The Art of Data Structures

Trees

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CSC 162: The Art of Data Structures

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Agenda

- To understand what a tree data structure is and how it is used
- To see how trees can be used to implement a map data structure
- To implement trees using a list
- To implement trees using classes and references

Trees

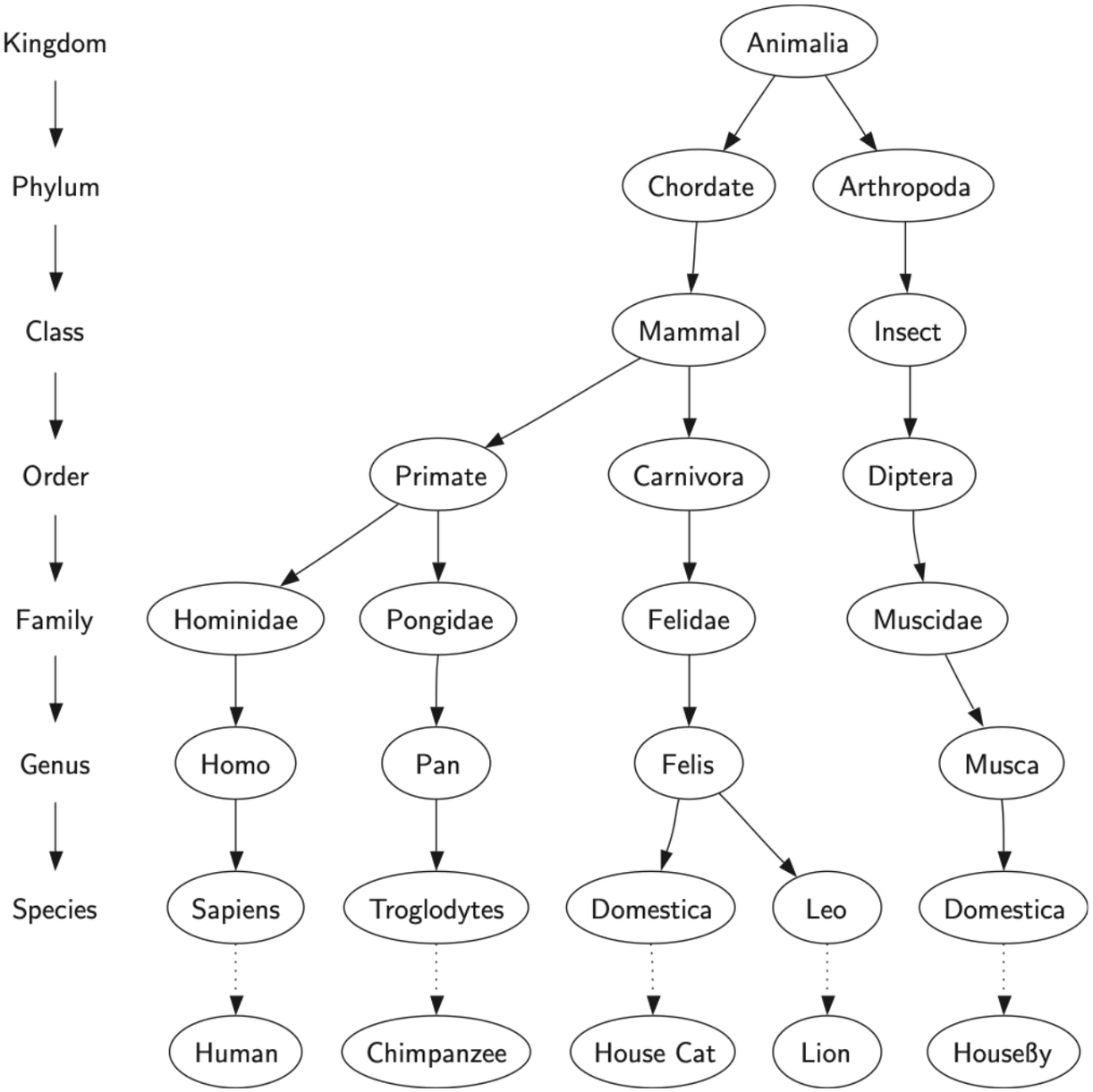
Trees

Properties

- Hierarchical
- Child nodes are all independent
- Path to leaf nodes are unique
- Subtrees

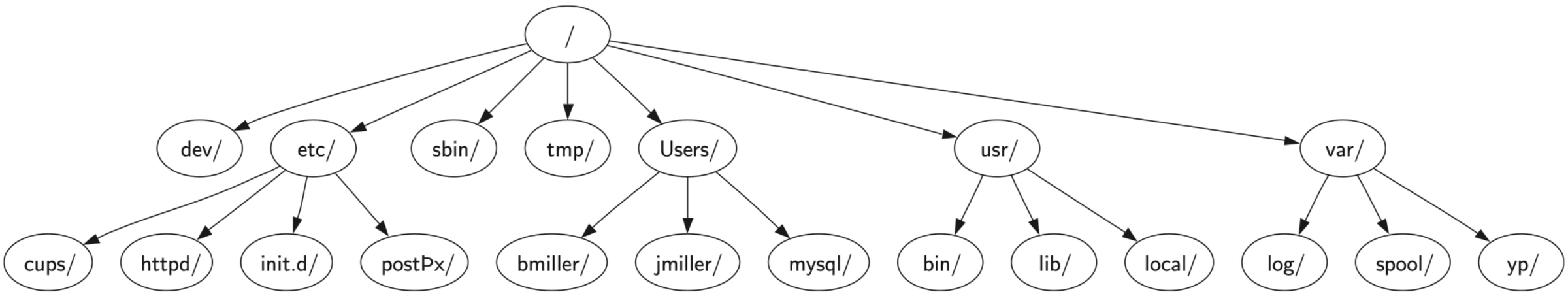
Trees

Example: Animal Taxonomy



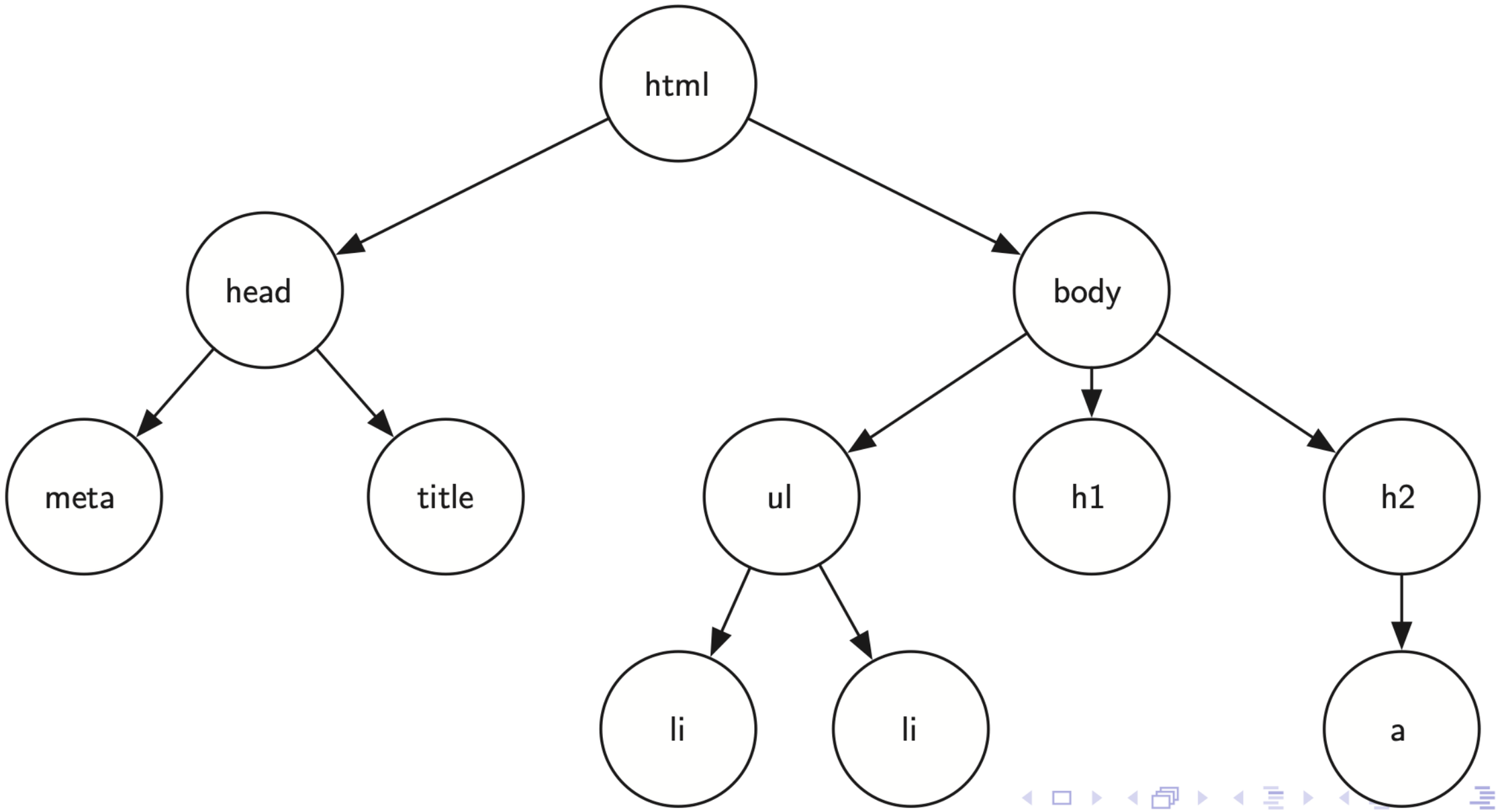
Trees

Example: UNIX File System



Trees

Example: HTML Markup Elements



Trees

Vocabulary

- **Node** A node is a fundamental part of a tree. It can have a name, which we call the “key"
- **Edge** An edge connects two nodes to show that there is a relationship between them (incoming/outgoing)
- **Root** The root of the tree is the only node in the tree that has no incoming edges

Trees

Vocabulary

- **Path** A path is an ordered list of nodes that are connected by edges
- **Children** The set of nodes c that have incoming edges from the same node to are said to be the children of that node
- **Parent** A node is the parent of all the nodes it connects to with outgoing edges

Trees

Vocabulary

- **Sibling** Nodes in the tree that are children of the same parent are said to be siblings
- **Subtree** A subtree is a set of nodes and edges comprised of a parent and all the descendants of that parent
- **Leaf Node** A leaf node is a node that has no children

Trees

Vocabulary

- **Level** The level of a node n is the number of edges on the path from the root node to n
- **Height** The height of a tree is equal to the maximum level of any node in the tree

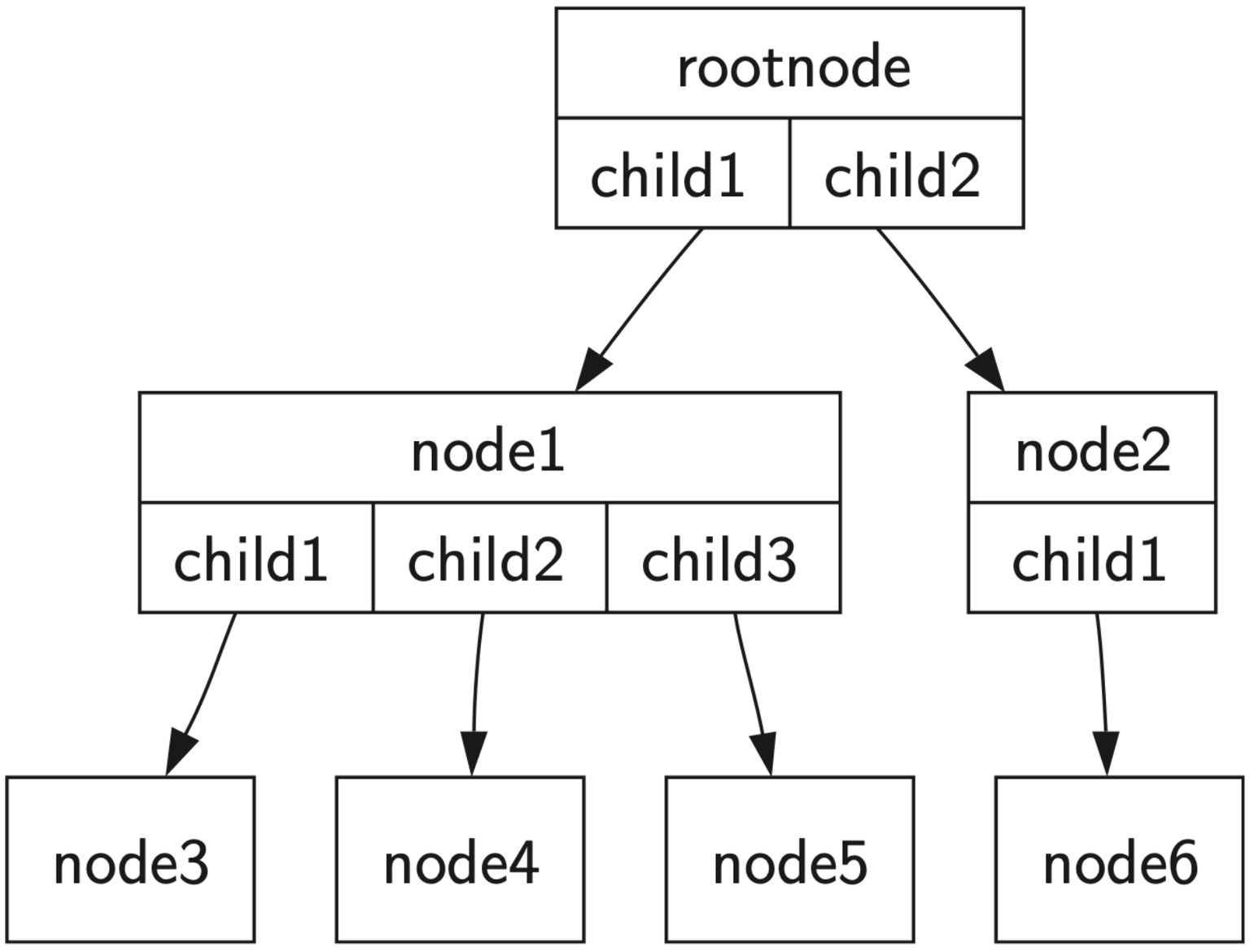
Trees

Defined as Nodes and Edges

- One node of the tree is designated as the root node
- Every node *n*, except the root node, is connected by an edge from exactly one other node *p*, where *p* is the parent of *n*
- A unique path traverses from the root to each node
- If each node in the tree has a maximum of two children, we say that the tree is a **binary tree**

Trees

A Tree with a Set of Nodes and Edges



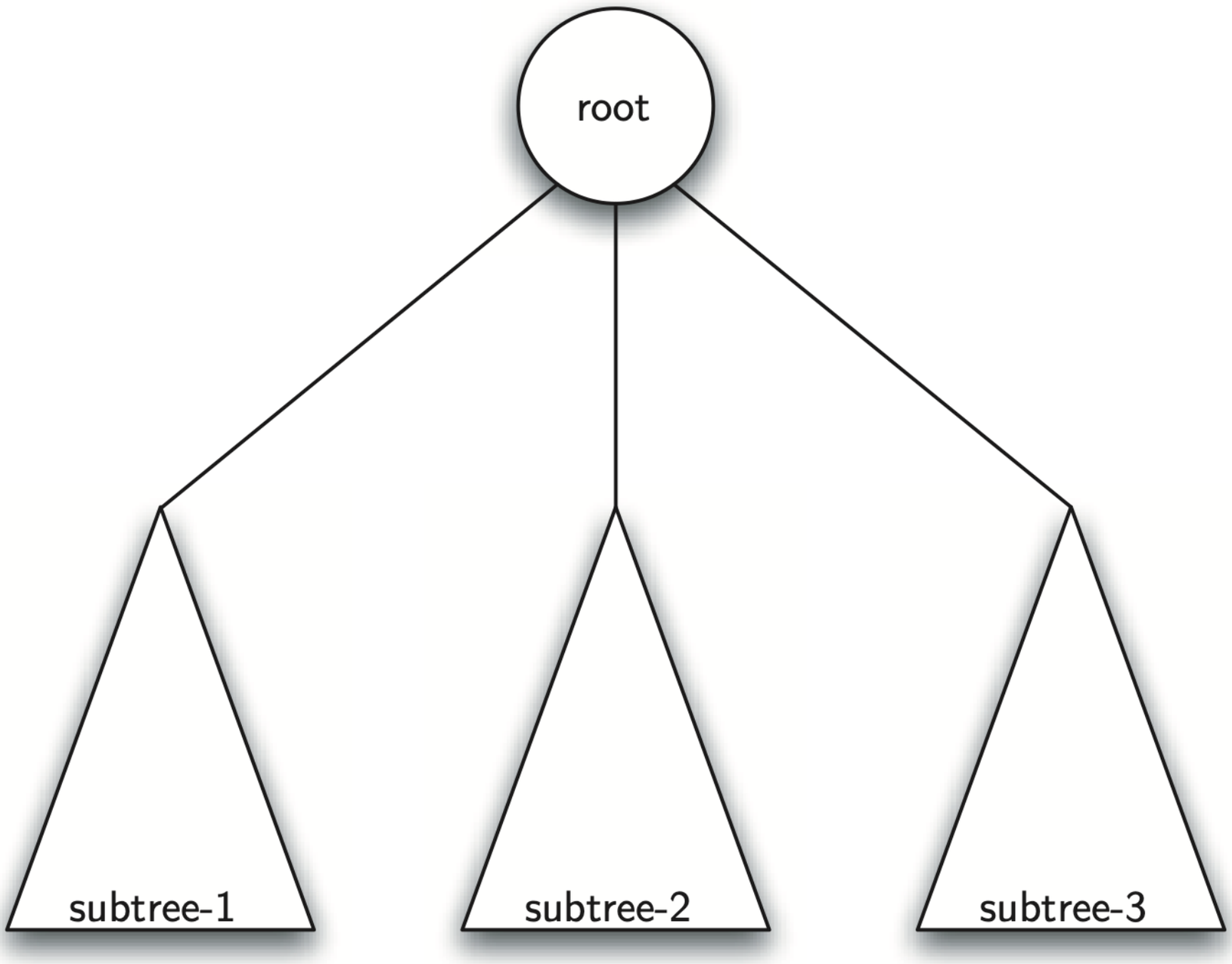
Trees

Defined Recursively

- A tree is either **empty** or consists of a **root and zero or more** subtrees, each of which is also a tree
- The root of each subtree is connected to the root of the parent tree by an edge

Trees

A Recursive Definition of a Tree



Trees
Specification

- `binary_tree()/BinaryTree()` creates a new instance of a binary tree using a procedural or OO method

- `get_left_child()` returns the binary tree corresponding to the left child of the current node

- `get_right_child()` returns the binary tree corresponding to the right child of the current node

Trees

Specification (cont.)

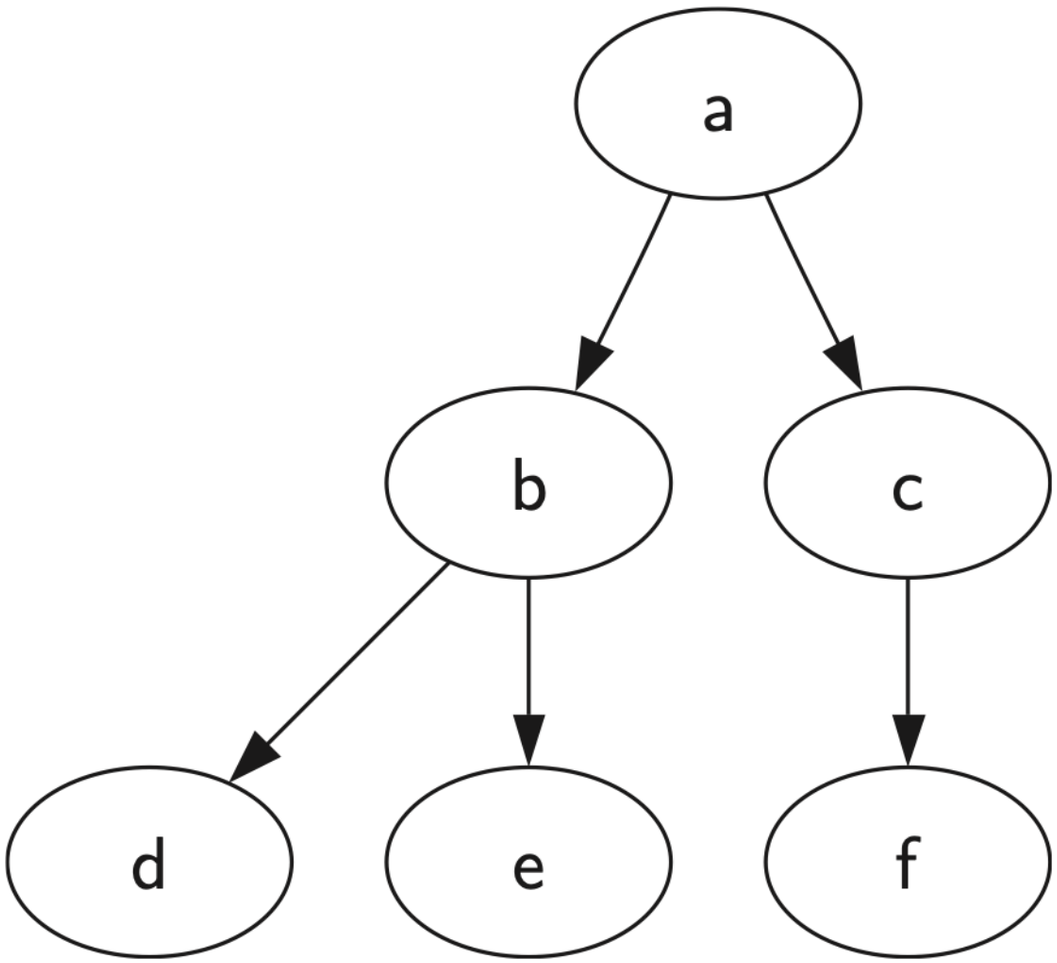
- `set_root_val(val)` stores the object in parameter val in the current node
- `get_root_val()` returns the object stored in the current node
- `insert_left(val)` creates a new binary tree and installs it as the left child of the current node
- `insert_right(val)` creates a new binary tree and installs it as the right child of the current node

Tree Implementation

Representing a Tree as a List of Lists

Tree Implementation

Representing a Tree as a List of Lists




```
mytree = ['a',    #root
          ['b',   #left subt
           ['d' [], []],
           ['e' [], []] ],
          ['c',   #right sub
           ['f' [], []],
           [] ]
          ]
```

Tree Implementation

Procedural Implementation

```
# This is an example of a binary tree data structure created
# with python lists as the underlying data structure.

def binary_tree(r):
    return [r, [], []]
```

Tree Implementation

Procedural Implementation

```
def insert_left(root, new_branch):
    t = root.pop(1)
    if len(t) > 1:
        root.insert(1, [new_branch, t, []])
    else:
        root.insert(1, [new_branch, [], []])

    return root
```

Tree Implementation

Procedural Implementation

```
def insert_right(root, new_branch):
    t = root.pop(2)
    if len(t) > 1:
        root.insert(2, [new_branch, [], t])
    else:
        root.insert(2, [new_branch, [], []])

    return root
```

Tree Implementation

Procedural Implementation

```
def get_root_val(root):
    return root[0]

def set_root_val(root, new_val):
    root[0] = new_val

def get_left_child(root):
    return root[1]

def get_right_child(root):
    return root[2]
```

Tree Implementation

Procedural Implementation (Usage 1)

```
r = binary_tree(3)
insert_left(r,4)
insert_left(r,5)
insert_right(r,6)
insert_right(r,7)
l = get_left_child(r)
print(l)

set_root_val(l,9)
print(r)
insert_left(l,11)
print(r)
print(get_right_child(get_right_child(r)))
```

Tree Implementation

Procedural Implementation (Usage 2)

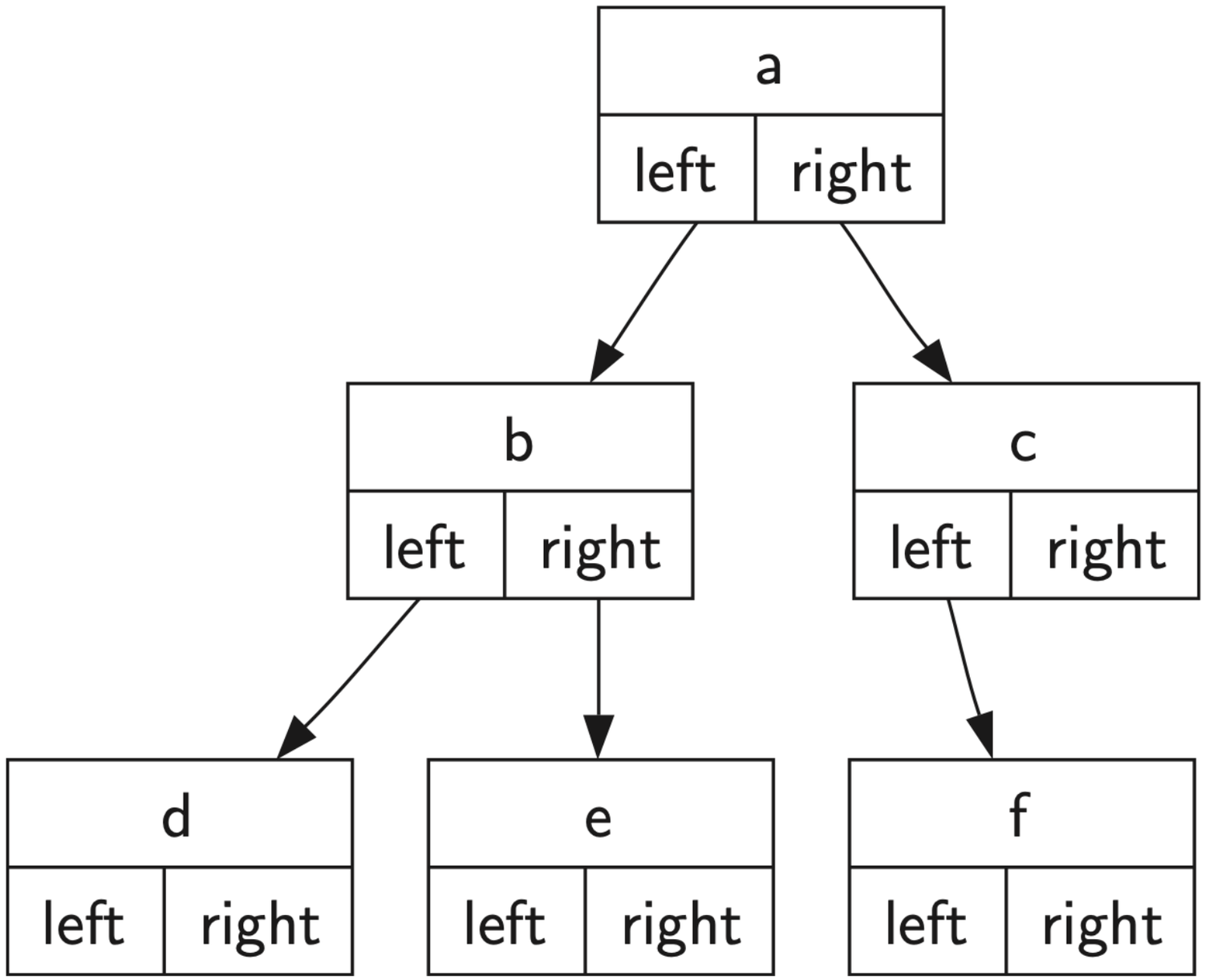
```
b = binary_tree(‘a')

# Build up the left side of this tree
insert_left(b,'b')
insert_right(get_left_child(b), 'd')

# Build up the right side of this tree
insert_right(b,'c')
insert_left(get_right_child(b), 'e')
insert_right(get_right_child(b), 'f')

print(b)
```

Tree Implementation

Using an Object-Oriented Approach

Tree Implementation

Using a Nodes and References Approach



Tree Implementation
Using a Nodes and References Approach

```
# This is an example of a BinaryTree data structure
# built as a class
# This example will only work if the rootObj passed into the
# class is a python primitive data type.

class BinaryTree:
    def __init__(self, root_obj):
        self.key = root_obj
        self.left_child = None
        self.right_child = None

    def insert_left(self, new_node):
        if self.left_child == None:
            self.left_child = BinaryTree(new_node)
        else:
            t = BinaryTree(new_node)
            t.left_child = self.left_child
            self.left_child = t
```

Tree Implementation

Using a Nodes and References Approach

```
def insert_right(self, new_node):
    if self.right_child == None:
        self.right_child = BinaryTree(new_node)
    else:
        t = BinaryTree(new_node)
        t.right_child = self.right_child
        self.right_child = t
```

```
def get_right_child(self):
    return self.right_child

def get_left_child(self):
    return self.left_child

def set_root_val(self, obj):
    self.key =obj

def get_root_val(self):
    return self.key
```

Tree Implementation

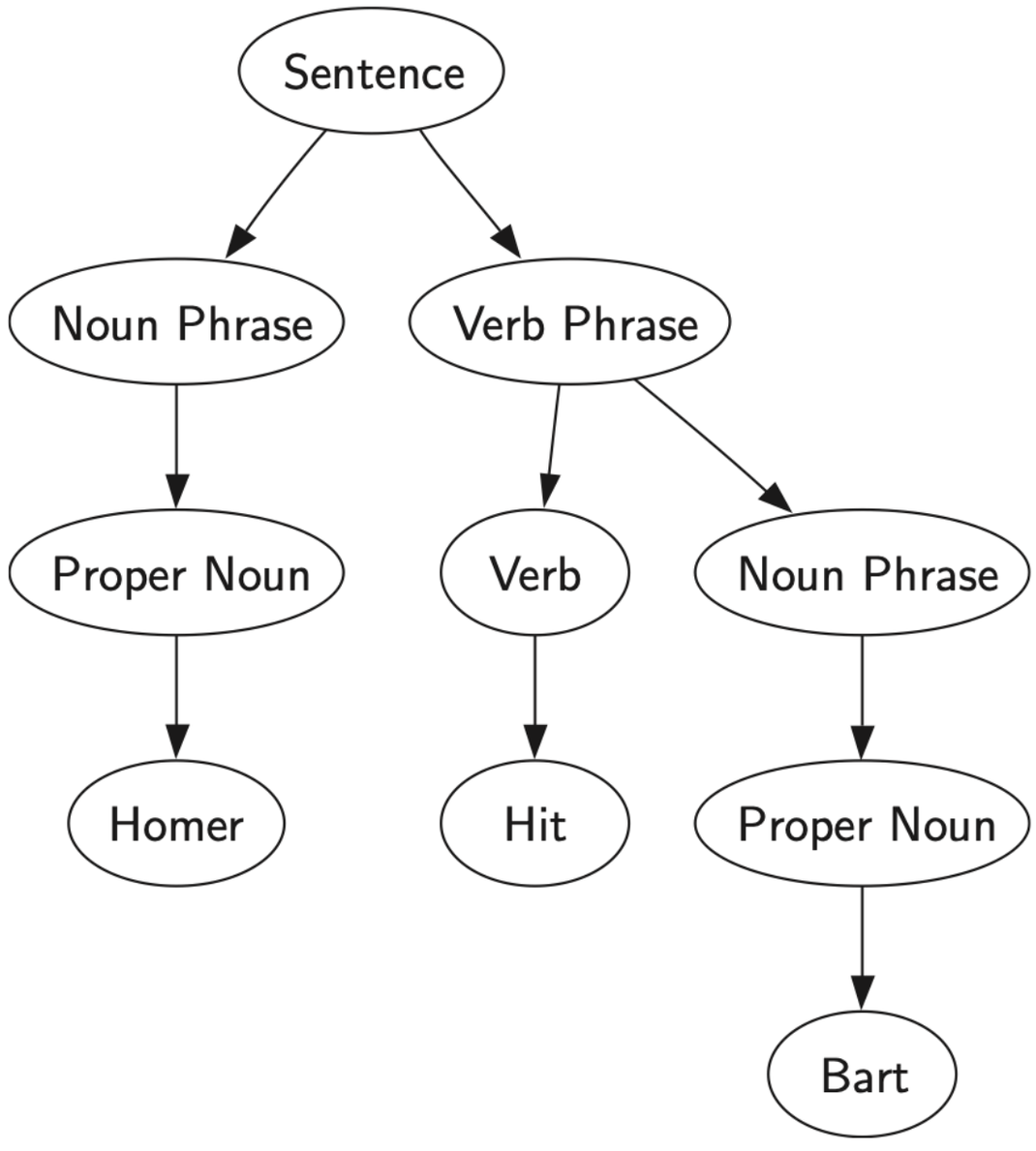
Using a Nodes and References Approach

```
r = BinaryTree('a')
print(r.get_root_val())
print(r.get_left_child())

r.insert_left('b')
print(r.get_left_child())
print(r.get_left_child().get_root_val())

r.insert_right('c')
print(r.get_right_child())
print(r.get_right_child().get_root_val())

r.get_right_child().set_root_val('hello')
print(r.get_right_child().get_root_val())
```

Binary Tree Applications

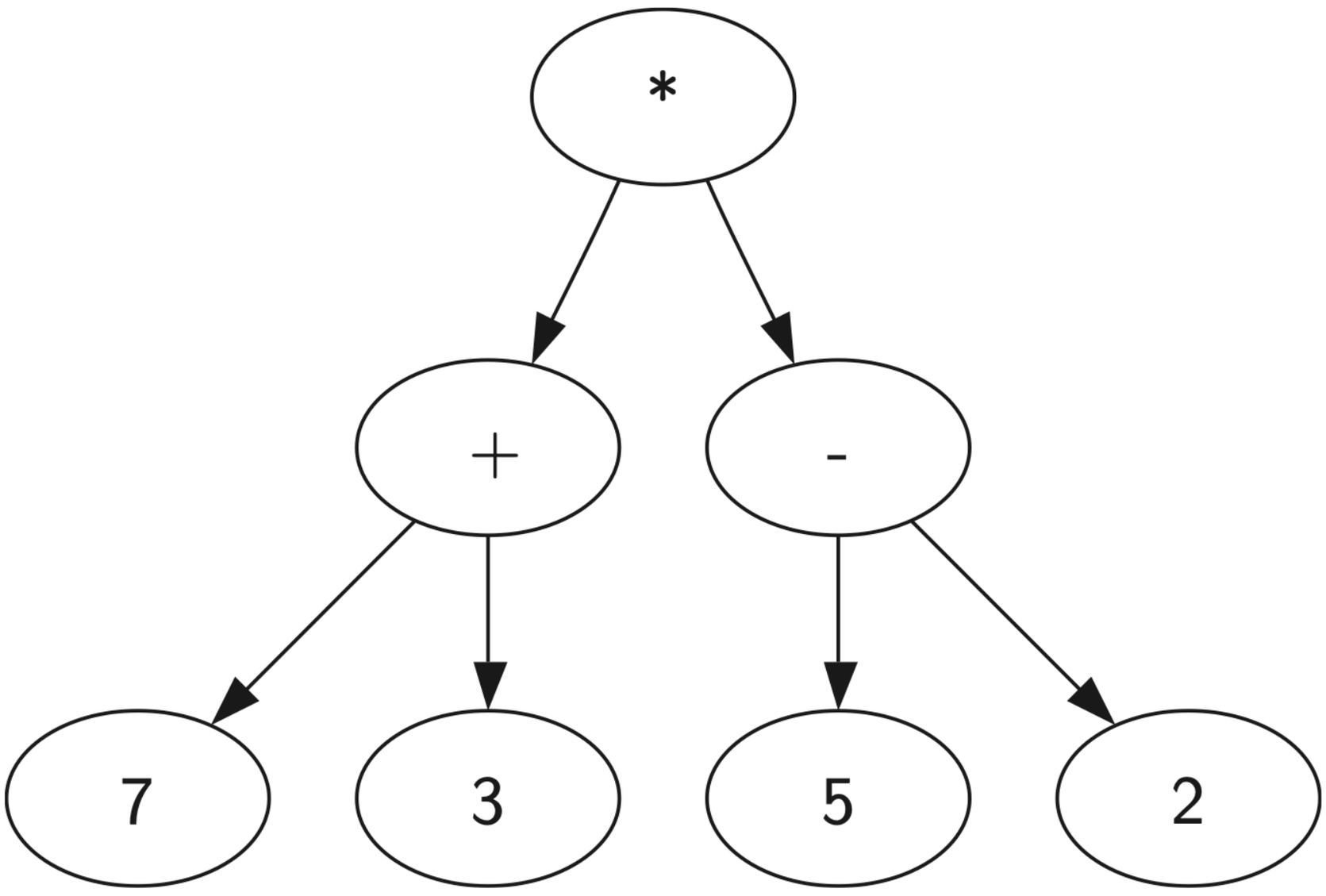
Binary Tree Applications

A Parse Tree for a Simple Sentence



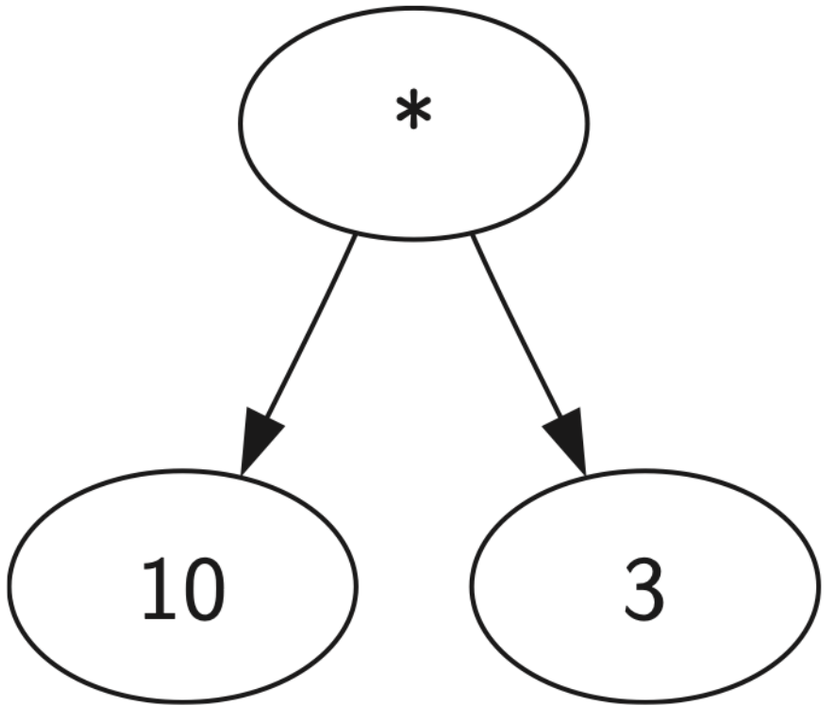
Binary Tree Applications

Parse Tree for ((7+3)∗(5−2))



Binary Tree Applications

Simplified parse tree for ((7 + 3) ∗ (5 − 2))



Binary Tree Applications

Simplified parse tree for ((7 + 3) ∗ (5 − 2))

- How to build a parse tree from a fully parenthesized mathematical expression
- How to evaluate the expression stored in a parse tree
- How to recover the original mathematical expression from a parse tree

Binary Tree Applications

Simplified parse tree for ((7 + 3) ∗ (5 − 2))

1. If the current token is a `"("`, add a new node as the left child of the current node, and descend to the left child
2. If the current token is in the list `['+','-','/','*']`, set the root value of the current node to the operator represented by the current token. Add a new node as the right child of the current node and descend to the right child

Binary Tree Applications

Simplified parse tree for ((7 + 3) ∗ (5 − 2))

3. If the current token is a number, set the root value of the current node to the number and return to the parent
4. If the current token is a `")"`, go to the parent of the current node

Binary Tree Applications

Tracing Parse Tree Construction

(3 + (4*5))

Binary Tree Applications

Tracing Parse Tree Construction

1. Create an empty tree
2. Read `"("` as the first token. By rule 1, create a new node as the left child of the root. Make the current node this new child
3. Read `"3"` as the next token. By rule 3, set the root value of the current node to `"3"` and go back up the tree to the parent

Binary Tree Applications

Tracing Parse Tree Construction

4. Read "+" as the next token. By rule 2, set the root value of the current node to "+" and add a new node as the right child. The new right child becomes the current node
5. Read a "(" as the next token. By rule 1, create a new node as the left child of the current node. The new left child becomes the current node

Binary Tree Applications

Tracing Parse Tree Construction

6. Read a "4" as the next token. By rule 3, set the value of the current node to "4". Make the parent of "4" the current node
7. Read "*" as the next token. By rule 2, set the root value of the current node to "*" and create a new right child. The new right child becomes the current node

Binary Tree Applications

Code to Create a Parse Tree

```
def build_parse_tree(fpexp):
    fplist = fpexp.split()
    p_stack = Stack()
    e_tree = BinaryTree('')

    p_stack.push(e_tree)
    current_tree = e_tree

    for i in fplist:
        if i == '(':
            current_tree.insert_left('')
             p_stack.push(current_tree)
             current_tree = current_tree.get_left_child()
        elif i not in ['+', '-', '*', '/', ')']:
            current_tree.set_root_val(int(i))
             parent = p_stack.pop()
             current_tree = parent

# Code continues on next slide...
```

Binary Tree Applications

Code to Create a Parse Tree (cont.)

```
        elif i in ["+", "-", "*", "/"]:
            # Create right child and descend
            current_tree.set_root_val(i)
            current_tree.insert_right('')
            p_stack.push(current_tree)
            current_tree = current_tree.get_right_child()
        elif i == ")":
            current_tree = p_stack.pop()
        else:
            raise ValueError("invalid expression given!")

    return e_tree

pt = buildParseTree("( ( 10 + 5 ) * 3 )")
```

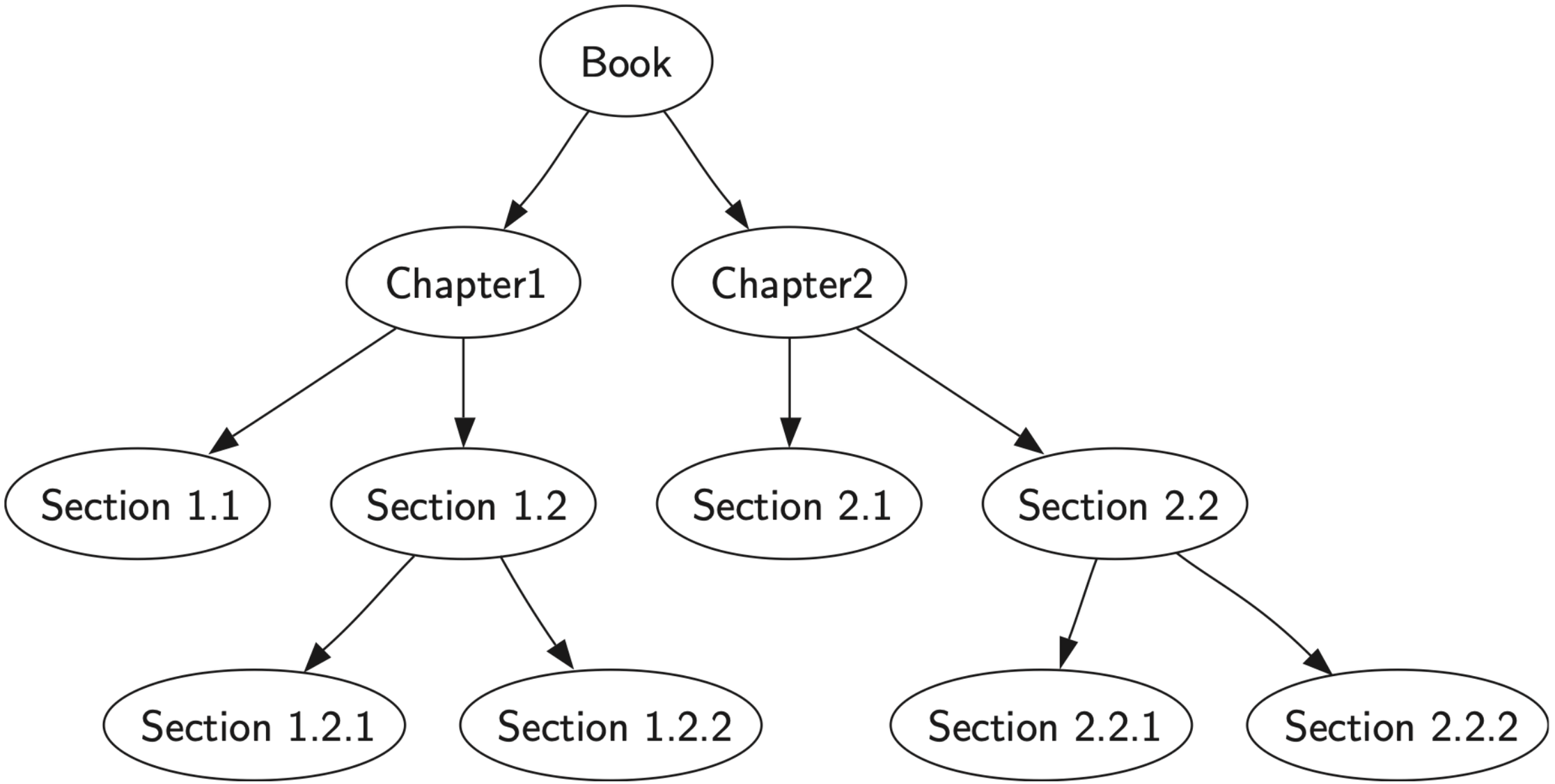
Binary Tree Applications

Recursive Function to Evaluate a Binary Parse Tree

```
def evaluate(parse_tree):
    opers = {
        '+': operator.add,
        '-': operator.sub,
        '*': operator.mul,
        '/': operator.truediv
    }

left_c = parse_tree.get_left_child()
right_c = parse_tree.get_right_child()

if left_c and right_c:
   fn = opers[parse_tree.get_root_val()]
    return fn(evaluate(left_c), evaluate(right_c))
else:
   return parse_tree.get_root_val()
```

Binary Tree Applications

Representing a Book As a Tree



Binary Tree Applications

External Function Implementing Preorder Traversal of a Tree I

```
def preorder(tree):
    if tree:
        print(tree.get_root_val())
        preorder(tree.get_left_child())
        preorder(tree.get_right_child())
```

Binary Tree Applications

Postorder Traversal Algorithm I

```
def postorder(tree):
    if tree:
        postorder(tree.get_left_child())
        postorder(tree.get_right_child())
        print(tree.get_root_val())
```

Binary Tree Applications

Postorder Evaluation Algorithm I

```
def evaluate_post(tree):
    opers = {
        '+': operator.add,
        '-': operator.sub,
        '*': operator.mul,
        '/': operator.truediv
    }

    res1 = None
    res2 = None

    if tree:
        res1 = evaluate_post(tree.get_left_child())
        res2 = evaluate_post(tree.get_right_child())
        if res1 and res2:
            fn = opers[tree.get_root_val()]
            return fn(res1, res2)
        else:
            return tree.get_root_val()
```

Binary Tree Applications

Inorder Traversal Algorithm I

```
def inorder(tree):
    if tree:
        inorder(tree.get_left_child())
        print(tree.get_root_val())
        inorder(tree.get_right_child())
```

Binary Tree Applications

Modified Inorder Traversal to Print Fully Parenthesized Expression I

```
def printexp(tree):
    s_val = ""
    if tree:
        s_val = '(' + printexp(tree.get_left_child())
        s_val += str(tree.get_root_val())
        s_val += printexp(tree.get_right_child()) + ')'

    return s_val
```

Binary Tree Applications

Recursive Function to Evaluate a Binary Parse Tree

```
in_string = "( ( 10 + 5 ) * 3 )"
print(in_string)
pt = build_parse_tree(in_string)

print(evaluate_post(pt))
print(preorder(pt))
print(postorder(pt))
print(inorder(pt))

print(printexp(pt))
```

Questions?