

1 Schedule

IMPORTANT: Homework is **due Dec 8, 1pm**. Bring it to CSB 620 or 621 anytime between 9am and 1pm (the TA is travelling—late homework will not be graded).

There is no **QUIZ** for this section.

2 List of algorithms covered in the class

(B-basic, I-intermediate, A-advanced):

- I: Vertex cover (p. 278, DSV).
- I: Clustering (p. 280, DSV).
- I: TSP (p. 282, DSV).
- A: Knapsack (p. 283, DSV).
- A: Set-cover.

3 Basic material

Important concepts, problems, theorems, and algorithms:

- NP, NP-hard, NP-complete, reduction.
- Basic NP-complete problems (SAT, 3SAT, vertex cover, clique, k -coloring, integer linear programming)

Homework):

6.1 (due Dec 8, 1pm) Give a reduction from 3-COLORING to integer linear programming.

6.2 (due Dec 8, 1pm) Suppose that you have a black-box which for any input graph G answers whether G is 3-colorable or not. Assume that the black-box takes time $O(1)$ to answer the queries.

Give an efficient algorithm which, for any 3-colorable input graph G , finds a 3-coloring of G . At the beginning of your solution clearly state:

- the (asymptotic) running time of your algorithm, and
- the (asymptotic) number of queries made to the black-box.

6.3 (due Dec 8, 1pm) A vertex cover of a graph $G = (V, E)$ is a subset S of vertices such that for each edge $e \in E$ at least one of its endpoints is in S . Consider the following optimization problem:

VERTEX-COVER

INSTANCE: A graph G .

SOLUTION: A vertex cover S of G .

OBJECTIVE: Minimize the size of S .

Consider the following reduction from VERTEX-COVER to INTEGER LINEAR PROGRAMMING. For each vertex $i \in V = \{1, \dots, n\}$ we will have a variable x_i and constraints $0 \leq x_i$ and $x_i \leq 1$. For each edge $\{i, j\} \in E$ we will have a constraint $x_i + x_j \geq 1$. Finally, the objective is to minimize $x_1 + \dots + x_n$. Thus our integer linear program is

$$\begin{aligned} \min \quad & \sum_{i \in V} x_i \\ & x_i \leq 1, \text{ for } i \in V, \\ & x_i \geq 0, \text{ for } i \in V, \\ & x_i + x_j \geq 1, \text{ for } \{i, j\} \in E, \\ & x_i \in \mathbb{Z}, \text{ for } i \in V \quad (\text{integrality}). \end{aligned} \tag{1}$$

Let O be the optimum of (1).

Now view (1) as a linear program (i.e., drop the integrality constraint). Let R be the optimum of this linear program.

a) Which of the following two is always true?

$$R \leq O \qquad O \leq R$$

b) Find a graph G for which the values of O and R are different.

6.4 (due Dec 8, 1pm)[BONUS PROBLEM] Prove that if the graph G in Problem 6.3 is bipartite, then $O = R$.

4 Additional problems from the book (do NOT turn in)

Solve the following problems from the book.

- 8.1, 8.3, 8.4, 8.6, 8.14, 8.18, 9.7, 9.9.