CSC2/455 Software Analysis and Improvement
Program Analysis II – Model Checking

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URCS

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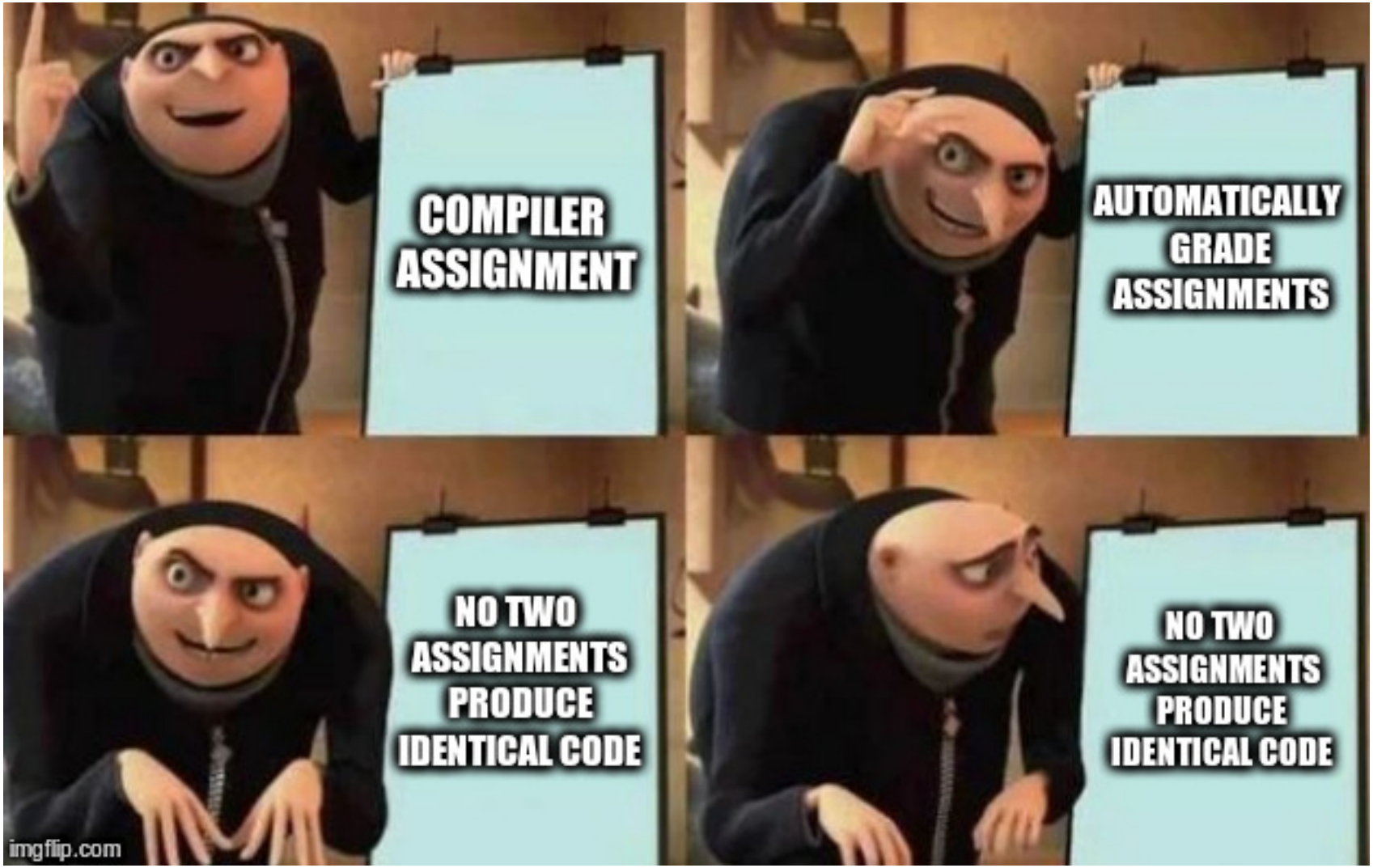
Outline

Outline

The Plan


COMPILER ASSIGNMENT
AUTOMATICALLY GRADE ASSIGNMENTS
NO TWO ASSIGNMENTS PRODUCE IDENTICAL CODE
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imgflip.com

Check for Equivalence

- A: Original source program
- B: Compiler-generated program (e.g. your 3-address code)
- Is $A = B$?
  - Program equivalence problem
  - Undecidable in general

Test?

- Develop test cases
- Run B with these test cases
  - Works
  - Tests may miss bugs
- Also, many programs harder to test
  - Don't have `main`
  - Accept input interactively
  - Buggy compilers may introduce infinite loops

Solution

- Ended up using bounded model checking for C
  - CBMC
- Allows me to check that certain properties hold across all executions
- Can still require manual inspection
  - And I manually inspected all your results – successful or not

Computing the minimum of three numbers

```
int min_of_3(int x, int y, int z) {
   int min3;

   if(x > y) {
      if(y > z) {
         min3 = z;
      } else {
         min3 = y;
      }
   } else {
     if(x > z) {
         min3 = z;
     } else {
         min3 = x;
     }
   }

   return min3;
}
```

Adding Assertions

```
int min_of_3(int x, int y, int z) {
   ...

   __CPROVER_assert(min3 == x || min3 == y || min3 == z,
                    "must be one of inputs");

   __CPROVER_assert(min3 <= x, "<= x");
   __CPROVER_assert(min3 <= y, "<= y");
   __CPROVER_assert(min3 <= z, "<= z");

   return min3;
}
```

(Note: CBMC can also use existing `assert` statements)

Verifying

```
$ cbmc --function min_of_3 filename.c
CBMC version 5.6 64-bit x86_64 linux
 ...
Removal of function pointers and virtual functions
Partial Inlining
Generic Property Instrumentation
Starting Bounded Model Checking
size of program expression: 64 steps
simple slicing removed 5 assignments
Generated 4 VCC(s), 4 remaining after simplification
Passing problem to propositional reduction
converting SSA
Running propositional reduction
Post-processing
 ...

** Results:
[min_of_3.assertion.1] must be one of inputs: SUCCESS
[min_of_3.assertion.2] <= x: SUCCESS
[min_of_3.assertion.3] <= y: SUCCESS
[min_of_3.assertion.4] <= z: SUCCESS

** 0 of 4 failed (1 iteration)
VERIFICATION SUCCESSFUL
```

Another implementation

```
int min_of_3(int x, int y, int z) {
   int min3;

   if(x > y && y > z) {
     min3 = z;
   } else {
     if(x > y)
       min3 = y;
     else
       min3 = x;
   }

   __CPROVER_assert(min3 == x || min3 == y || min3 == z,
                    "must be one of inputs");

   __CPROVER_assert(min3 <= x, "<= x");
   __CPROVER_assert(min3 <= y, "<= y");
   __CPROVER_assert(min3 <= z, "<= z");

   return min3;
}
```

Verifying

```
CBMC version 5.6 64-bit x86_64 linux
 ...
Partial Inlining
Generic Property Instrumentation
Starting Bounded Model Checking
size of program expression: 58 steps
simple slicing removed 5 assignments
Generated 4 VCC(s), 4 remaining after simplification
Passing problem to propositional reduction
converting SSA
Running propositional reduction
Post-processing
Solving with MiniSAT 2.2.1 with simplifier
 ...
Runtime decision procedure: 0.018s

** Results:
[min_of_3.assertion.1] must be one of inputs: SUCCESS
[min_of_3.assertion.2] <= x: SUCCESS
[min_of_3.assertion.3] <= y: SUCCESS
[min_of_3.assertion.4] <= z: FAILURE

** 1 of 4 failed (2 iterations)
VERIFICATION FAILED
```

What! My code, wrong?

```
$ cbmc --trace --function min_of_3 file.c
  ...
State 17 file min3_2.c line 1 thread 0
------------------------------------------------
  INPUT x: -1412553063 (10101011110011100010011010011001)

State 19 file min3_2.c line 1 thread 0
------------------------------------------------
  INPUT y: -1151925590 (10111011010101110000001010101010)

State 21 file min3_2.c line 1 thread 0
------------------------------------------------
  INPUT z: -1949367656 (10001011110011110000001010011000)

 ...

State 30 file min3_2.c line 10 function min_of_3 thread 0
------------------------------------------------
  min3=-1412553063 (10101011110011100010011010011001)

Violated property:
  file min3_2.c line 17 function min_of_3
  <= z
  min3 <= z
```

Loops: Definite Bounds

```
for(i = 0; i < 10; i++) {
  ...
}
```

CBMC will unroll loop.

Loops: Symbolic Bounds

```
for(i = 0; i < N; i++)
   B;
```

gets unrolled by a fixed number (B is body), with unroll assert:

```
i = 0;
if(i < N) {
  B;
  i++;

  if(i < N) {
     B;
     i++;

     assert(N == 2);
  }
}
```

- If `assert` fails, unrolling was insufficient.
  - Not sound!
  - Otherwise, conclusion is sound

Other complications

- Pointers, arrays, dynamic memory allocation, etc.
- See CPROVER manual for more details

Outline

Basic Ideas

- Formula φ
  - Correctness (Safety) property
  - Propositional logic
  - Example: first argument of all the `__CPROVER_assert` statements
- Intepretation $\mathcal{K}$
  - More on this later
- We ask: $\mathcal{K} \models \varphi$?
  - Is φ true in $\mathcal{K}$?

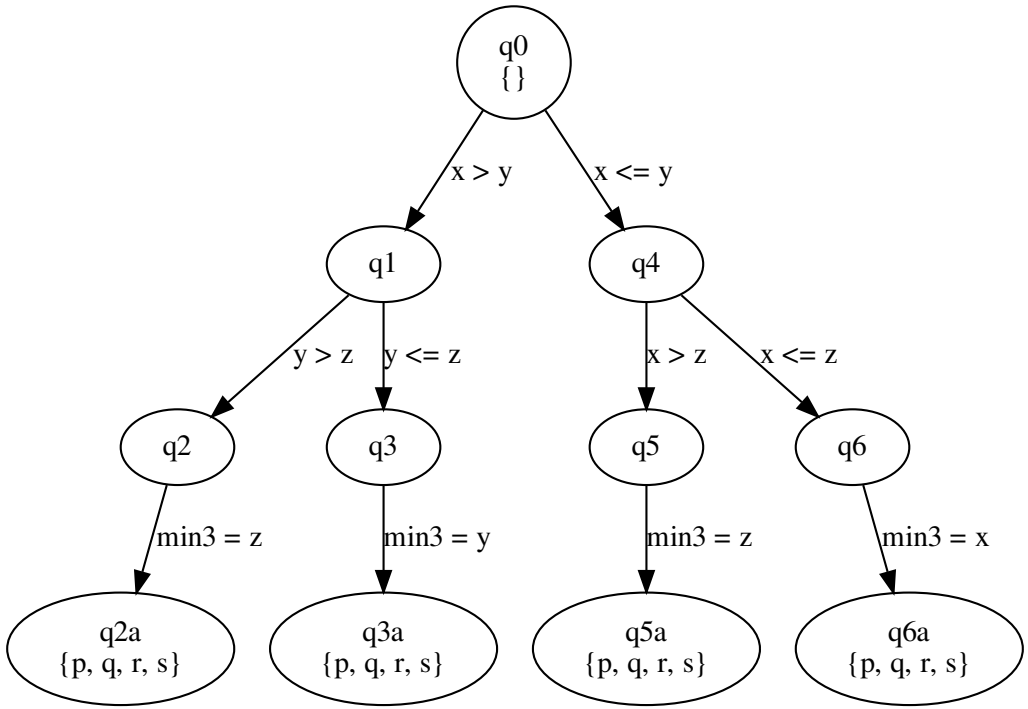
Transition System

- $\mathcal{T} = (Q, I, E, \delta)$
  - set of states Q (e.g. values of all variables)
  - initial states $I \in Q$
  - action labels E (e.g. program statements)
  - (total) transition relation $\delta \subset Q \times E \times Q$
- A run of $\mathcal{T}$ is the same as a trace of states
  - $s0e0s1...$ where $(s0, e0, s1) \in \delta$, and $s0 \in I$
- A reachable state is a state that exists in some run.

Kripke Structures

- Let $\mathcal{V}$ be a set of propositions
  - e.g. `min3 <= x`
  - e.g. `min3 <= y`
- A Kripke structure $\mathcal{K} = (Q, I, E, \delta, \lambda)$ is a transition system where:
  - $\lambda : Q \rightarrow 2^{\mathcal{V}}$
- λ is a function that maps a state q to the (subset) of propositions from $\mathcal{V}$ that are true in that state
  - $q \models P$ where $P \in \mathcal{V}$

Kripke structure for our min-of-3 example



- Let p be the "must be one of inputs" proposition
- Let q, r, s be the $<= x, <= y, <= z$ proposition
- (Note: True propositions in internal states not shown)

Invariants

- An invariant is a safety property for the system that holds in every reachable state
- An inductive invariant holds in the initial state, and is preserved by all transitions
  - including transitions from unreachable states
  - more on this when we discuss Hoare Logic

Invariant Checking Algorithm: High level details

- Assume finite Kripke structure
- Given an invariant to check,
  - Enumerate all reachable states
  - Check that invariant holds in all of them

Invariant Checking Algorithm: Pseudocode

```
def verify_inv(ks, inv):
    done = set()
    todo = set()

    for s in ks.initial_states():
        if s in done: continue

        todo.add(s)

        while len(todo) > 0:
            ss = todo.pop()
            done.add(ss)

            if not ss.satisfies(inv): return False

            for succ in ss.successors():
                if succ not in done: todo.add(succ)

    return True
```

based on Figure 3.3 in S. Merz, An introduction to Model Checking.

Outline

Progress

- Does something "good" eventually happen?
- Does the system ever deadlock?
- Does the system livelock?
  - An action e is no longer possible after a particular state q_i
- These require reasoning over *sequences* of states
  - These can be infinite even in a finite Kripke structure

These properties need a *temporal* logic, that incorporates notions of (logical) "time points" into formulae we want to check.

Specifying temporal properties in PTL

- Let $\sigma = q_0 q_1 \ldots$ be a sequence of states
  - σ_i is the state i
  - $\sigma|_i$ is the suffix $q_i q_{i+1} \ldots$ of σ
- Let φ be a formula
- $\sigma \models \varphi$ if $\varphi \in \lambda(\sigma_0)$
- $\mathrm{X}\varphi$ (also a formula), read as "next φ",
  - $\sigma \models \mathrm{X}\varphi$ if $\sigma|_1 \models \varphi$
- $\varphi \mathrm{U} \psi$ (also a formula), read as "φ until ψ"
  - $\sigma \models \varphi \mathrm{U} \psi$ if and only if there exists $k \in N$
  - $\sigma|_k \models \psi$
  - for all $1 \leq i < k$, $\sigma|_i \models \varphi$
  - Note: φ can continue to hold after k

More temporal properties

- $\mathrm{F}\varphi$, "eventually φ"
  - $\mathrm{true}\mathrm{U}\varphi$
- $\mathrm{G}\varphi$, "always φ"
  - $\neg\mathrm{F}\neg\varphi$
- $\varphi\mathrm{W}\psi$, "φ unless ψ"
  - $(\varphi\mathrm{U}\psi) \vee G\psi$
- $\mathrm{GF}\varphi$
- $\mathrm{FG}\varphi$

Some examples of invariants

- $\mathrm{G}\neg(own_1 \wedge own_2)$
  - where own_1 and own_2 are propositions representing states in which locks for resource are obtained by process 1 and 2
- Other properties (see the reading)
  - weak and strong fairness
  - precedence
  - etc.

Existential and Universal Properties: CTL

- Branching time logic for properties of systems
  - Computation Tree Logic (CTL)
- $\mathrm{EX}\varphi$, there exists a transition where φ holds from current state
- $EG\varphi$, exists a path from current state where φ holds on all states
- EU, exists a path until...
- Also Ax properties, properties that hold on all possible paths from current state

Verifying PTL and CTL invariants?

- State sequences of infinite length possible
- How do we check invariants?

Büchi Automata

- ω-automaton
  - run on infinite strings
- strings represent state sequences (actually $\lambda(q_0)\lambda(q_1)...$)
- non-deterministic as well as deterministic
  - but non-deterministic Büchi automata more powerful

Büchi Automata Example

Stephan Merz, An Introduction to Model Checking

Outline

Further Reading and Links

- Stephan Merz, An Introduction to Model Checking
  - Accessible and good introduction, with links to other material
- Spin Model Checker
- Selected industrial applications
  - CACM, "How Amazon Web Services Uses Formal Methods
  - CACM, "A Decade of Software Model Checking with SLAM
- A segue into compiler verification
  - Ken Thompson, Reflections on Trusting Trust, Turing Award Lecture 1984
  - The COMPCERT project