

On the Representation of Vague and Uncertain Knowledge

L. K. Schubert
University of Alberta

Summary

The present investigation arose within the context of research on a language understanding and reasoning system based on a semantic net (Schubert, Goebel, & Cercone, to appear). Any belief system designed for nondeductive plausible inference must be capable of treating propositions as more or less credible. Perception, inference, and informants are all unreliable sources of information; also informants may explicitly qualify propositions "probabilistically," as in the sentence "I probably will not be in my office tomorrow." For a given propositional representation (e.g., logic or semantic nets), credibility information can be added systematically by the introduction of two propositional functions $C(p)$ and $D(x,p)$ with values ranging over some subset of the interval $[0,1]$, including its endpoints. $C(p)$ denotes the credibility of proposition p , and $C(p) = 1$, $C(p) = 0$ express respectively that p is certainly true and that p is certainly false. $D(x,p)$ denotes individual x 's degree of belief in proposition p , and $D(x,p) = 1$, $D(x,p) = 0$ express respectively that x regards p as certainly true, and that x regards p as certainly false. These functions permit the expression of both absolute and relative credibilities. Relative credibilities derive from statements involving such phrases as "more likely" or "relatively unlikely."

Where statements concern gradable attributes such as age, height, or temperature, simple credibilities can be replaced by credibility distributions over base variables for these attributes (Zadeh, 1975). Such distributions over mutually exclusive, jointly exhaustive "spectra" of base variable values generalize credibility distributions over incompatible pairs p , $\neg p$ of propositions. Consideration of such spectra supports the interpretation of credibilities as (subjective) probabilities, since the total credibility of the disjunction of alternatives is then 1 and the credibility of the conjunction of any 2 alternatives is 0, as it should be.

In general, the credibility assigned to a proposition, or credibility distribution assigned to a base variable, as a result of an informant's statement depends not only on the form of the statement (and its context) but also on prior beliefs about the proposition and about the informant. If credibilities are regarded as probabilities, then the required adjustments can be made on Bayesian principles.

The main thesis of the paper is that not only uncertainty but also the pragmatics of vagueness can be modelled in terms of credibilities and credibility distributions. The pragmatic analysis contrasts sharply with the usual semantic analyses of vagueness (e.g., see the articles on vagueness in Synthese, vol. 30, 1975).

For vague predicates the principle of bivalence fails, i.e., there are "borderline cases" where the predicates are neither definitely true nor definitely false. A key issue in semantic theories of vagueness concerns compounds of contradictories. For example, Loth Fine (1975) and Dummett (1975) regard the conjunction "b is pink and b is red" as definitely false of any uniformly coloured blob b , even when "b is pink" and "b is red" have indefinite truth

values (as they might if b is intermediate in colour between pink and red). The reason is the supposed incompatibility of the predicates "pink" and "red." This intuition finds formal expression in the "precisification" theory of vagueness, in which a formula is regarded as true if all possible classifications of the borderline cases of the predicates as definite instances or definite noninstances of the predicates make it true.

In the precisification theory "indefinite" does not behave truth-functionally. By contrast, "fuzzy logic" theories of vagueness (Zadeh, 1972, 1975a,b, 1977a,b; Lakoff, 1973) treat "indefinite" as a range of intermediate truth values or "possibility" values. Under the truth-value arithmetic of such theories, the conjunction of statements with indefinite truth values is again indefinite in truth value.

The author conducted a casual experiment in a class of 40 computing science sophomores to determine whether there is any psychological basis for preferring one theory over the other. The experiment involved classifying statements about disks of various shades from pink to red as true (T), false (F), or neither true nor false (U). Those cases were examined where the precisification theory predicts F for a UU conjunction while the fuzzy logic theory predicts U for the same conjunction. The predominant response was neither F nor U, but T! Furthermore, most FF conjunctions were rated T. Thus, two non-truths can make a truth; e.g. "The disk is pink," and "The disk is red" may be considered individually indefinite or false for a given disk, but true in conjunction.

In any adequate computational theory of vagueness, it is necessary to take into account not only failure of the principle of bivalence, but also variations in application of vague terms by language users (e.g., Black, 1937). This pragmatic phenomenon introduces an element of uncertainty into the communication process. When an informant states that "There are several apples on the table," it is uncertain to what range of numbers the informant considers the term "several" applicable. This kind of example motivates the following approach to the analysis of vague adjectives expressing gradable attributes. The hearer is assumed to possess (or be capable of generating by plausible inference) prior credibility distributions over base variable values, e.g., a distribution for the prior credibility of various numbers of apples on the table. The pragmatic profile of such a vague term is taken to be a "compatibility function" of exactly the form stipulated by Zadeh (e.g., 1977a,b), i.e., a function assuming values in $[0,1]$ over a base variable. However, these values are not interpreted as possibilities, as by Zadeh, but as conditional probabilities that a language user will consider the vague term applicable, if he knows the value of the base variable. The accommodation of a vague predication is then simple: it consists essentially of taking the normalized product of the prior credibility distribution with the pragmatic profile according to Bayes' formula. Conversely, a language user is willing to employ a vague term whenever he is sufficiently confident that other language users would consider the term applicable. The required measure of confidence can again be computed from the prior credibilities and pragmatic profile.

From this probabilistic point of view, the puzzle of true UU and FF conjunctions seems solvable. If the "tails" of the pragmatic profiles for "pink" and "red" overlap in the "pinkish-red" region, then, even if the two predicates are not considered applicable in

themselves, their conjunction has the effect of making pinkish-red shades in the region of overlap more probable than any others; i.e., the conjunction places the shade of colour exactly where it is perceived to lie. Hence the conjunction may reasonably be considered true.

The semantic consequences of the proposed pragmatic theory are as yet unclear. The pragmatic account appears to obviate the need for "degrees of truth" in the semantics. The account does seem compatible with a semantic theory in which vagueness amounts to "deficiency of meaning", as in precisification theories.

REFERENCES

- Black, M. (1937). Vagueness: an exercise in logical analysis. Philos. Sci., 4, 427-455.
- Fine, K. (1975). Vagueness, truth and logic. Synthese 30, 265-300.
- Dummett, M.A.E. (1975). Wang's paradox. Synthese 30, 301-324.
- Lakoff, G. (1973). Hedges: a study in meaning criteria and the logic of fuzzy concepts. J. Philos. Logic 2, 458-508.
- Schubert, L. K., Goebel, R., & Cercone, N. (to appear). The structure and organization of a semantic net for comprehension and inference. In Findler, N.V. (ed.), Associative Nets - The Representation and Use of Knowledge by Computers, Academic Press.
- Zadeh, L. A. (1972). A fuzzy set interpretation of linguistic hedges. J. Cybernetics 2, 4-34.
- Zadeh, L.A. (1975a). The concept of a linguistic variable and its application to approximate reasoning - I. Inf. Sciences 8, 199-249.
- Zadeh, L. A. (1975b). The concept of a linguistic variable and its application to approximate reasoning - II. Inf. Sciences 8, 301-357.
- Zadeh, L. A. (1977a). A theory of approximate reasoning. Memorandum No. UCB/ERL M77/58, Electronics Research Lab., College of Engineering, University of California, Berkeley, Ca.
- Zadeh, L. A. (1977b). PRUF - A meaning representation language for natural languages. Memorandum No. ERL-M77/61, Electronics Research Lab., College of Engineering, University of California, Berkeley, Ca.