sim$  105, 109 - relationships between nondeterministic  $\sim$  classes 12 - study of nondeterministic  $\sim$  15 – understanding of  $\sim$ 9 weak ~ 105, 112selector 52, 64, 81, 97, 99  $-\sim$  function see function, selector – associative  $\sim$  111–112 – commutative  $\sim$  111–112  $- \mathcal{F} - \sim -9,119$ - more powerful  $\sim s$  109 - NPMV- $\sim$  11, 15, 74, 92 - NPMV<sub>t</sub>- $\sim$  11, 12, 81, 90 - NPSV- $\sim$  32, 34, 38, 50, 52, 55, 56 - NPSV<sub>t</sub>-~ 14, 15 - P-∼ 7,92 - symmetric  $\sim 4,86$ symmetric P- $\sim$  100 Selman v, vii, 1, 2, 4, 5, 15, 16, 39, 40, 59, 76-78, 103sequence – characteristic  $\sim$  102 set  $-\sim s \leq_r$ -equivalent to some set in  $\mathcal{C}$ 118 $(a, b)_p$ -recursive  $\sim s$  109 - advice complexity of the P-selective  $\sim s$  111 advice for P-selective  $\sim s = 20$ best currently known upper bounds for extended lowness of selective  $\sim s$ 50- best currently known upper bounds for lowness of selective  $\sim s = 51$ - cheatable  $\sim s$  109, 113 - class containing Turing self-reducible complete  $\sim s = 75$ - closure properties of the P-selective  $\sim s = 6$ closure under complement of NPMV-selective  $\sim s = 15$ closure under complement of  $NPMV_t$ selective  $\sim s = 15$ 
  - collection of  $\sim$ s 19, 20, 79, 81, 117
  - complement of a  $\sim$  11
- Salomaa viii Schnorr 16 Schöning 15, 16, 58, 59 segment selectivity 9, 16, 79, 106  $-(A,k)-\sim 113$  $-(i, j) \sim 106$ - broadening of  $\sim$  9 – deterministic  $\sim -90$  $-\mathcal{F}-\sim 9,119$ - FEXP-∼ 109 – forms of  $\sim$  105 - FPP-∼ 109, 113  $-\mathrm{FP}_t^L \sim 88$
- 91

– complement of an NPSV-selective  $\sim$ – P-superterse  $\sim s$  109 12– P-verbose  $\sim s$  109 - complete  $\sim s = 13, 38$ polynomial-time semi-rankable  $\sim$ s - complete characterization of 110 P-selective  $\sim s = 6$ - polynomial-time Turing equivalent - complete characterization of  $\sim s 8$ semi-recursive  $\sim s = 6$ - query  $\sim s$  69 computationally simple  $\sim$  in a – refinement of the semi-feasible  $\sim s - v$ natural sense v - self-reducible  $\sim s$  79,88  $-\Delta_k^p$  extended low  $\sim s$  118 – self-reducible P-selective  $\sim$  88 – semi-feasible  $\sim s v, vi, 1, 121$ - disjunctively self-reducible  $\sim$ s 103,105, 118 semi-recursive  $\sim s = v, 6, 15, 83, 96,$ - easily-countable  $\sim s = 108, 109$ 109, 122  $\Sigma_k^p$  extended low ~s 118 - FPP-selective  $\sim$  109 - sparse  $\sim$ s 2,122 – implicitly membership-testable  $\sim s$ 2, 15, 119– standard left cut  $\sim$ see cut – left cut  $\sim$ 28. see cut- superterse  $\sim s$  113 - tally  $\sim$  8, 17 lowness for nondeterministically selective  $\sim s$  49  $-\Theta_k^p$  extended low  $\sim s = 118$ – lowness of P-selective  $\sim s$  46 – Turing self-reducible  $\sim s$  123 – membership comparable  $\sim s = 107$ , – verbose  $\sim s = 108$ – weakly P-rankable  $\sim s$  110 108– membership complexity of  $\sim s = 1$ – weakly P-selective  $\sim s$ 105- near-testable  $\sim s = 2, 15, 108, 109, 121$ – weakly-P-rankable  $\sim$ s 110- nearly near-testable  $\sim s = 2,119$ weakly-P-selective  $\sim s$ 105 nondeterministic version of semiset-f = 9-12, 14, 32-34, 38, 39, 50, 52,55, 56, 74, 90, 92, 93, 119, 120, 122 feasible  $\sim$  vi – nondeterministically selective  $\sim s$ setting - satisfiability  $\sim s$  73 10, 61, 67, 73, 90 - nonempty finite sub~ of a P-selective Sewelson 76 set 21, 24, 25 SH 106, 107 nonempty finite sub~ of an Shamir 59 NPMV-selective set 34 Sheu 58, 59 – nonrecursive  $\sim$  $\Sigma^*$ 1, 4-7, 12, 17-20, 23, 28, 29, 39, 53,v - NP  $\sim s$  2 56, 77, 80, 85, 89, 91, 95, 108, 109, 111,  $- \text{NP-} \leq_{tt}^{p} \text{-hard} \sim 68$  $- \text{NP-} \leq_{T}^{p} \text{-complete} \sim 68$ 117, 124  $\begin{array}{c} \Sigma^p_0 \\ \Sigma^p_1 \\ \Sigma^p_2 \\ \Sigma^p_3 \\ \Sigma^p_k \end{array}$ 41,12241 - NP-complete  $\sim 5, 8, 14, 15, 42, 61$ 35-38, 40, 41, 47, 67, 75, 111 - NPMV-selective  $\sim s = 34, 54$ - NPMV<sub>t</sub>-selective  $\sim s = 16, 34, 54$ 36, 4137, 38, 42, 43, 45, 54, 58, 118, 119, - NPSV-selective  $\sim s = 16, 49$ - NPSV<sub>t</sub>-selective  $\sim s = 14, 16, 88$ 121, 122  $\Sigma^{p,A}_{\iota}$  42, 45, 118, 119, 122, 123 - oracle  $\sim 5$ - P-close  $\sim s = 2, 15, 121$ simplicity 8 – P-enumerable  $\sim$  77 - lowness is a notion of organizational - P-selective  $\sim$  v, 1-9, 17, 18, 20,  $\sim 43$ 21, 23-32, 39, 46-49, 61-68, 70, 73,organizational  $\sim$  of selectivity classes 76, 79-83, 85, 86, 88, 94-96, 98-103, 43105-111, 113, 121 - structural  $\sim$  8,9 – P-selective  $\sim$ s whose intersection is simulation 12, 27, 53 not semi-recursive 82 Sipser vii – P-selective nonrecursive  $\sim s - v$ Sivakumar vii, 76, 77 – P-semi-rankable  $\sim$ s 105, 110 sorting

1*C* T

 $-\sim$  of queries according to a selector  $- \sim$  of semi-feasible algorithms iii 64, 65, 92, 94-96, 98, 99, 101, 102  $-(A,k)-\sim 113$ vii space 19 – deterministic  $f(n) \sim 118$ - physical  $\sim$  taken by a circuit 19 – polynomial  $\sim$  121 SPARSE 46, 48, 50, 107, 121, 122  $- R_T^p(\sim) = 35$ spinach 41  $\sim$  is not low for Popeve 41 Stearns 76 step 26, 31, 32, 51, 53, 56, 57, 62, 64, 86, 87, 89, 91, 93, 98, 99, 116 - computation  $\sim s$  62 Stephan 77, 103, 113  $\Theta_k^{\check{p}}$ Stol vii Stricker viii string - advice  $\sim 20, 25, 33$ time - collection of  $\sim s = 21, 57, 116$ - concatenation of  $\sim s$  17 – easily decodable advice  $\sim$  23 117- evil  $\sim$ 52study 9  $-\sim$  of associative selectivity 112  $-\sim$  of nondeterministic selectivity 9, 15 $- \sim \text{ of P-selectivity} \quad 1,7$ - complexity-theoretic  $\sim$  of one-way functions 16 complexity-theoretic  $\,\sim\,$  of semimembership complexity 1 TALLY 6, 30, 47, 122  $- \mathrm{R}_{tt}^p(\sim) = 35$ Tantau vii, 102, 103 tape 116  $\sim$  oracle  $\sim$ 116technique - minimum path  $\sim 61, 62, 68, 75$ - parallel census  $\sim 68$ test 1,49 - classic simplicity  $\sim 3$ testability 109, 113– near- $\sim$ Thakur vii Theorem - Karp-Lipton  $\sim 35$  relativized version of the Karp-–  $\sim$ 's Lemma 76 Lipton  $\sim 38$ theory token  $-\sim$  of positive relativization 59 - advice  $\sim 24-27, 31, 32, 34, 39, 117$ 

 $- \sim$  of semi-feasible computation v, – advice  $\sim$  43 – basic lowness  $\sim$  41  $- \operatorname{coding} \sim 97$ - complexity  $\sim$  1, 3, 16, 41, 46, 103 - computational complexity  $\sim$  vii – extended-lowness  $\sim$ 46- lowness  $\sim$  41, 43, 46 - P-selectivity  $\sim 68$ - recursive function  $\sim v, 2, 58, 103$ selectivity  $\sim 3,105$ - tournament  $\sim$  63  $\Theta_0^{p,A}$  123 43, 118, 123 $\Theta_{\iota}^{n,A}$  45, 118, 123 Thierauf 16, 39, 40, 76–78, 103 thresholds 106 - almost polynomial  $\sim$  (APT) 2,16 co-nondeterministic polynomial  $\sim$ deterministic "polynomial exponential"  $\sim$  119 – deterministic exponential  $\sim$  118 - deterministic  $f(n) \sim -118$ – deterministic polynomial  $\sim$  121 deterministic polynomial  $\sim$  relative to an oracle 121 - deterministic polynomial  $\sim$  relative to an oracle with a bounded number of queries 121 – exponential  $\sim$  vi, 26 - linear  $\sim$  98,100 – linear exponential  $\sim$  27 nondeterministic "polynomial exponential"  $\sim$  119 nondeterministic exponential  $\sim$ 119- nondeterministic  $f(n) \sim 121$ nondeterministic polynomial  $\sim$ 24,119.120 - nondeterministic running  $\sim$  121 – probabilistic polynomial  $\sim$  121 - quasipolynomial  $\sim$  (qP) 2, 16, 121 - random polynomial  $\sim$  122 unambiguous nondeterministic polynomial  $\sim 123$ Toda 75,76  $-\sim$  Ordering Lemma 76

Torán 77 Torenvliet iii, iv, viii, 39, 77, 103 tournament 63, 64, 76 – king of a  $\sim$  24 tree 26  $-\sim$  of possible queries 100 - self-reduction  $\sim 90, 91, 93$ union 7, 65, 96, 98, 99, 102, 105, 123 UP 66, 67, 123 variable 5, 9, 72, 79, 84 - logical and of  $\sim s$  124 - logical or of  $\sim s$  124 Veltman viii Verbeek vii verboseness 113 Vereshchagin 58 verification – polynomial-time  $\sim$  of a certificate 33 Vyskoč 59

Wagner 76 Wang vii, 16, 39, 40, 76–78, 103, 113 Watanabe vii, 40, 58, 112weakly-FP<sup> $\Sigma_2^p$ </sup>-rankable 111 weakly-P-rankable 110 – the weakly P-rankable sets 110 Wechsung 40, 76, 77 West 76 worktape 10,120 - semi-infinite  $\sim$  10, 120 world - real  $\sim$  49 - relativized  $\sim$  46, 48, 49, 76, 103, 108, 118, 119 Wössner viii Young 15, 16, 113 Zaki vii, 16, 113 Zimand vii, 16, 59, 113 ZPP 37-40, 67, 68, 74, 78, 108, 122,  $\begin{array}{c} 123 \\ {\rm ZPP}^{\rm NP} & 40, 67, 68, 78, 122 \end{array}$