CSC2/455 Software Analysis and Improvement
Foundations of Data Flow Analysis - II

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Outline

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Part I of Foundations

- Methods to solve dataflow analysis equations
  - IDEAL
  - Meet over paths (MOP)
  - Maximum Fixed Point (MFP)
  - $\mathrm{IDEAL} \subseteq \mathrm{MOP} \subseteq \mathrm{MFP}$
- (Semi)lattice-based framework
  - $(D, V, \wedge, F)$
  - Monotone framework
- Greatest Lower Bound
  - If $z = x \wedge y$, then $z \leq x$ and $z \leq y$

Monotone Framework

- A given $(D, V, \wedge, F)$ is monotone if for all $x, y \in V$, and $f \in F$:
  - $x \leq y \rightarrow f(x) \leq f(y)$
  - equivalently, $x \leq y \rightarrow f(x \wedge y) \leq f(x) \wedge f(y)$
  - The proof of equivalence is in the textbook.
- In addition, the framework is *distributive* if:
  - $f(x \wedge y) = f(x) \wedge f(y)$
- Note that these properties do not necessarily arise automatically, F must be designed to have these properties
  - And proofs must be written to show that F does.
  - We'll see this for a complicated example today.

General Iterative Algorithm

```
forwards(IN, OUT, meet, top, v_entry, f_transfer)
   OUT[entry] = v_entry

   for each basic block B except ENTRY:
      OUT[B] = top

   do {
      for each basic block B except ENTRY:
          # this calculates the meet over predecessors, /\p OUT[p]
          IN[B] = reduce(meet, [OUT[p] for p in B.predecessors])
          OUT[B] = f_transfer(IN[B])

   } while(some OUT changes value)
```

- Does this calculate the solution to the dataflow problem?
- Does this algorithm terminate?
- Does this algorithm calculate the *maximum* fixed point – i.e. the most precise solution admissible?

This class

- Proofs that answer these three questions
- Relationships between IDEAL, MOP and MFP in terms of the framework
- Examples of:
  - a non-distributive framework (from Dragon 9.4, Constant Propagation)
  - lattices containing infinite values
  - possibly some proof writing exercises (from Dragon 9.3)

Outline

Proof #1

The iterative algorithm computes the solution to the dataflow problem.

- The iterative algorithm performs an unbounded number of iterations as long as IN and OUT change
- *When it terminates*, IN and OUT have not changed for an iteration
- The values of IN and OUT therefore satisfy the equations
  - Hence they are solutions of the dataflow problem

Proof #2

The iterative algorithm terminates (i.e. converges to a fix point).

- When we apply the $\wedge$ operator, we obtain the glb
  - i.e. $z = x \wedge y$ and $z \leq x$ and $z \leq y$
- Since the framework is monotone:
  - $f(x) \leq f(y)$ if $x \leq y$
  - i.e. OUT values are no greater than the IN values
- At each step, these values decrease or remain the same
  - When they all remain the same, we terminate
- If values decrease, recall the lattice has finite height
  - Implies a finite number of steps before we reach $\perp$
  - $x \wedge \perp = \perp$ and $f(\perp) = \perp$ (i.e once a value becomes $\perp$, it no longer changes)
  - We terminate in this case as well

Proof #3

The fixed point solution computed by the iterative algorithm is the *maximum* fixed point.

Proof By induction, for forward analyses

(BASIS) After the first iteration, values of $\mathrm{IN}[B]$ and $\mathrm{OUT}[B]$ are $\leq$ their initial values.

- At initialization, $\mathrm{OUT}[B]$ is $\top$ for all blocks B except ENTRY
- After the first iteration, in a monotone framework, all values will be $\leq$ those at initialization by definitions of the $\wedge$ and transfer functions

Proof #3: Inductive step

Assume that:

- $\mathrm{IN}[B]^k \leq \mathrm{IN}[B]^{k-1}$
- $\mathrm{OUT}[B]^k \leq \mathrm{OUT}[B]^{k-1}$

Show that:

- $\mathrm{IN}[B]^{k+1} \leq \mathrm{IN}[B]^k$
- $\mathrm{OUT}[B]^{k+1} \leq \mathrm{OUT}[B]^k$

Proof #3: Continued

- To obtain $\mathrm{IN}[B]$ we must apply $\wedge$ to all $\mathrm{OUT}[P]$
  - P is a predecessor of B
  - This implies $\mathrm{IN}[B] \leq \mathrm{OUT}[P]$ ($\wedge$ yields glb)
  - From our inductive hypothesis, $\mathrm{OUT}[P]^k \leq \mathrm{OUT}[P]^{k-1}$
  - applying $\wedge$ on both sides over all P, $\mathrm{IN}[B]^{k+1} \leq \mathrm{IN}[B]^k$
- Now, $\mathrm{OUT}[B] = f(\mathrm{IN}[B])$
  - In the monotone framework, $f(x) \leq f(y)$ when $x \leq y$
  - We have shown $\mathrm{IN}[B]^{k+1} \leq \mathrm{IN}[B]^k$
  - Therefore, after applying f to both sides, by monotonicity, we have $\mathrm{OUT}[B]^{k+1} \leq \mathrm{OUT}[B]^k$

Properties of the IDEAL solution

- Any solution greater than IDEAL is incorrect (or unsafe)
- Any solution less than or equal to IDEAL is conservative[1], or safe.

To see why, consider IDEAL solution $x = p_1 \wedge p_2 \wedge ... \wedge p_n$:

- How can we obtain a value $z = p_1 \wedge ...$ *greater than* x?
- How can we obtain a value $y = p_1 \wedge ...$ *less than* x?

(recall the relationship between the results of the meet operator and its operands)

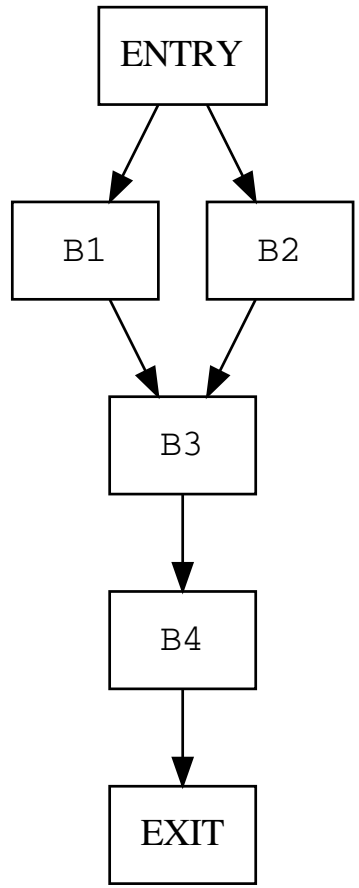
[1] In the English sense

Relationship between IDEAL and MOP

- MOP considers a superset of all executable paths
  - MOP solution $y = p_1 \wedge p_2 \wedge ... \wedge p_n \wedge p_{n+1} ...$
- What is the relationship between MOP (y) and IDEAL (z)?

Relationship between MOP and MFP

- $\mathrm{MOP}[B_4] = ((f_{B_3} \circ f_{B_1}) \wedge (f_{B_3} \circ f_{B_2}))(v_{entry})$
- $\mathrm{IN}[B_4] = f_{B_3}(f_{B_1}(v_{entry}) \wedge f_{B_2}(v_{entry}))$



In a distributive framework, MOP = MFP

- $\mathrm{MOP}[B_4] = ((f_{B_3} \circ f_{B_1}) \wedge (f_{B_3} \circ f_{B_2}))(v_{entry})$
- $\mathrm{IN}[B_4] = f_{B_3}(f_{B_1}(v_{entry}) \wedge f_{B_2}(v_{entry}))$

If $f(x \wedge y) = f(x) \wedge f(y)$ (i.e. distributive):

- $\mathrm{IN}[B_4] = f_{B_3}(f_{B_1}(v_{entry})) \wedge f_{B_3}(f_{B_2}(v_{entry}))$

- If the framework is distributive, then MOP solution = MFP solution
  - Otherwise by monotonicity MFP $\leq$ MOP
- In either case,
  - MFP $\leq$ MOP $\leq$ IDEAL
  - So all methods produce "safe" solutions

Outline

Analyses so far

- Live variable analysis
- Available Expressions
- Reaching Definitions
- These are all distributive (implies monotonicity)
- Their lattices contain a finite number of values
- Their lattices have finite height

Constant Propagation

- Does this variable have a constant value at this point in the program?
  - Used to perform constant folding (i.e. evaluate constant expressions at compile time)
- Data flow analysis framework
  - Direction?
  - Values?
  - Meet operator?
  - Transfer function?

Constant Propagation

- Direction: Forward
- Values:
  - UNDEF: variable is undefined so far
  - c: variable is constant value c
  - NAC: variable is not a constant
- Meet operators and transfer functions are slightly more complicated.

Meet for Constant Propagation

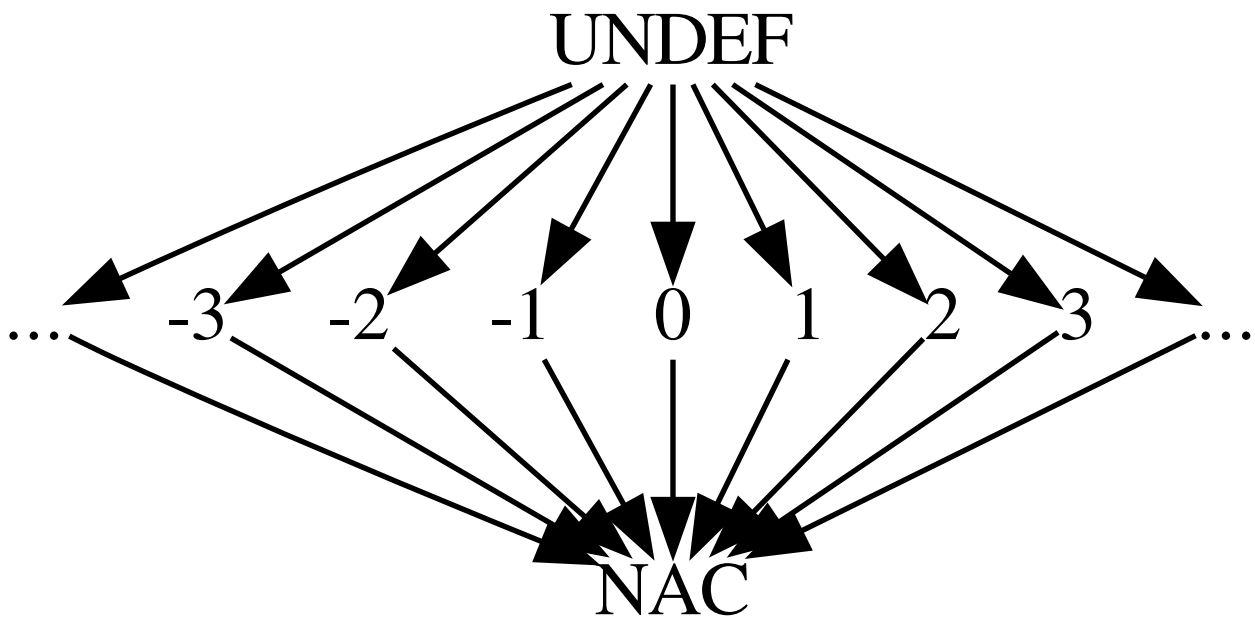
- $\text{UNDEF} \wedge v = ?$
- $\text{NAC} \wedge v = ?$
- $c \wedge c = ?$
- $c_1 \wedge c_2 = ? \ (c_1 \neq c_2)$

Meet for Constant Propagation

- $\text{UNDEF} \wedge v = v$
  - UNDEF is $\top$
- $\text{NAC} \wedge v = \text{NAC}$
  - NAC is $\bot$
- $c \wedge c = c$
- $c_1 \wedge c_2 = \text{NAC}$

What does the lattice for constant propagation look like?

The lattice for constant propagation



The Transfer Function

- $\mathrm{OUT}[s] = f(\mathrm{IN}[s])$ for a statement s
  - Slightly easier to understand if we use statements instead of basic blocks
- Observe that non-assignment statements do not change values
  - f is simply the identify function $f(x) = x$ for such statements
- What about assignment statements?
  - `x = c`, where `x` is a variable, and `c` is a constant
  - `x = y + z`, where `+` is any binary operator
  - `x = *y` or `x = f(...)`, where `f` is a function call

The Transfer Function - II

- Note that IN (and OUT) are maps (i.e. dictionaries)
  - From variables to their current dataflow values (UNDEF, c, or NAC)
  - Let's call this map m, so that $m(x)$ returns the dataflow value for variable x
- `x = c`, changes $m(x) \leftarrow c$
- `x = y + z`, where + is any binary operator (not just addition)
  - $m(x) \leftarrow m(y) + m(z)$ if $m(y)$ and $m(z)$ are constants
  - $m(x) \leftarrow \mathrm{NAC}$ if either $m(y)$ or $m(z)$ is NAC
  - $m(x) \leftarrow \mathrm{UNDEF}$ otherwise
- `x = *y` or `x = f(...)`, $m(x) \leftarrow \mathrm{NAC}$ (conservatively)
- Note that $m(v) \leftarrow m(v)$ for all $v \neq x$
  - I.e. the other values of the map remain unchanged

Note that I use slightly different notation than the textbook, which uses m' on the LHS

Is this monotonic?

Is $\mathrm{OUT}[s] \leq \mathrm{IN}[s]$ for every s?

- For the two cases below, it is "surely ... monotone":
  - $m(x) \leftarrow c$
  - $m(x) \leftarrow \mathrm{NAC}$
- What about `x = y + z`?
  - Need to show that $m(x)$ does not get greater as $m(y)$ (and/or) $m(z)$ get smaller
  - Show by case analysis and symmetry

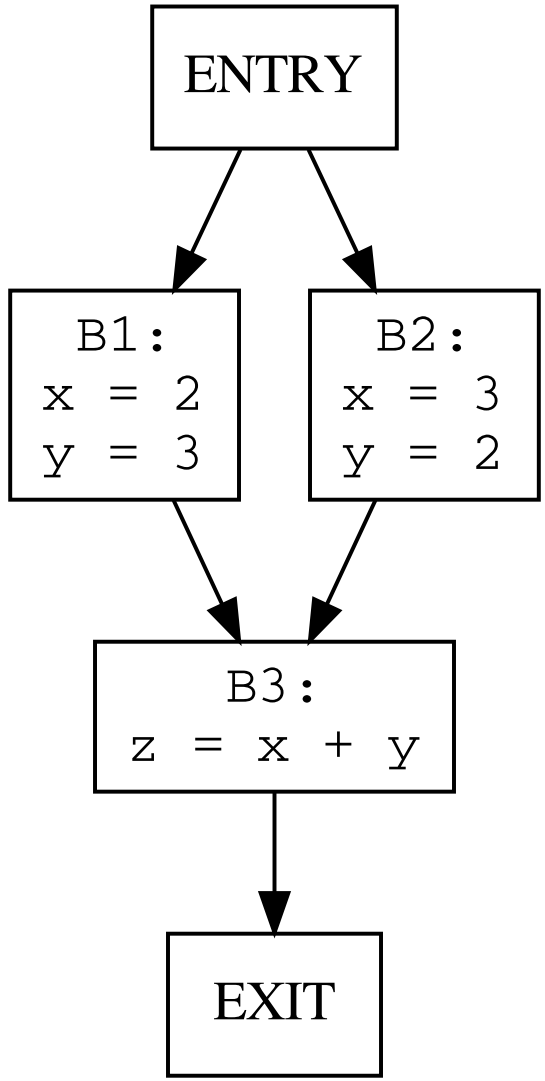
`x = y + z` as $m(z)$ gets smaller

$m(y)$	$m(z)$	output $m(x)$
UNDEF	UNDEF	UNDEF
	c_2	
	NAC	
c_1	UNDEF	
	c_2	
	NAC	
NAC	UNDEF	
	c_2	
	NAC	NAC

`x = y + z` as $m(z)$ gets smaller (answers)

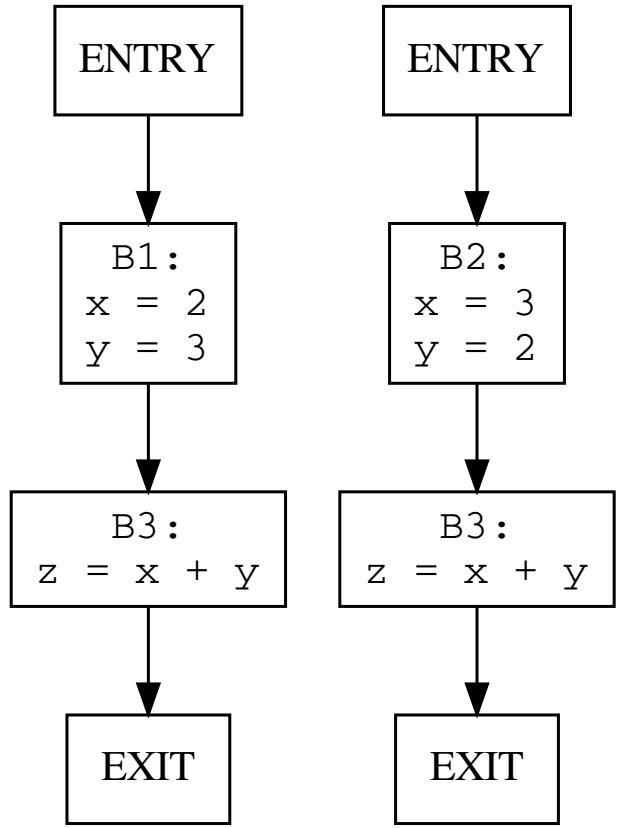
$m(y)$	$m(z)$	output $m(x)$
UNDEF	UNDEF	UNDEF
	c_2	UNDEF
	NAC	NAC
c_1	UNDEF	UNDEF
	c_2	$c_1 + c_2$
	NAC	NAC
NAC	UNDEF	NAC
	c_2	NAC
	NAC	NAC

Is it distributive?



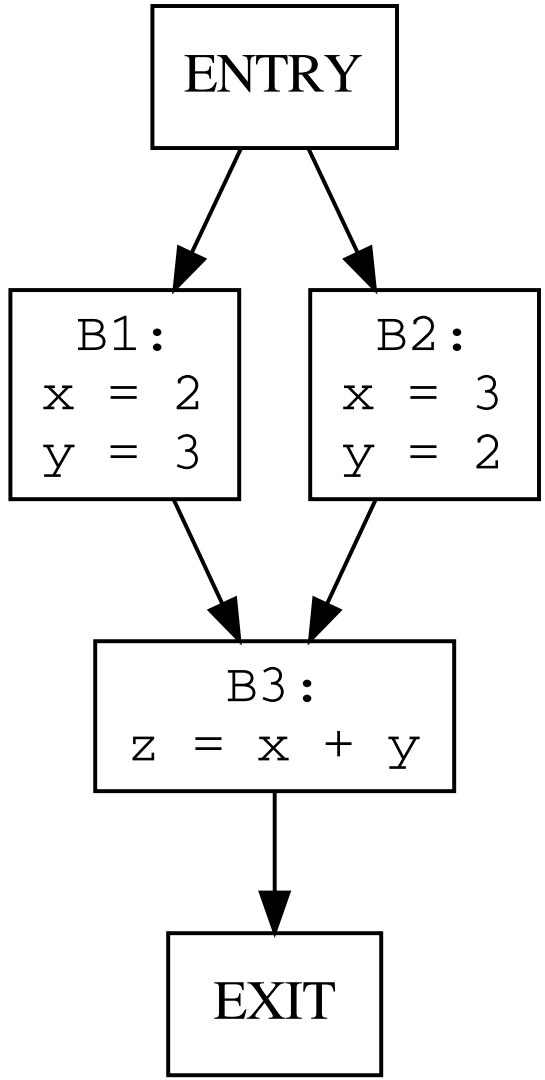
MOP solution

- Path 1 (`x = 2; y = 3; z = x + y`)
  - $m(z) = 5$, so z is a constant
- Path 2 (`x = 3; y = 2; z = x + y`)
  - $m(z) = 5$, so z is a constant
- Meet over Path 1 and Path 2
  - $m(z) = 5 \wedge 5$, so z is a constant



MFP solution

- At end of block B_1
  - $m(x) = 2$ and $m(y) = 3$
- At end of block B_2
  - $m(x) = 3$ and $m(y) = 2$
- Meet before block B_3
  - $m(x) = 2 \wedge 3$ (i.e. case $c_1 \wedge c_2$)
  - $m(y) = 3 \wedge 2$
- Conclusion?



Constant Propagation is not distributive

- For constant propagation, in most non-trivial programs
  - $\mathrm{MFP} < \mathrm{MOP}$

Outline

References

- Chapter 9 of the Dragon book
  - Section 9.3, 9.4