

1 Schedule

Problem sessions:

Mon, Nov. 19, 7pm-8pm

Mon, Nov. 26, 7pm-8pm

Wed, Nov. 28, 8:15pm-9:15pm

(There will be no problem session on Wed, Nov. 21)

The quiz will be on Thursday, Nov. 29.

2 List of algorithms covered in the class

(B-basic, I-intermediate, A-advanced):

I: Gaussian elimination (p. 219, DSV).

I: Computing the dual of a linear program (p. 206, DSV).

A: The simplex algorithm (p. 213, DSV).

B: Zero-sum games using linear programming (p. 208, DSV).

A: Max-flow using linear programming (p. 198, DSV).

3 Basic material

Important concepts, problems, theorems, and algorithms:

- system of linear equations, rank of a matrix,
- linear program, dual linear program,
- basic matrix notation.

Testing method:

- Solve a small linear program (2-3 variables).
- Solve a system of equations using Gaussian elimination (up to 4 variables).
- Given a system of equations, write it in a matrix form.
- Compute rank of a matrix.
- Solve a zero-sum game.

Example problems:

7.1 (due Nov 27, 2007) Solve the following linear program:

$$\begin{aligned} \max x_1 + x_2 \\ x_1 + 2x_2 &\leq 3 \\ 3x_1 + x_2 &\leq 4 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

7.2 (due Nov 20, 2007) Solve the following system of equations using Gaussian elimination:

$$\begin{aligned}y_1 + y_2 + y_4 &= 4, \\y_1 + y_3 &= 10, \\y_3 + 4y_4 &= 3, \\y_2 + y_4 &= 1.\end{aligned}$$

7.3 (due Nov 20, 2007) Write the following system of equations in the matrix form $Ax = b$:

$$\begin{aligned}y_1 + y_2 + y_4 &= 4, \\y_1 + y_3 &= 10, \\y_3 + 4y_4 &= 3, \\y_2 + y_4 &= 1.\end{aligned}$$

7.4 (due Nov 20, 2007) Compute the rank of the following two matrices

$$\begin{pmatrix} 2 & 4 & 9 \\ 1 & -3 & 5 \\ 0 & 10 & -1 \end{pmatrix}, \begin{pmatrix} 2 & 4 & 9 & 3 \\ 1 & -3 & 5 & -1 \\ 0 & 10 & -1 & 5 \end{pmatrix}$$

Does the system $2x + 4y + 9z = 3, x - 3y + 5z = -1, 10y - z = 5$ have a solution?

7.5 (due Nov 20, 2007) Write the following system of equations in the matrix form $Ax = b$:

$$\begin{aligned}y_1 + y_2 + y_4 &= 4, \\y_1 + y_3 &= 10, \\y_3 + 4y_4 &= 3, \\y_2 + y_4 &= 1.\end{aligned}$$

4 Additional homework

7.6 (due Nov 27, 2007) [Problem rating: I.] Solve the following linear program:

$$\begin{aligned}\max \sum_{i=1}^9 x_i, \\x_1 + x_2 &\leq 1, \\x_2 + x_3 &\leq 1, \\x_3 + x_4 &\leq 1, \\x_4 + x_5 &\leq 1, \\x_5 + x_6 &\leq 1, \\x_6 + x_7 &\leq 1, \\x_7 + x_8 &\leq 1, \\x_8 + x_9 &\leq 1, \\x_9 + x_1 &\leq 1, \\x_i &\geq 0, i = 1, \dots, 9.\end{aligned}$$

Use a **linear programming solver** to obtain the solution (for example you can use freeware `lpsolve` or function `Maximize` in `MATHEMATICA` (installed in most labs)).

7.7 (due Nov 20, 2007) [Problem rating: I.] There are n bottles which contain different mixtures of three chemicals called A, B, C . The i -th bottle contains the chemicals in ratio $a_i : b_i : c_i$ (thus, $a_i/(a_i + b_i + c_i)$ fraction of the i -th

bottle is chemical A , $b_i/(a_i + b_i + c_i)$ fraction of the i -th bottle is chemical B , and $c_i/(a_i + b_i + c_i)$ fraction of the i -th bottle is chemical C). We want to know whether it is possible to obtain a mixture containing the chemicals A, B, C in ratio $a : b : c$ by mixing various amounts from the bottles. Give an efficient algorithm for this problem.

For example, if the input is $n = 2$, the ratios in the bottles are $1 : 1 : 2$ and $3 : 3 : 1$, and we want to obtain mixture with ratio $1 : 1 : 1$ then the answer is YES (we can take 2 parts from the first bottle and 1 part from the second bottle).

7.8 (due Nov 27, 2007) [Problem rating: I.] Construct the linear program dual to the following linear program:

$$\begin{aligned} \max x_1 + 2x_2 + 3x_3 + 4x_4 \\ x_2 + x_3 + x_4 &\leq 1 \\ x_1 + x_3 + x_4 &\leq 2 \\ x_1 + x_2 + x_4 &\leq 3 \\ x_1 + x_2 + x_3 &\leq 4 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0. \end{aligned}$$

Find the optimal solution of the primal and the dual problem. **Use a linear programming solver** to obtain the solutions.

5 Additional problems from the book (do not turn in)

Try to solve the following problems. A few of them will be on the quiz. We will go over the ones that you choose in the problem sessions.

- 7.1, 7.2, 7.3, 7.4, 7.7, 7.8, 7.11, 7.12, 7.13, 7.15, 7.19, 7.27.