CSC2/455 Software Analysis and Improvement
Dominators and SSA Form - II

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URCS

Outline

Review

Dominance Frontiers and Dominator Trees

Emitting code for SSA form

The SSA Form and Functional Programming

Postscript

Outline

Dominators

- A node n in the CFG dominates a node m iff:
  - n is on all paths from entry to m
  - by definition, a node n always dominates itself
  - if $n \neq m$, then n *strictly* dominates m
- Computed using a dataflow-style analysis
  - Each node annotated with a set of its dominators

Static Single Assignment Form

- Simple algorithm to generate SSA form
  - Introduce ϕ functions
  - Rename variables using Reaching Definitions
- Algorithm can generate excessive ϕ functions
  - TODAY: Use *dominance frontiers* to place the minimal number of ϕ functions
- Also today: Removing ϕ functions
  - Machines don't support ϕ functions, so we must emulate them

Maximal SSA Form

- Insert ϕ nodes for each definition at every join node
- Rename LHS
- Rename RHS using reaching definitions

Reducing the number of *phi* nodes

- Why insert ϕ nodes at only join nodes?
- Can we skip inserting ϕ nodes for a definition at some join node?

Outline

Dominance Frontiers

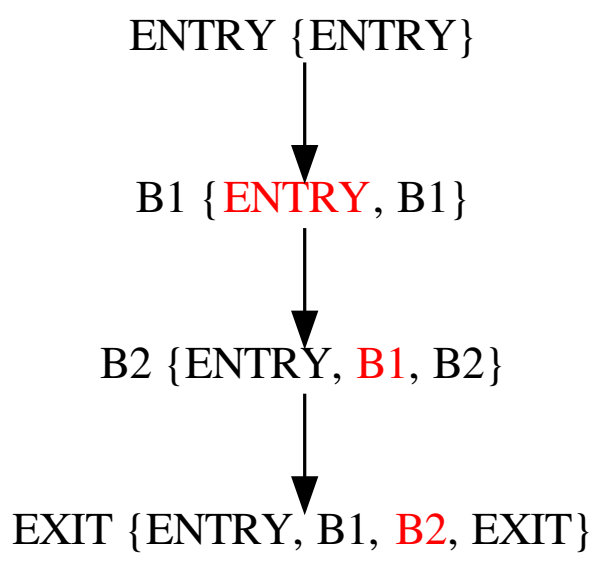
- The dominance frontier of a node n ($\mathrm{DF}(n)$) is a *set* of nodes
- A node $m \in \mathrm{DF}(n)$ iff:
  - n does not strictly dominate m
  - n dominates q where $q \in \mathrm{pred}(m)$
- Note that dominance frontiers only contain *join* nodes
  - I.e. nodes with multiple predecessors
- Computing the dominance frontier of each node:
  - Iterative Data-flow analysis?

Dominance Frontiers: Direct algorithm

Direct calculation of dominance frontiers using *dominator trees*.

Immediate Dominators

- The *immediate* dominator of a node m ($\mathrm{IDOM}(m)$) is the node n:
  - such that n strictly dominates m, and
  - n does not strictly dominate o where $o \in (\mathrm{DOM}(m) - \{m\})$
  - in some sense, n is the "closest" dominator in the CFG to m.
- By definition, ENTRY has no immediate dominator

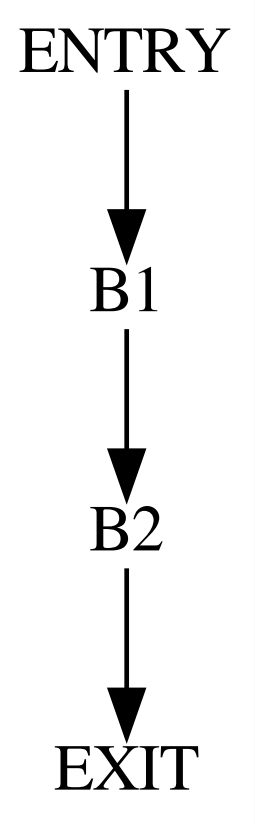


Not Strictly Dominates

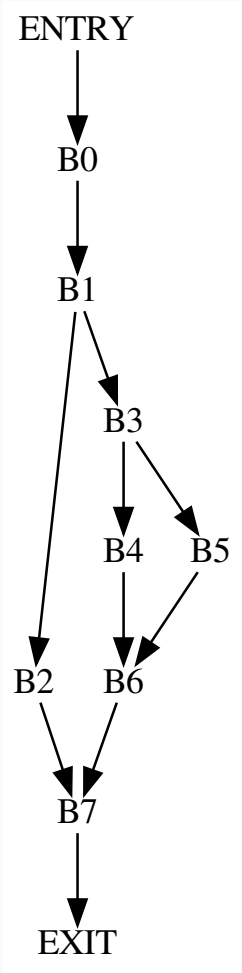
- n strictly dominates m
  - $SDOM(n, m) = n \in DOM(m) \wedge n \neq m$
- n does not strictly dominate m
  - $\neg SDOM(n, m) = n \notin DOM(m) \vee n = m$

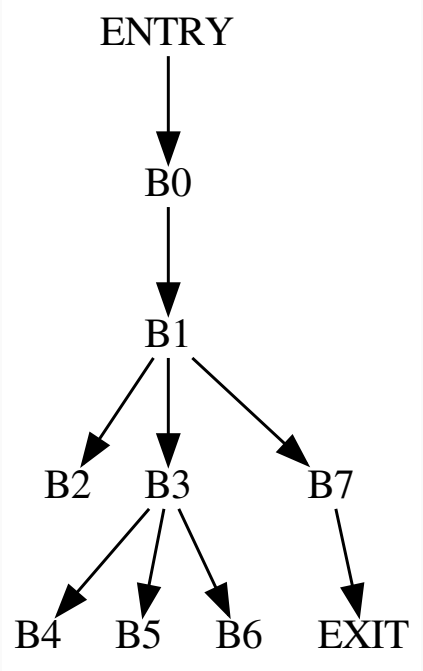
Dominator Tree

- Note that each node in the CFG can have only one immediate dominator
  - Can you see why?
- Create a graph $G = (V, E)$, where:
  - V is the set of basic blocks
  - There is an edge (n, m) in E if n is the immediate dominator of m (i.e. $\mathrm{IDOM}(m) = n$)



Example: CFG and its dominator tree

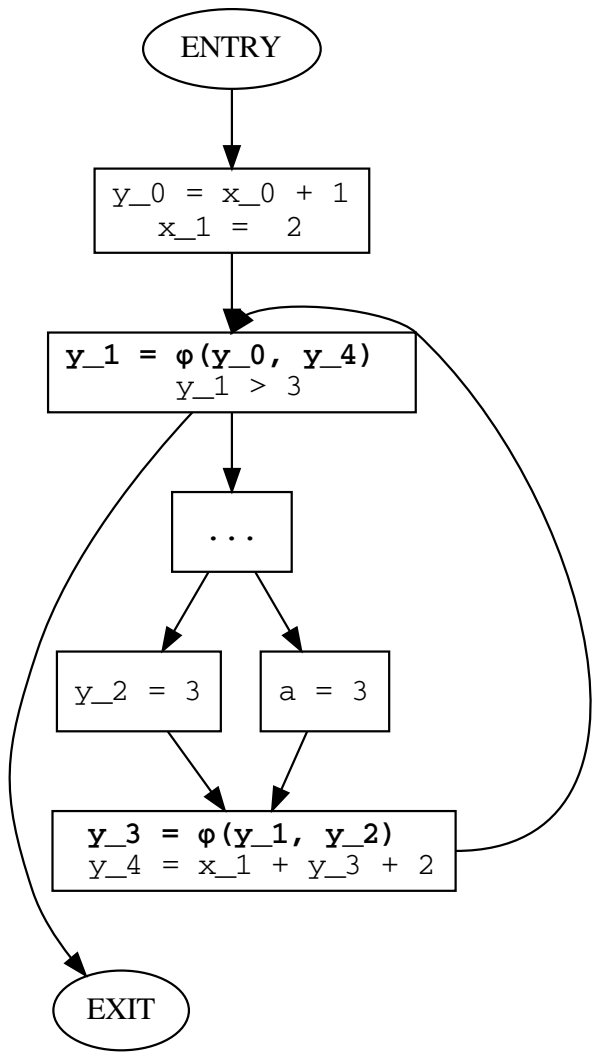




Computing the dominance frontier

- Find all join nodes in CFG, e.g. j
- For all nodes n that dominate predecessors of j (in the CFG)
  - If n does not strictly dominate j, add j to $\mathrm{DF}(n)$
- This last step can be operationalized over all predecessors p of j *in the CFG*:
  - Start traversing the dominator tree at p
  - If p is $\mathrm{IDOM}(j)$, stop. Otherwise add j to $DF(p)$
  - Repeat by moving up the dominator tree until you reach $\mathrm{IDOM}(j)$

Example: Non-redundant ϕ functions



Placing ϕ functions

- For each definition d in basic block n:
  - Place a ϕ function for d in all nodes m where $m \in DF(n)$
  - Note that each ϕ function is also a definition!
  - Repeat, until no more ϕ functions need to be inserted
- This is the minimal number of ϕ functions for a definition d structurally
  - Can we further reduce the overall number of ϕ functions?
- (Figure 9.9 in Cooper and Turczon)

Other optimizations

- Dead definitions
  - Definitions that are not read (i.e. overwritten) do not need ϕ functions
- Two forms:
  - *Semi-pruned* SSA form, using "globals" names (those variables that are live in to a block)
  - *Pruned* SSA form, using LIVEOUT information

Outline

Renaming variables

- SSA form introduced "subscripts" for each variable
- Should we drop them when generating code?

```
a_0 = x_0 + y_0
b_0 = a_0
a_1 = 17
c_0 = a_0
```

Problem with dropping subscripts

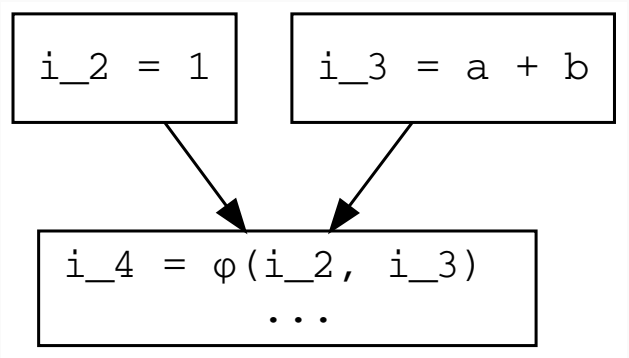
```
a = x + y
b = a
a = 17
c = a    # WRONG!
```

Handling subscripts

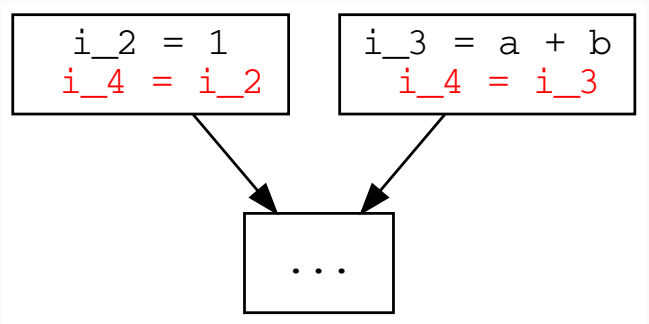
- Each definition becomes a new variable
  - I.e. Do NOT drop subscripts
- Preserves data dependences
  - Esp. important when we aggressively move code from basic blocks (e.g. very busy expressions, loop invariant code motion, etc.)

Code for ϕ functions

- Introduce copies along each incoming edge to a join node



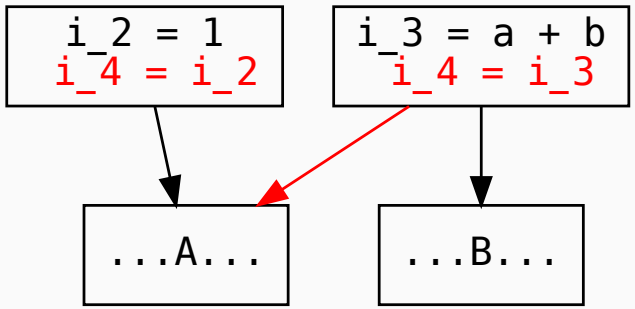
Inserting appropriate copies along incoming edges



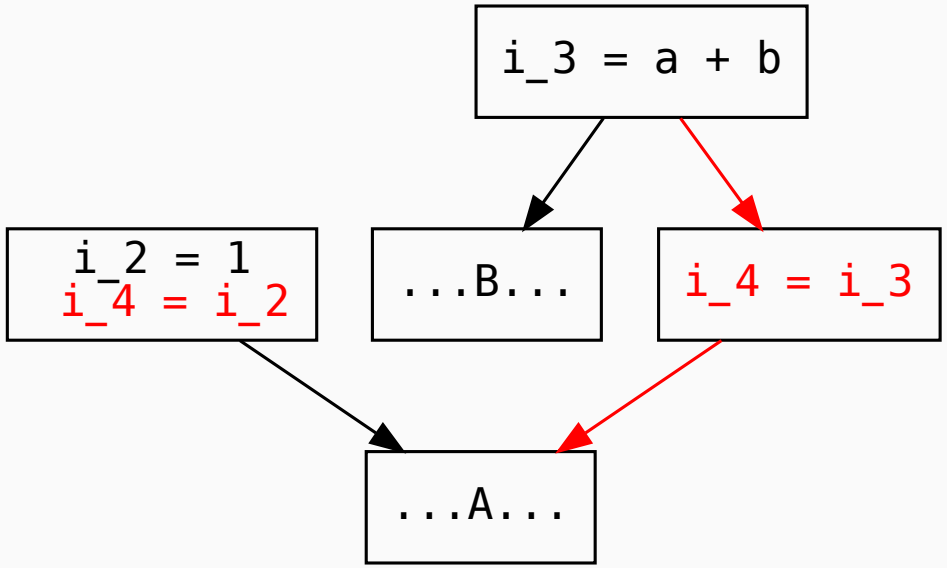
Critical edges

- Executing ϕ functions by inserting copies into predecessor blocks is not always correct
- If such a predecessor block has multiple successors, then the ϕ function may execute when it shouldn't
  - This *may* be harmless, but not always
- Edges connecting such predecessors to the block containing the ϕ function are called *critical* edges

Critical Edges: Example



Splitting critical edges



- Such edges need to be *split* by inserting a block on that edge
- See the discussion in Cooper and Turczon for more details and an example

Outline

Purely Functional Programs

- Everything is a value
- No “assignment”, just binding values to names
- No control flow such as jumps
    - Must be emulated using functions

Example: Factorial

```
def fact(N):
    res = 1
    for i in range(1, N+1):
        res *= i

    return res

def fac(N):
    return 1 if N <= 1 else N * fac(N - 1)
```

Factorial: 3 Address Code

```
def fact(N):
    res = 1
    i = 1
    if i > N goto loop_end

    loop_head:
       res = res * i
       i = i + 1
       if i <= N goto loop_head

    loop_end:
       return res
```

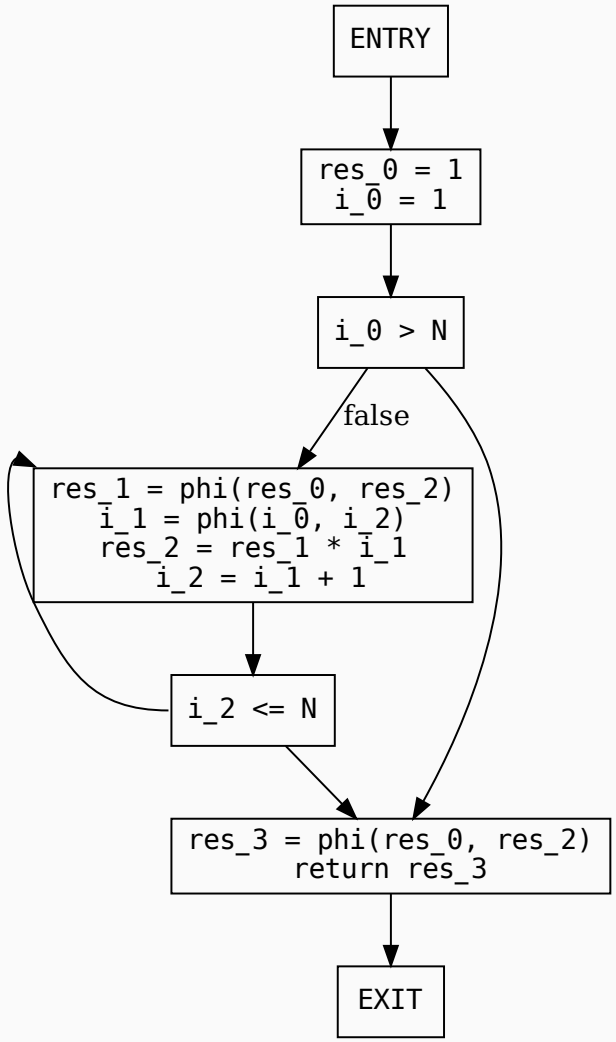
Factorial: SSA form

```
def fact(N):
    res_0 = 1
    i_0 = 1
    if i_0 > N goto loop_end

    loop_head:
       res_1 = phi(res_0, res_2)
       i_1 = phi(i_0, i_2)

       res_2 = res_1 * i_1
       i_2 = i_1 + 1
       if i_2 <= N goto loop_head

    loop_end:
       res_3 = phi(res_0, res_2)
       return res_3
```



Factorial: Function Conversion

```
def fact(N):
  res_0 = 1
  i_0 = 1

  def loop_head(res_1, i_1):
      res_2 = res_1 * i_1
      i_2 = i_1 + 1
      return loop_head(res_2, i_2) if i_2 <= N else loop_end(res_2)

  def loop_end(res_3):
      return res_3

  return loop_end(res_0) if i_0 > N else loop_head(res_0, i_0)
```

- Each basic block is converted to a function
- Parameters to this function are the LHS of the ϕ functions in that BB
- Arguments picked from arguments of ϕ function depending on the path the BB was on.

Outline

References

- Chapter 9 of Cooper and Turczon
  - Section 9.2.1
  - Section 9.3
- Andrew W. Appel, SSA is functional programming *
- Optional:
  - Various authors, The SSA book