

RESOLUTION

Resolution + paramodulation provide a simple basis for mechanized deduction & Q.A.

- dominant technique since A. Robinson '65
- basis of Prolog

1. Add denial of desired conclusion φ to premises Δ , forming $\Delta \cup \{\neg\varphi\}$
2. Convert $\Delta \cup \{\neg\varphi\}$ to clause form
3. Repeatedly resolve (+factor +paramodulate) till \square is derived.

Simple example

- 1. All dogs bark
- 2. Snoopy is a dog
- Prove: Something barks
- Denial: Nothing barks

$$(\forall x (\text{Dog}(x) \Rightarrow \text{Barks}(x)))$$

$$\text{Dog}(\text{Snoopy}) \quad \Delta$$

$$(\exists x \text{Barks}(x))$$

$$\neg(\exists x \text{Barks}(x)) \quad \neg\phi$$

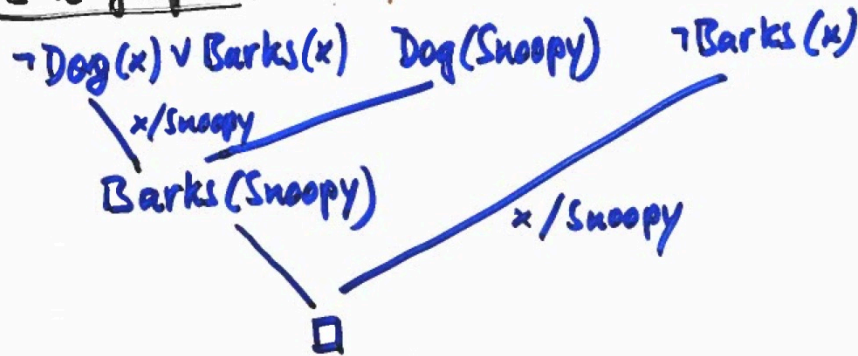
Conversion :

- 1. $\neg \text{Dog}(x) \vee \text{Barks}(x)$ convert \Rightarrow to \vee , drop \forall
- 2. $\text{Dog}(\text{Snoopy})$ no change
- 3. $\neg \text{Barks}(x)$ change $\neg \exists$ to $\forall \neg$, drop \forall

Refutation

- r[2, 1a] 4. $\text{Barks}(\text{Snoopy})$ (x/Snoopy)
- r[4, 3] 5. \square (x/Snoopy)

As a graph :



CONVERSION TO CLAUSE FORM

cf, CNF in satisfiability testing

- 1. Eliminate \Rightarrow and \Leftrightarrow
 $\phi \Rightarrow \psi \dots \dots \rightarrow \neg \phi \vee \psi$
 $\phi \Leftrightarrow \psi \dots \dots \rightarrow (\phi \Rightarrow \psi) \wedge (\psi \Rightarrow \phi)$
- 2. Move \neg inward (5 rules)
 $\neg(\phi \vee \psi) \rightarrow \neg \phi \wedge \neg \psi$
 $\neg(\phi \wedge \psi) \rightarrow \neg \phi \vee \neg \psi$
 $\neg \neg \phi \rightarrow \phi$
 $\neg(\forall x \phi) \rightarrow \exists x \neg \phi$
 $\neg(\exists x \phi) \rightarrow \forall x \neg \phi$
- 3. Rename duplicate bound variables
- 4. Skolemize (getting rid of \exists 's)
- 5. Drop \forall 's

E.g., $(\exists x \text{Robot}(x)) \dots \dots \rightarrow \text{Robot}(c)$
 give name to "the x that exists"

E.g., $(\forall x (\exists y \text{Bigger-than}(y, x)))$
 \downarrow Make up a function that picks an appropriate y for each x
 $\text{Bigger-than}(f(x), x)$
 we drop \forall because it makes all literals literal

Skol(ϕ) is satisfiable iff ϕ is satisfiable
 unsatisfiable iff ϕ is unsatisfiable

E.g., "For every pair of persons in the world, there is a 3rd person who speaks all of the languages of the first two"

$$\forall x \forall y \exists z. S(x, y, z)$$

<u>x</u>	<u>y</u>	<u>f(x, y)</u>
Smith	Lafrance	Trudeau
Smith	Glukov	Yevtushenko
Kohl	Yevtushenko	Yevtushenko
Trudeau	Yevtushenko	Nabokov
	etc.	

$$S(x, y, f(x, y)) \quad \text{dropping } \forall: \text{ literal}$$

6. Move \forall inward (distribute \forall over \wedge)

$$(\varphi \wedge \psi) \forall x \dots \rightarrow (\varphi \forall x) \wedge (\psi \forall x)$$

$$x \forall (\varphi \wedge \psi) \dots \rightarrow (x \forall \varphi) \wedge (x \forall \psi)$$

7. Drop \wedge

E.g.

$$(\forall x [(\exists y [P(x, y) \vee Q(x, y)]) \Rightarrow (\exists y R(x, y))])$$

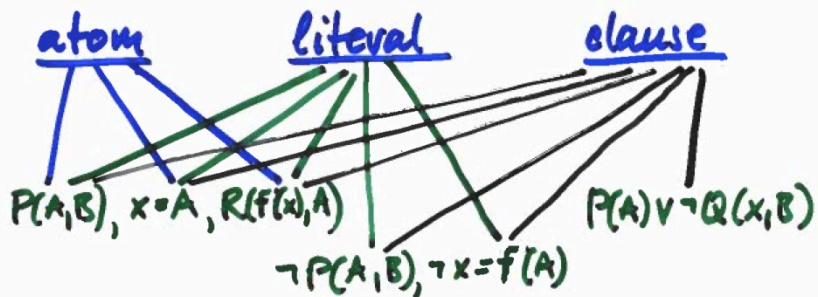
negate

yields a Skolem function of x, after renaming y to z

See if you can get 2 clauses,

$$\neg P(x, y) \vee R(x, f(x)), \quad \neg Q(x, y) \vee R(x, f(x))$$

Some terminology



Narrow resolution (without unification)

$\frac{\textcircled{\varphi} \vee \psi, \textcircled{\neg\varphi} \vee \chi}{\psi \vee \chi}$ more gen: $\frac{\textcircled{\varphi} \vee \psi_1 \vee \dots \vee \psi_m, \textcircled{\neg\varphi} \vee \chi_1 \vee \dots \vee \chi_n}{\psi_1 \vee \dots \vee \psi_m \vee \chi_1 \vee \dots \vee \chi_n}$

cf., a syllogism:

$\frac{P \Rightarrow Q, Q \Rightarrow R}{P \Rightarrow R}$

Let P be $\neg\psi$
Q " φ
R " χ

$\frac{\neg\psi \Rightarrow \varphi, \varphi \Rightarrow \chi}{\neg\psi \Rightarrow \chi}$

or $\frac{\psi \vee \textcircled{\varphi}, \textcircled{\neg\varphi} \vee \chi}{\psi \vee \chi}$ as above

soundness?

Narrow resolution, with unification:

$$P(x, f(x), z) \quad P(A, f(A), z) \quad P(A, f(A), z)$$

$$\neg P(A, y, w) \xrightarrow{x/A} \neg P(A, y, w) \xrightarrow{y/f(A)} \neg P(A, f(A), w) \xrightarrow{z/w}$$

most general unifier (mgu) $(x/A)(y/f(A))(z/w)$
i.e., constraints $x=A, y=f(A), z=w$

Example

- $\neg P(x, y, u) \vee \neg P(y, z, v) \vee \neg P(u, z, w) \vee \frac{P(x, v, w)}{1d}$
- $\frac{\neg P(i(x), E, w) \vee P(u, x, E)}{2a}$

Want to resolve 1d, 2a

Rename x, u in (2) $\dots \rightarrow s, t$

- $\neg P(x, y, u) \vee \neg P(y, z, v) \vee \neg P(u, z, w) \vee \frac{P(x, v, w)}{1d}$
- $\frac{\neg P(i(s), E, t) \vee P(t, s, E)}{2a}$

mgu: $(x/i(s))(v/E)(t/w)$
 $\leftarrow \text{or } (w/t)$

Unify (not normally shown separately):

$$1. \neg P(i(s), y, u) \vee \neg P(y, z, E) \vee \neg P(u, z, w) \vee \underline{P(i(s), E, w)}$$

$$2. \underline{\neg P(i(s), E, w)} \vee P(w, s, E)$$

$r[1d, 2e]$:

$$3. \neg P(i(s), y, u) \vee \neg P(y, z, E) \vee \neg P(u, z, w) \vee P(w, s, E)$$

As part of a refutation graph:

$$1. \neg P(x, y, u) \vee \neg P(y, z, v) \vee \neg P(u, z, w) \vee \underline{P(x, v, w)}$$

$$2. \underline{\neg P(i(s), E, t)} \vee P(t, s, E)$$

$(x/i(s))(v/E)(t/w)$

$$3. \neg P(i(s), y, u) \vee \neg P(y, z, E) \vee \neg P(u, z, w) \vee P(w, s, E)$$

Factoring

E.g., $\underline{P(x, A)} \vee Q(x, y) \vee \underline{P(f(y), z)}$
 $(x/f(y))(z/A)$

factor $\underline{P(f(y), A)} \vee Q(f(y), y)$

E.g., 1. $\underline{\neg P(x, y, u)} \vee \neg P(y, z, v) \vee \underline{\neg P(u, z, w)} \vee P(x, v, w)$
 $(w/x)(z/y)(w/x)$ N.B.

factor $f[1a, e]: \underline{\neg P(x, y, x)} \vee \neg P(y, y, v) \vee P(x, v, x)$

Why we need factoring:

1. $P(x) \vee P(y)$
 2. $\neg P(u) \vee \neg P(w)$ } clearly unsatisfiable!

3. $P(y) \vee \neg P(w)$ $r[1a, 2a]$

4. $\neg P(w) \vee \neg P(w)$ $r[2a, 3a]$, after renaming

etc.

can't derive \square !