

# Lecture 8: Color From Absorption & Scattering: Painting, Dyeing, and Printing

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CSC 292/572, Fall 2022  
Mobile Visual Computing



# Logistics

Written assignment 2 is due Sept. 23 11:30 AM.

- Can work in groups of 2.

# The Roadmap

Theoretical Preliminaries

Human Visual Systems

**Color in Nature, Arts, Tech**  
(a.k.a., the birth, life, and death of light)

Digital Camera Imaging

Modeling and Rendering

Applications



Color Source

Display and Lightning

**Material Colors**





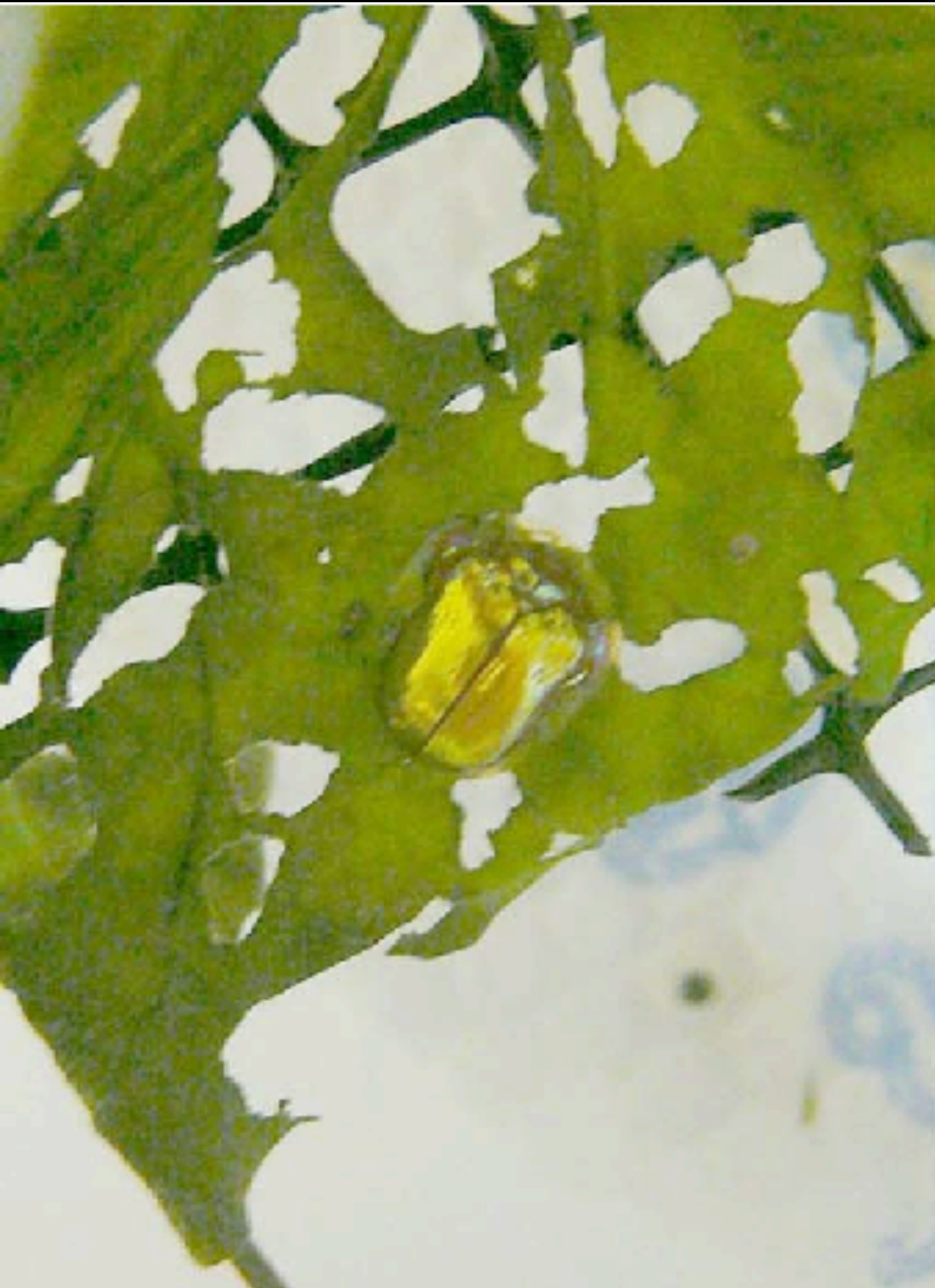






# Panamanian Tortoise Beetle changing its color

[Vigneron et al., 2007]





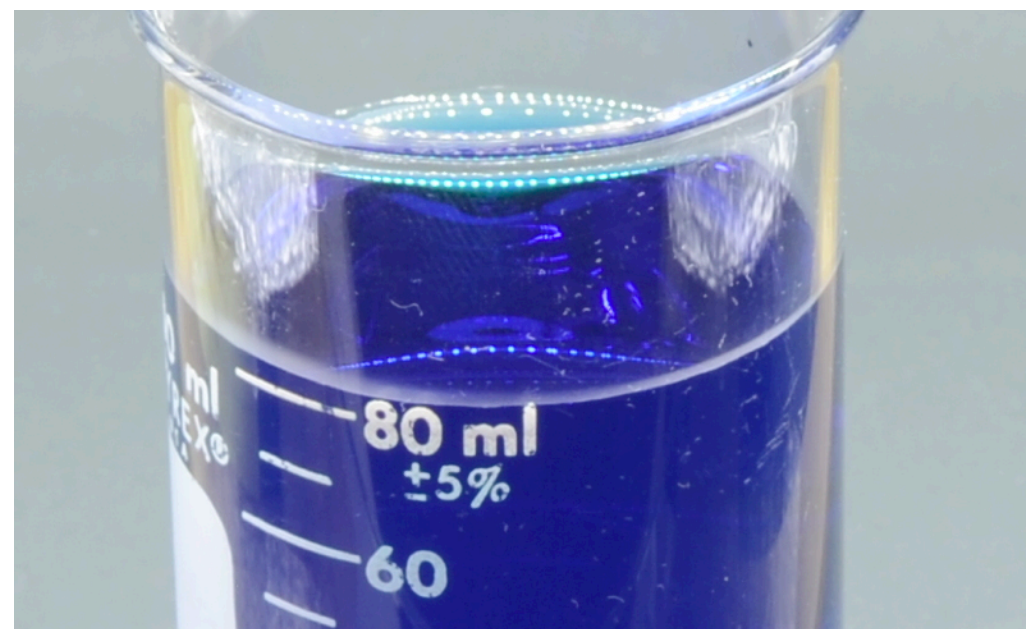
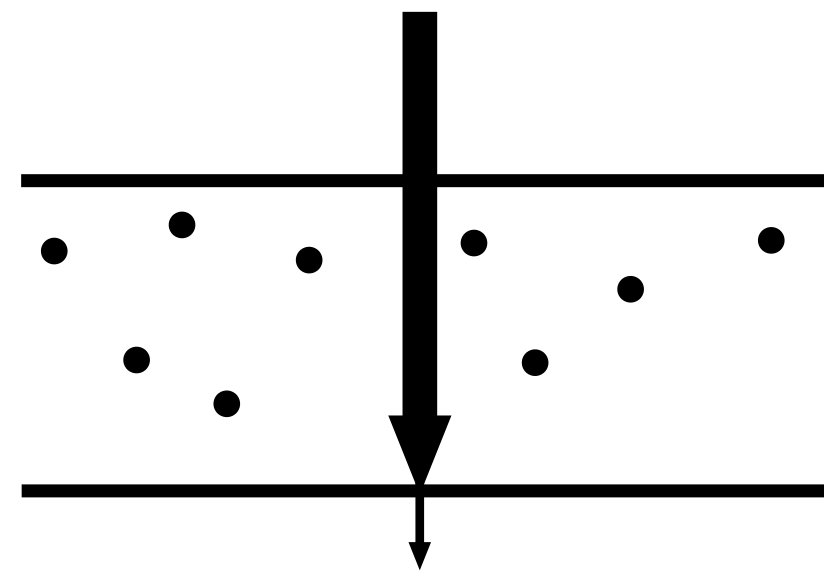
Iridescence from Anorthite (a common mineral) depends on the viewing angle. Both photos were taken at the Royal Ontario Museum early this year.



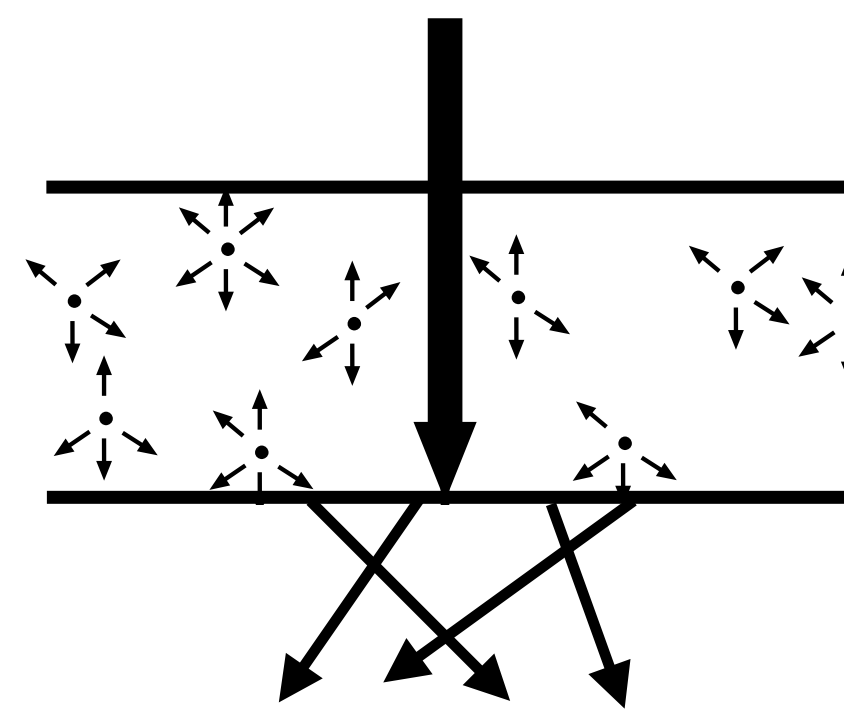


# Color Schemes

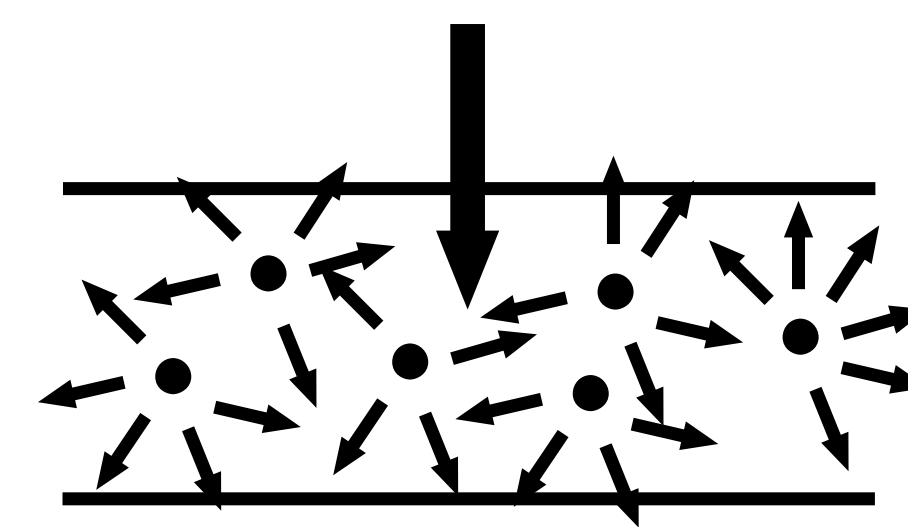
Pure absorption  
(Transparent)



A little scattering + absorption  
(Translucent)



A lot of scattering + absorption  
(Opaque)

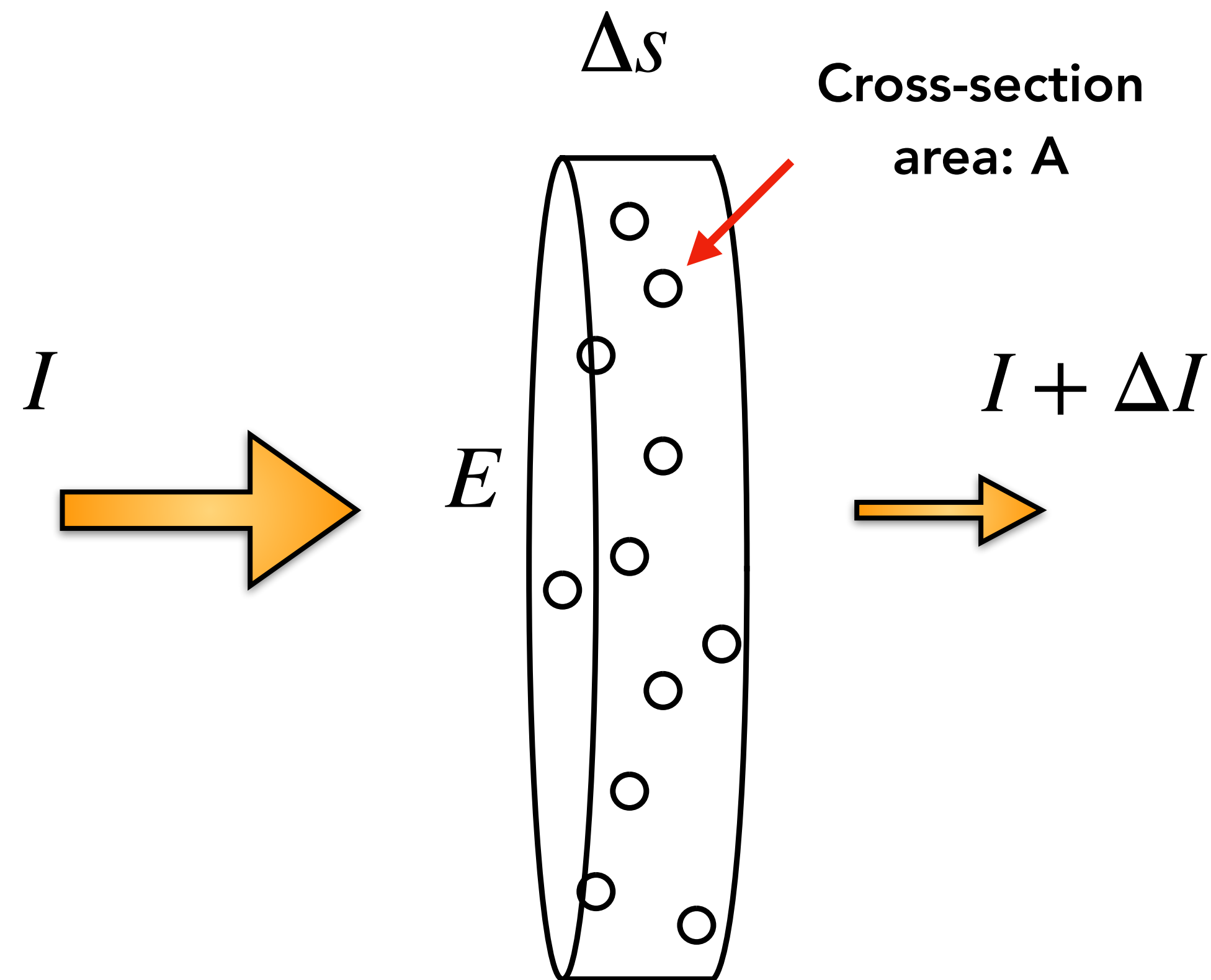


# Pure Absorption



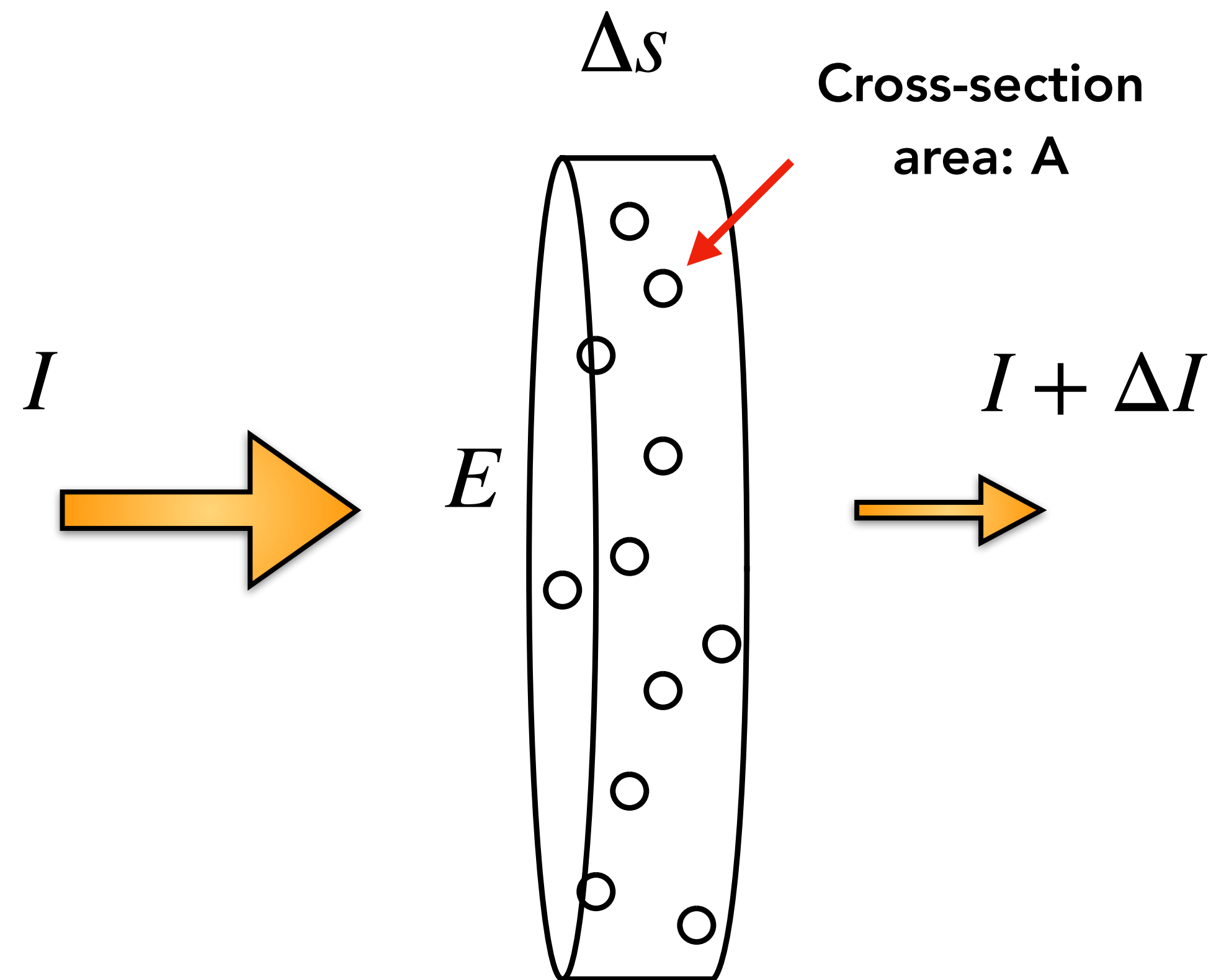
# Modeling Pure Absorption

Consider a very thin slice (so thin that no particles are covering each other)



# Modeling Pure Absorption

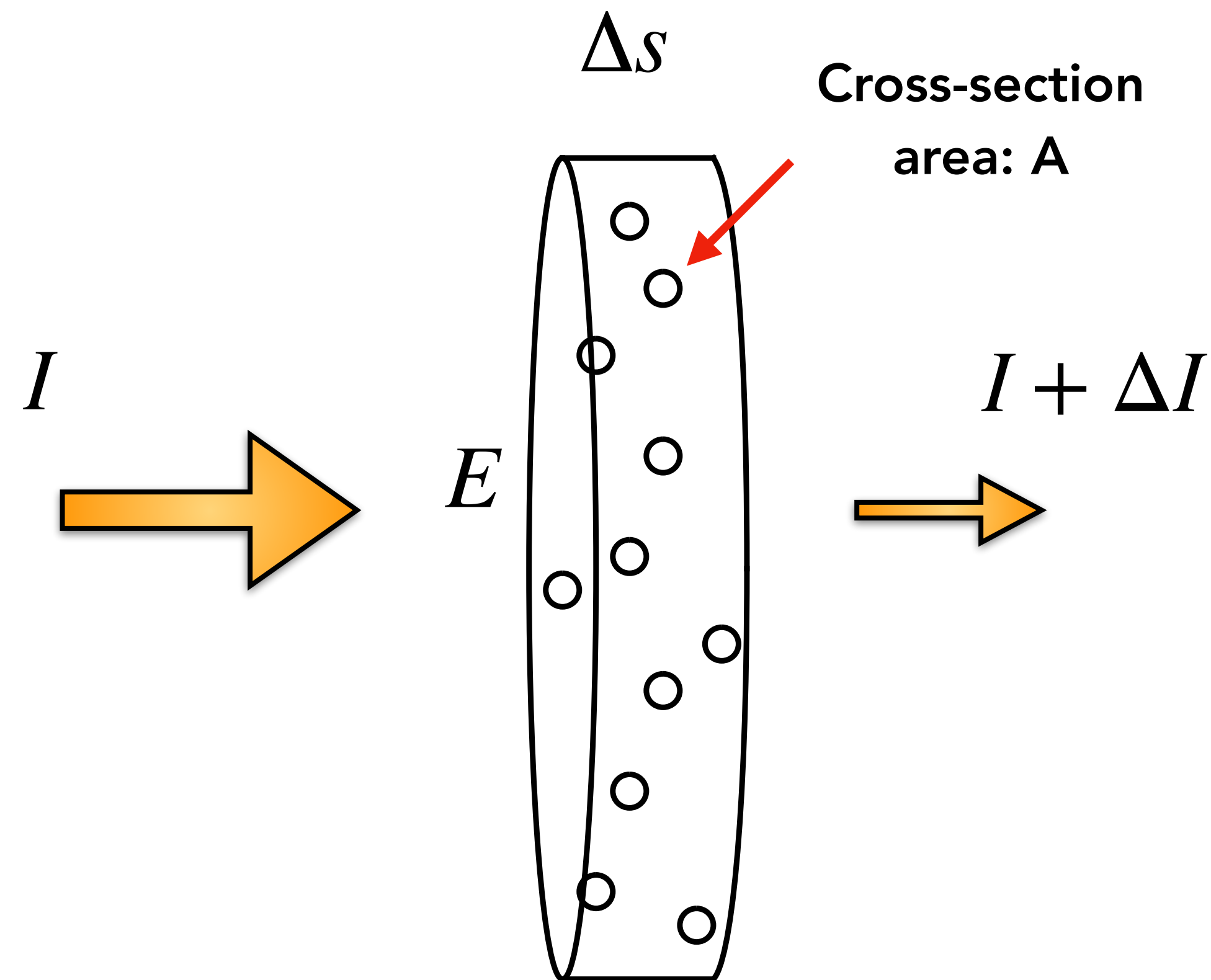
Consider a very thin slice (so thin that no particles are covering each other)



$$\frac{\rho E \Delta s A}{E}$$

# Modeling Pure Absorption

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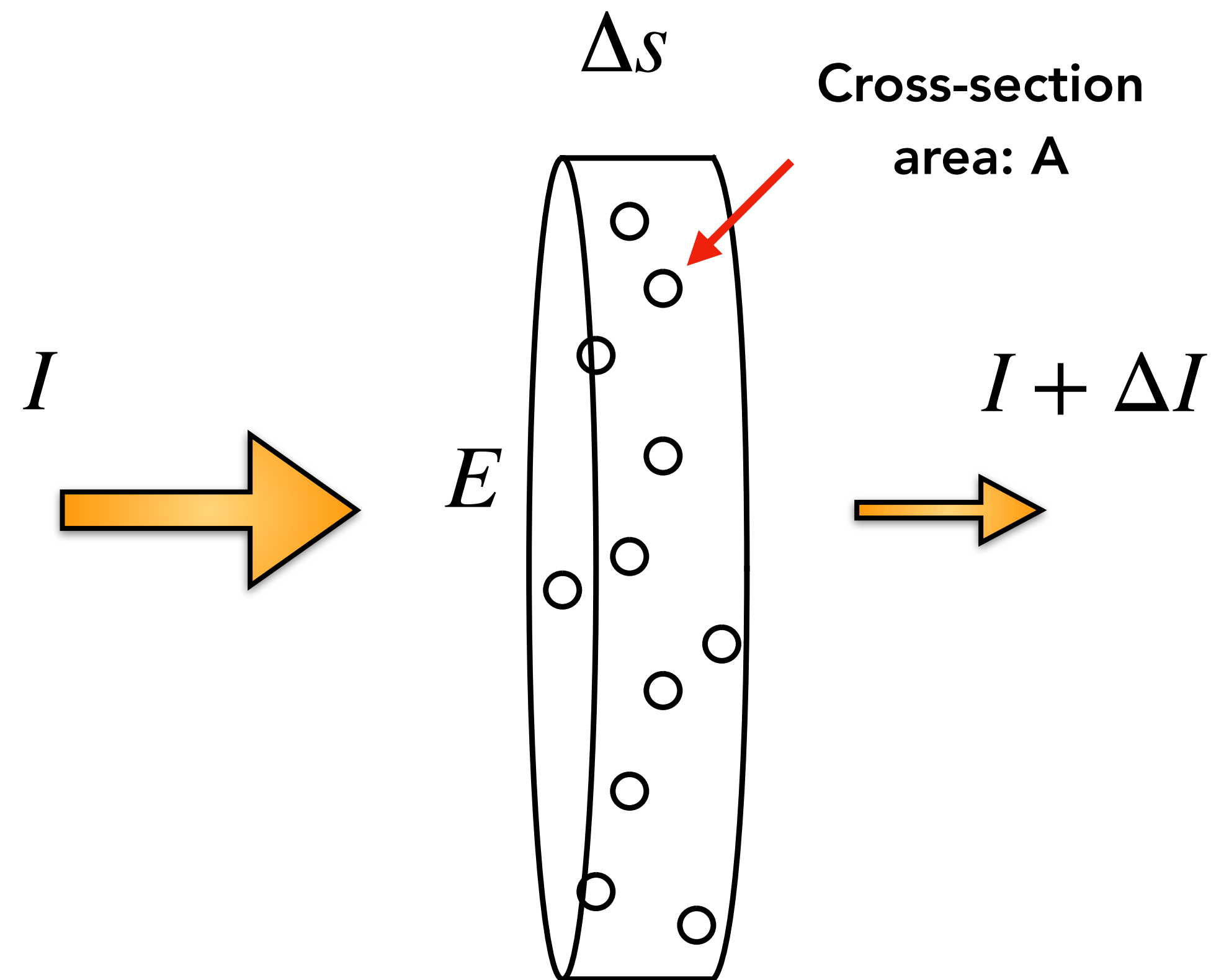


Concentration (# of particles / unit volume)

$$\frac{\rho E \Delta s A}{E}$$

# Modeling Pure Absorption

Consider a very thin slice (so thin that no particles are covering each other)



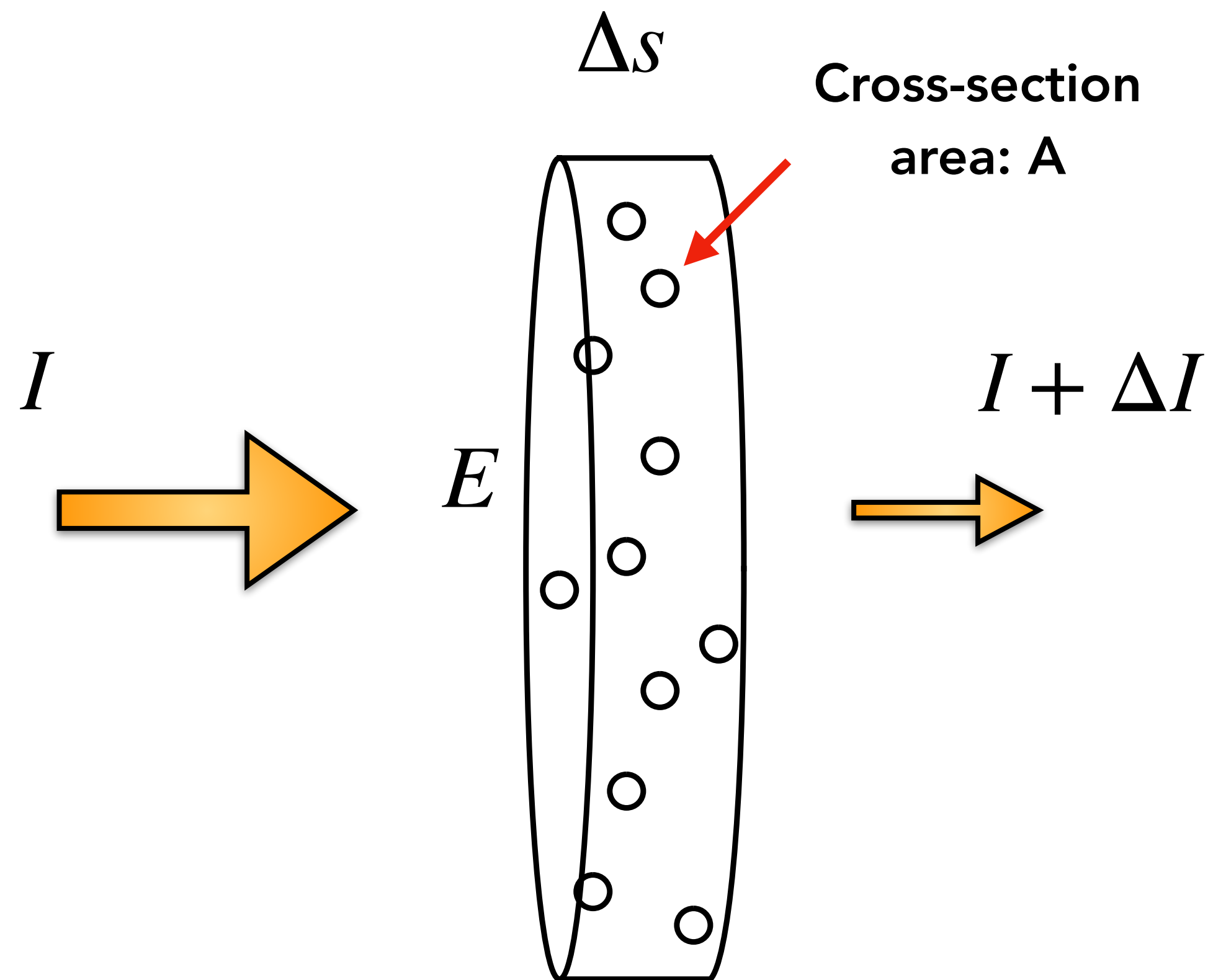
Concentration (# of particles / unit volume)

Total area covered by the particles

$$\frac{\rho E \Delta s A}{E}$$

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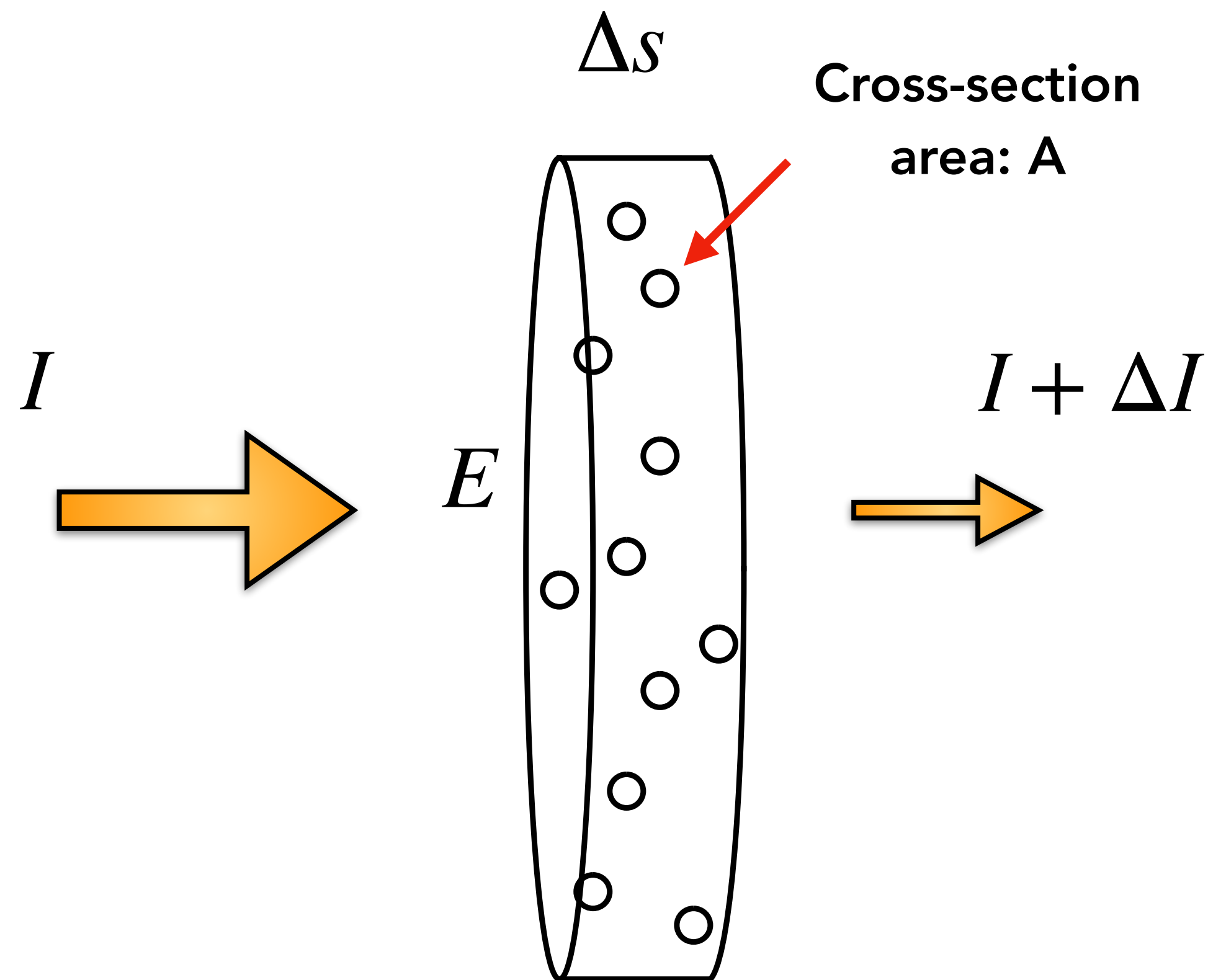
Concentration (# of particles / unit volume)

Total area covered by the particles

A diagram showing the equation for the percentage of area occluded by particles. The equation is 
$$\frac{\rho E \Delta s A}{E}$$
 enclosed in a blue box. A pink box highlights the numerator  $\rho E \Delta s A$ . A teal box highlights the term  $\rho$  in the numerator. A blue line points from the text "Percentage of area occluded by particles" to the entire equation. A pink line points from the text "Total area covered by the particles" to the pink box. A teal line points from the text "Concentration (# of particles / unit volume)" to the teal box.

# Modeling Pure Absorption

Consider a very thin slice (so thin that no particles are covering each other)

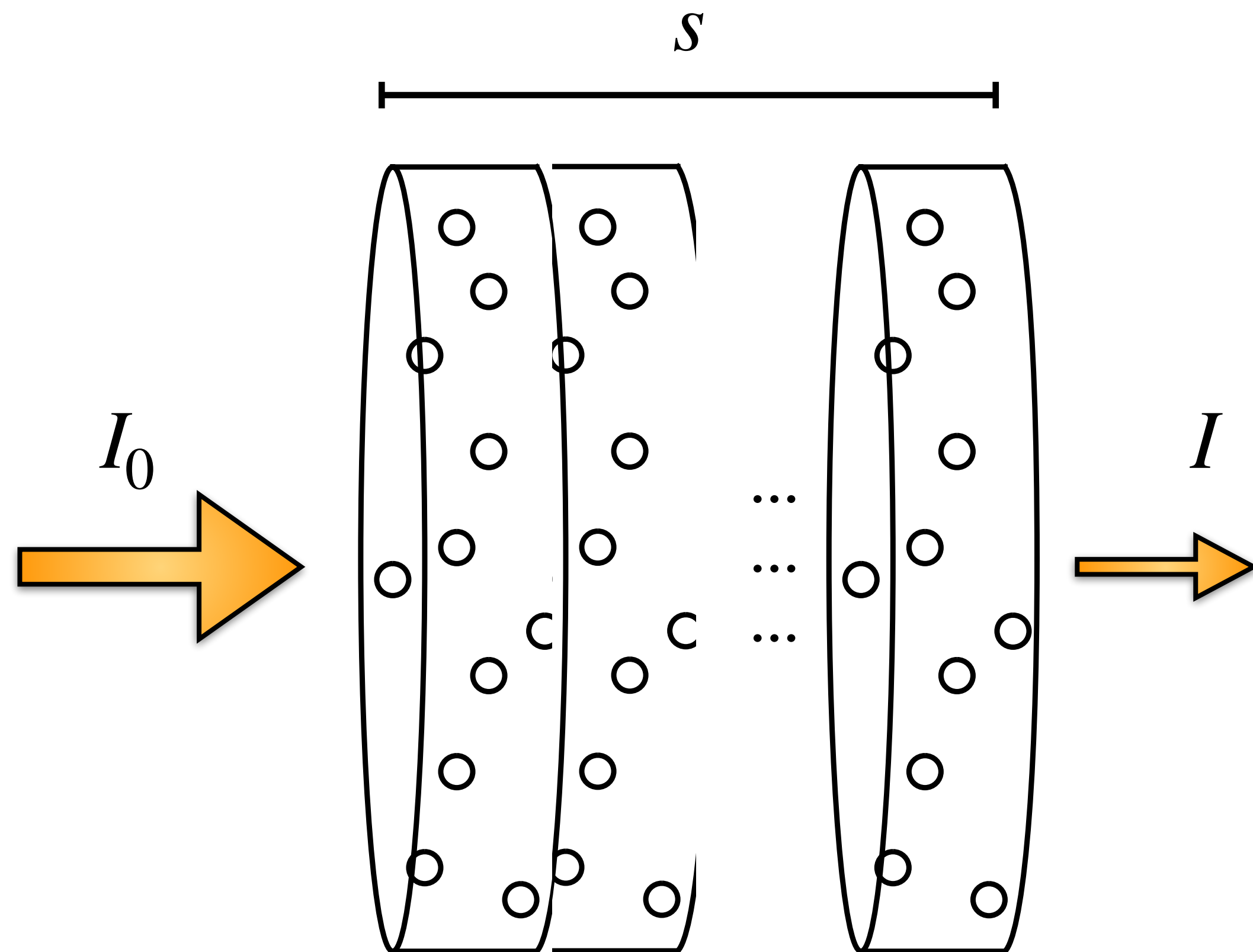


Percentage of  
absorbed photons

$$\frac{\rho E \Delta s A}{E} = - \frac{\Delta I}{I}$$

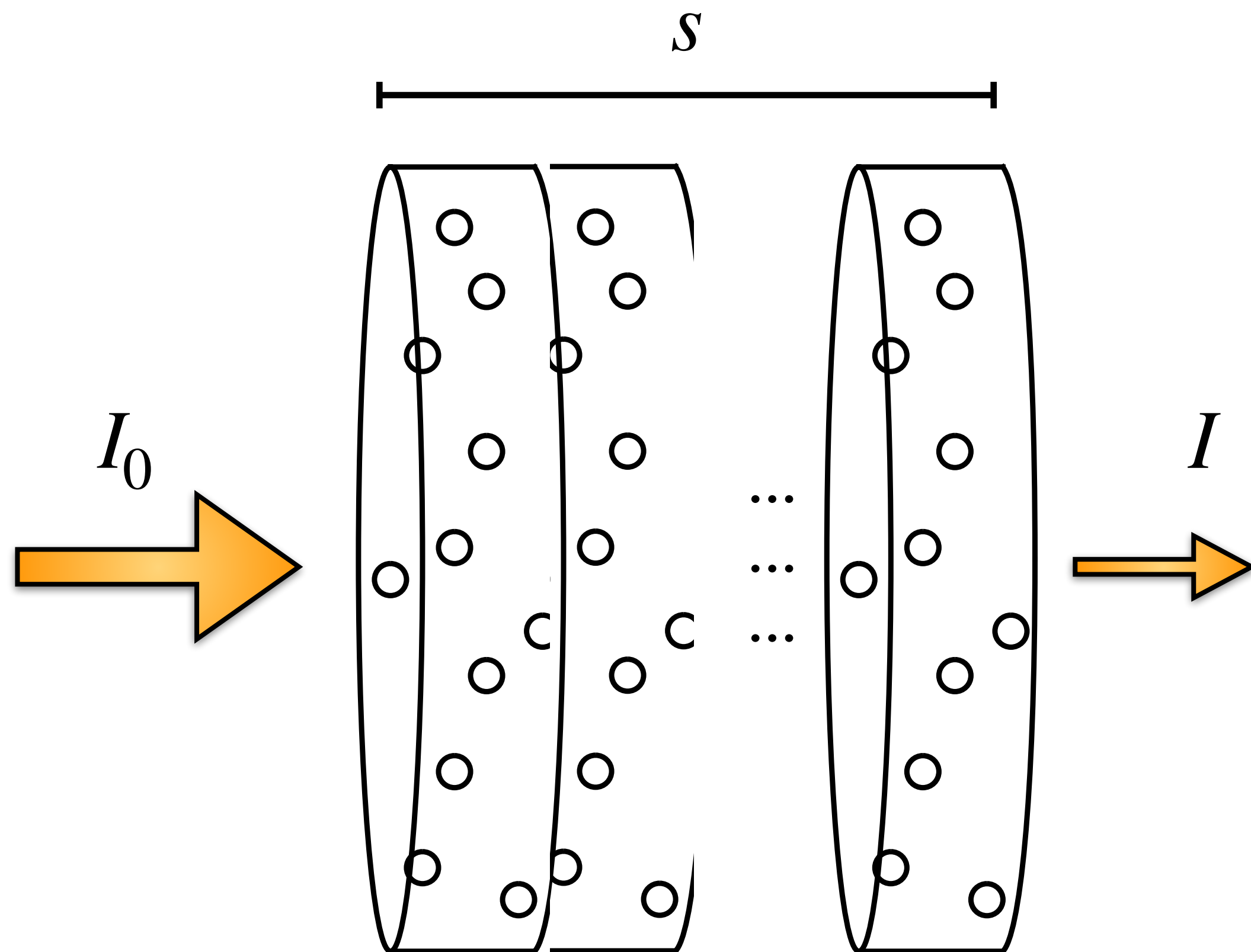
$$\frac{\Delta I}{\Delta s} = -\rho I A$$

# Beer-Lambert Law

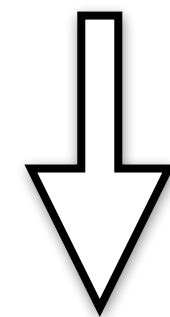


$$\frac{dI}{ds} = -\rho I A$$

# Beer-Lambert Law



$$\frac{dI}{ds} = -\rho l A$$

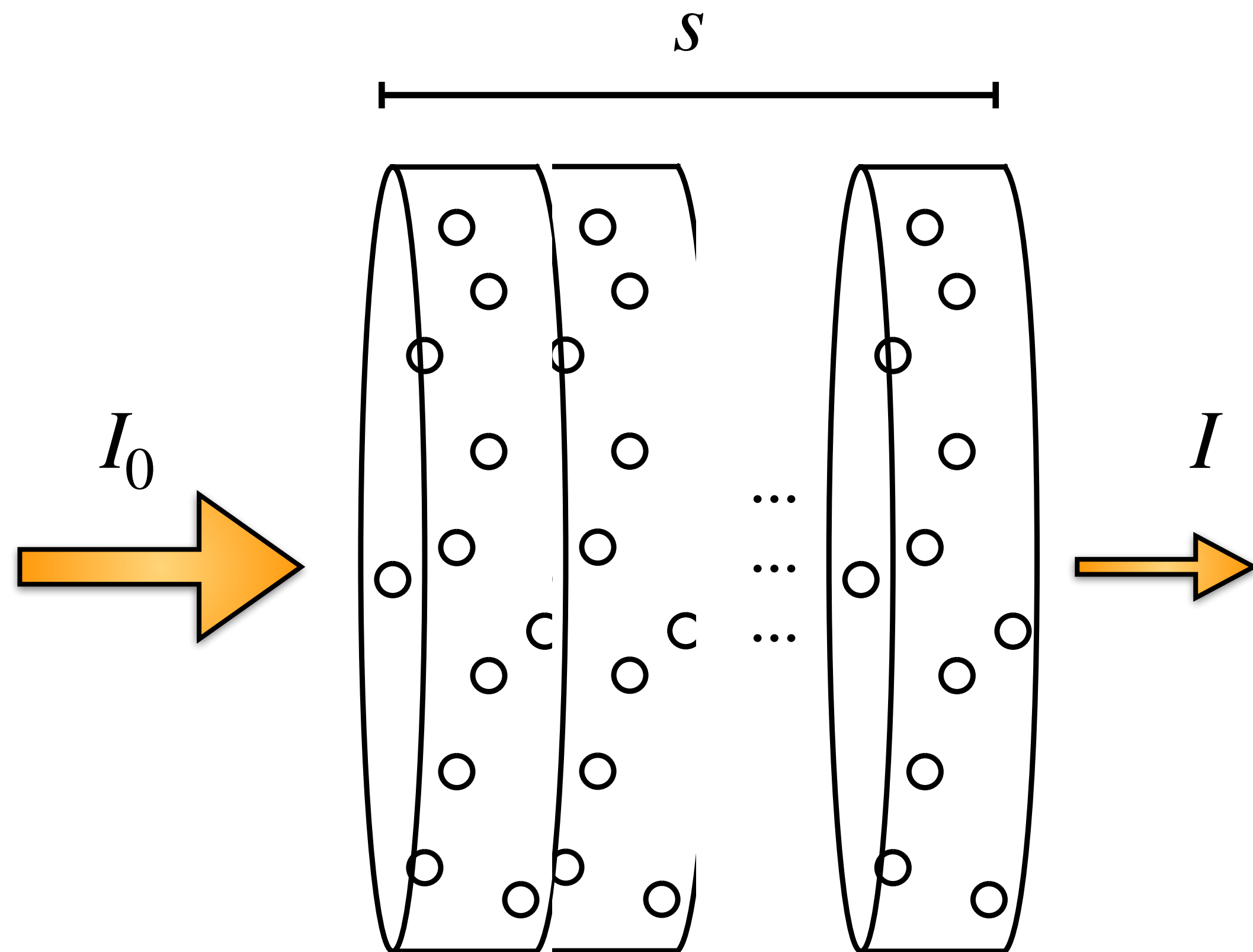


$$I = I_0 e^{-\rho A s}$$

If density is uniform

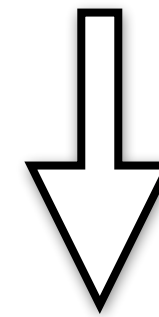


# Beer-Lambert Law



As  $s$  increases (light travel farther), the intensity decreases **exponentially**!

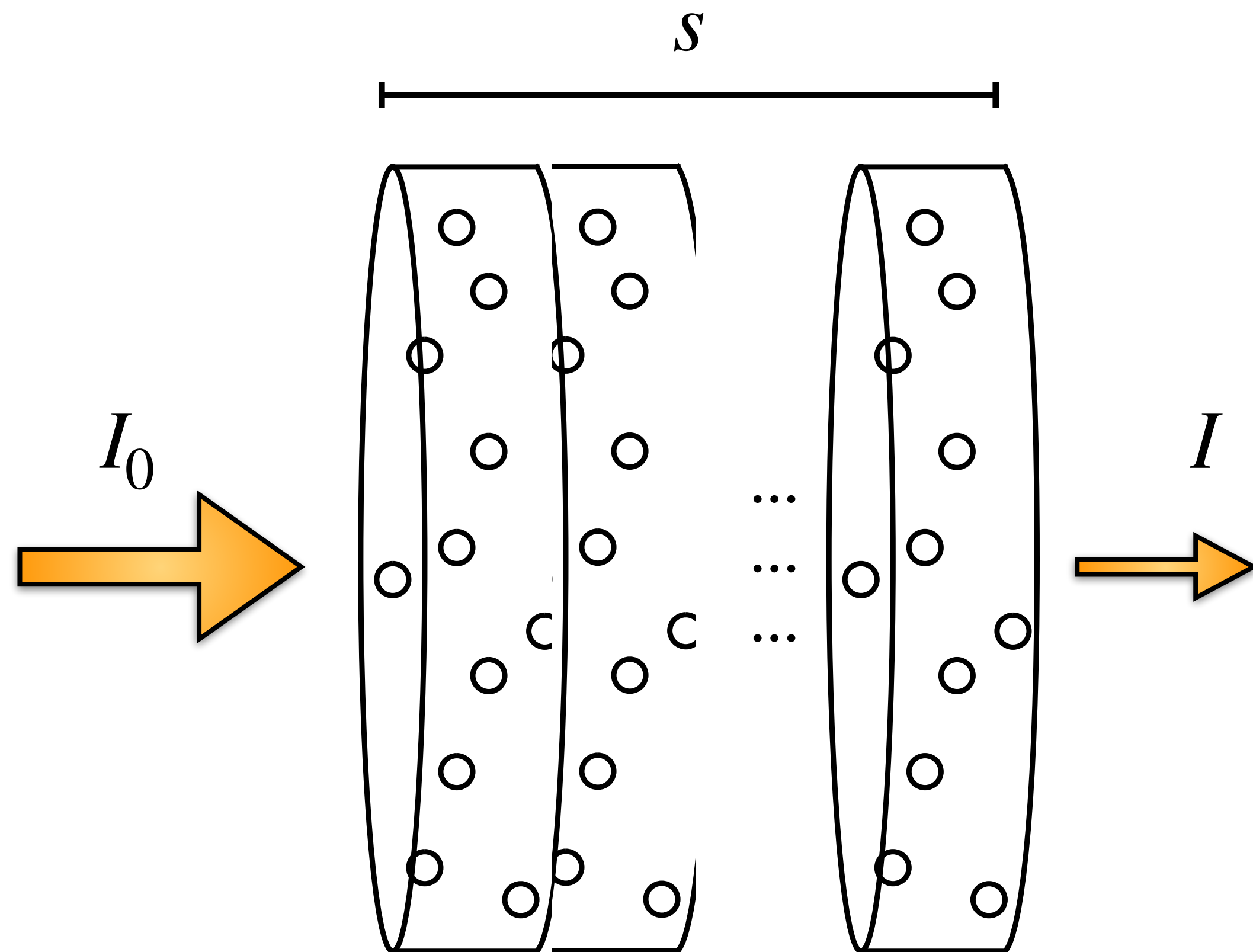
$$\frac{dI}{ds} = -\rho I A$$



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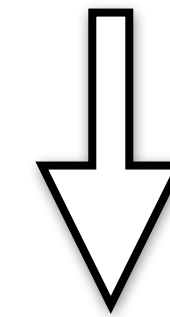
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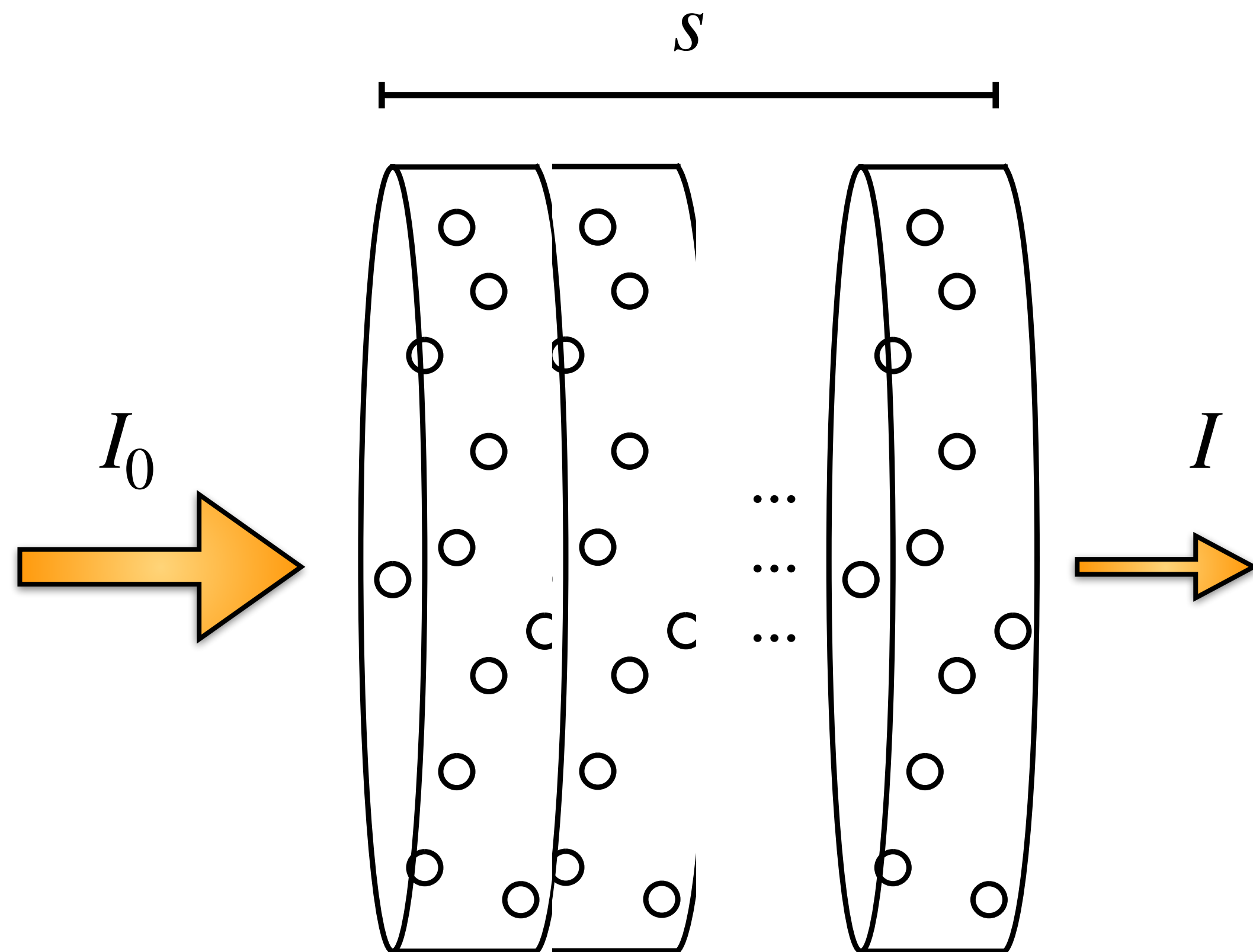
$$I = I_0 e^{-\rho A s}$$

If density is uniform

$$I = I_0 e^{-\int_0^s \rho(s) A ds}$$

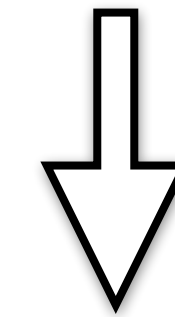
If density is non-uniform

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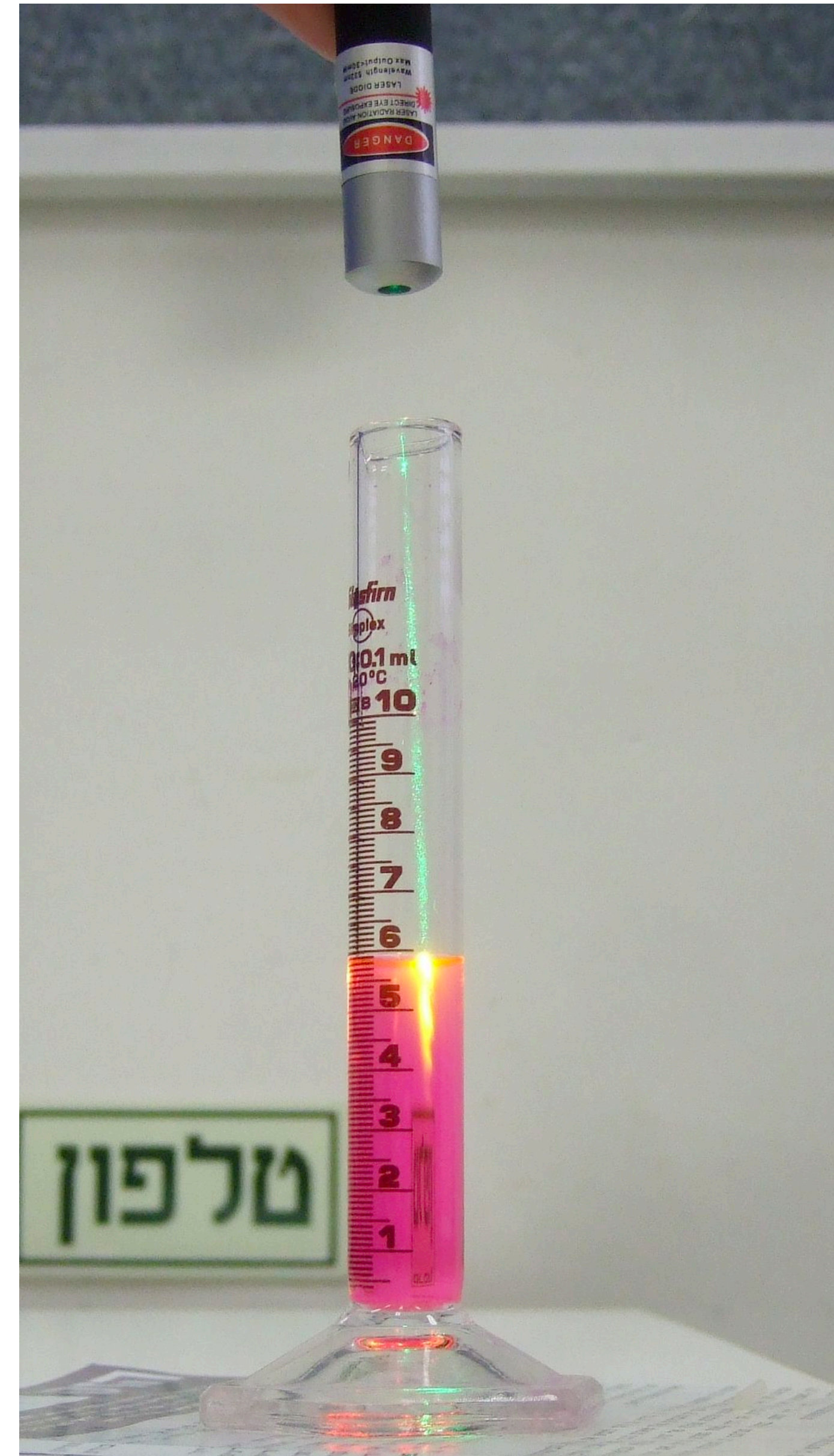
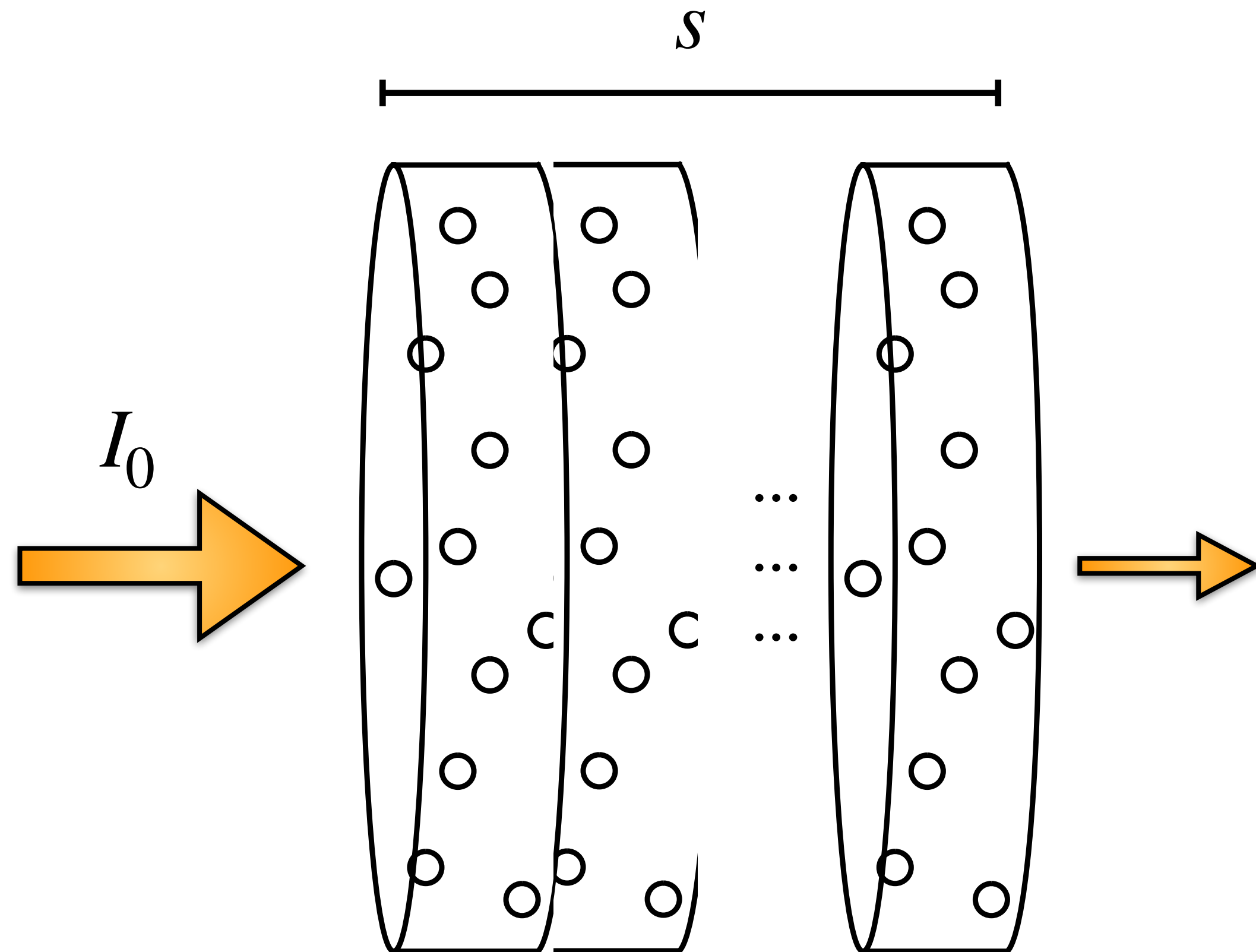
If density is non-uniform

*"The person credited with a scientific law is almost certainly not the one who discovered it."*

Bouguer was the first to discover this law.



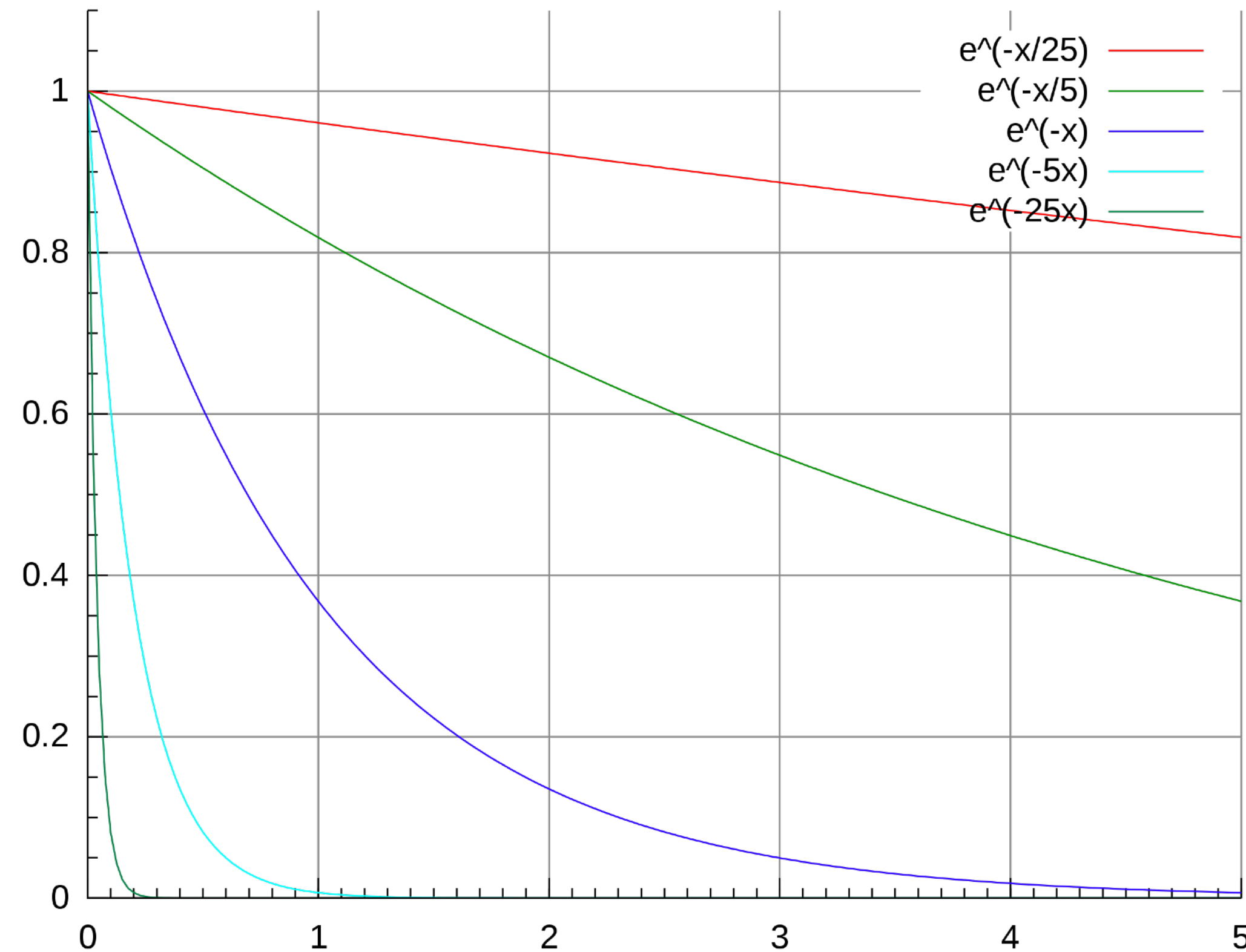
# Beer-Lambert Law





# Exponential Decay

$$I = I_0 e^{-\rho A s}$$



Exponential decay happens when a quantity decreases at a rate proportional to its current value.

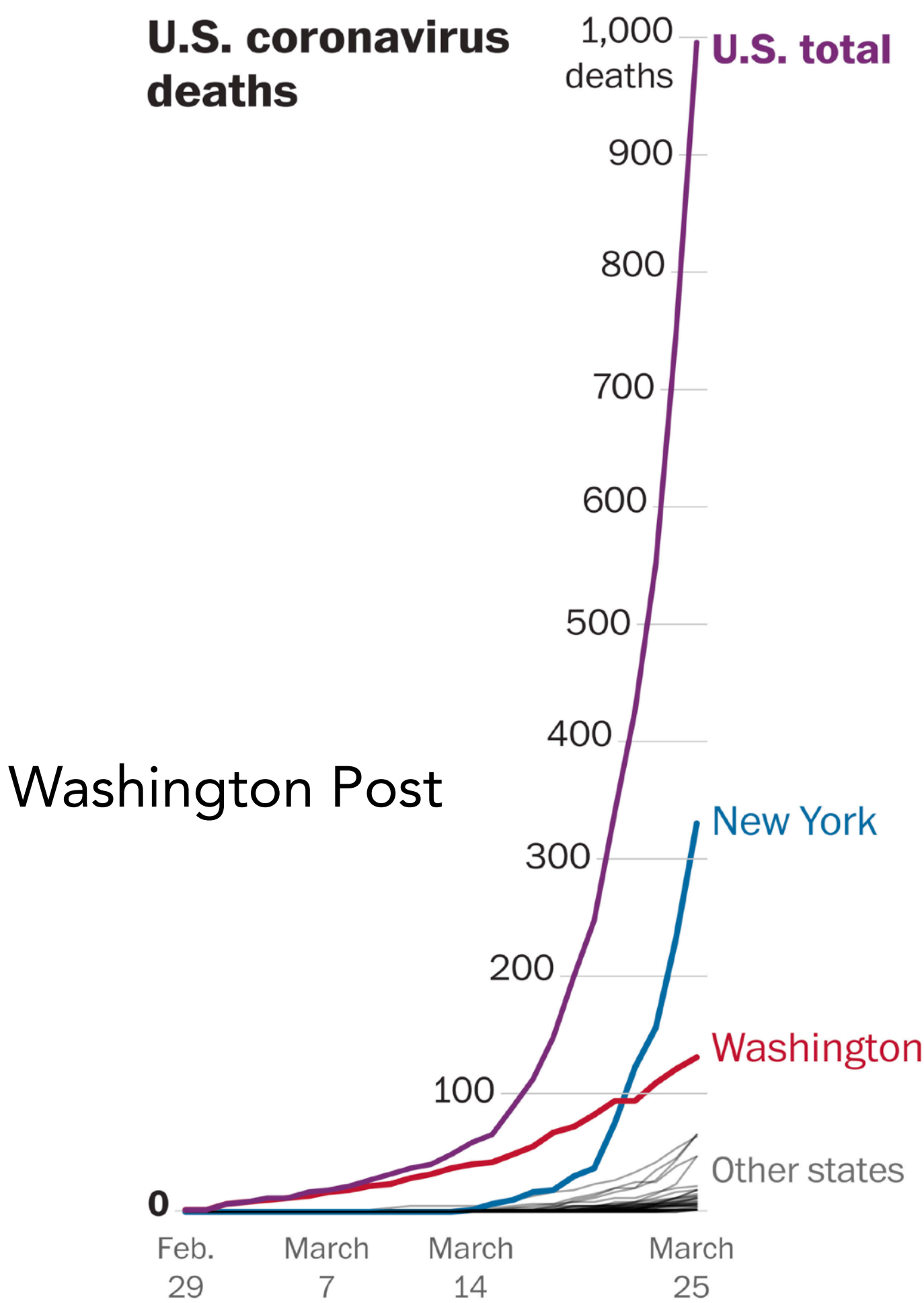
- Opposite of exponential growth.

Exponential decay means lights are hard to be completely absorbed.

- 10% of light @ 70 m deep under ocean, but you can still see lights at 850 m deep.

Many real-world scenarios exhibit exponential growth/decay.

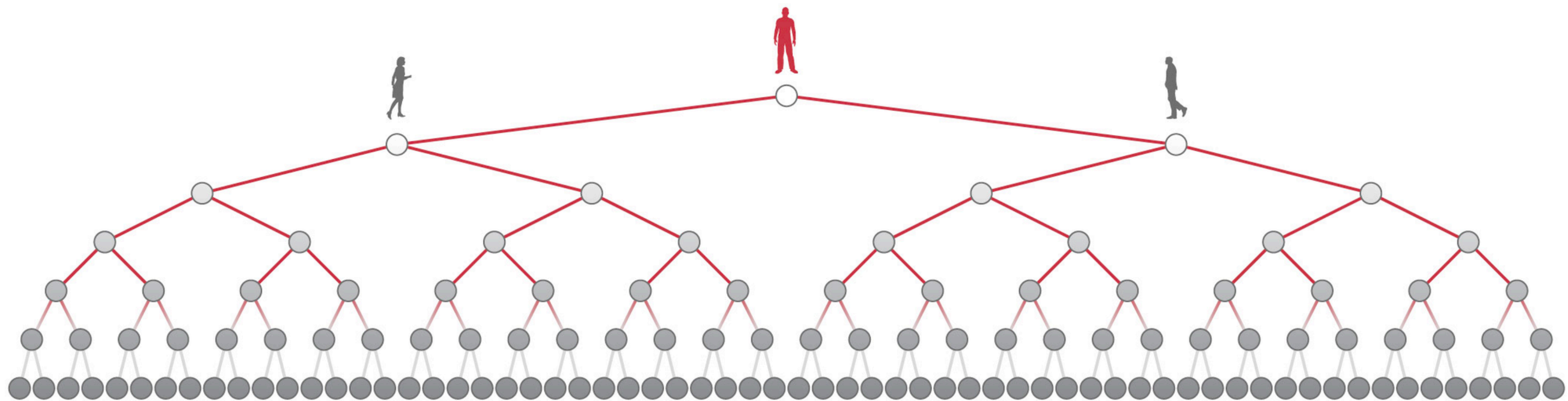
# Exponential Growth



Source: Post reporting from state health department data

Every exponential growth will come to an end when...

Coronavirus Chain of Transmission



New York Times

# Absorption Efficiency and Coefficient

Density/concentration      Cross-section  
(Unit: 1/m<sup>3</sup>)                  area (Unit: m<sup>2</sup>)

$$I = I_0 e^{-\rho A s}$$

\* As always, absorption efficiency and coefficient are **wavelength-dependent**.

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More generally



$$I = I_0 e^{-\rho Q_a A s}$$

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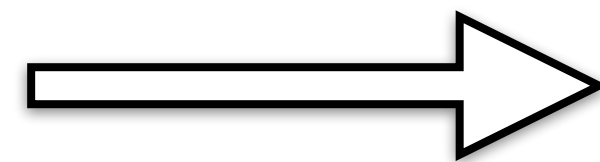


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**Absorption coefficient:**  
Unit: 1/m

$$\mu = \rho Q_a A$$



$$I = I_0 e^{-\mu s}$$

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# Absorption Efficiency and Coefficient

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Percentage of light left

$$I = I_0 e^{-\mu s}$$

$$\frac{I}{I_0} = e^{-\mu s}$$

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# Absorption Efficiency and Coefficient

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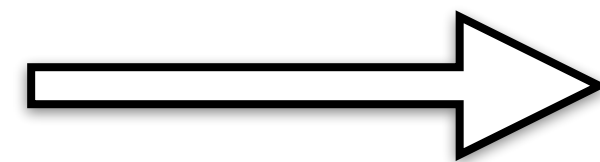


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$$I = I_0 e^{-\rho Q_a A s}$$

**Absorption coefficient:**  
Unit: 1/m

$$\mu = \rho Q_a A$$



Percentage of light left

$$I = I_0 e^{-\mu s} \qquad \frac{I}{I_0} = e^{-\mu s}$$

\* As always, absorption efficiency and coefficient are **wavelength-dependent**.

$C_a = Q_a A$  is sometimes called **effective absorption cross section**


# Absorption Efficiency and Coefficient

Informally,  $\mu$  quantifies how much light is absorbed per unit distance.

- $\mu = 1$  means after passing through 1 m the radiance is reduced by a factor of  $e$ .
- Another way to understand  $\mu$  is that  $\mu = 0.1$  means a photon can travel about 10 m before being absorbed.

Absorption coefficient:

Unit: 1/m

$$\mu = \rho Q_a A$$


$$I = I_0 e^{-\mu s}$$



# Photoreceptor Absorption

Percentage of light left  $\frac{I}{I_0} = e^{-\mu s}$

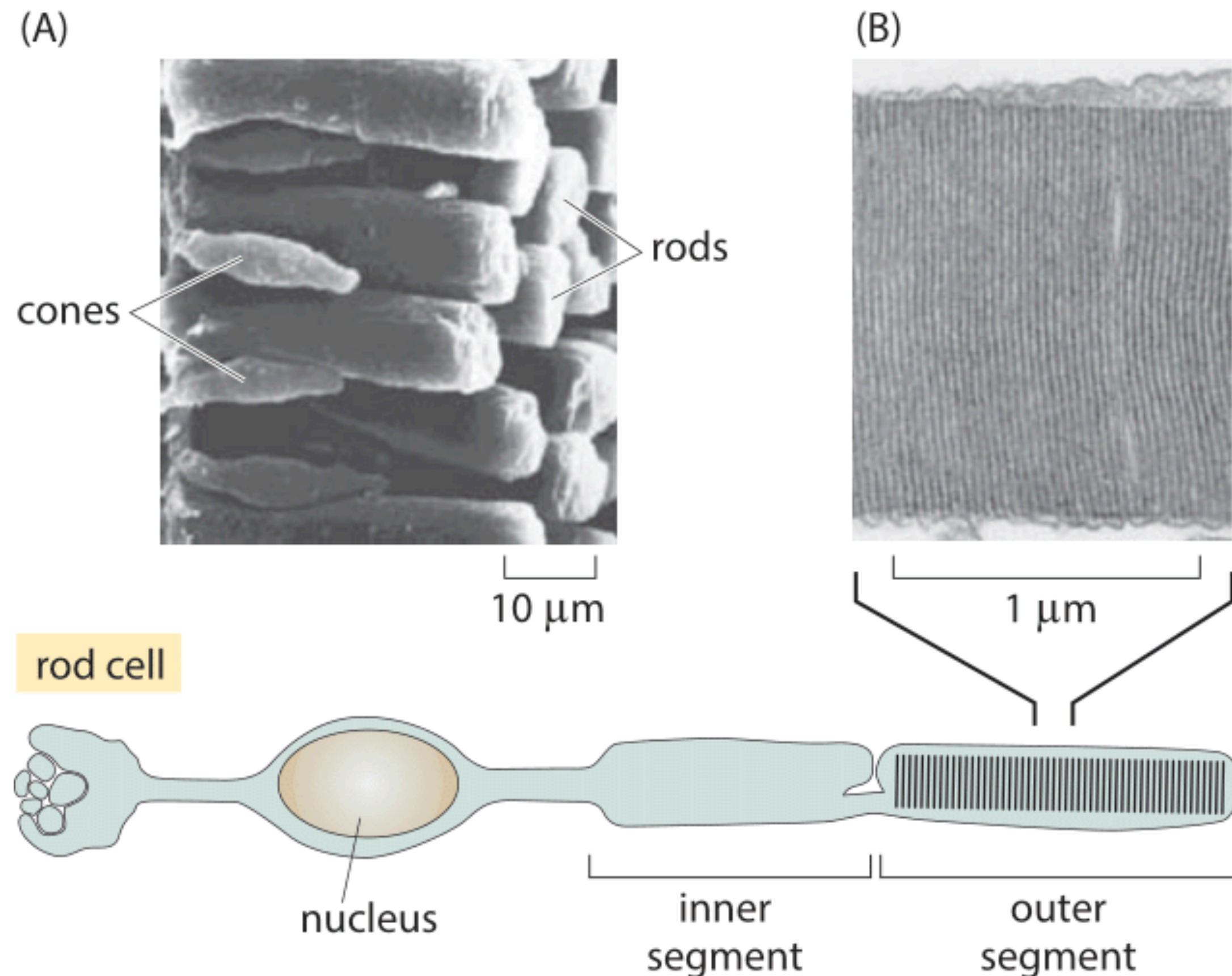
Outer segment is light-sensitive part of rods. It's about  $25 \mu\text{m}$  long.

- It absorbs about 3% of the incident light for every  $\mu\text{m}$ .
- How much light is absorbed by a rod cell? (Answer 53%)

$$0.97 = e^{-\mu s}, s = 1 \mu\text{m}$$

$$\Rightarrow \mu \approx 0.03 / \mu\text{m}$$

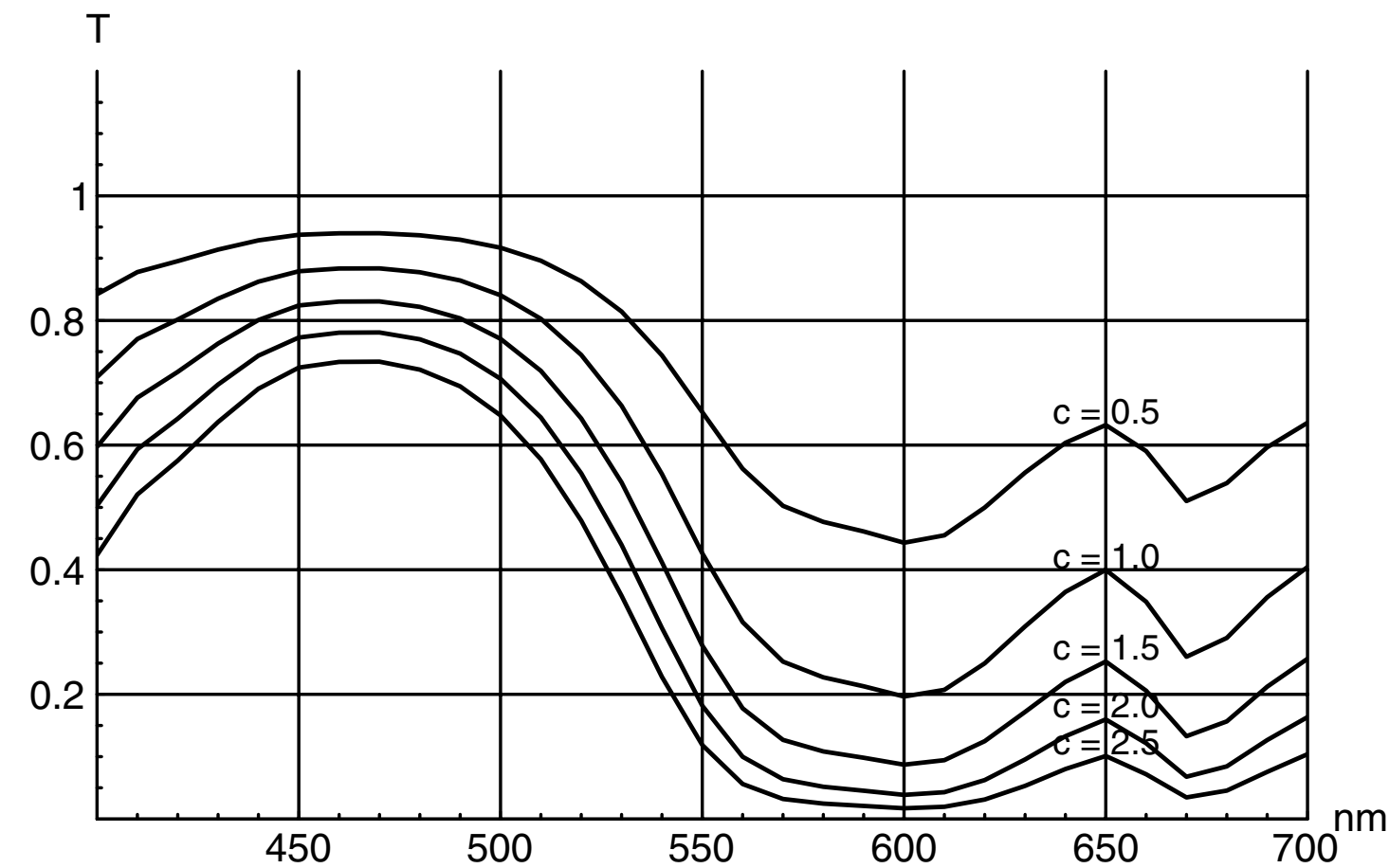
$$\Rightarrow e^{-0.03 \times 25} \approx 0.47$$



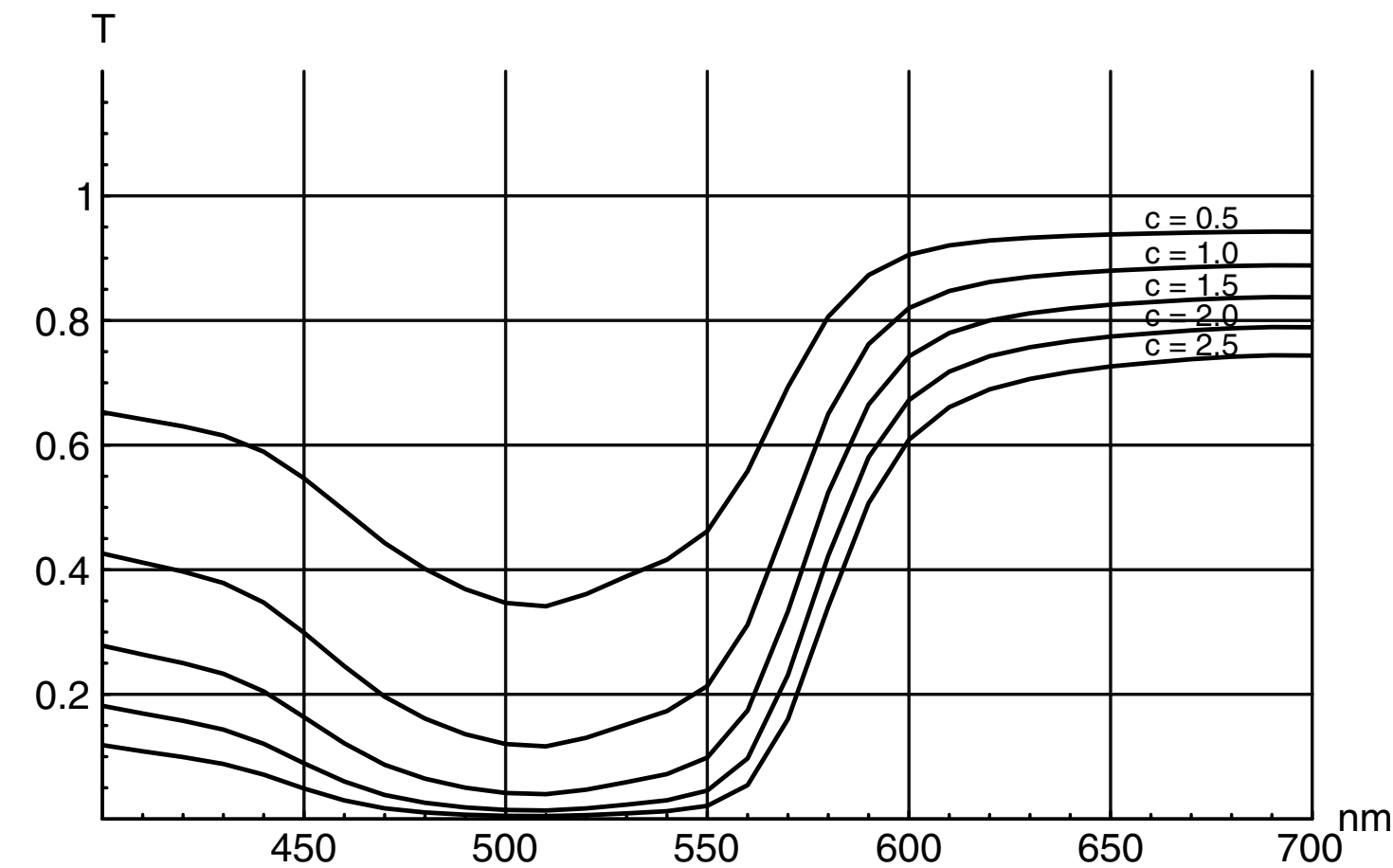
# Absorption/Transmission vs. Concentration

$$I = I_0 e^{-\rho Q_a A s}$$

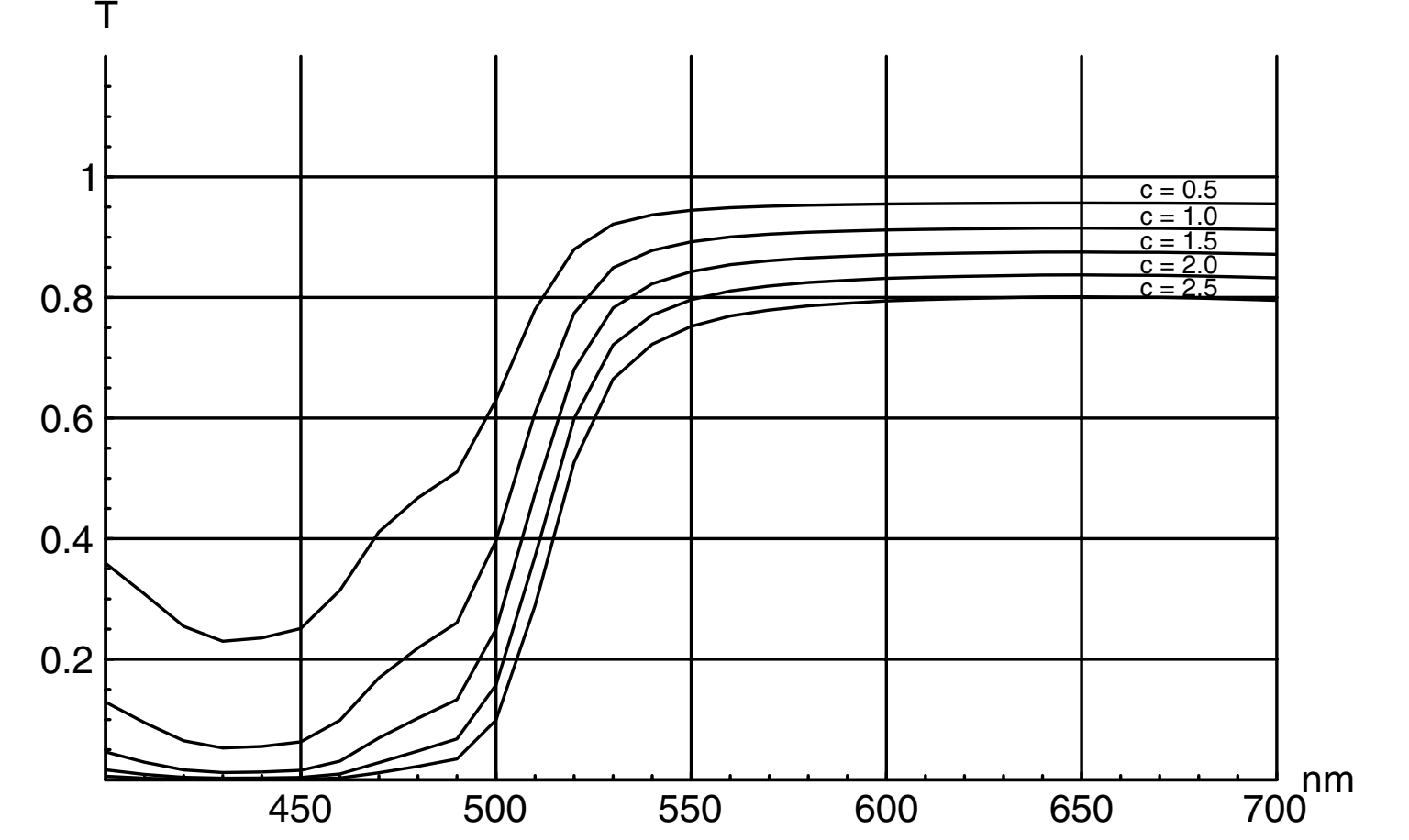
Cyan



Magenta



Yellow



Transmittance (1 - absorption) of three colored-ink at various concentrations (**c**).

# Pure Scattering

(from a single particle)



# Modeling Scattering

We need to model two aspects of scattering:

- How much is light scattered. This is captured by **scattering coefficient**.
- In which direction is a photon scattered. This is captured by the so-called “**phase function**”.

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Importantly, people have worked out approximations under different conditions, using which we can readily explain many scattering behaviors.

# Modeling Scattering

We will first consider cases where a photon is scattered by one particle.

- This happens when the particles are reasonably apart of each other.
- A photon can be scattered by multiple particles if the particle concentration is high or when the material is thick.

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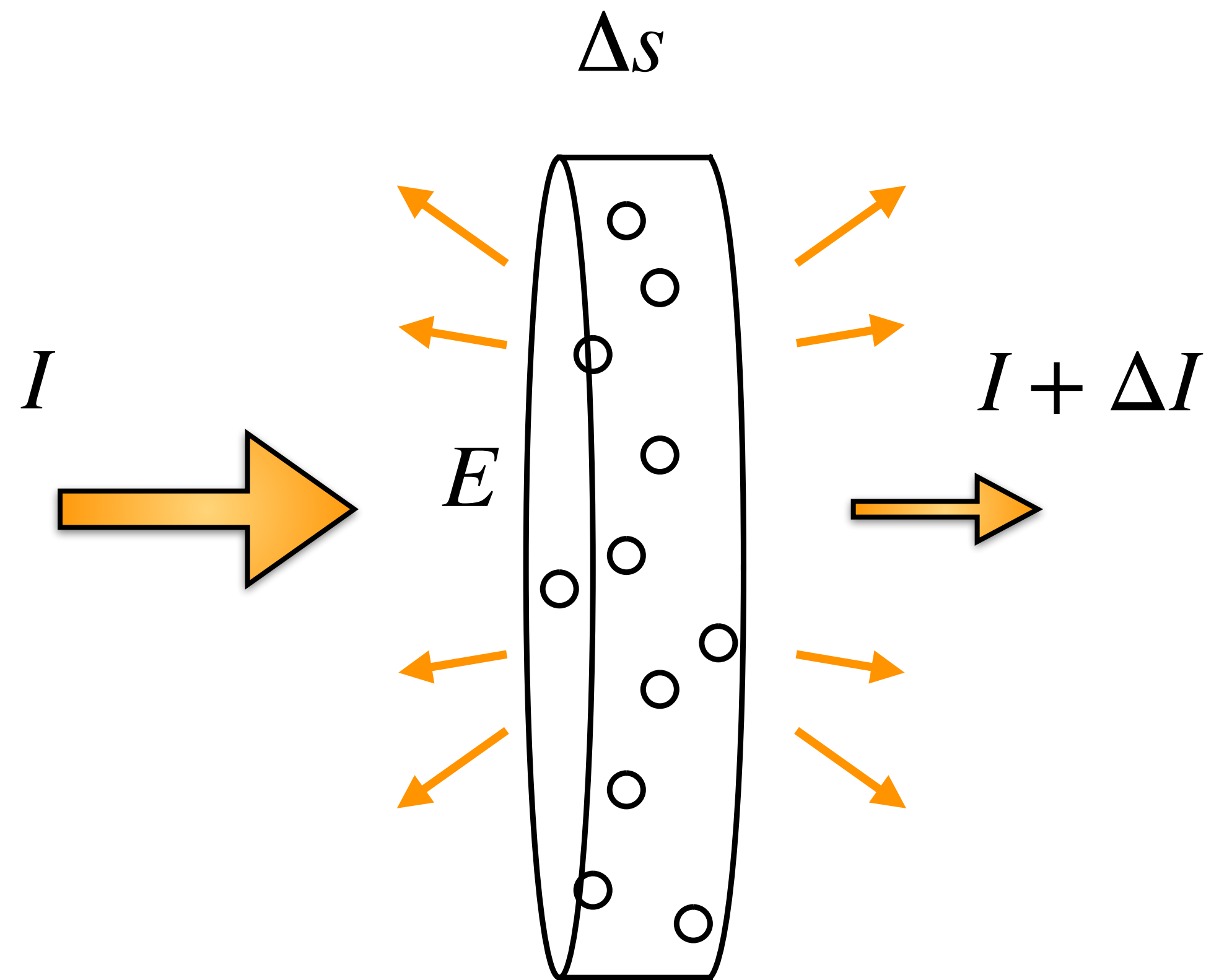
- This happens when the particles are reasonably apart of each other.
- A photon can be scattered by multiple particles if the particle concentration is high or when the material is thick.

We will then see general cases of multiple scattering + absorption.

We also assume elastic scattering, where scattered photon has the same wavelength as the incident photon.

- There are inelastic scattering, which does happen in nature but we won't consider here.

# Modeling Scattering



\* even though here we assume a photon is scattered by only a single particle, multiple particles can be scattering different photons.

**Scattering efficiency** (unitless): the probability that a photon is actually scattered when it hits a particle.

Concentration

Cross-section area

Distance traveled

$$I = I_0 e^{-\rho Q_s A s}$$

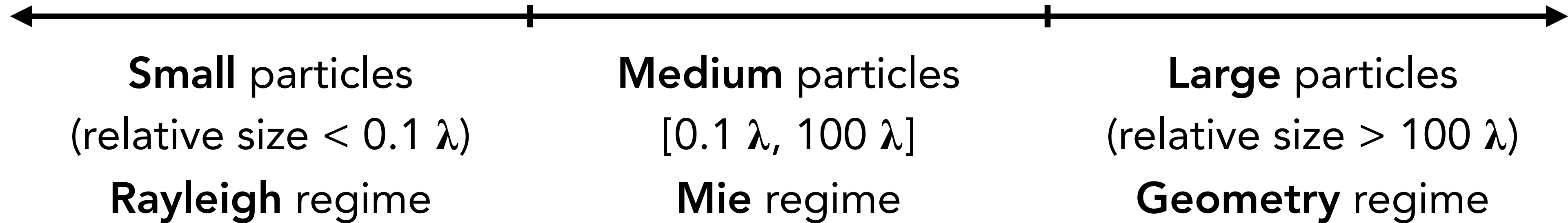
$$\mu = \rho Q_s A$$

is called **scattering coefficient**: Unit: 1/m

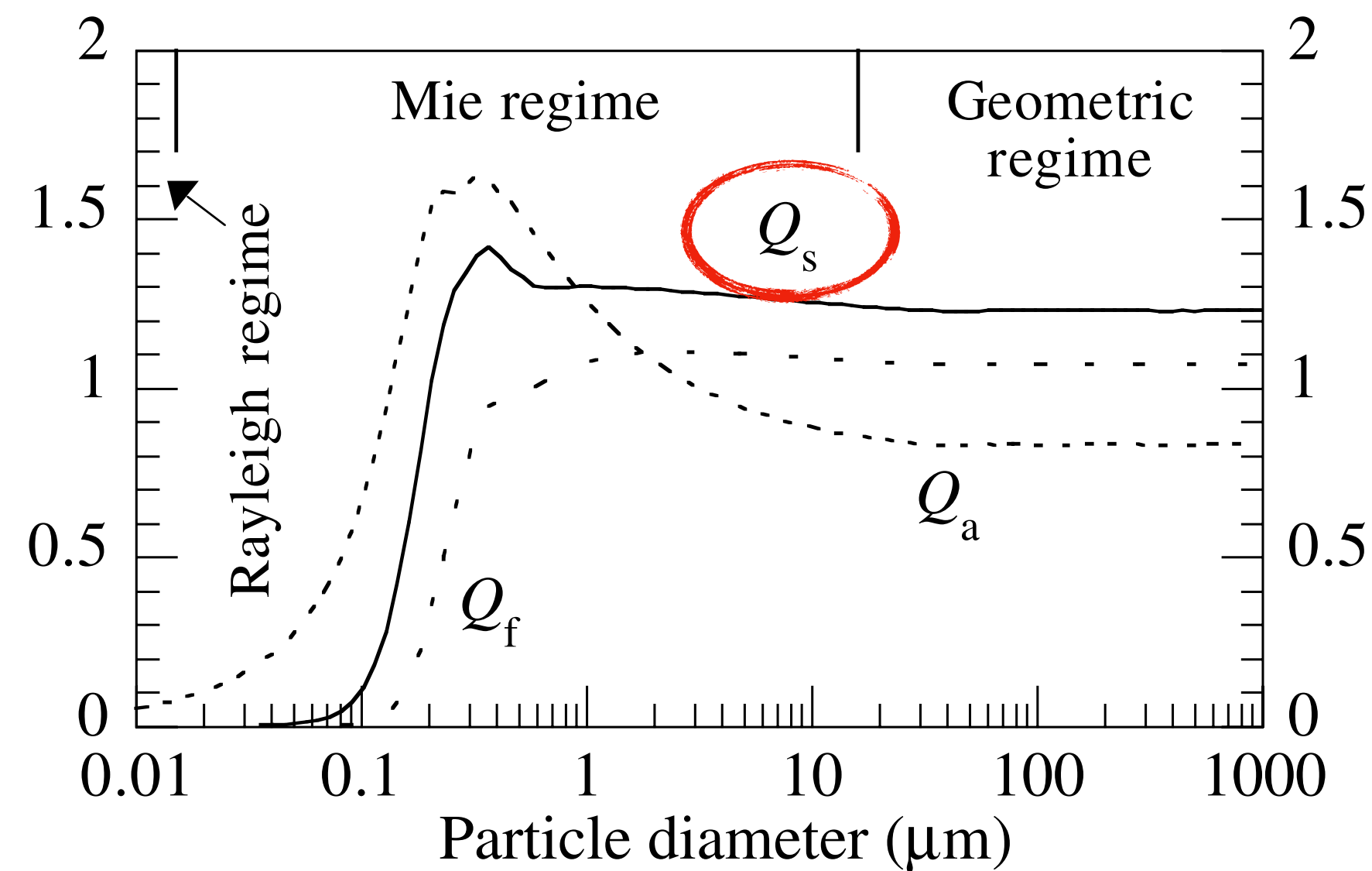
$$C_s = Q_s A$$

is called **effective scattering cross section**

# Three Scattering Regimes

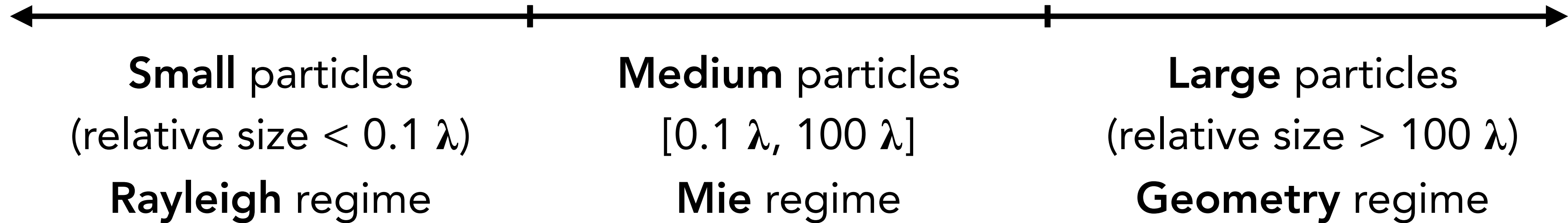


**Soot** particle  
scattering 500  
nm photons

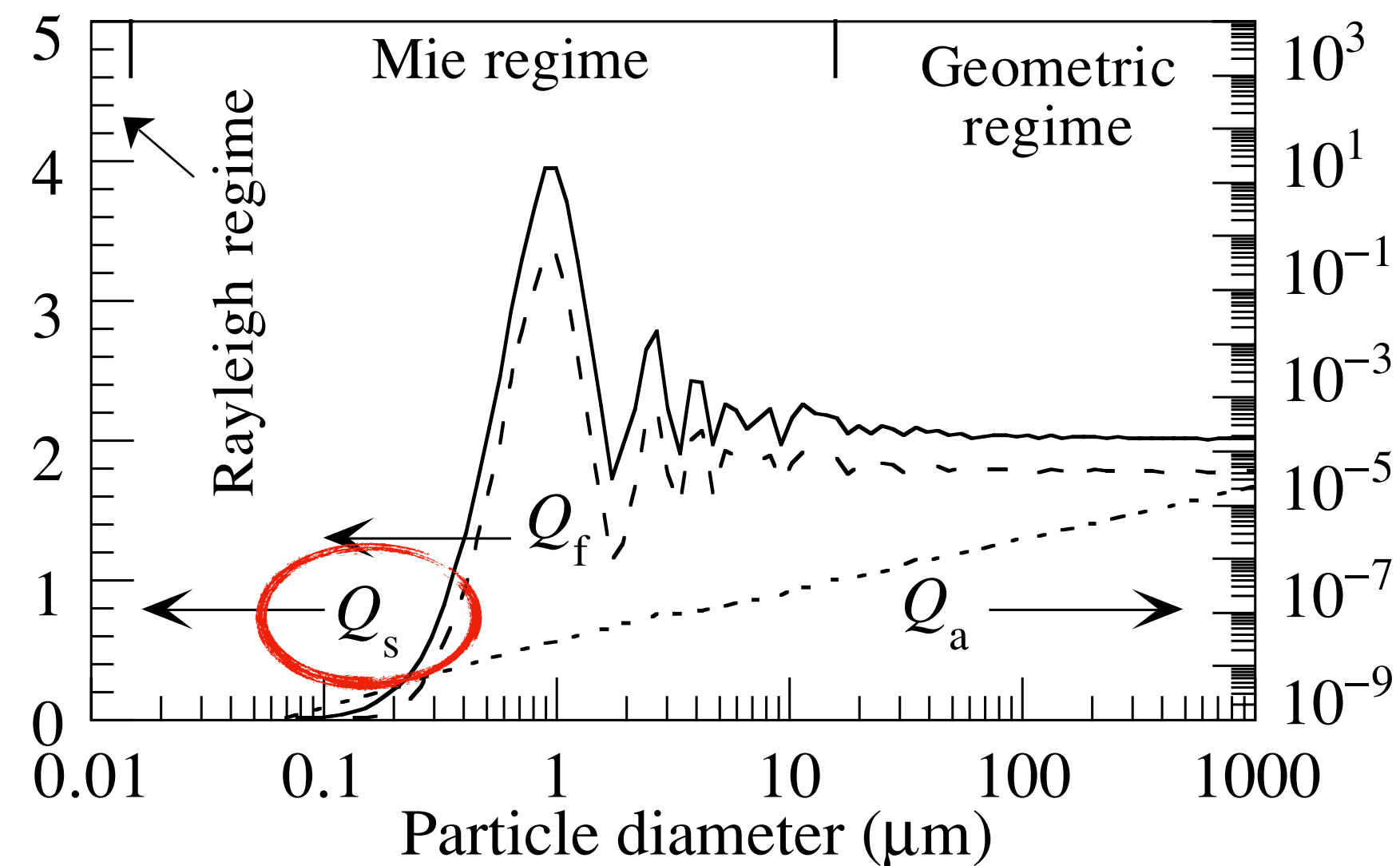




# Three Scattering Regimes



**Water drop**  
scattering 500  
nm photons



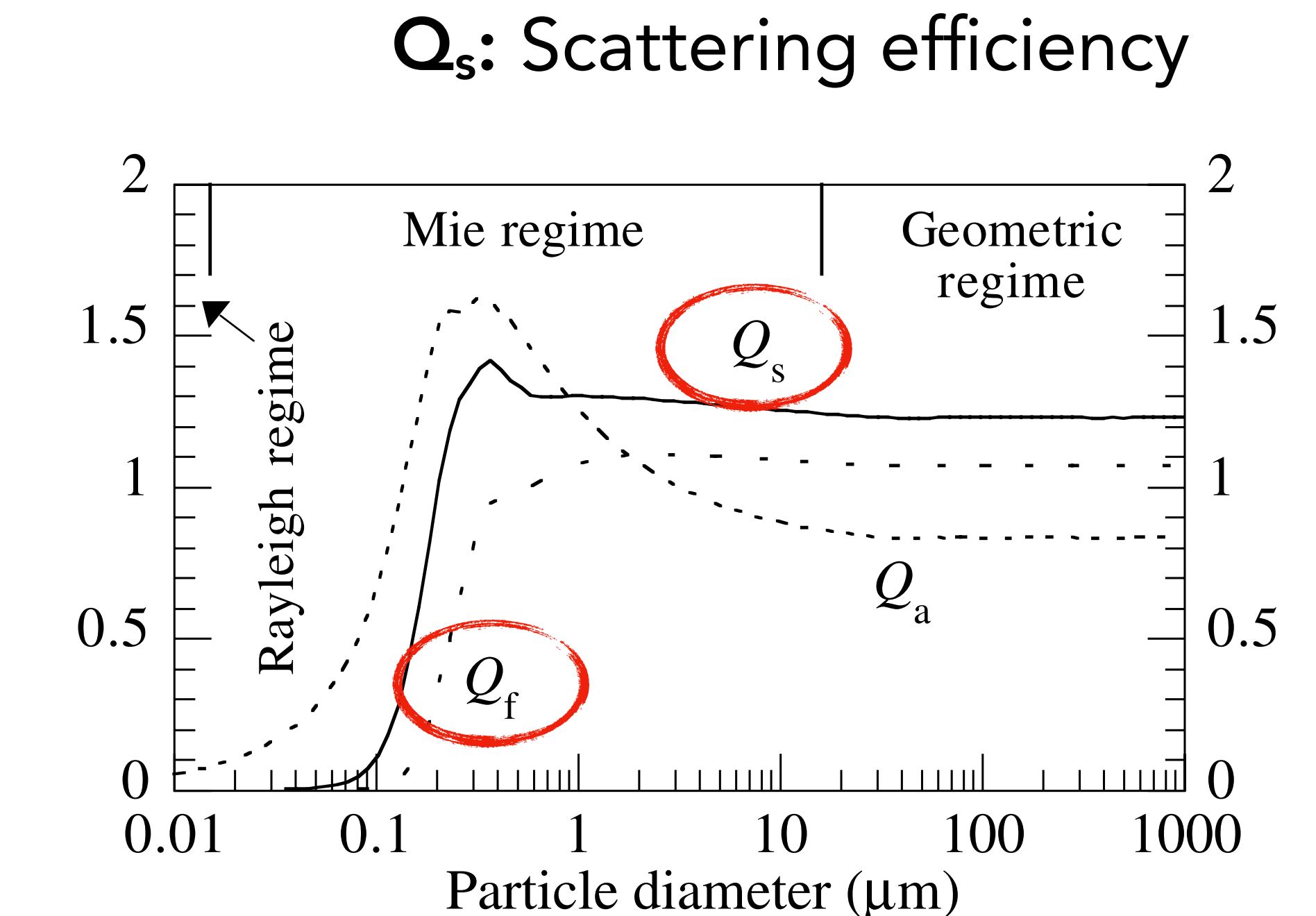
# General Trends

## Scattering efficiency

- Small particles scatter little; large particles scatter a lot.

## Scattering direction

- Small particles scatter lights both forward and backward.
- More forward direction as particle size increases.

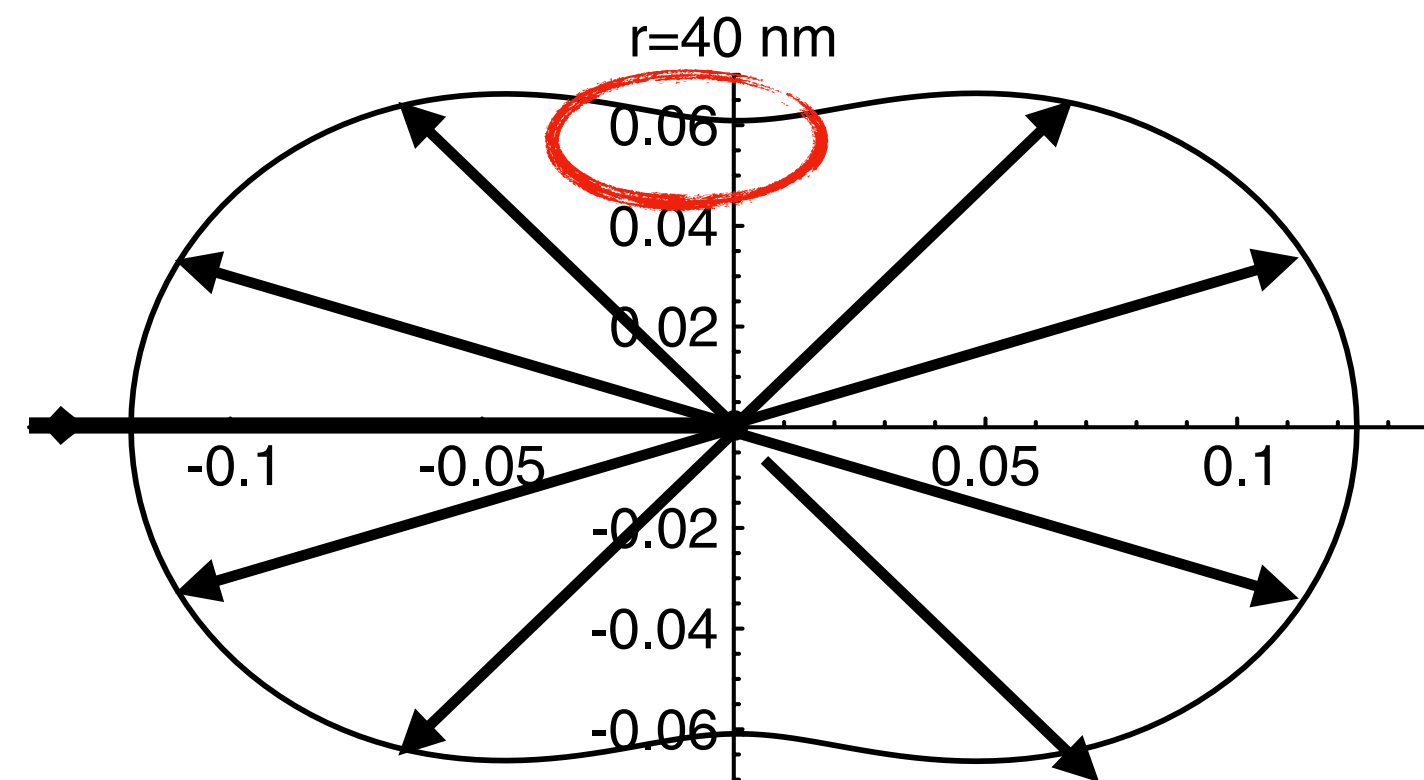


**$Q_f$ : Forward** scattering efficiency.  $Q_f/Q_s$  shows how much light is scattered forward.

# Scattering vs. Size

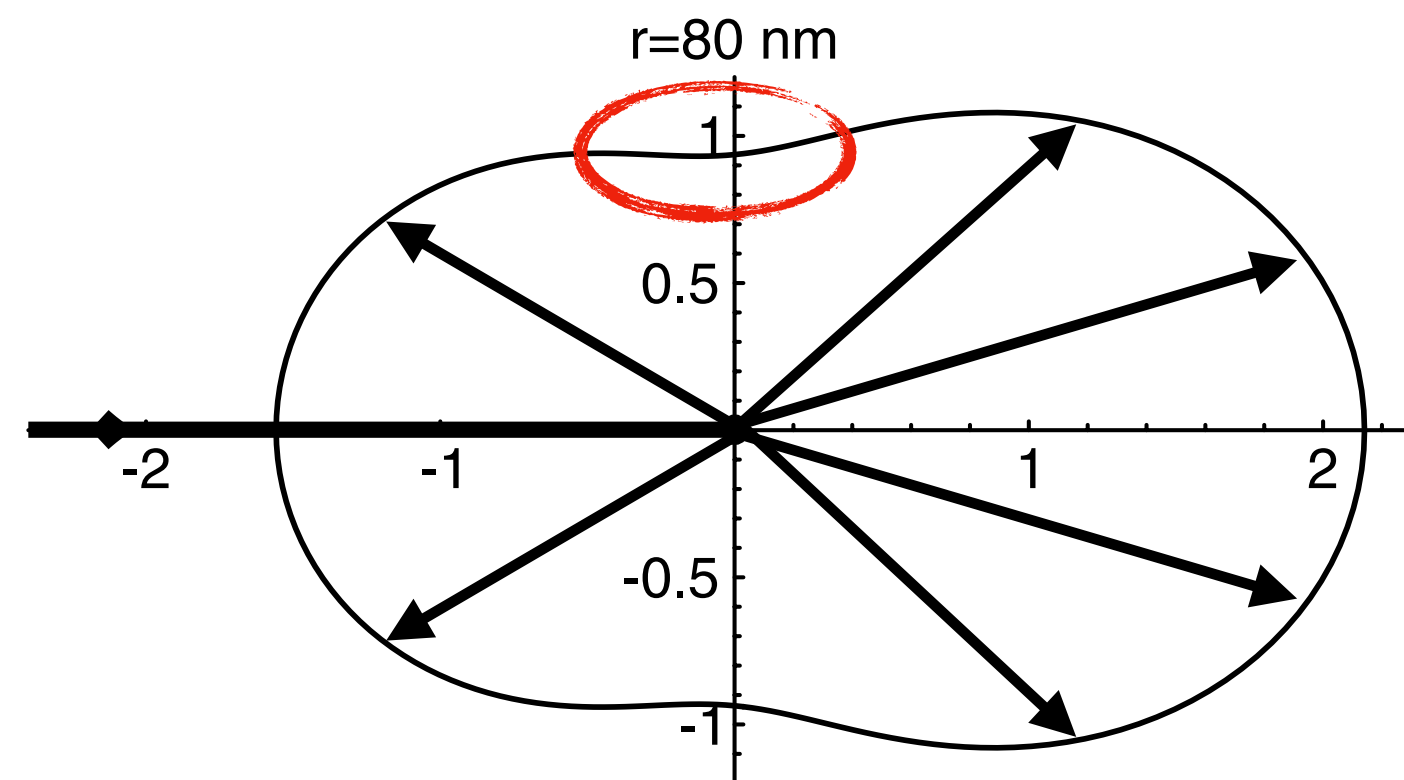
$r = 40 \text{ nm}$

$(< 0.1 \lambda)$



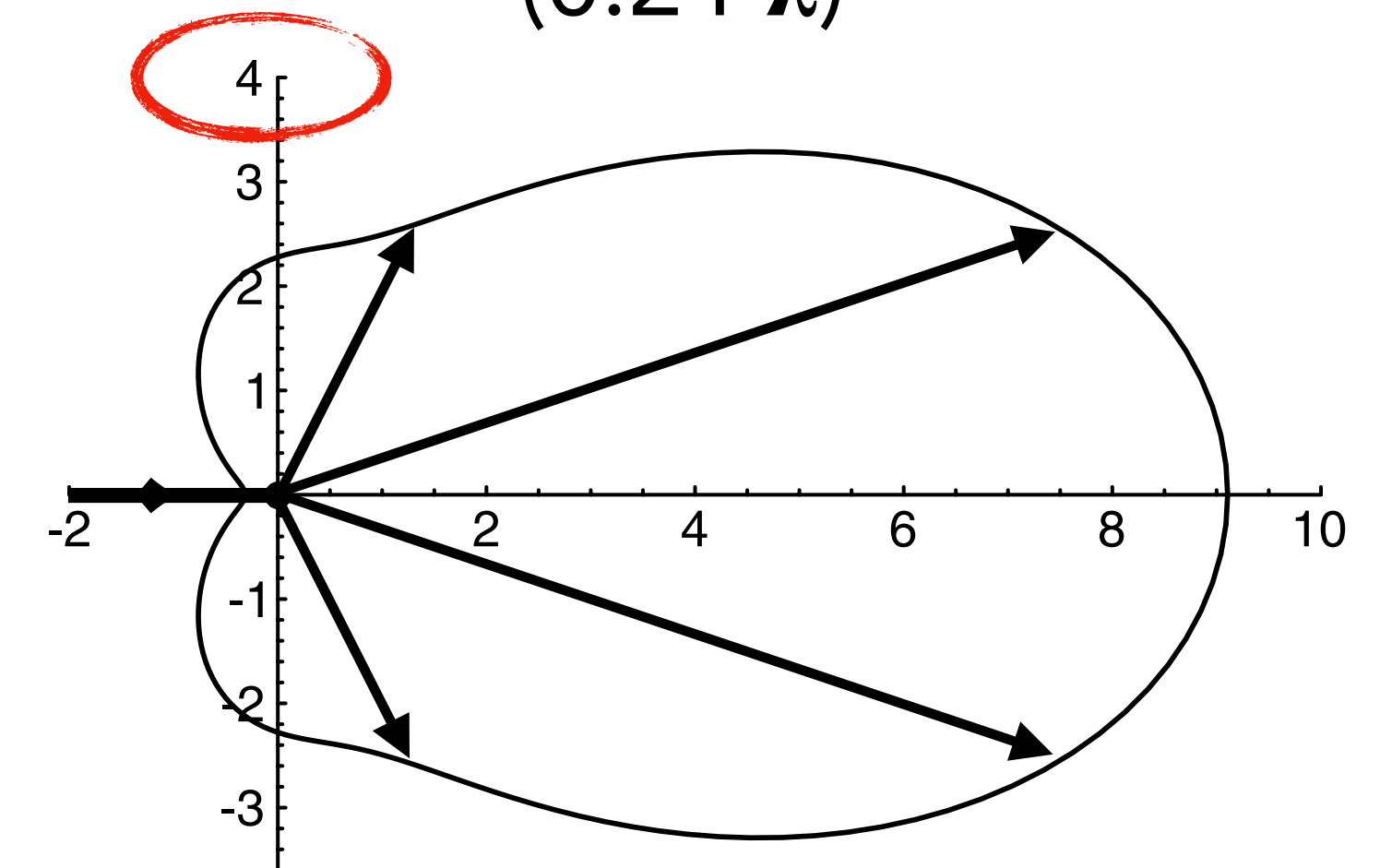
$r = 80 \text{ nm}$

$(0.16 \lambda)$



$r = 120 \text{ nm}$

$(0.24 \lambda)$

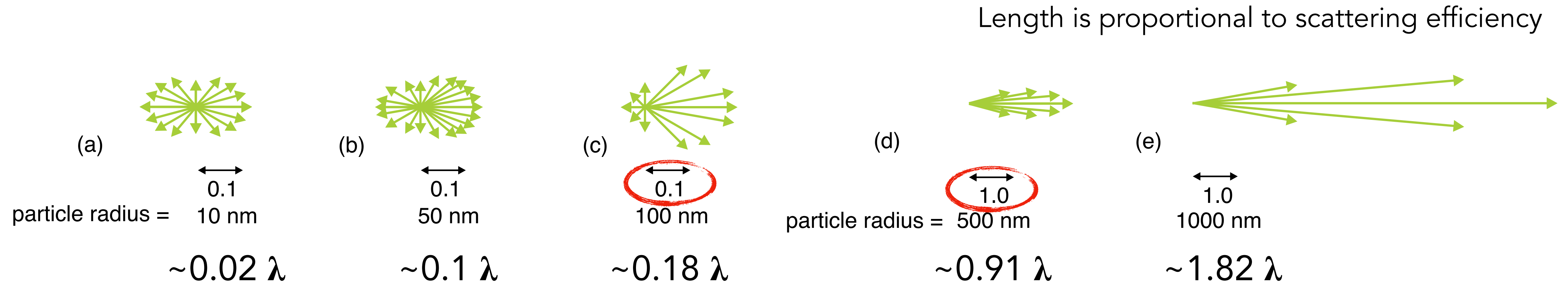


Spherical gold particle scattering **500 nm** photons.

As particle size increases

- significantly more forward scattering + higher scattering efficiency

# Scattering vs. Size

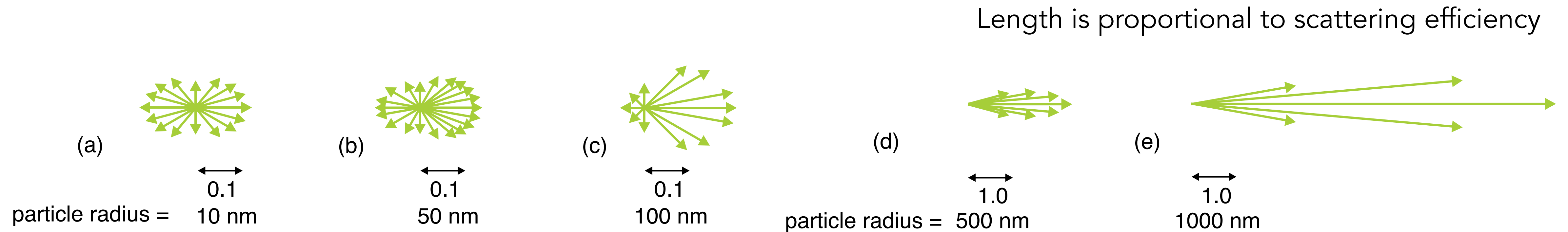


Scattering **550 nm** photons.

As particle size increases

- significantly more forward scattering + higher scattering efficiency

# Scattering vs. Size



Small

Large

Small cross section area

Low scattering efficiency

Even backward and  
forward scattering

Large cross section area

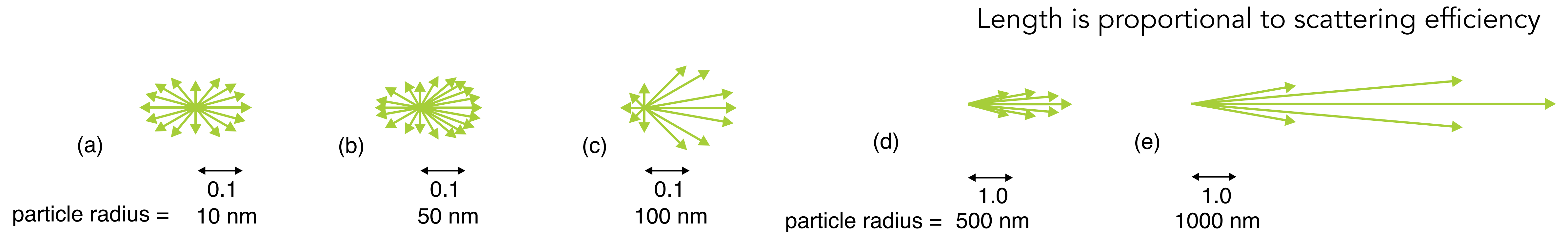
High scattering efficiency

Mostly forward scattering

Increase  
transparency



# Scattering vs. Size



Small

Large

Small cross section area

Low scattering efficiency

Even backward and forward scattering

Increase  
opacity

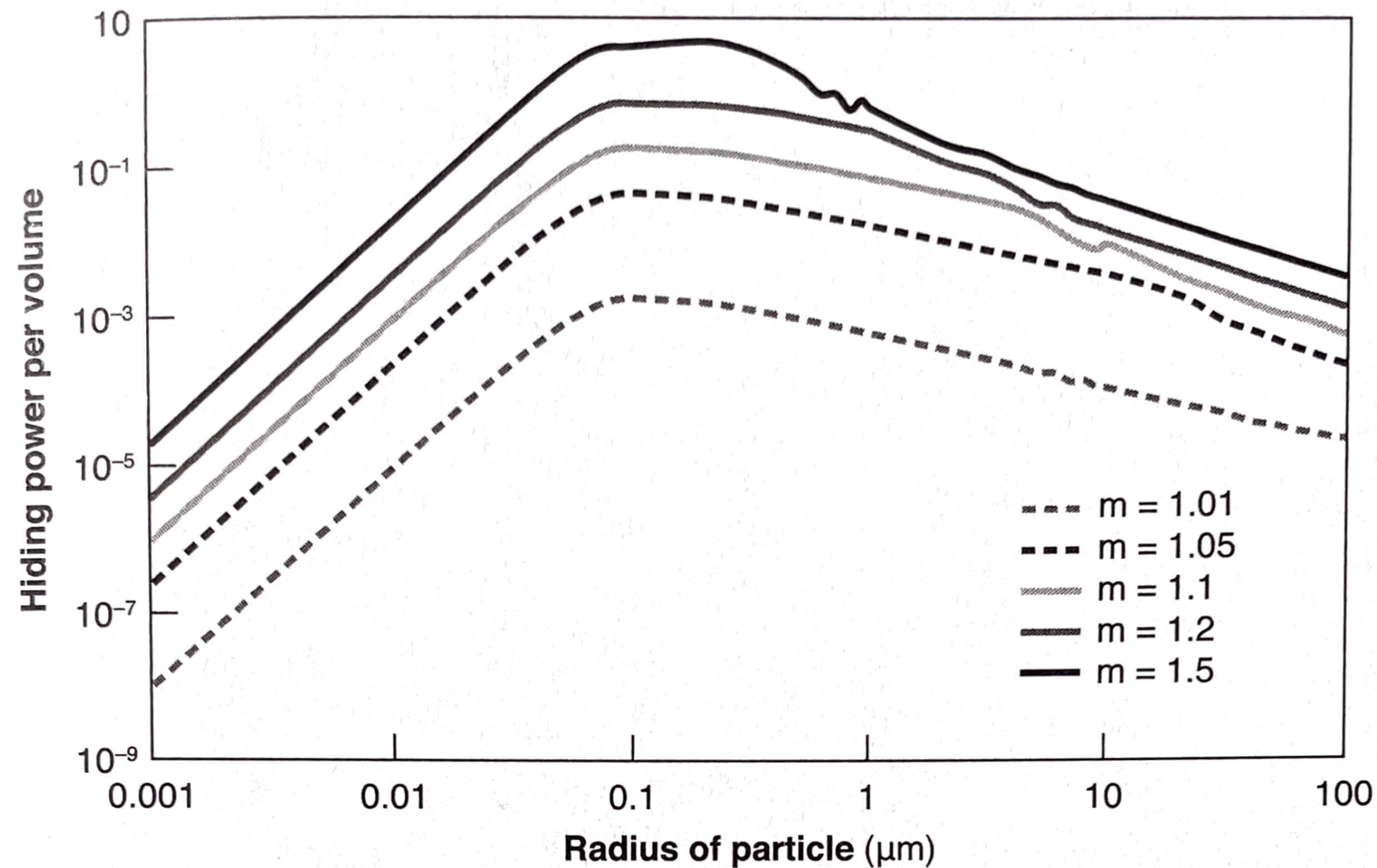
Large cross section area

High scattering efficiency

Mostly forward scattering

# "Hiding Power" vs. Size

$$\frac{H}{V} = Q_s A(1 - g) \propto \frac{Q_s(1 - g)}{r}$$



There is a sweet spot in particle size that provides the highest opacity (hiding light behind it).

- Accidentally the sweet spot falls roughly within the visible range.



**Hippopodius hippopus** is normally transparent, but when disturbed its protein precipitate and form larger particles. It then becomes opaque.





# Impact of Refractive Index

Part of scattering is caused by the refractive index mismatch between the particle and the surrounding medium.

- Water droplets surrounded by air in fog; pigments surrounded by oil in paint, etc.
- It's the relative index, ***m***, between the particle and the medium that matters.

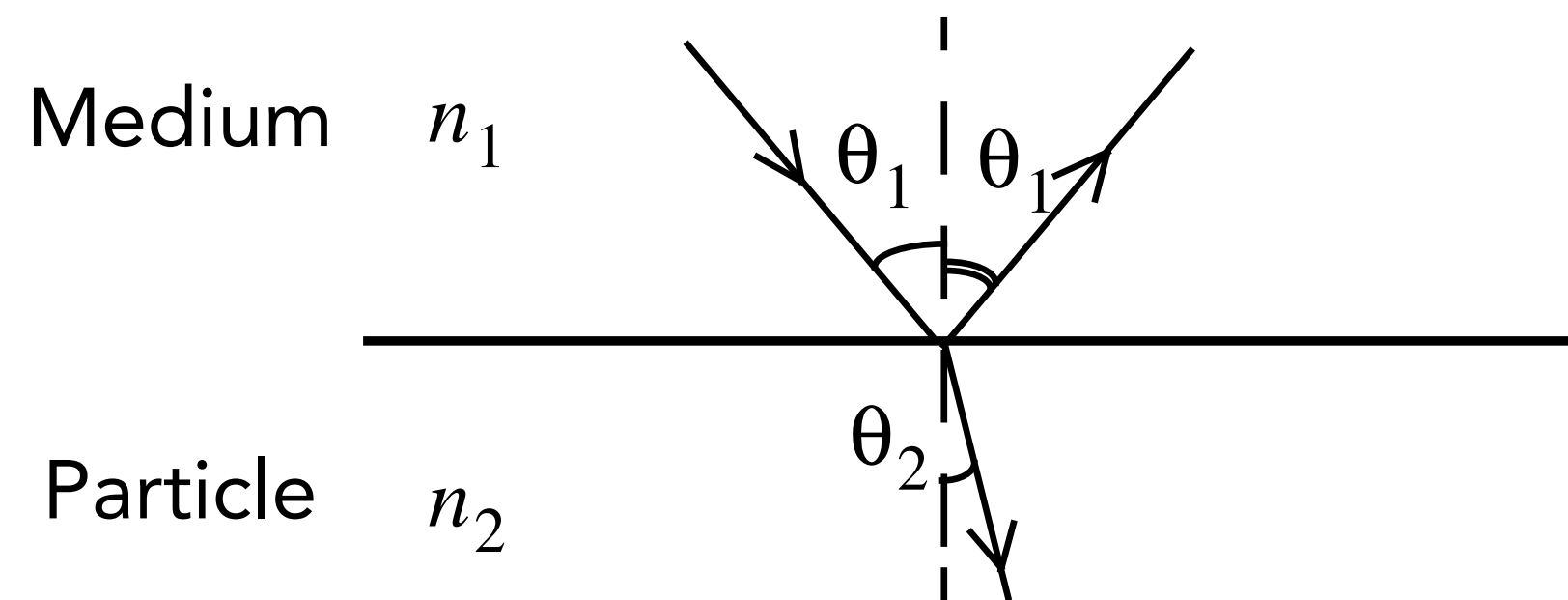
$$m = \frac{n_{particle}}{n_{medium}}$$

$$N_{air} \approx 1$$

$$N_{water} \approx 1.33$$

$$N_{glass} \approx 1.51$$

$$N_{SiO_2} \approx 1.46$$

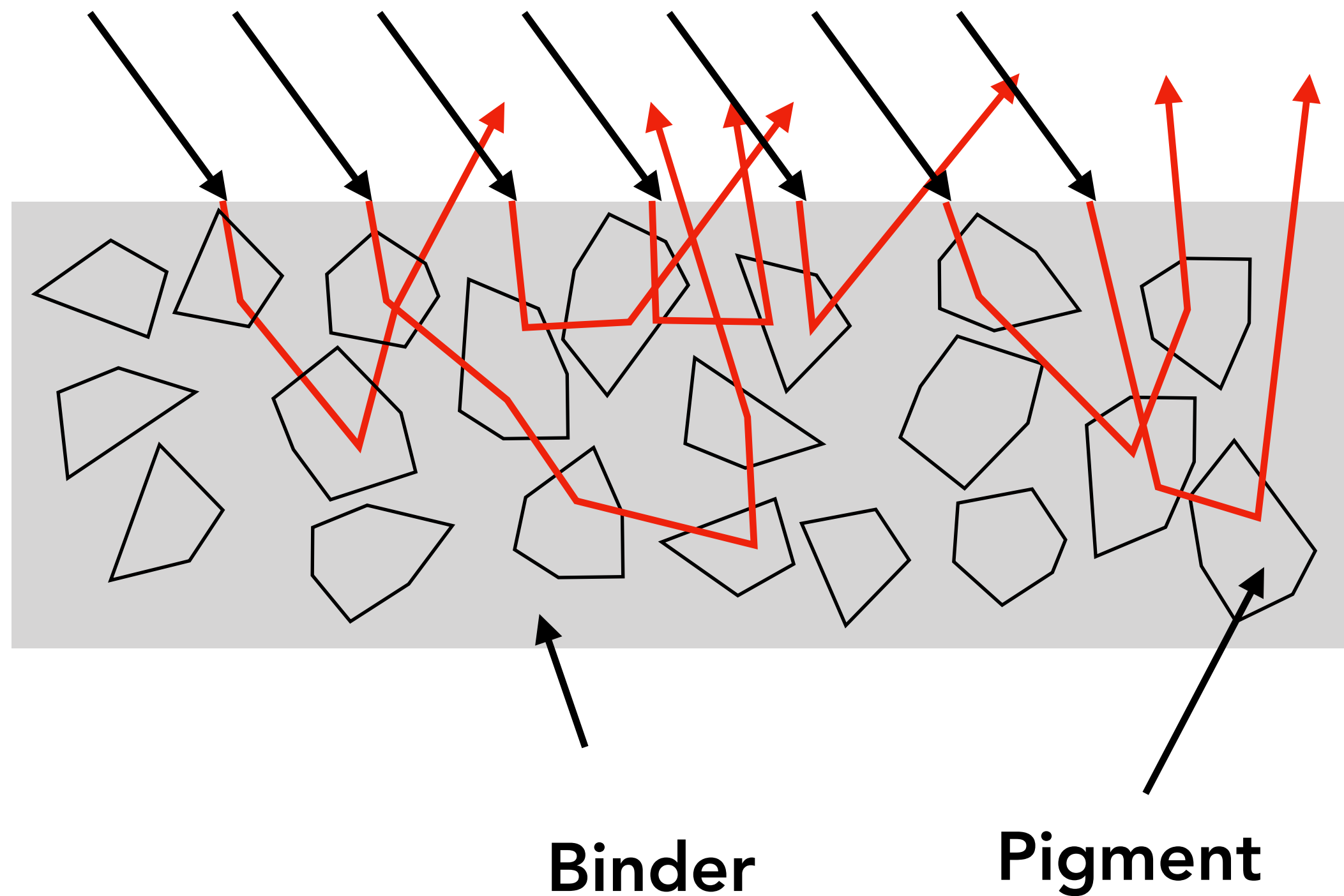


$$\frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2}$$

Snell's law

# **Aside: How Paints are Made**

# Paint $\sim$ = Pigment + Binder

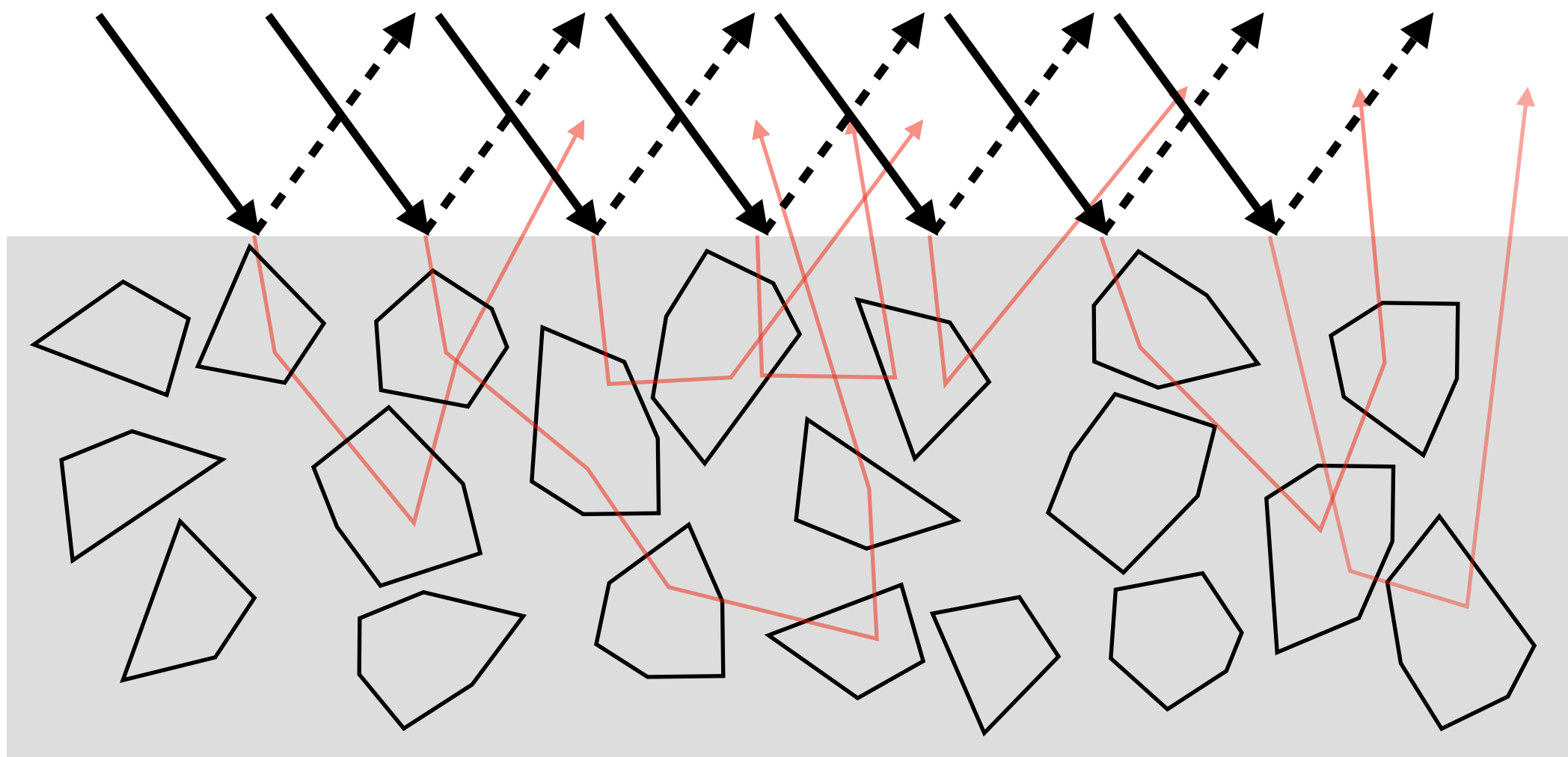


Binder (medium/vehicle) provides the adhesion needed to hold the pigments together.

Usually some additives are added:

- Glaze
- Extender to increase durability
- Glitter for shimmer

# Paint $\sim$ Pigment + Binder



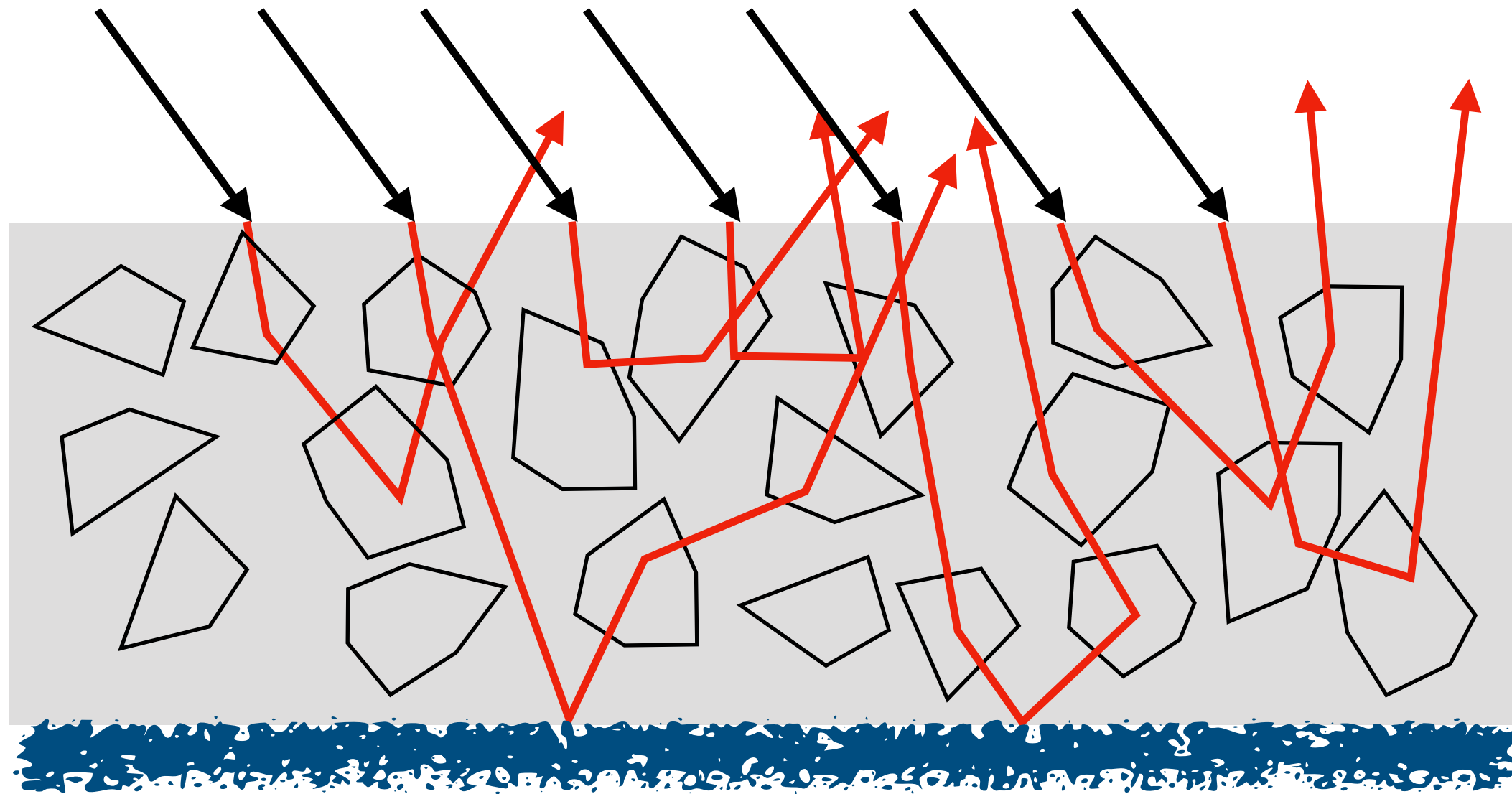
The incident lights are also reflected straight away at the air-medium surface (dashed lines)

- We will ignore this for now and come back to it later

Note that surface reflection is just the macroscopic behavior of photon scattering off of the binder particles (scattering with interference).

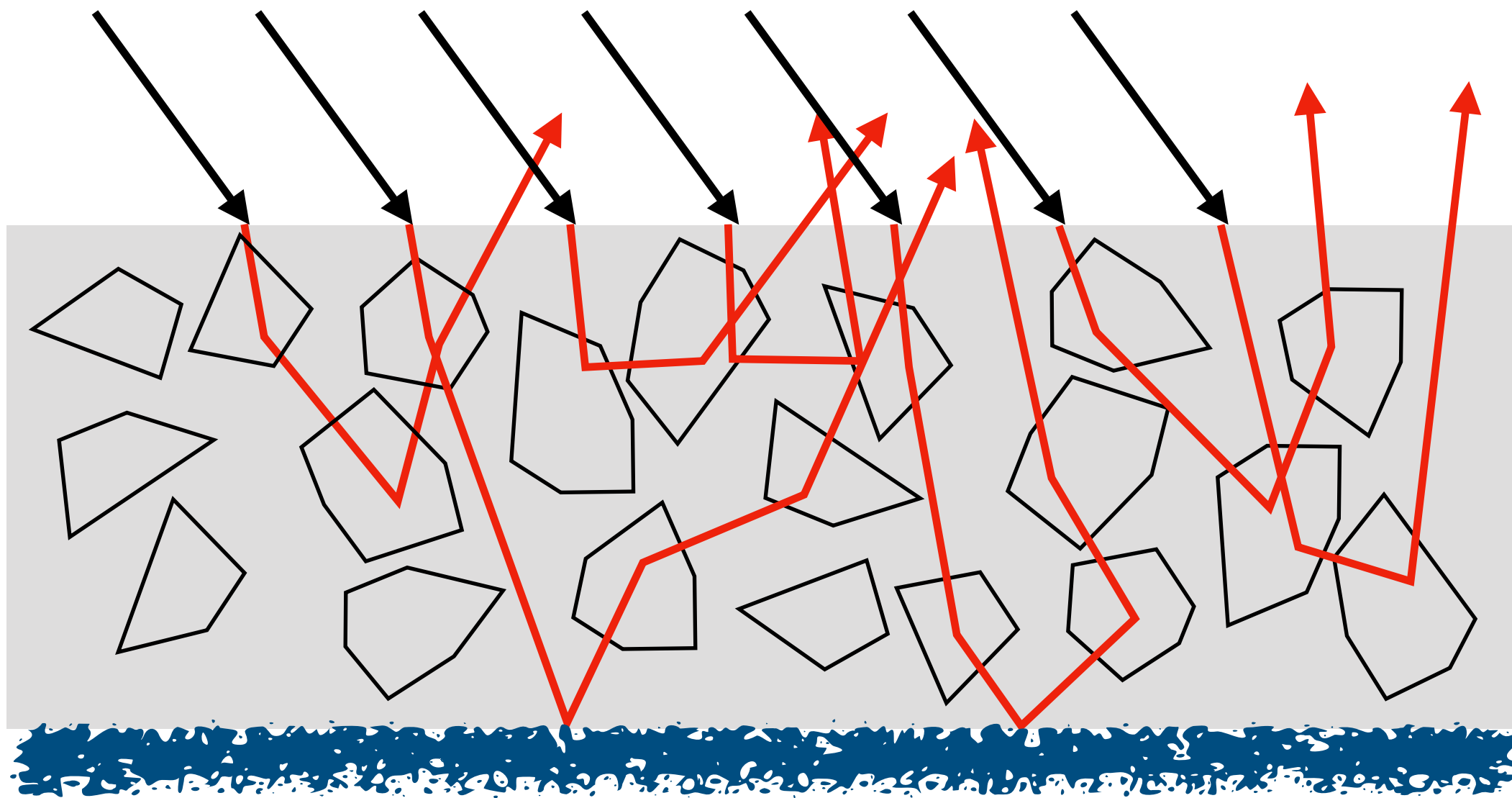


# Paint on Substrate





# Paint on Substrate

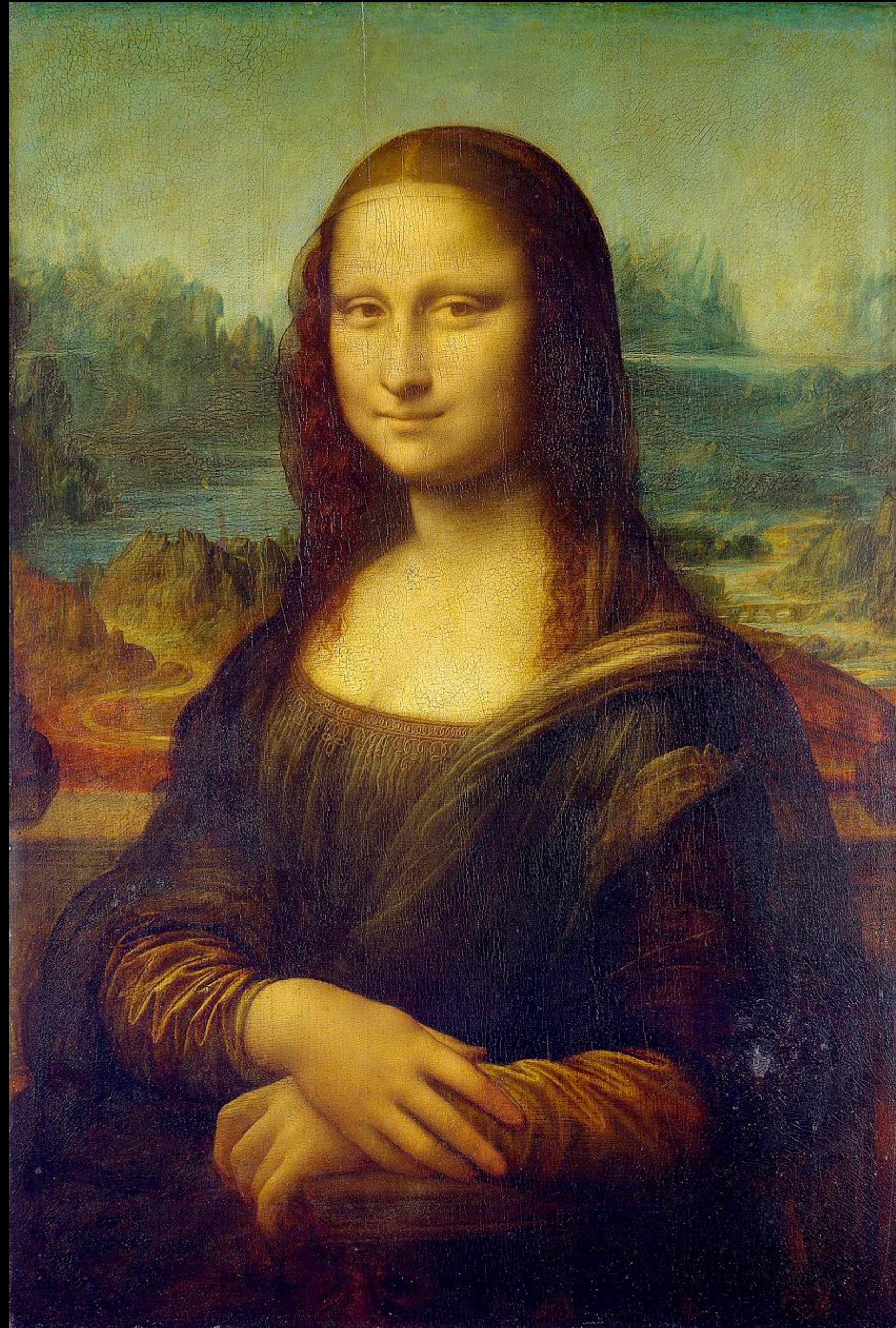


The paint is applied to a substrate, e.g., a canvas or paper. The substrate is nothing more than another layer of scattering/absorbing material.

- We will show how layered material is modeled later.



*Mona Lisa*, Leonardo Da Vinci, 1506. The Louvre.

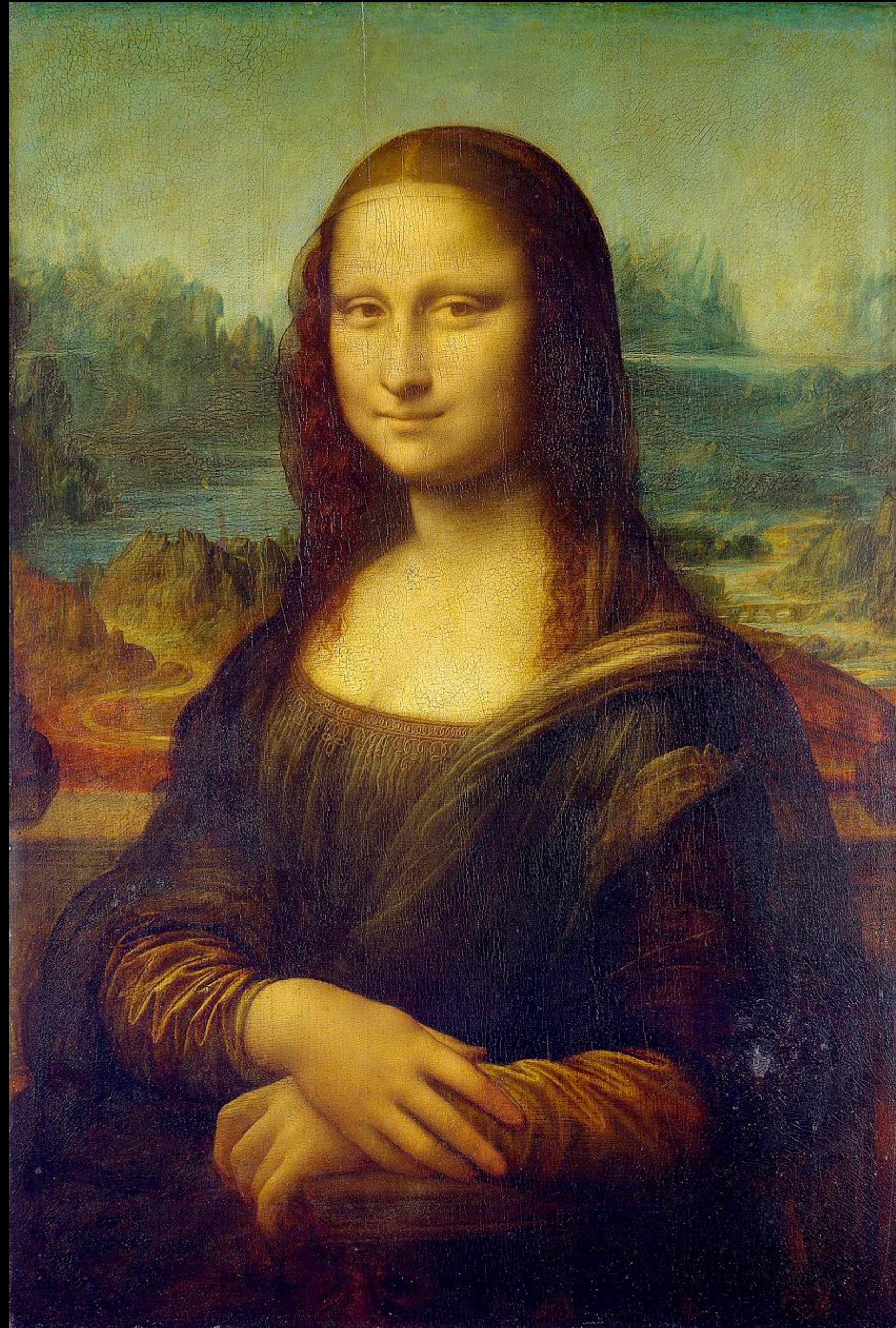


*The Starry Night*, Vincent van Gogh, 1889. MOMA.





*Mona Lisa*, Leonardo Da Vinci, 1506. The Louvre.



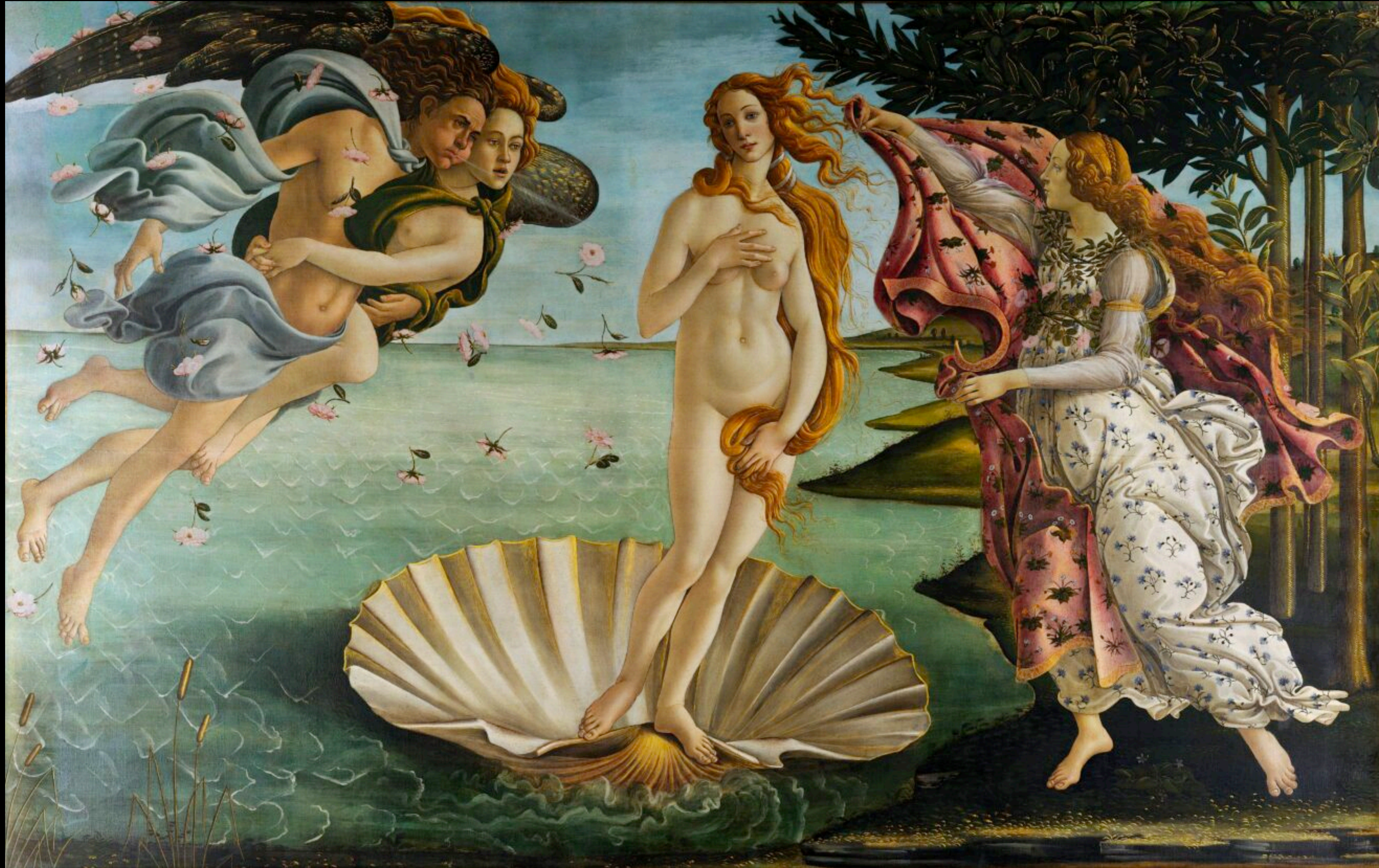
*The Starry Night*



Oil painting.  
Binder is (linseed) oil.

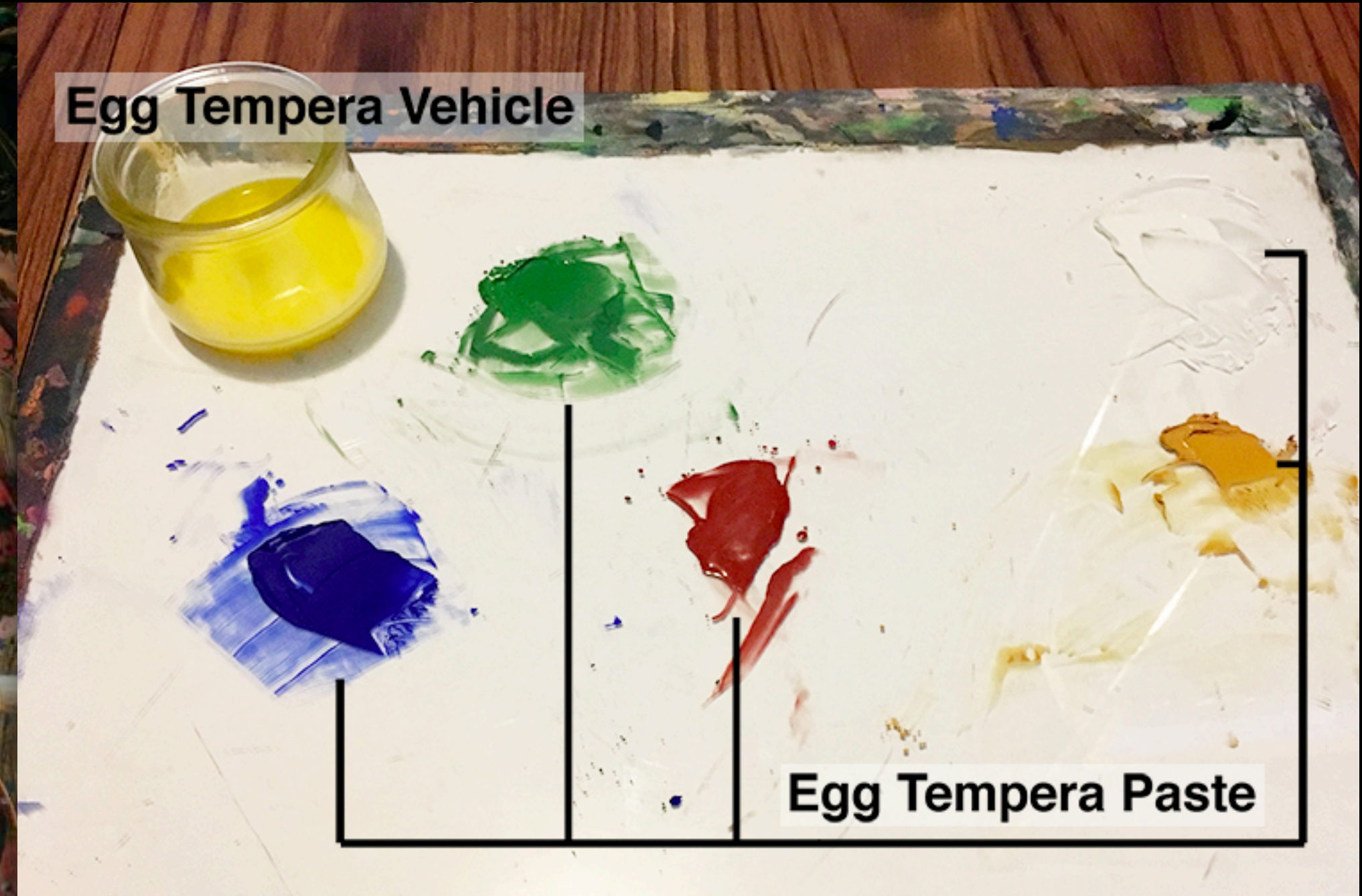
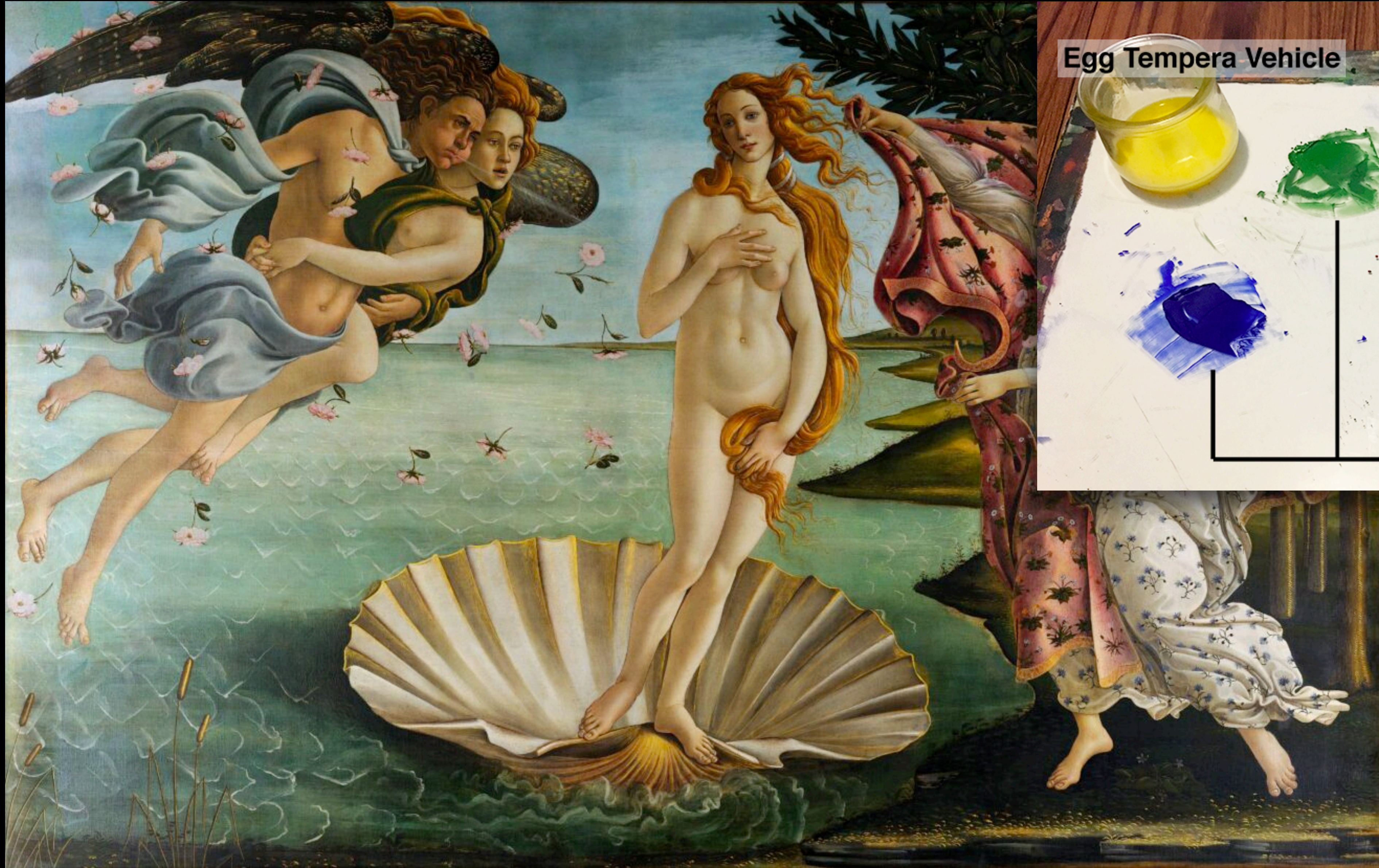


*The Birth of Venus*, Sandro Botticelli, c 1485. Uffizi Galley, Florence.





*The Birth of Venus*, Sandro Botticelli, c 1485. Uffizi Galley, Florence.



Tempera painting.  
Binder is egg yolk.



*Portrait of a Young Woman in Red,*  
A.D. 90–120. The Met.



*Mummy Portrait of a Woman, A.D. 100. Getty.*

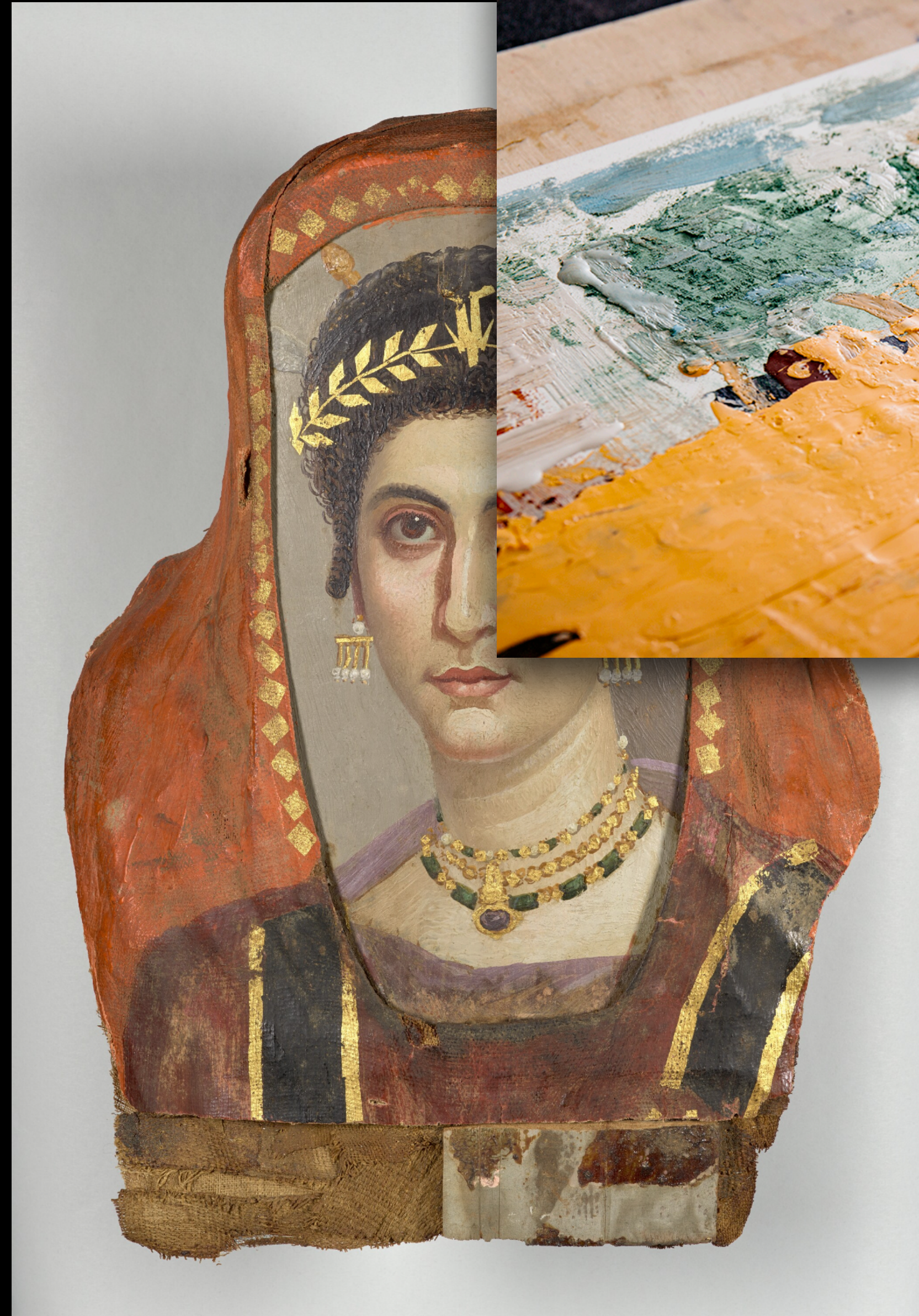




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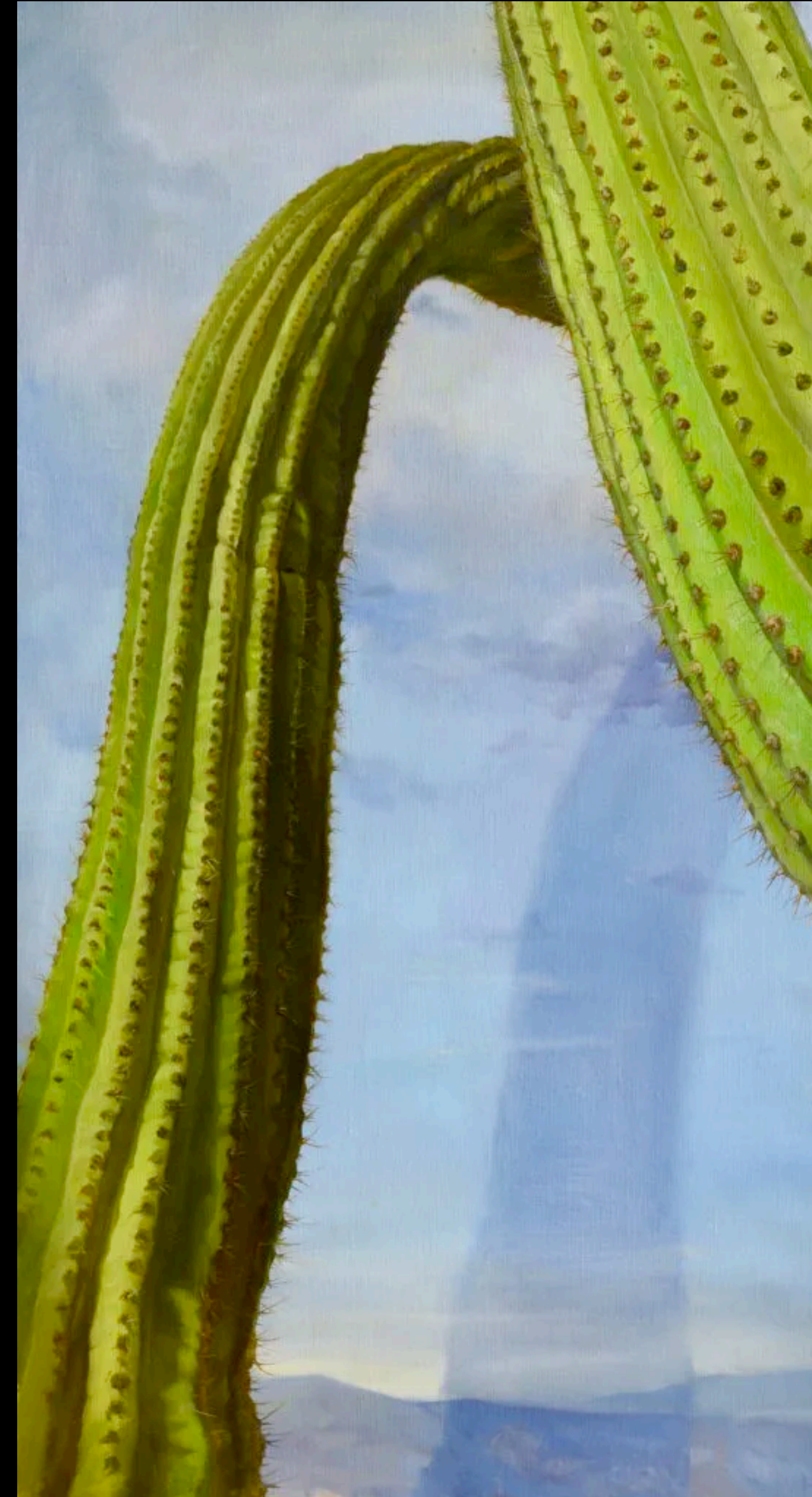
Encaustic painting.  
Binder is (bees)wax.



*Keep Moving Forward, Darien Bogart*



*Artificial Nature, Jingyi Wang*

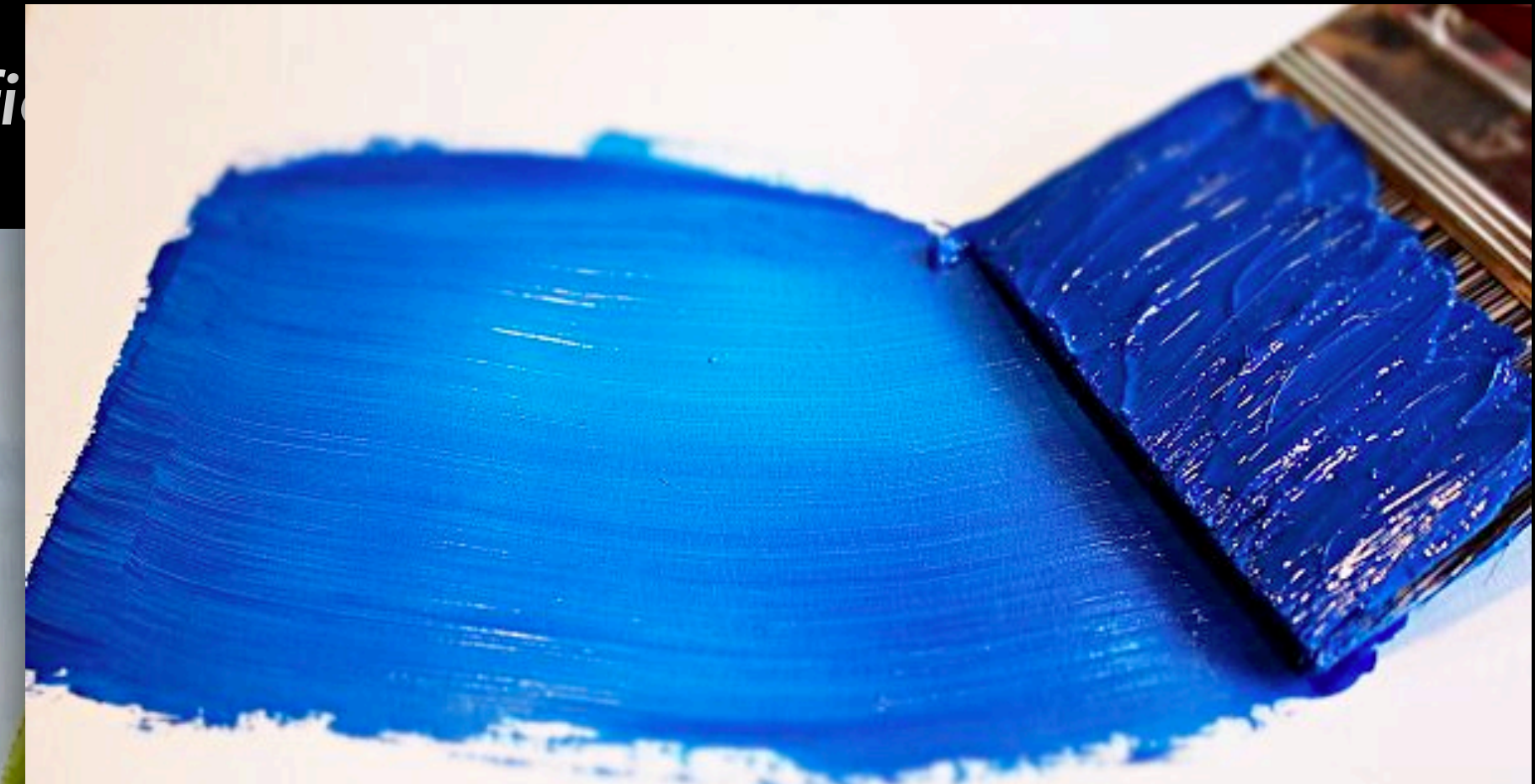




## Keep Moving Forward, Darien Bogart



## Artifi



Acrylic painting.  
Binder is "milk" + water.





## Stephen Quiller



<https://www.outdoorpainter.com/what-is-casein-paint/>



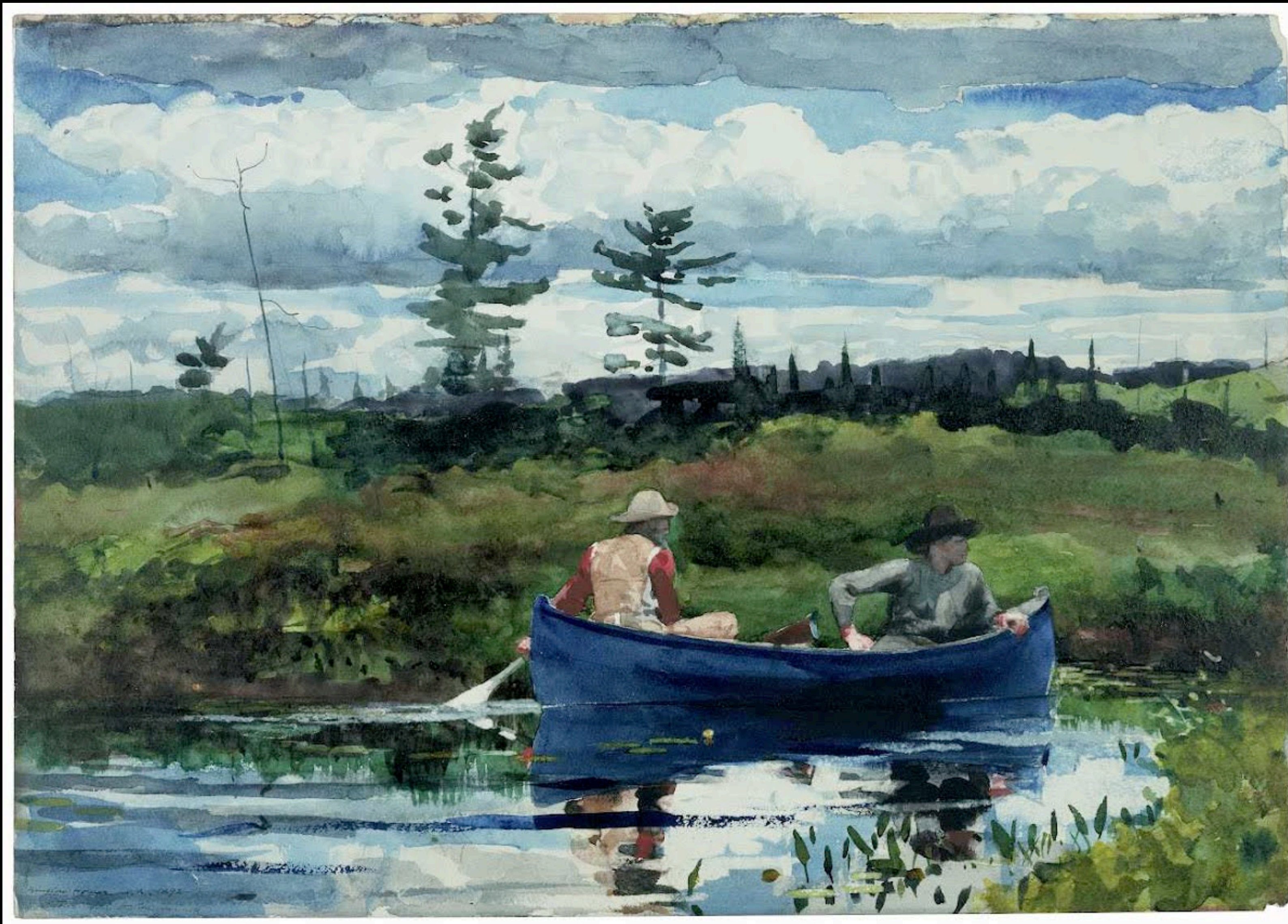
## Stephen Quiller



Casein painting.  
Binder is "milk" + water.

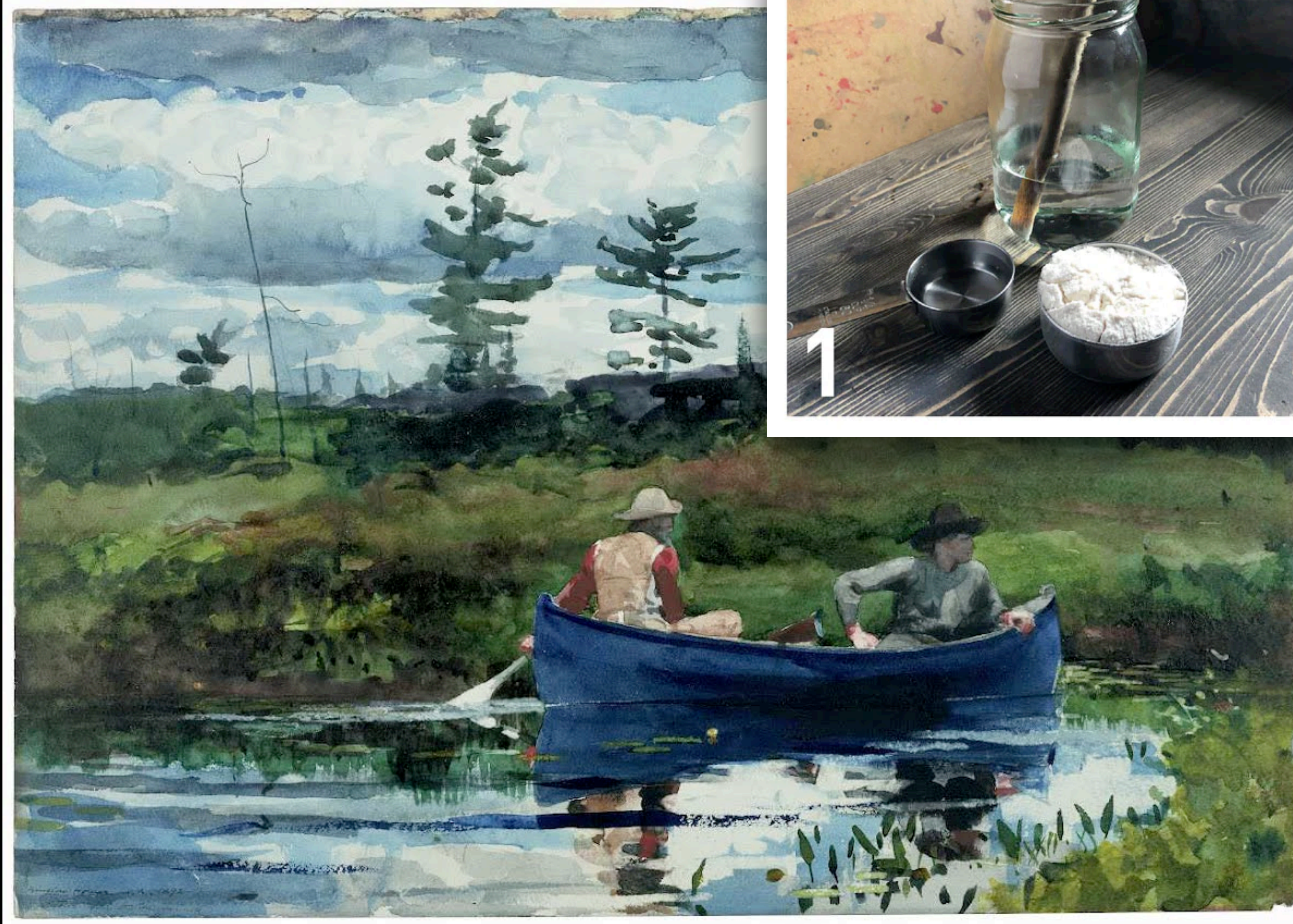


*The Blue Boat*, Winslow Homer, 1892. Museum of Fine Arts, Boston.





*The Blue Boat*, Winslow Homer, 1892. Museum



**"Watercolor" painting.  
Binder is Gum Arabic + water.**



# Pigment Size

average pigment particle size		
<i>microns</i>	<i>meters</i>	<i>representative pigments</i>
1000µm	10 <sup>-3</sup>	<b>= 1 millimeter</b>
500µm		.
100µm	10 <sup>-4</sup>	<i>coarse historical mineral pigments</i> modern luster & iridescent pigments
50µm		<b>smallest particles visible without magnification</b> cobalt violet manganese blue
10µm	10 <sup>-5</sup>	cobalt green cobalt turquoise cerulean blue manganese violet black iron oxides
5µm		ultramarine blue (RS) viridian cobalt blue violet (brown) iron oxides yellow iron oxides

1µm	10 <sup>-6</sup>	ultramarine blue (GS) red iron oxides cadmium red cadmium orange <i>semiopaque synthetic organics</i> diarylides pyrroles naphthols perinone orange
0.5µm		<b>= 500 nanometers</b> <b>= wavelength of "blue green" light</b> chromium oxide green cadmium yellow bismuth yellow titanium white transparent red iron oxides transparent yellow iron oxides <i>semitransparent synthetic organics</i> arylides benzimidazolones dioxazines perylenes
0.1µm	10 <sup>-7</sup>	zinc white iron [prussian] blue <i>transparent synthetic organics</i> quinacridones phthalocyanines
0.05µm		carbon black
0.01µm	10 <sup>-8</sup>	
0.0015µm	10 <sup>-9</sup>	<i>glucose (corn syrup) molecule</i>
0.0003µm	10 <sup>-10</sup>	<i>water molecule</i>



# Disperse Pigments



Pigments are very tiny but usually agglomerate, which is the powders we buy.

We need to separate the pigment particles from the agglomerates and disperse them into the binder.









<https://www.jacksonsart.com/blog/2022/05/02/making-a-paint-grinding-slab/>



## A Dutch Studio in the Sixteenth Century





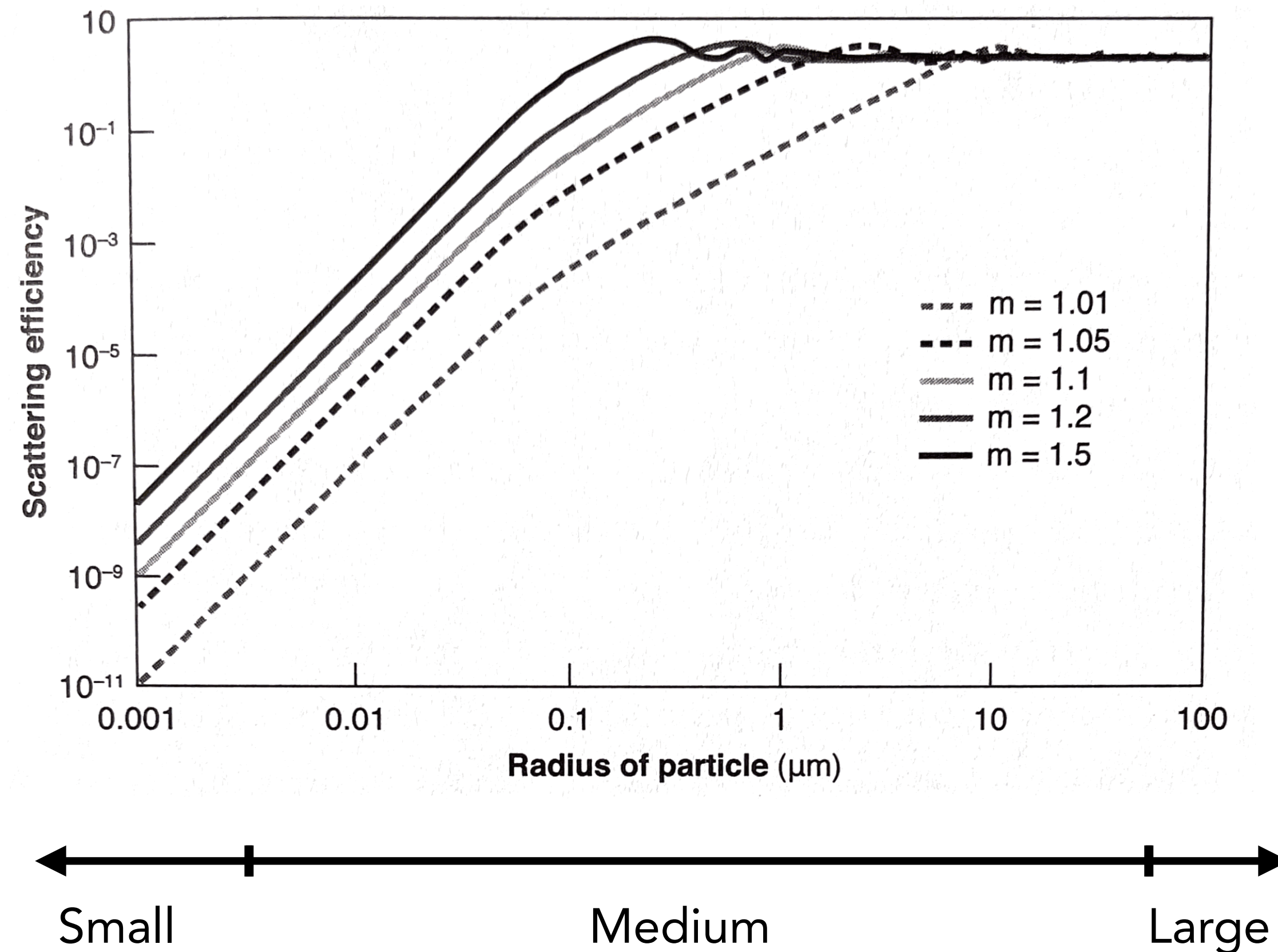
# Back From Aside



# Refractive Index vs. Scattering Efficiency

$$m = \frac{n_{particle}}{n_{medium}}$$

wavelength = 500 nm



## Observations:

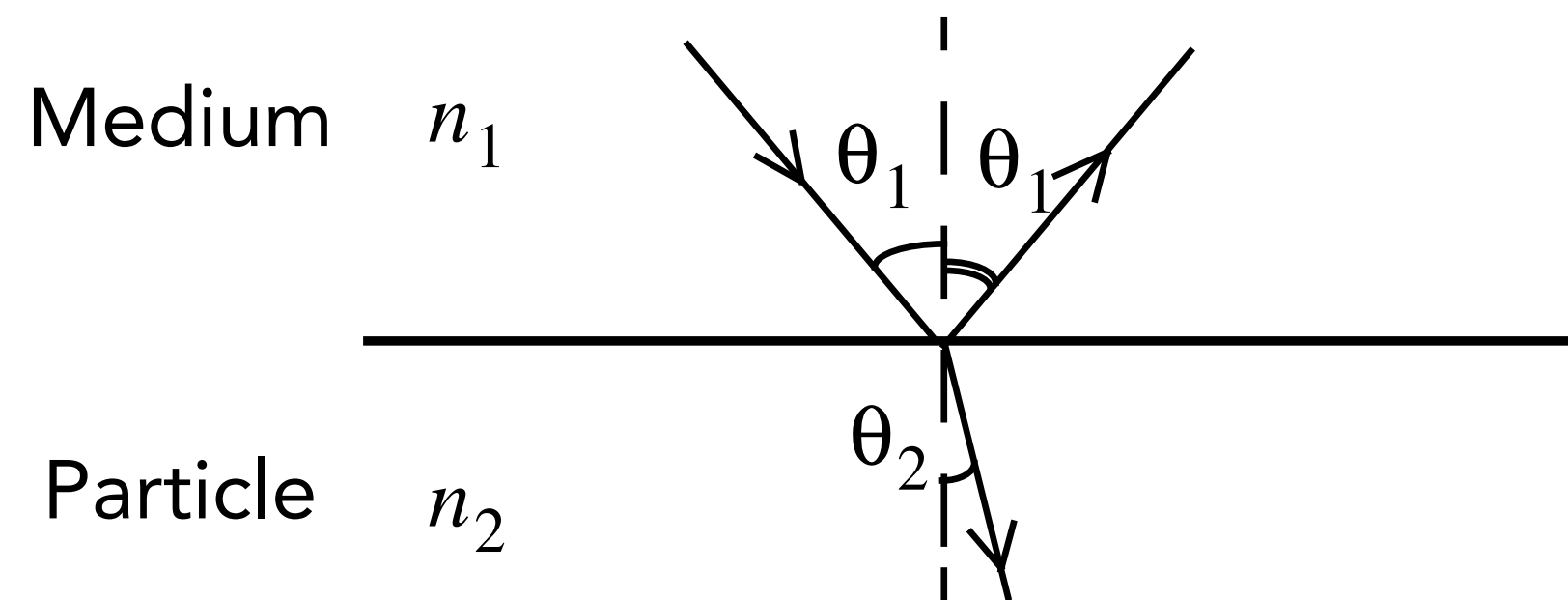
- Large relative refractive index ( $m$ ) increases scattering efficiency
- When  $m$  is close to 1, very little scattering until closer to the geometric (large) regime
- The scattering efficiency in the geometric regime is independent of refractive index  $m$ ,



# Refractive Index vs. Scattering Direction

As  $m$  approaches 1 (i.e., the refractive indices of the medium and particle become more similar), forward scattering is more significant.

- Extreme case is when the medium and the particle are the same material, then scattering is completely forward ( $\theta_1 = \theta_2$ )!

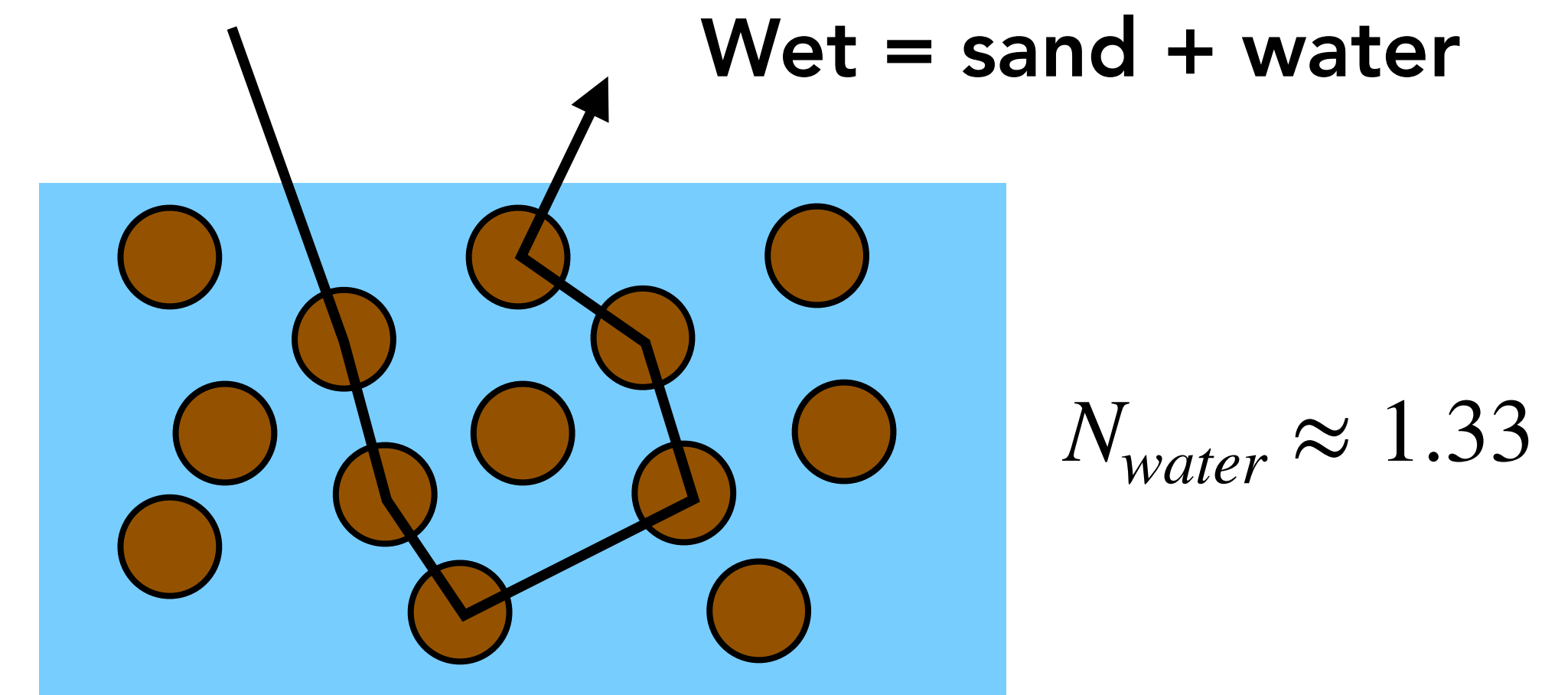
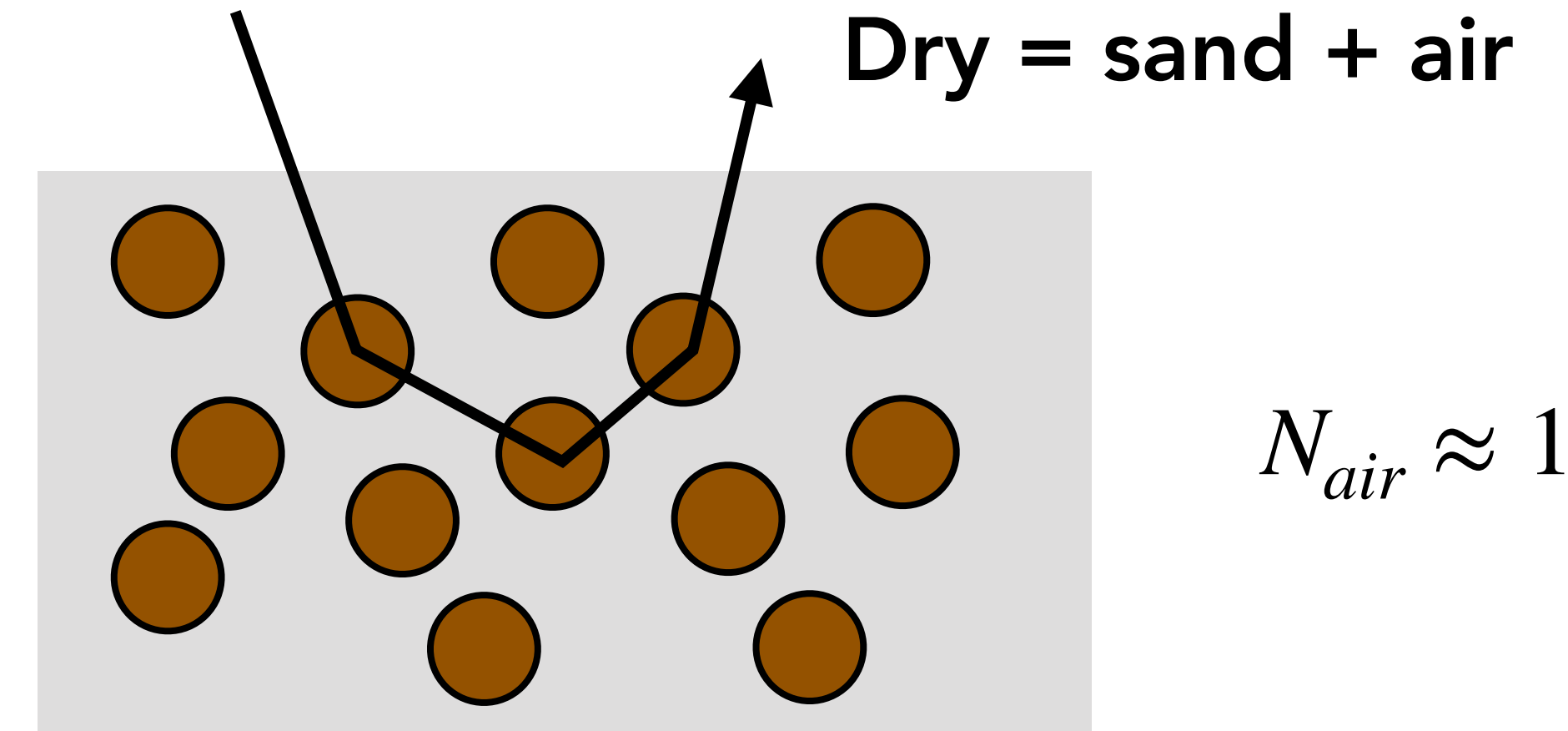


$$\frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2}$$



# Why Are Wet Things Darker?

$$N_{SiO_2} \approx 1.46$$

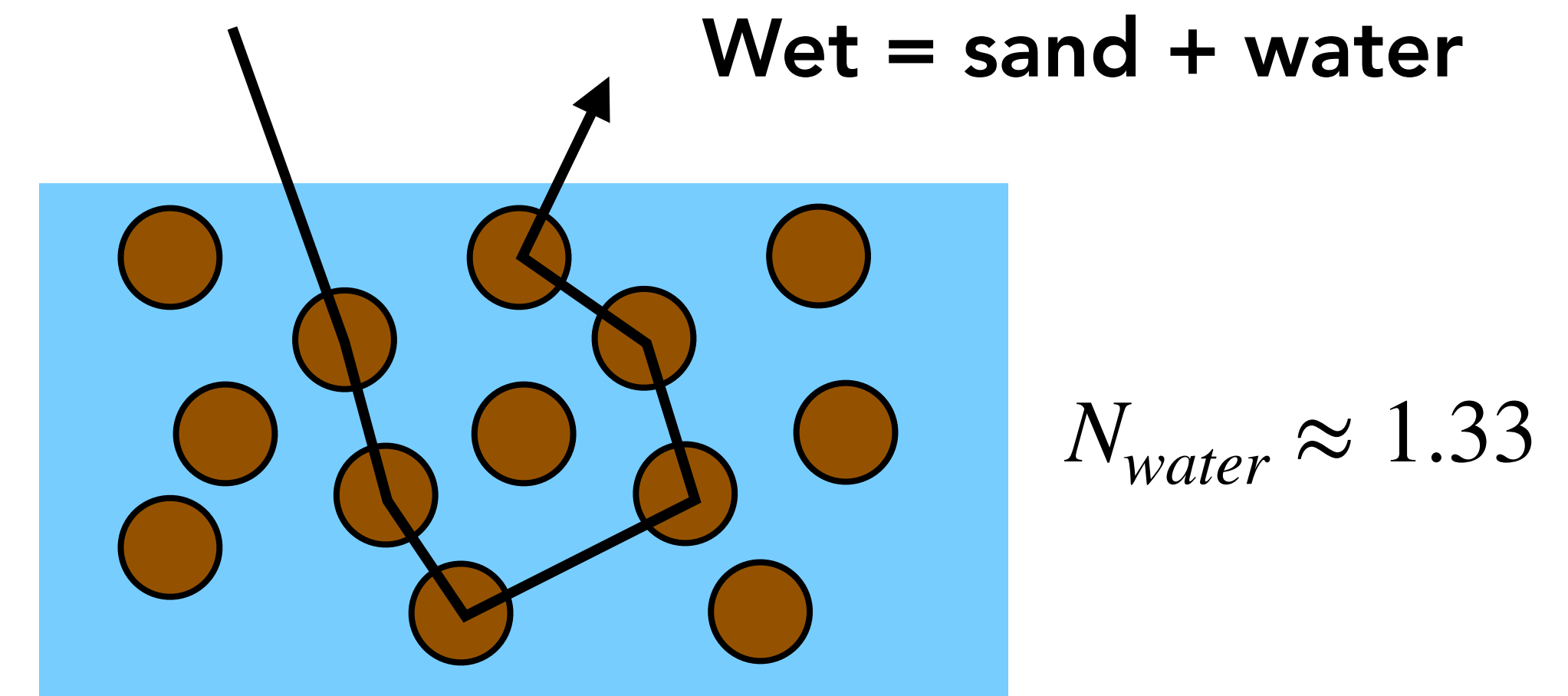
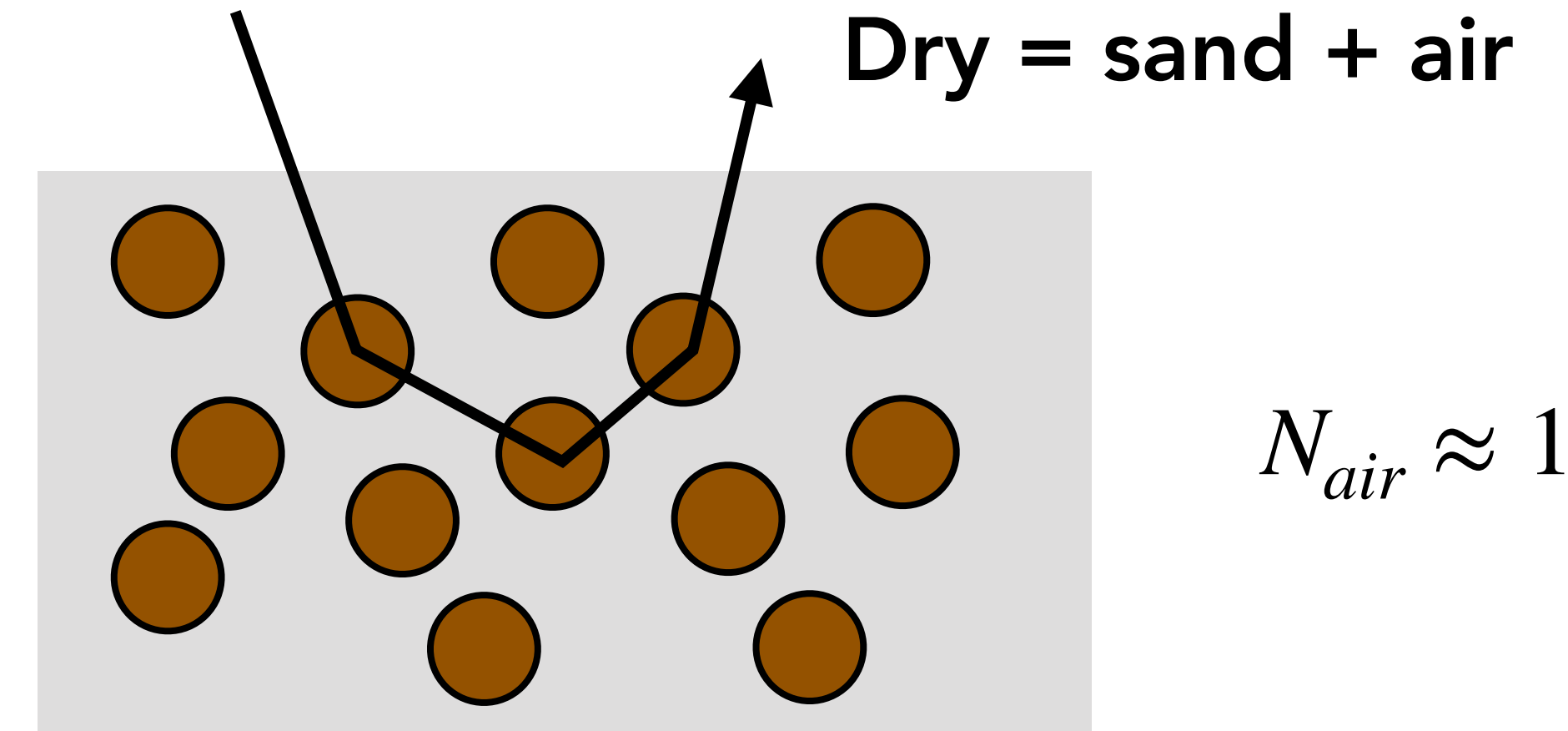




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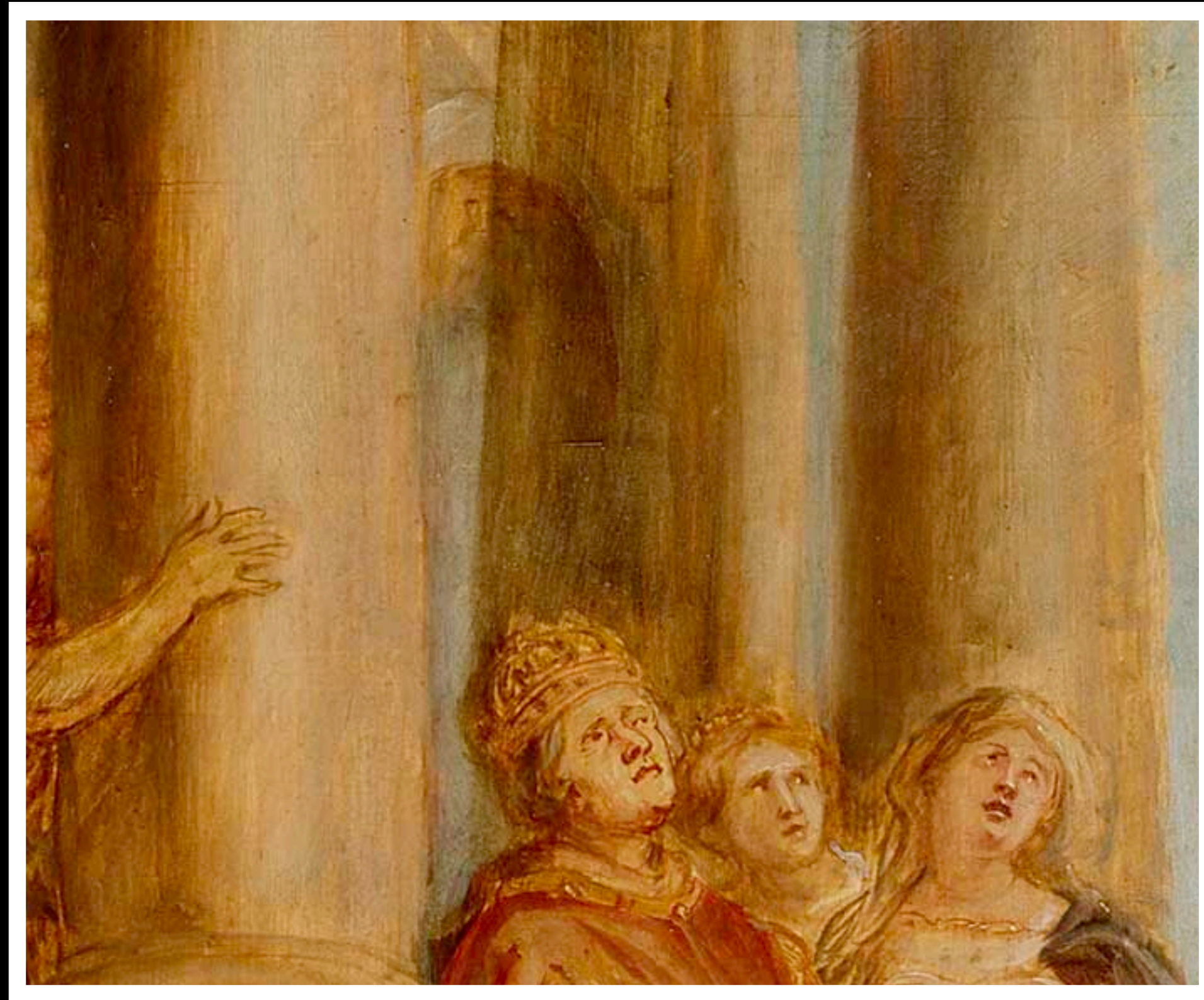
The RI of sand is much closer to water than to air. So light scattered forward more and travels longer distance and has more opportunities to be absorbed.

Note that scattering by sand is in the geometric regime (typical sand size is  $[62.5 \mu\text{m}, 2 \text{ mm}]$ ), so scattering efficiency is high regardless of RI. The question is where photons scatter.

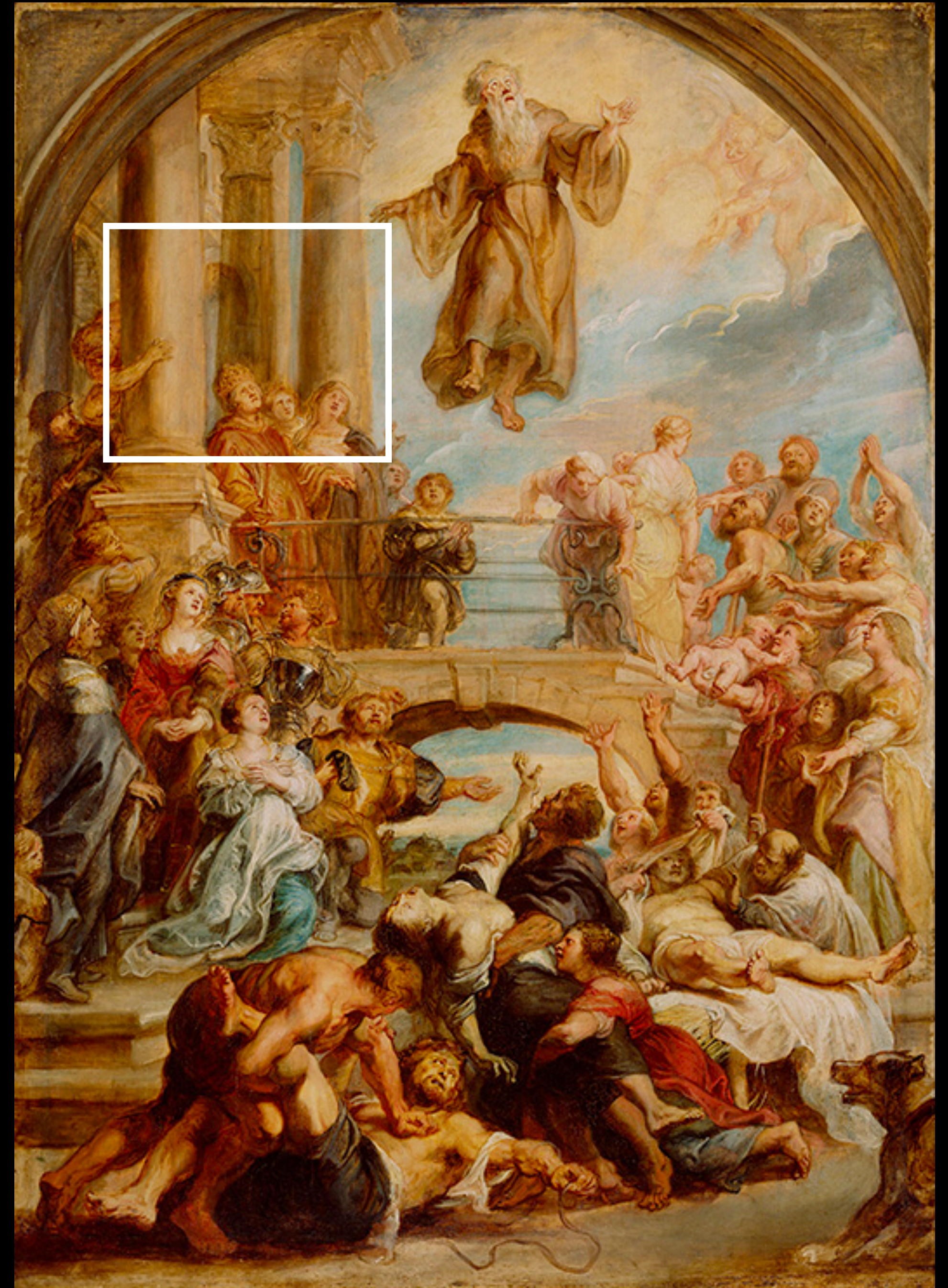




*The Miracles of Saint Francis of Paola, P. P. Rubens, 1627–8. J. Paul Getty Museum.*



The refractive index of the linseed oil in the paint increases over time and matches that of the pigment. The paint layer becomes transparent, revealing the pentimento underneath.





# Aside: Pentimento

The presence/emergence of earlier paints that have been painted over.

- Painter changing mind (pentimento is from “pentirsi” in Italian, which means repentance)
- Underdrawing

Can be identified using imaging techniques (e.g., X-ray), or from paint composition changes (previous slide).



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*The Old Guitarist*, Pablo Picasso,  
1903-1904. The Art Institute of Chicago.





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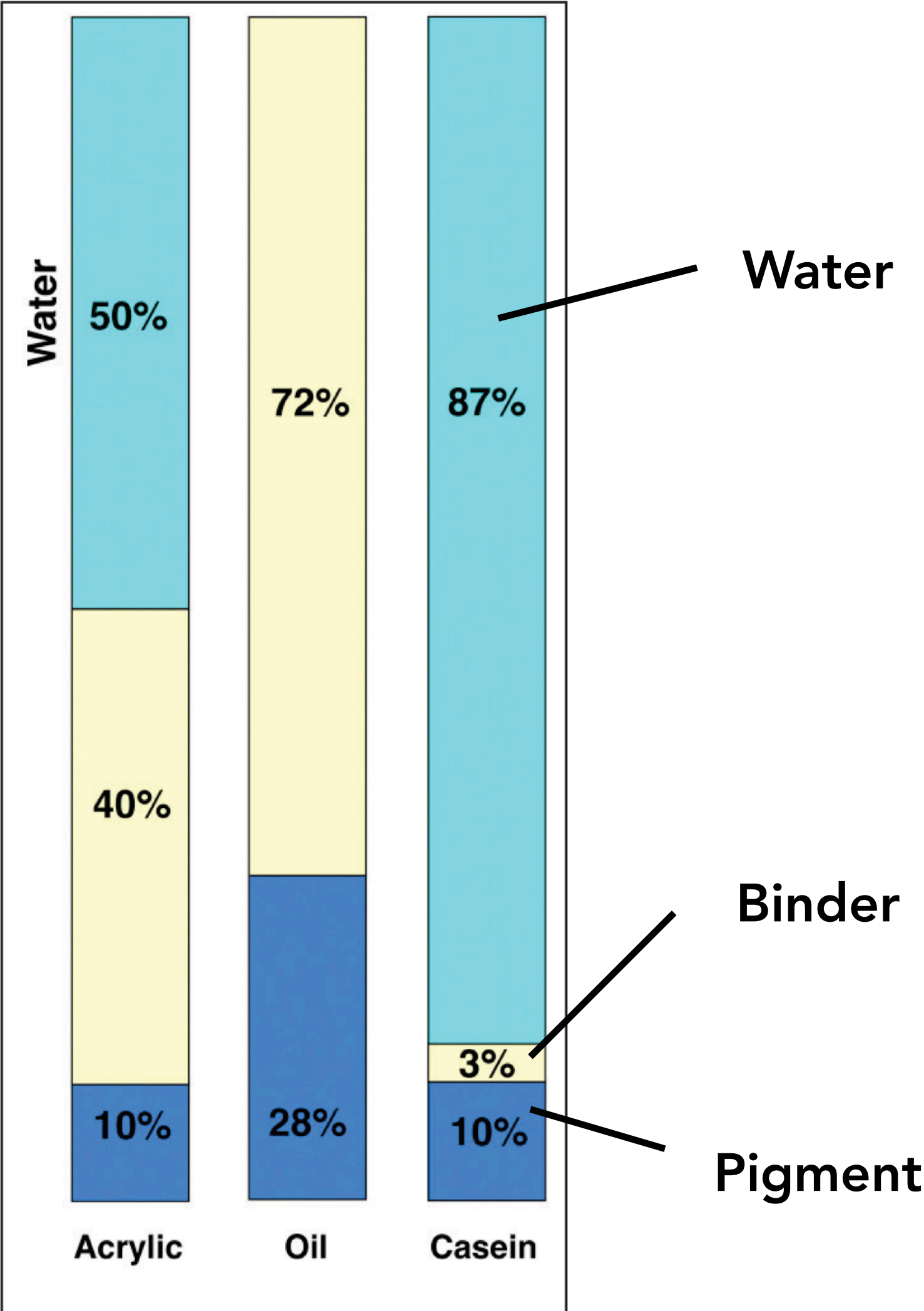
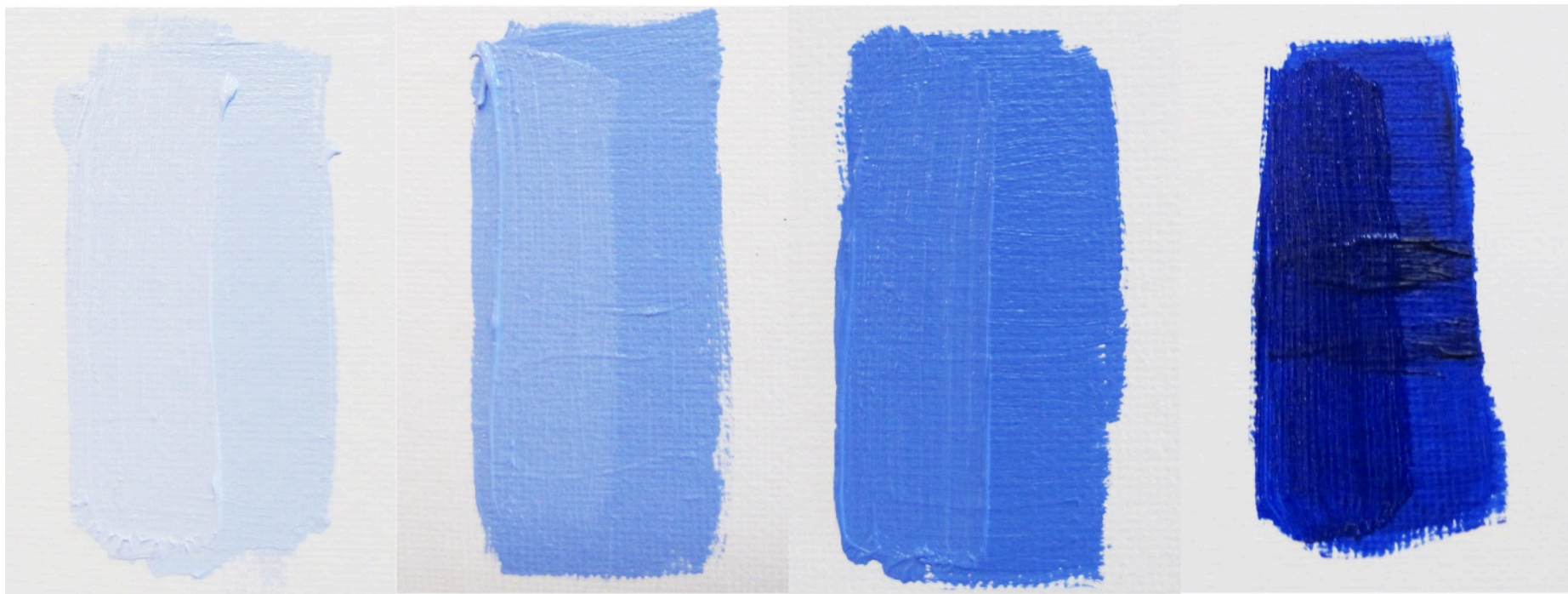
# Acrylic Paints Dry Dark

Wet  Dry

Naphthol Red



Ultramarine Blue





# Acrylic Paints Dry Dark

Wet  $\longrightarrow$  Dry

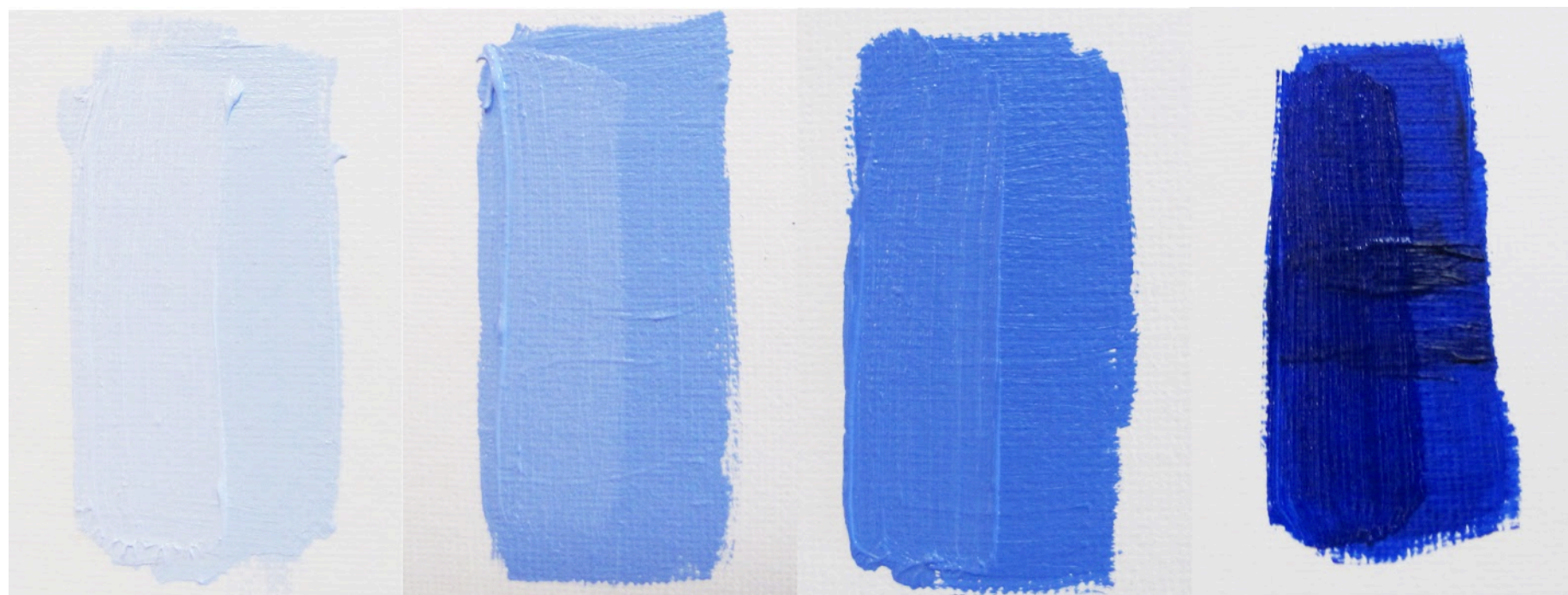
Naphthol Red

$$N_{\text{naphthol reds}} \approx 1.60$$



Ultramarine Blue

$$N_{\text{ultramarine-blue}} \approx 1.50$$



$$N_{\text{water}} \approx 1.33 \quad N_{\text{acrylic binder}} \approx 1.49$$

**Reason:** when water evaporates, the refractive indices of pigments and binder are closer, which encourages more forward scattering, which increases total absorption.



# General Scenarios

**(Absorption + Multiple Scattering)**



# Multiple Scattering

So far we assume that a photon is either absorbed or scattered by at most one particle before entering our eyes.

- Photon scattered by a particle will find its way out without encountering another scattering particle.



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- A photon scattered might be later re-scattered.



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- A photon scattered might be later re-scattered.

Multiple scattering + absorption **is** the common case in nature, arts, and tech.

- We've seen examples before (e.g., wet sand is darker and acrylic paints dry darker)



# Goal and Assumptions

Our goal is to estimate the (spectral) **reflectance** of a material (colorant) in the presence of multiple scattering and absorption.



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General case amounts of solving the so-called **Radiative Transfer Equations**, for which analytical solutions are hard to obtained. Again, we make simplifications to allow modeling special, but common cases.



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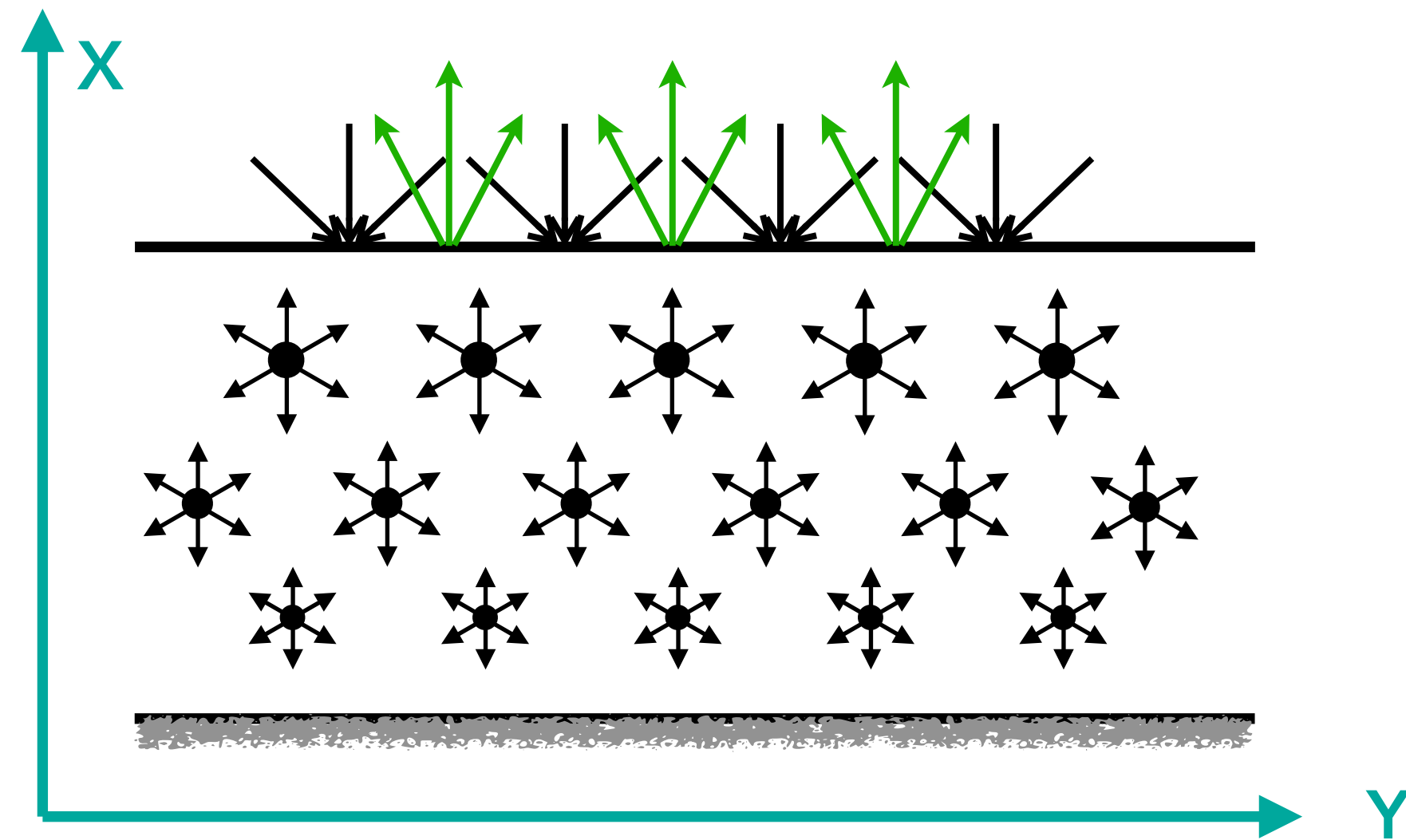
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The assumptions we make are:

- Particles have the same scattering and absorption coefficients (homogeneity)
- Particles are randomly oriented (special case: they are all spheres)
- Incident light is uniform at all directions at all surface points (diffuse illumination)



# Perfect Diffuse Model

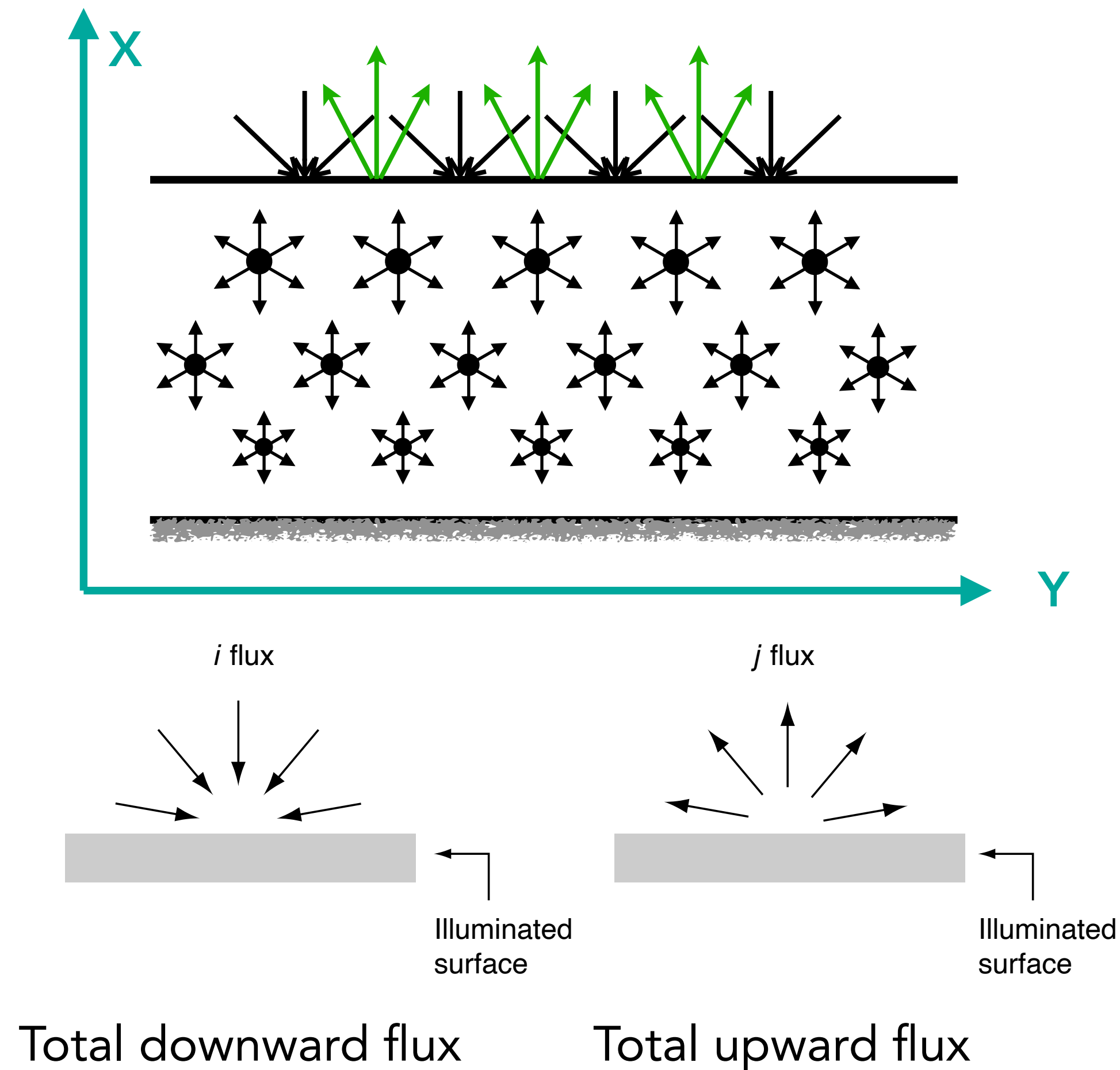


Given the assumptions, there is no difference between different horizontal points ( $Y$ ) at the same depth ( $X$ ): the downward (and upward) flux for all  $Y$ s at the same  $X$  is the same.



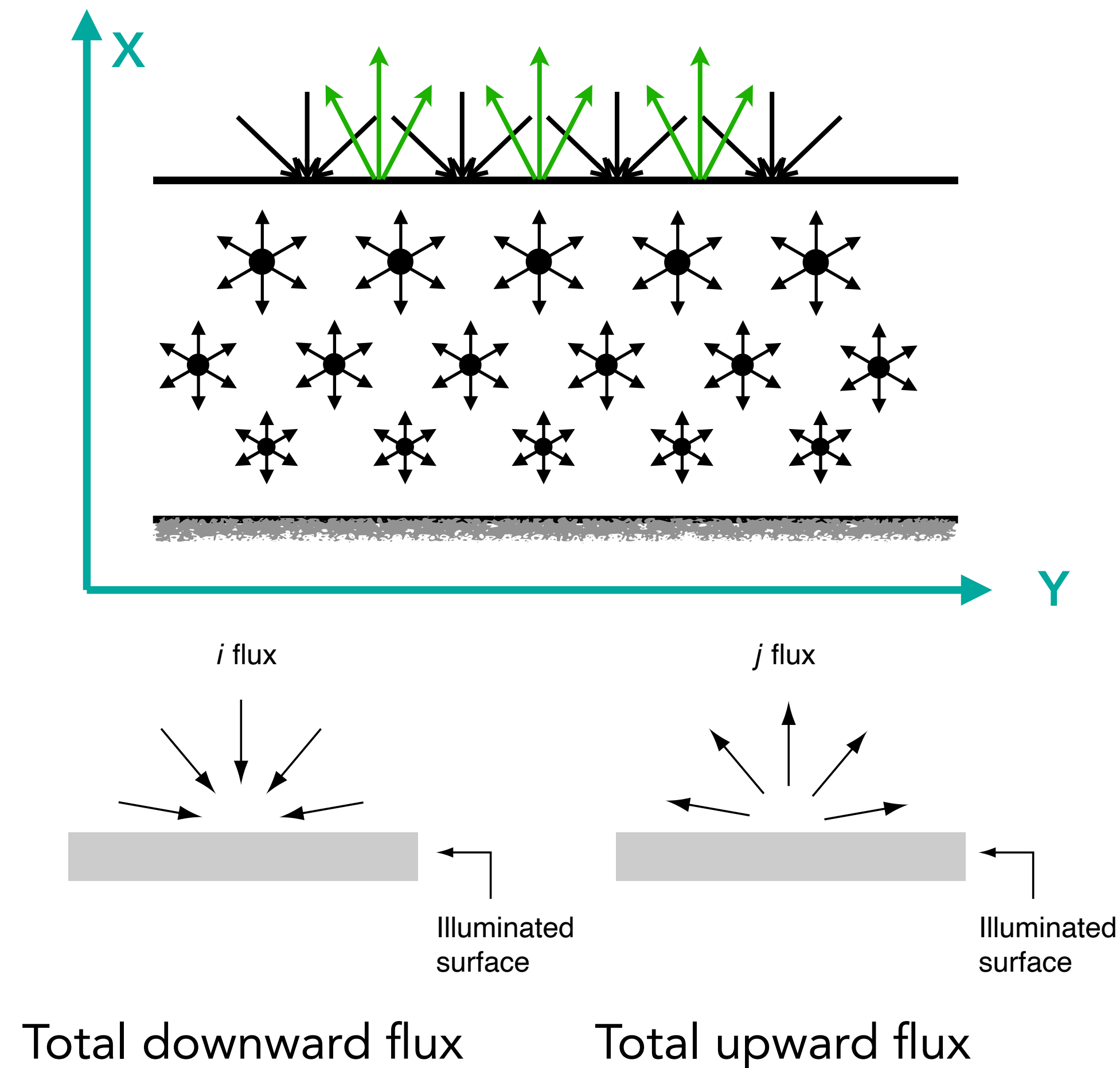
# Perfect Diffuse Model

Given the assumptions, there is no difference between different horizontal points (**Y**) at the same depth (**X**): the downward (and upward) flux for all **Y**s at the same **X** is the same.





# Perfect Diffuse Model



Given the assumptions, there is no difference between different horizontal points ( $Y$ ) at the same depth ( $X$ ): the downward (and upward) flux for all  $Y$ s at the same  $X$  is the same.

So the reflectance of any point is equivalent to the ratio of the total downward flux and the total upward flux at the depth where the point is.

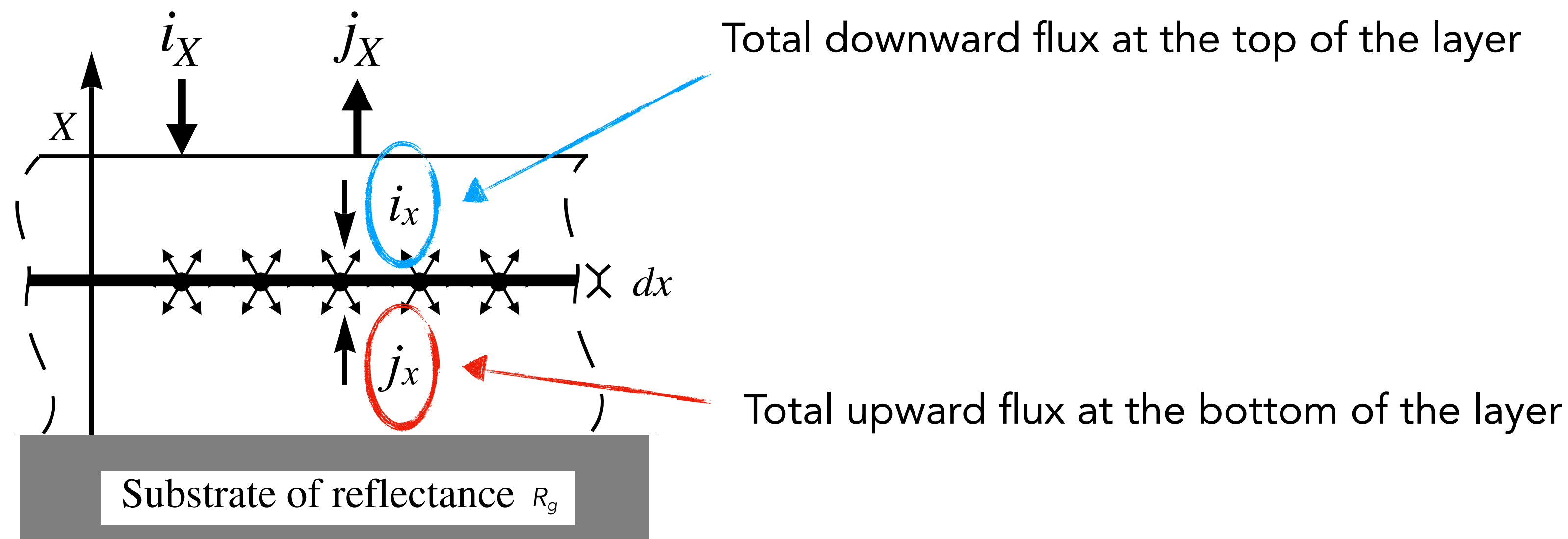


# Kubelka-Munk Equation

$K$ : absorption coefficient

$S$ : back-scattering coefficient

$X$ : thickness



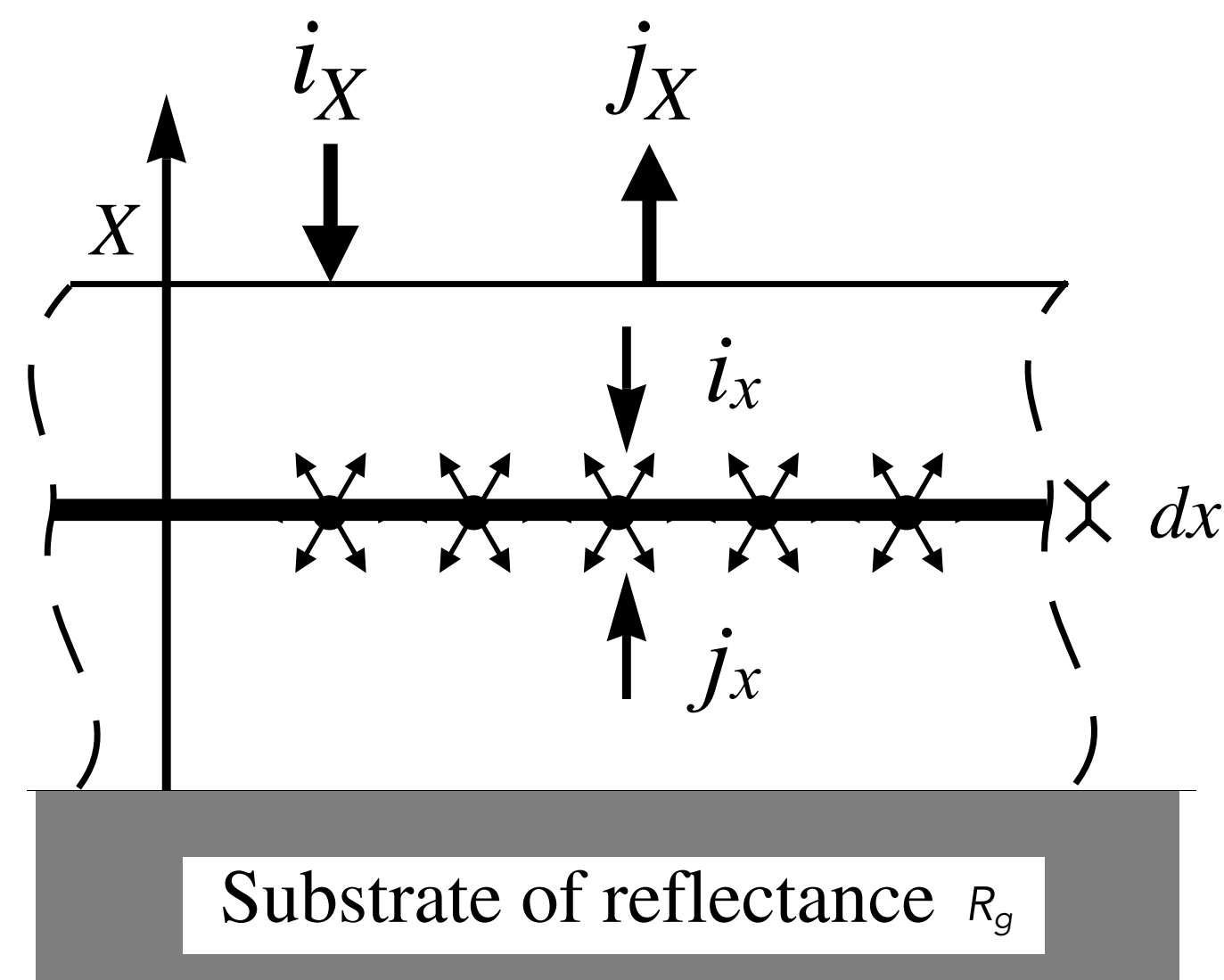


# Kubelka-Munk Equation

**K**: absorption coefficient

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**X**: thickness



$$di_x = -Ki_x dx - Si_x dx + Sj_x dx$$

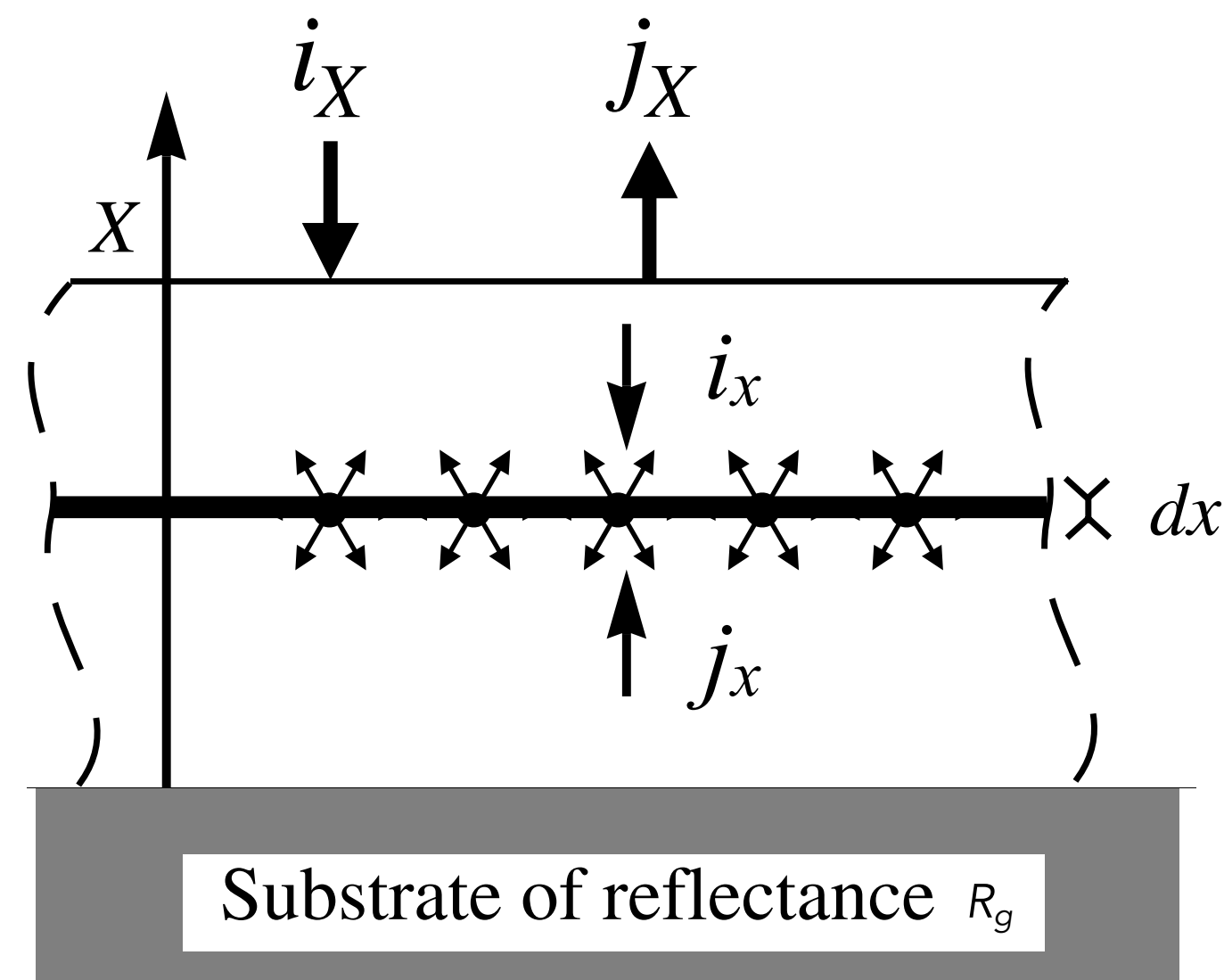


# Kubelka-Munk Equation

K: absorption coefficient  
 S: **back-scattering** coefficient  
 X: thickness

Total downward  
 flux gained

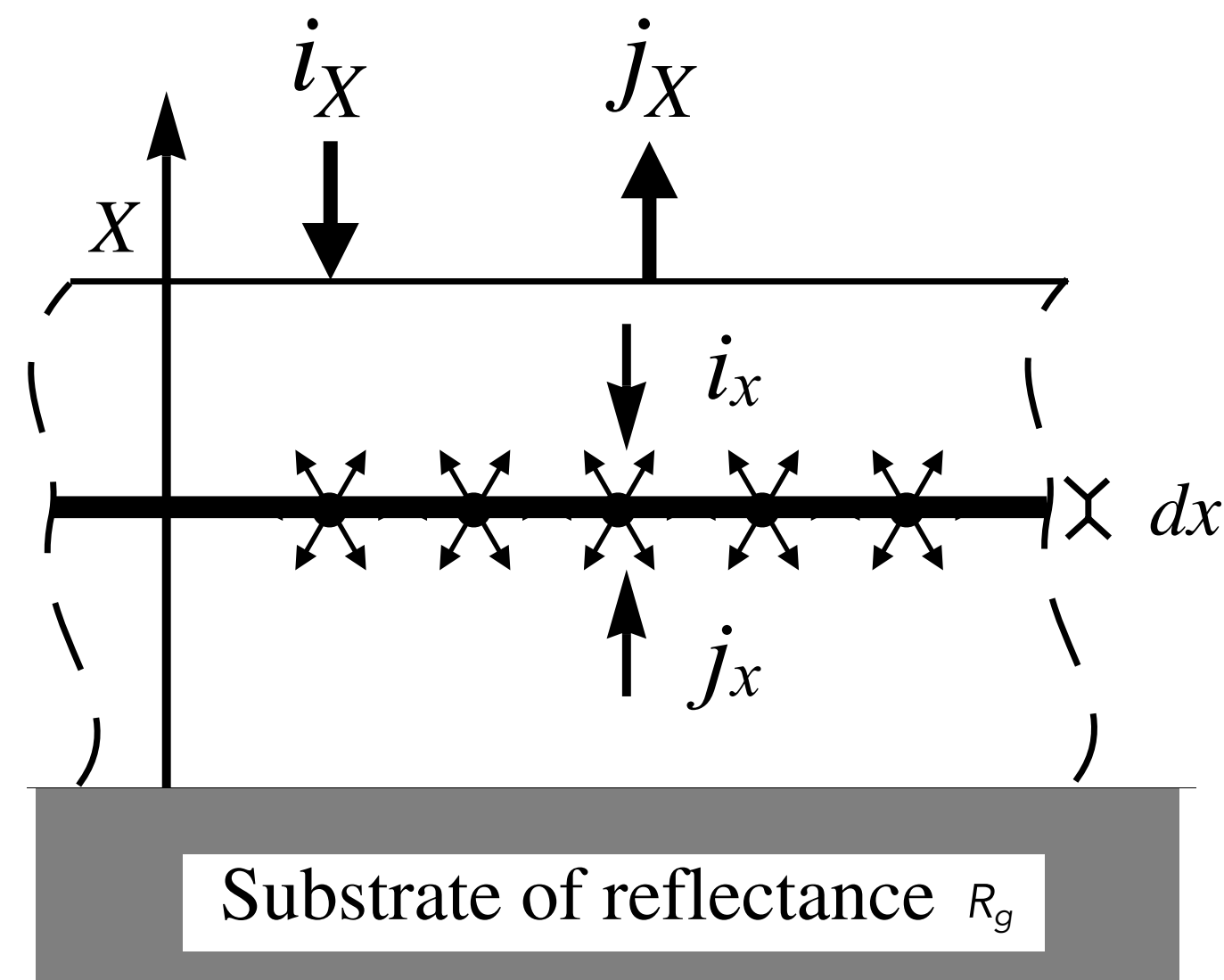
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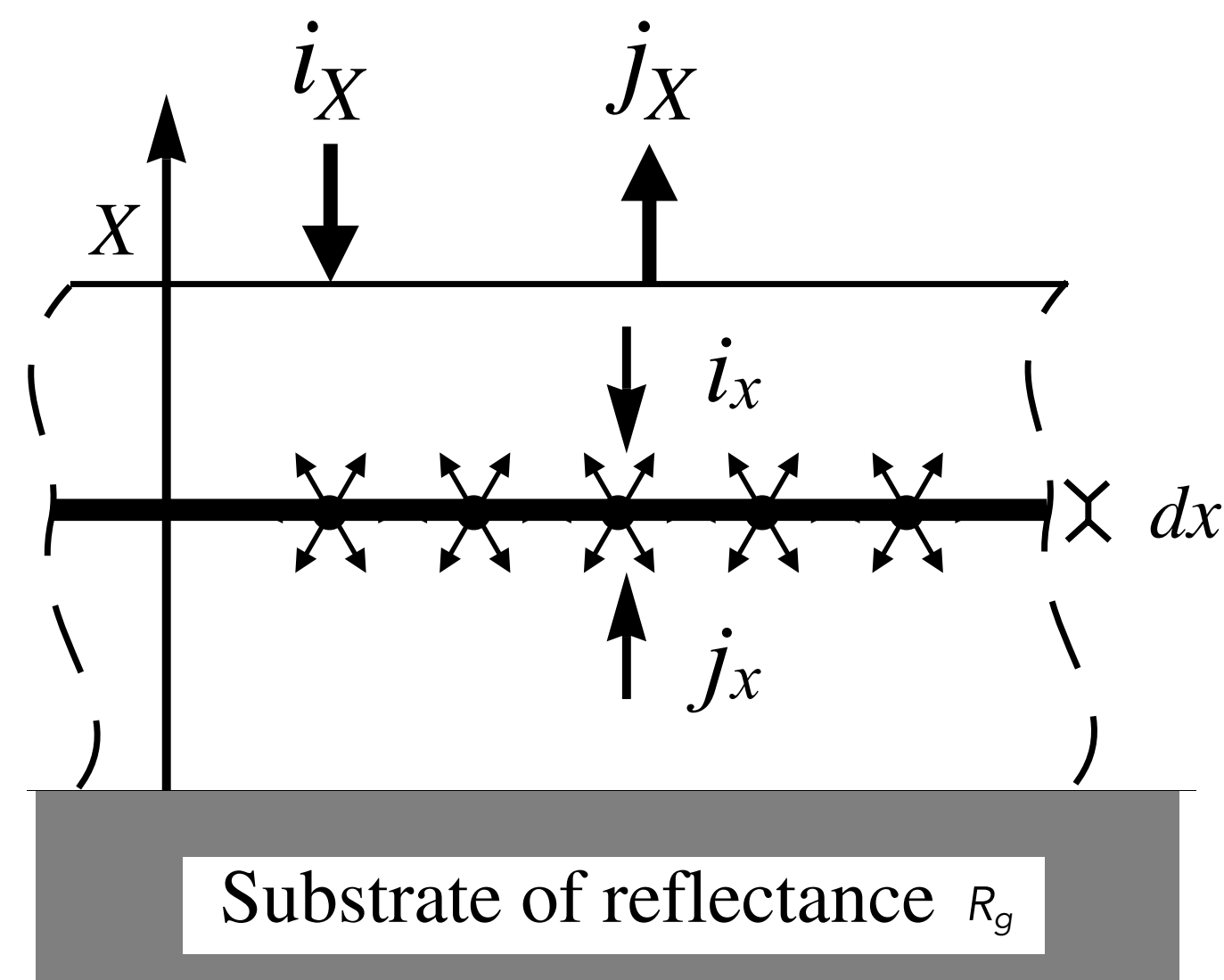
Flux absorbed

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Total downward  
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Flux absorbed

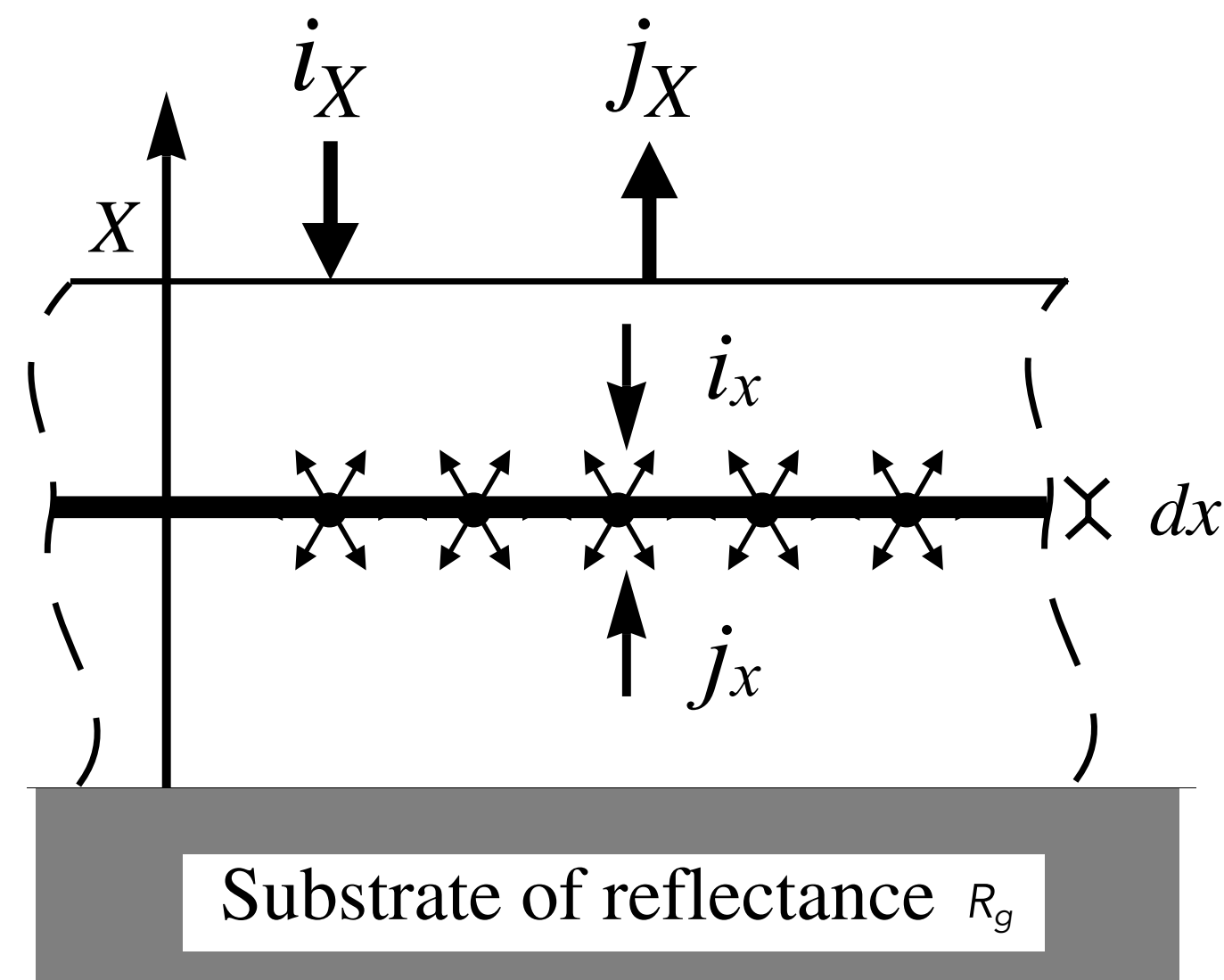
Flux scattered  
upward from  $i$

$$di_x = -Ki_x dx - Si_x dx + Sj_x dx$$



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**K**: absorption coefficient  
**S**: **back-scattering** coefficient  
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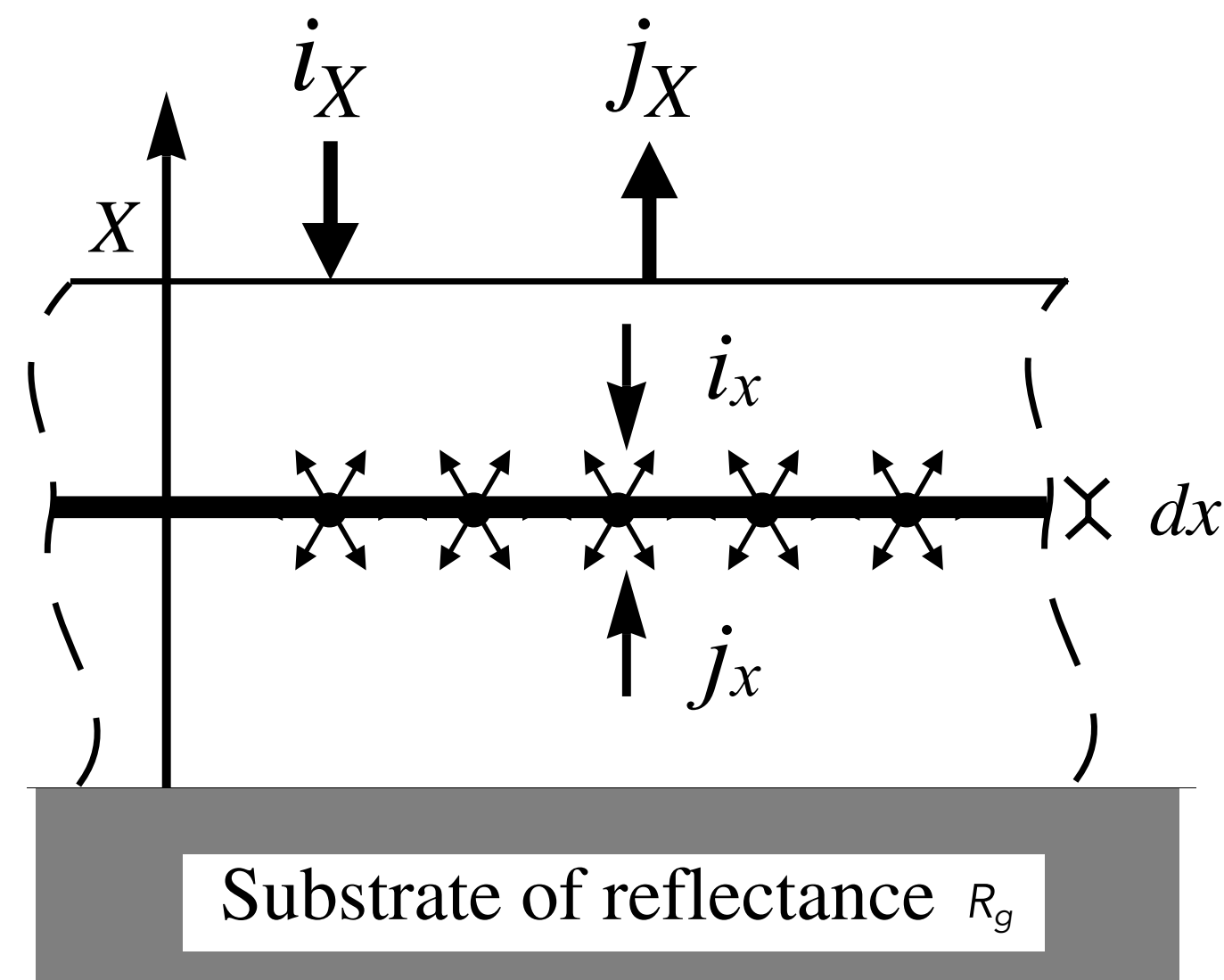
Total downward flux gained      Flux absorbed      Flux scattered upward from  $i$       Flux scattered downward from  $j$

$$di_x = -Ki_x dx - Si_x dx + Sj_x dx$$



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Total downward flux gained  
 Flux absorbed  
 Flux scattered upward from  $i$   
 Flux scattered downward from  $j$

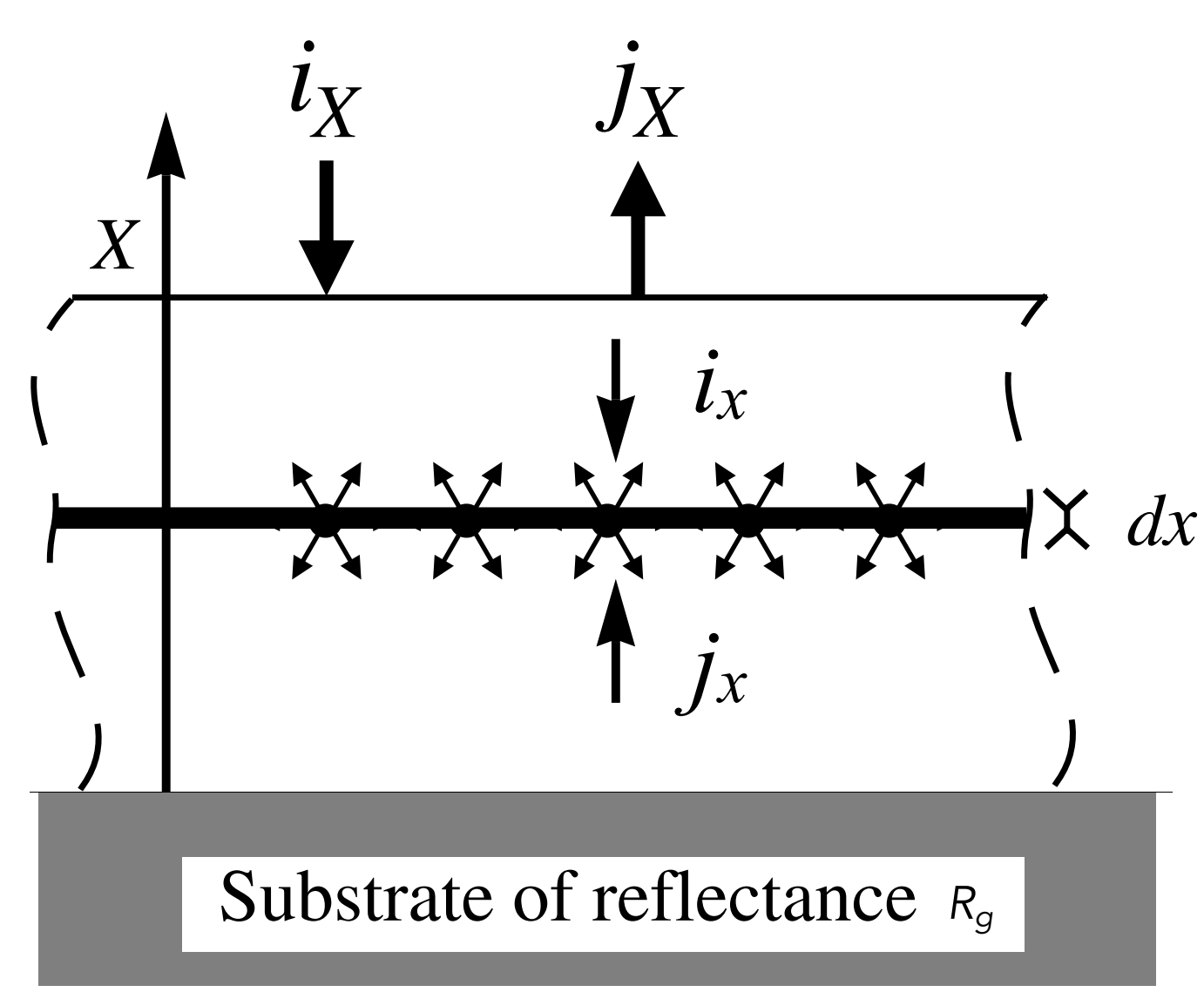
$$di_x = -Ki_x dx - Si_x dx + Sj_x dx$$

$$-dj_x = -Kj_x dx - Sj_x dx + Si_x dx$$



# Kubelka-Munk Equation

**K**: absorption coefficient  
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**X**: thickness



Total downward flux gained      Flux absorbed      Flux scattered upward from  $i$       Flux scattered downward from  $j$

$$di_x = -Ki_x dx - Si_x dx + Sj_x dx$$

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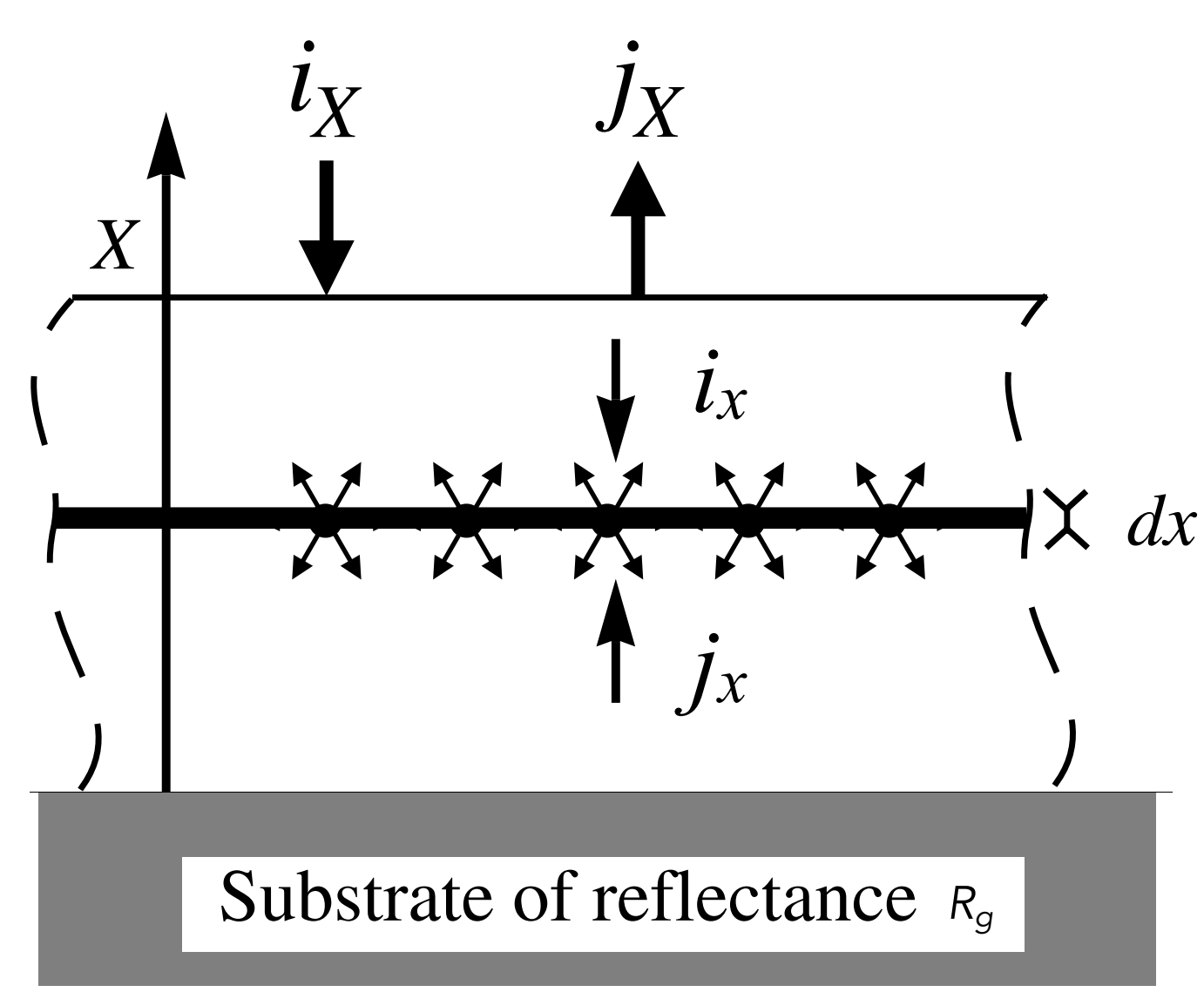
Boundary conditions:

$$R_g = \frac{j_0}{i_0}$$



# Kubelka-Munk Equation

**K**: absorption coefficient  
**S**: **back-scattering** coefficient  
**X**: thickness



Total downward flux gained      Flux absorbed      Flux scattered upward from *i*      Flux scattered downward from *j*

$$\begin{aligned}
 di_x &= -Ki_x dx - Si_x dx + Sj_x dx \\
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 \end{aligned}$$

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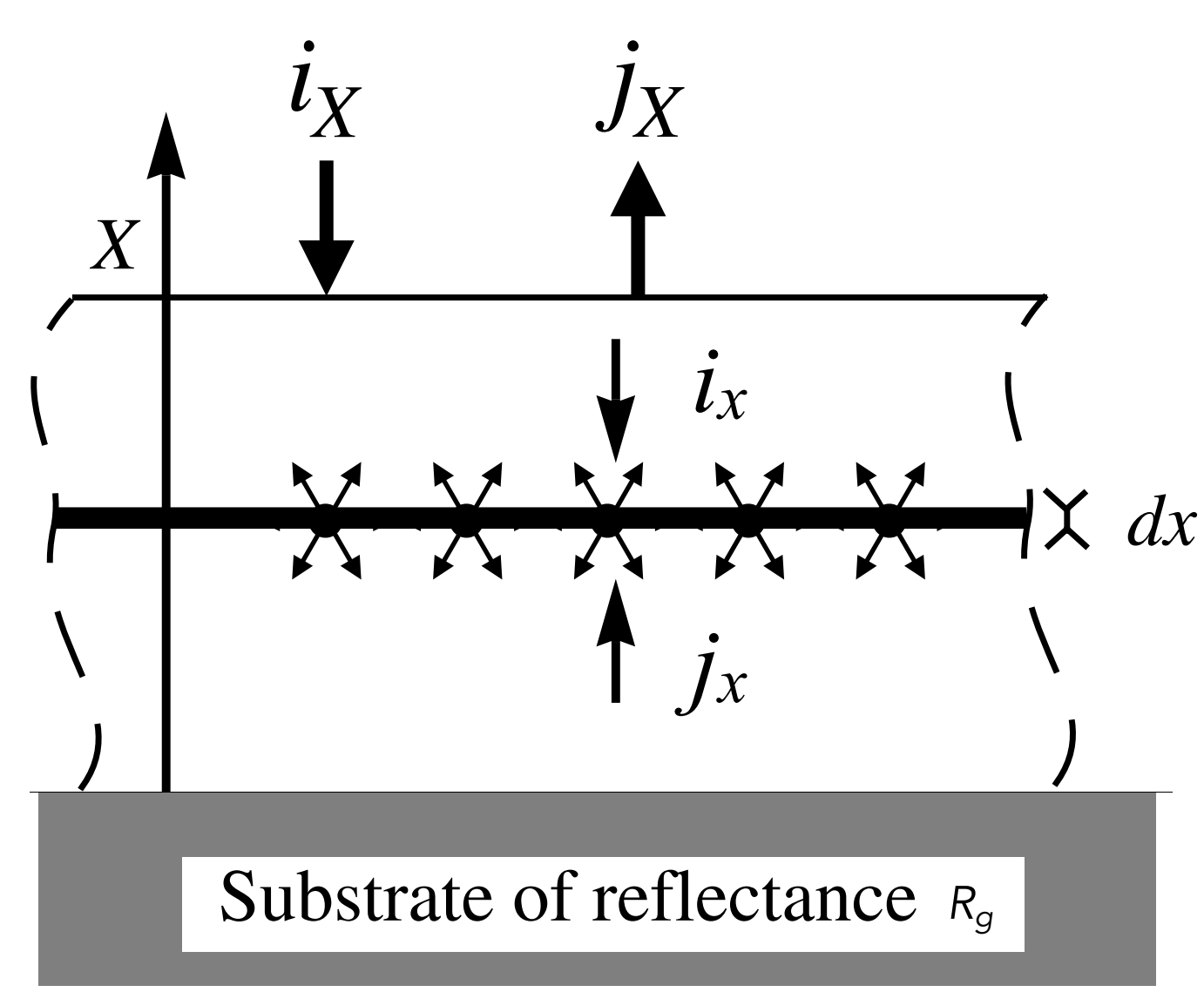
Reflectance of the layer:

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# Kubelka-Munk Equation

**K**: absorption coefficient  
**S**: **back-scattering** coefficient  
**X**: thickness



Total downward  
flux gained

$di_x$

Flux absorbed

$-Ki_xdx$

Flux scattered  
upward from  $i$

$-Si_xdx$

Flux scattered  
downward from  $j$

$+Sj_xdx$

$$di_x = -Ki_xdx - Si_xdx + Sj_xdx$$

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Boundary conditions:

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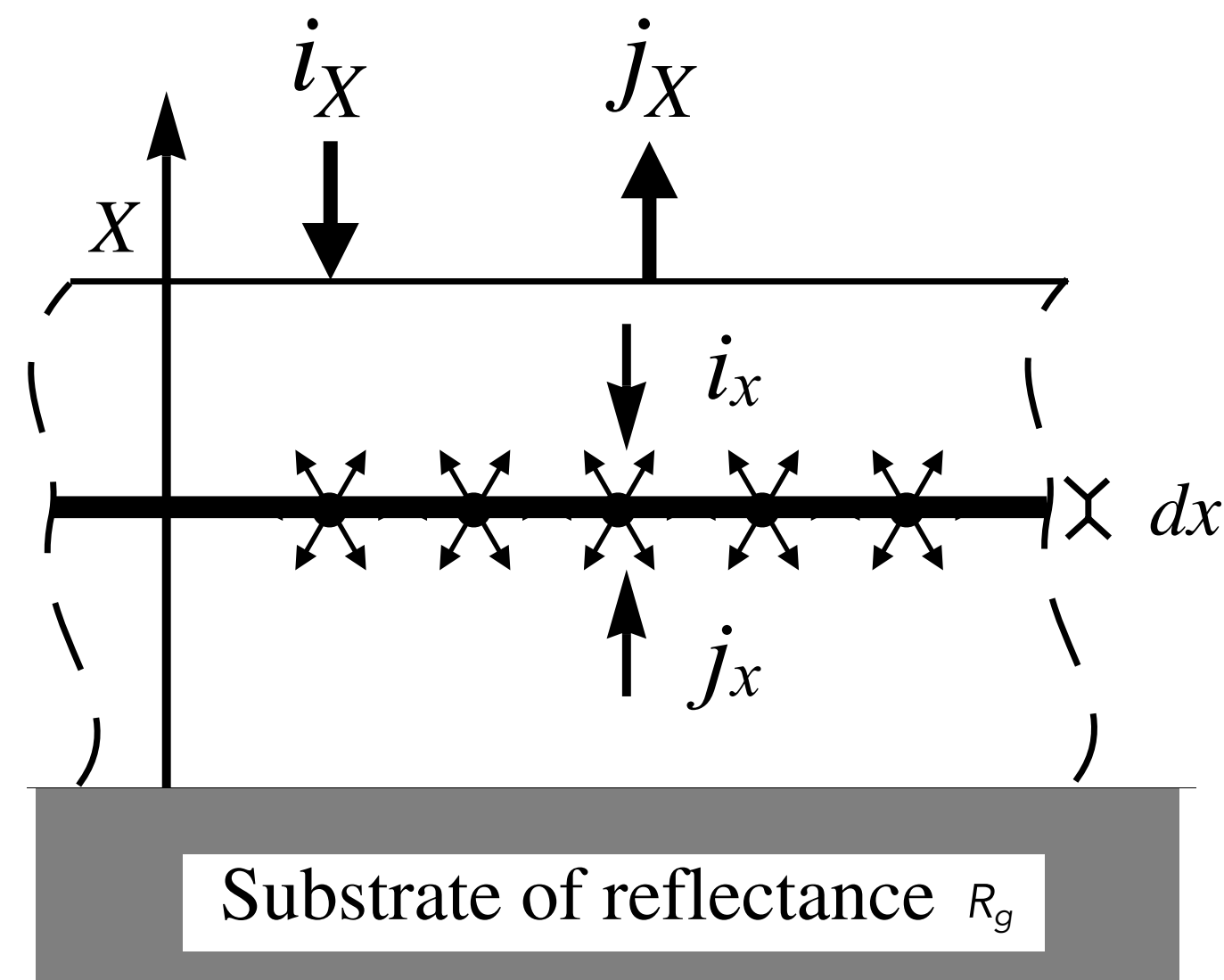
$$R_X = \frac{j_X}{i_X}$$

← This is what we want to calculate!



# Kubelka-Munk Equation: General Solution

$$R_X = \frac{1 - R_g[a - b \coth(bSX)]}{a - R_g + b \coth(bSX)} \quad a = \frac{K + S}{S} \quad b = \sqrt{a^2 - 1} \quad \coth x = \frac{e^{2x} + 1}{e^{2x} - 1}$$



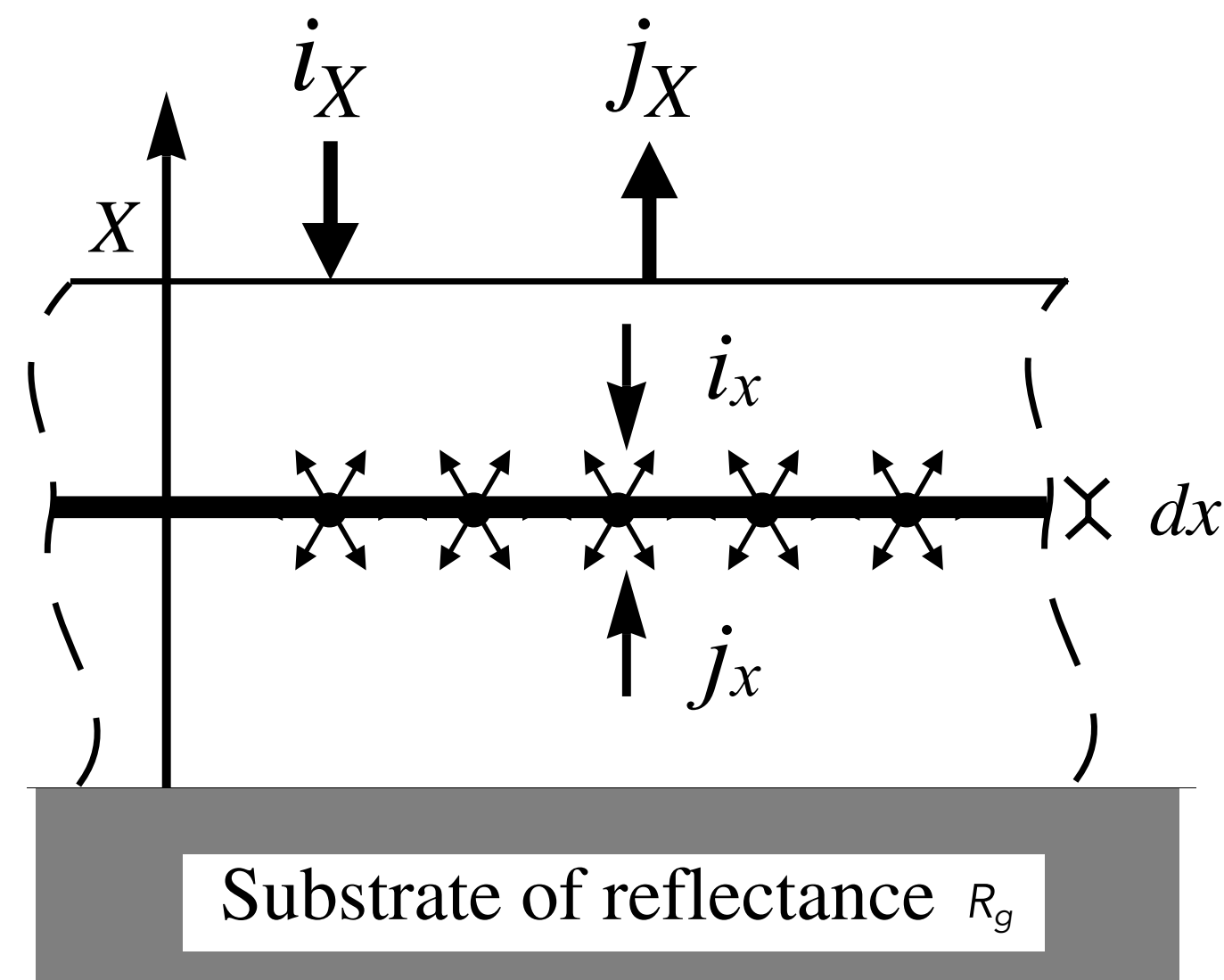
Reflectance depends on:

- Layer thickness **X**
- Absorption and scattering coefficients **K** and **S**
- Background reflectance: **R<sub>g</sub>**



# Kubelka-Munk Equation: No Scattering

$$S = 0 \quad \Rightarrow \quad R_X = R_g e^{-2KX} = e^{-KX} R_g e^{-KX}$$

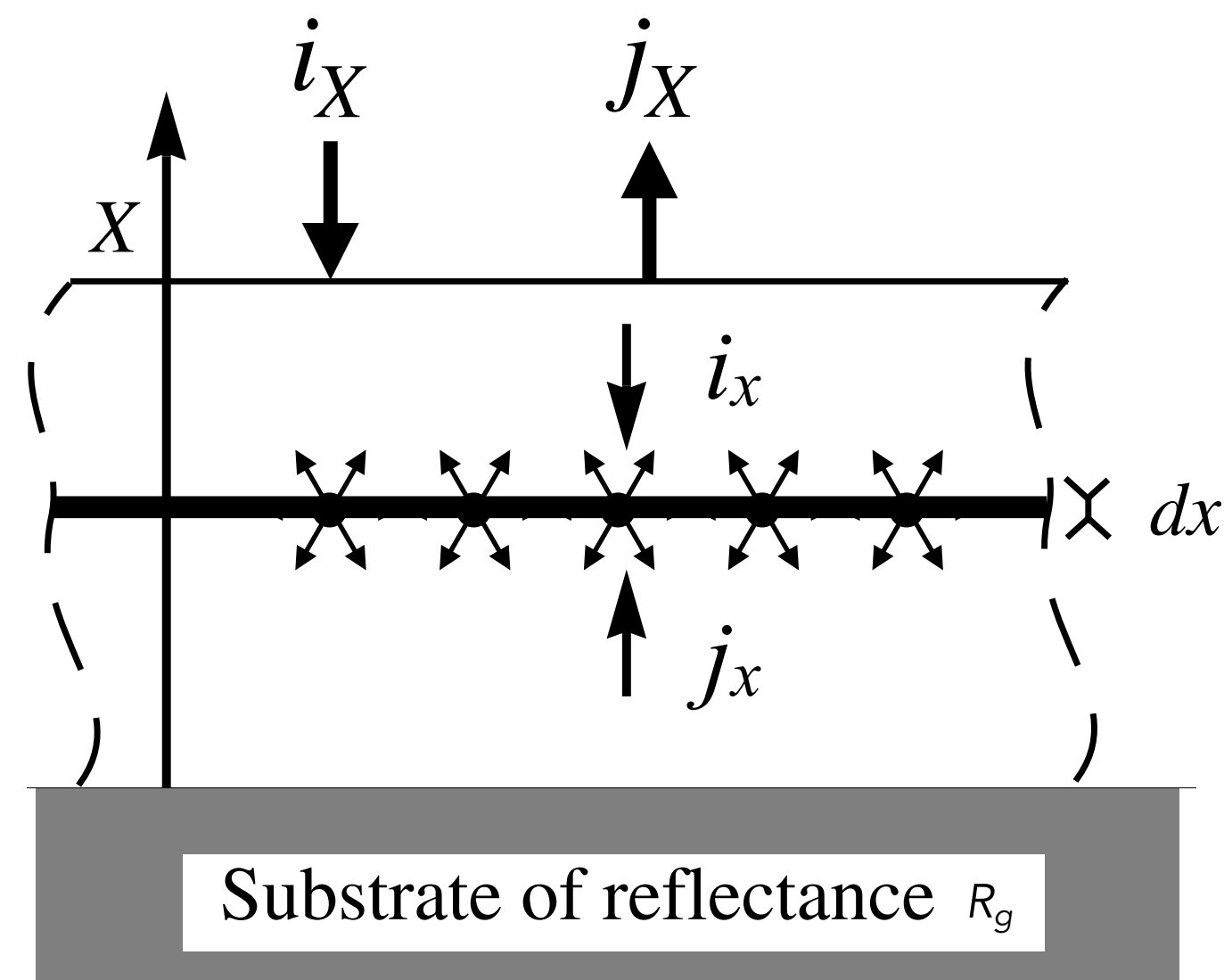




# Kubelka-Munk Equation: No Scattering

$$S = 0 \quad \Rightarrow \quad R_X = R_g e^{-2KX} = e^{-KX} R_g e^{-KX}$$

% of photons left after  
they reach the bottom



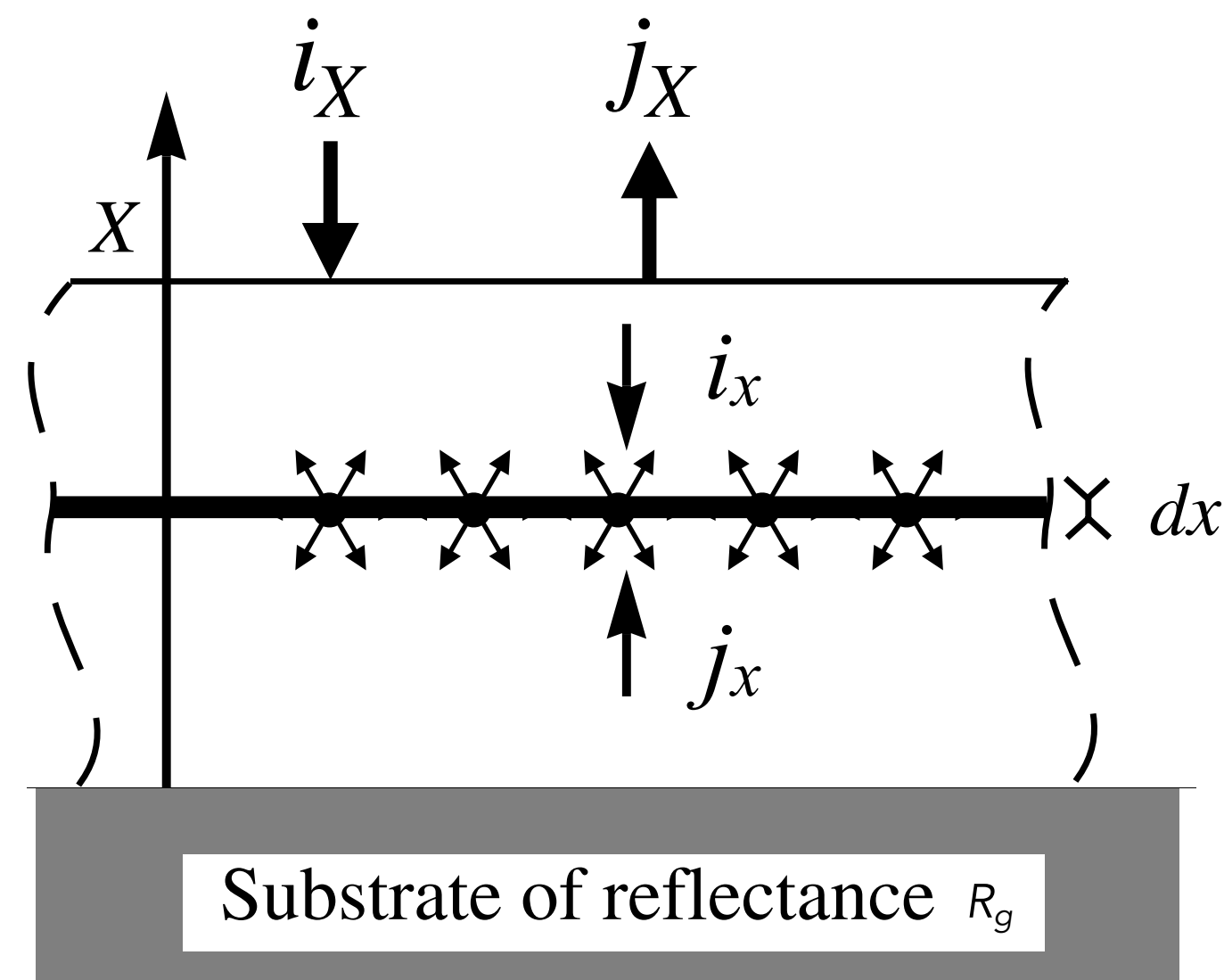


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% of photons left after  
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% of photons reflected  
back by the substrate





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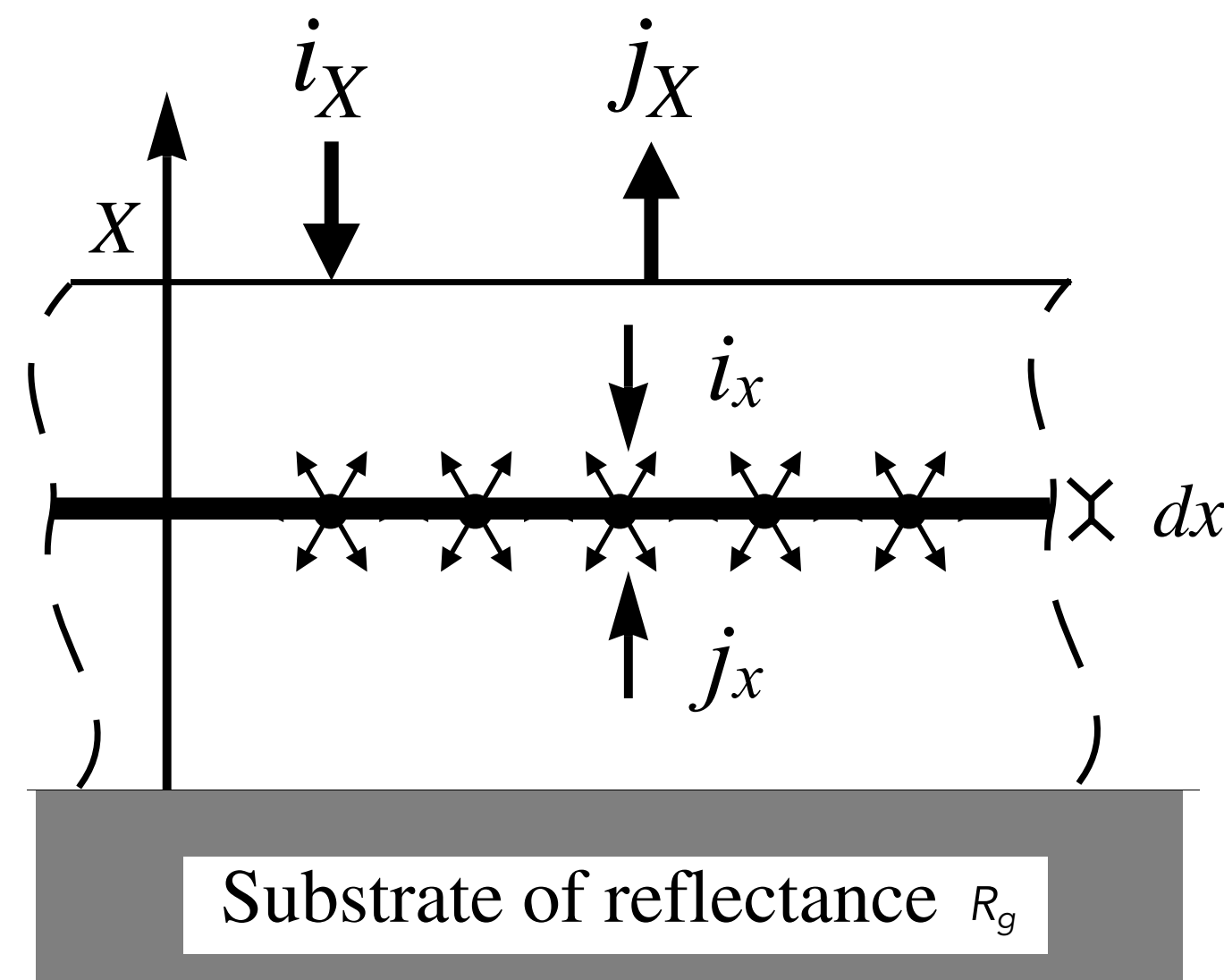
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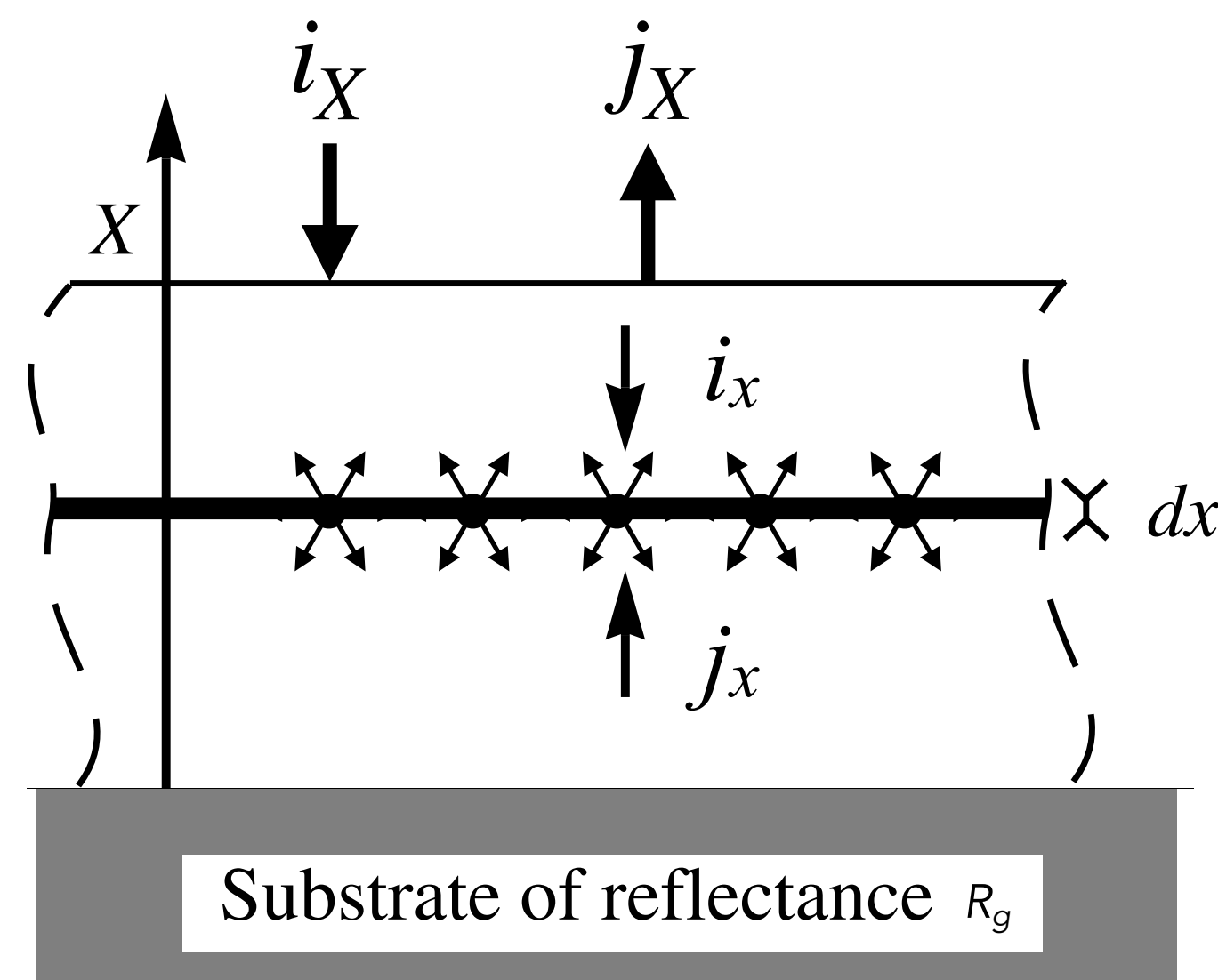
% of photons left after  
they reach the top again





# Kubelka-Munk Equation: No Substrate

$$R_g = 0 \Rightarrow R_X = \frac{1}{a + b \coth(bSX)} \quad a = \frac{K + S}{S} \quad b = \sqrt{a^2 - 1}$$



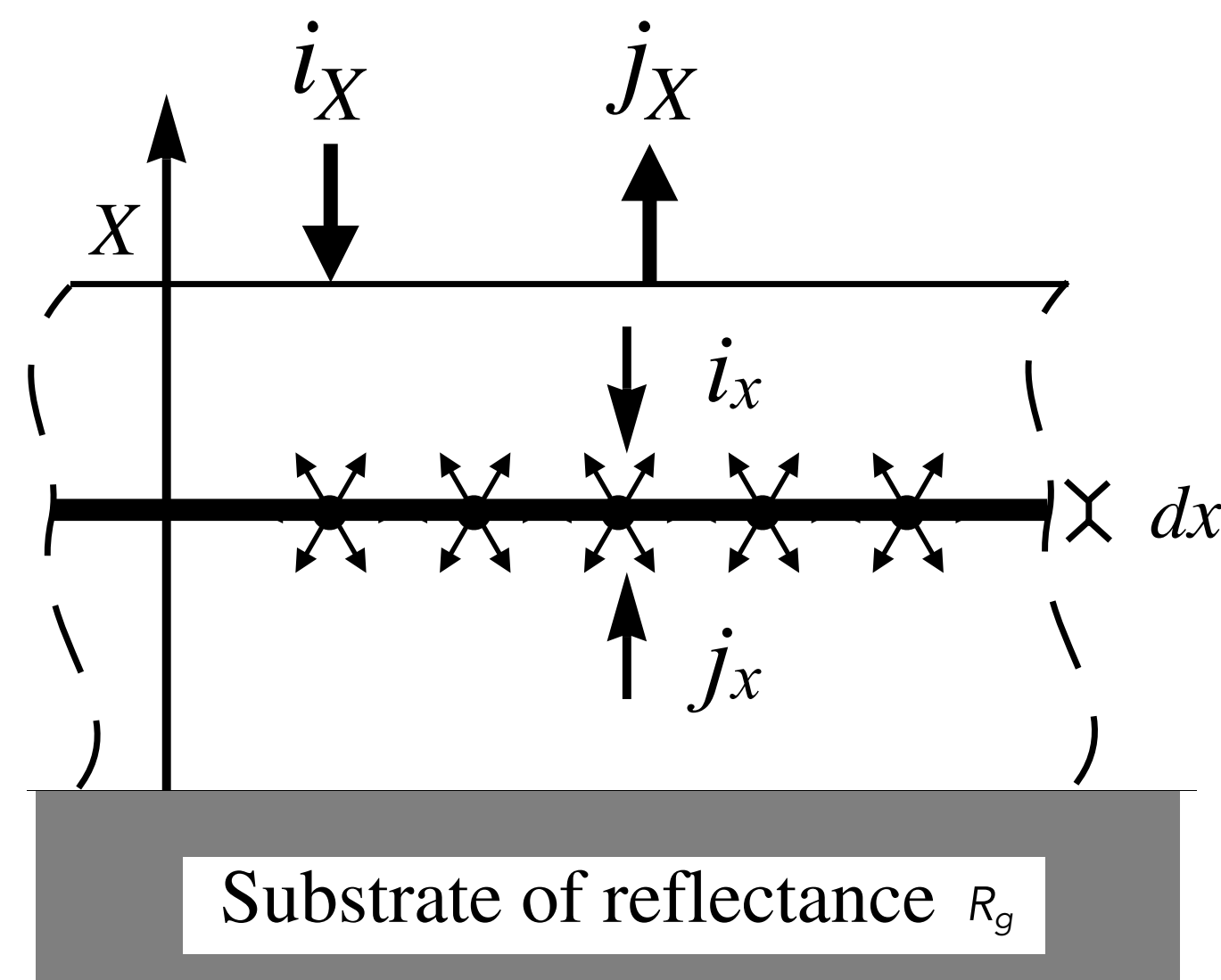
Substrate reflectance  **$R_g$**  is 0 now. Alternatively this can be thought of as having a **black substrate**. The effect is the same: whatever gets out of the layer never comes back.



# Kubelka-Munk Equation: Complete Hiding

$$\lim_{X \rightarrow \infty} R_X = R_\infty = \frac{1}{a + b} = 1 + \frac{K}{S} - \sqrt{\left(\frac{K}{S}\right)^2 + 2\frac{K}{S}}$$

$$\frac{K}{S} = \frac{(1 - R_\infty)^2}{2R_\infty}$$

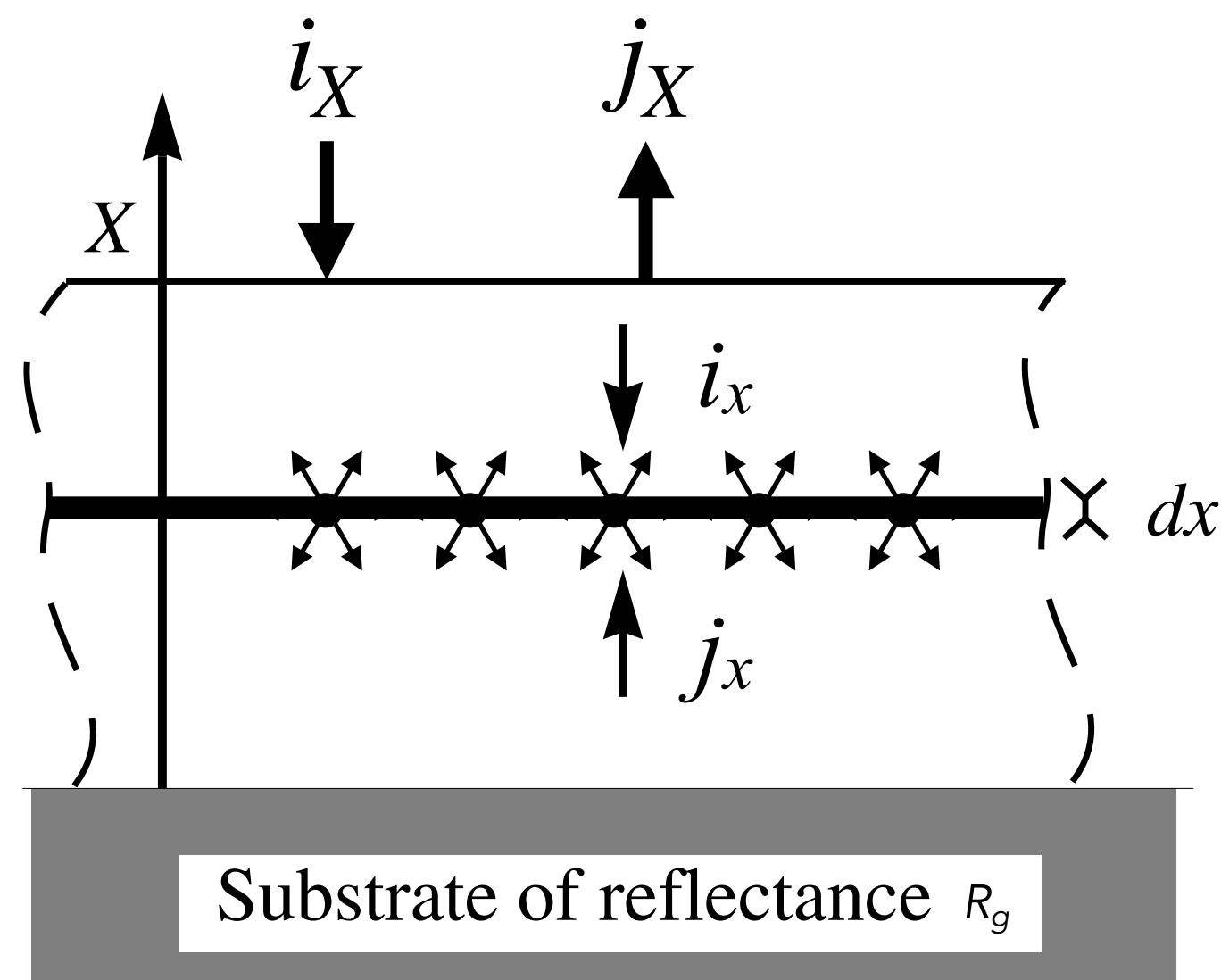


When the layer is so thick that the background reflectance is irrelevant, the resulting reflectance depends only on the absorption and scattering coefficients.

We say the "hiding is complete".



# Hiding Power in Painting



$$HP = \frac{R_{black}}{R_{white}}$$

Reflectance when substrate is black.

Reflectance when substrate is white.

In paint industry, the way they quantify hiding power is to see how the surface reflectance changes when the background is white vs. black.

If the layer's hiding is complete, the reflectance ratio should 1, i.e., the background color doesn't matter and the colorant is **opaque**.

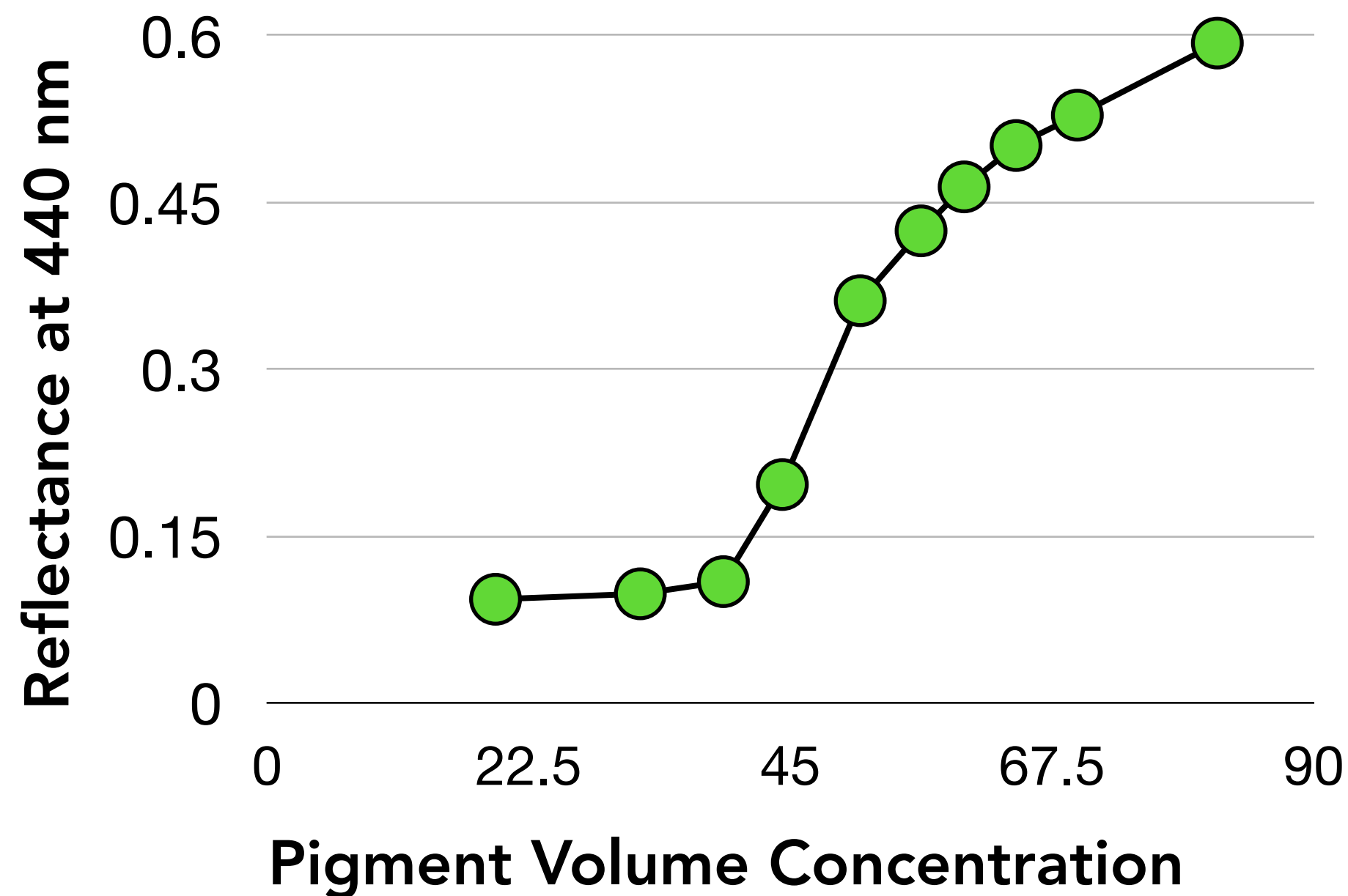


# Reflectance vs. Concentration

$$\frac{K}{S} = \frac{(1 - R_{\infty})^2}{2R_{\infty}}$$

When hiding is complete, the reflectance of the colorant doesn't depend on the concentration.

- Until when critical concentration is reached, where the assumptions of K-M model break.



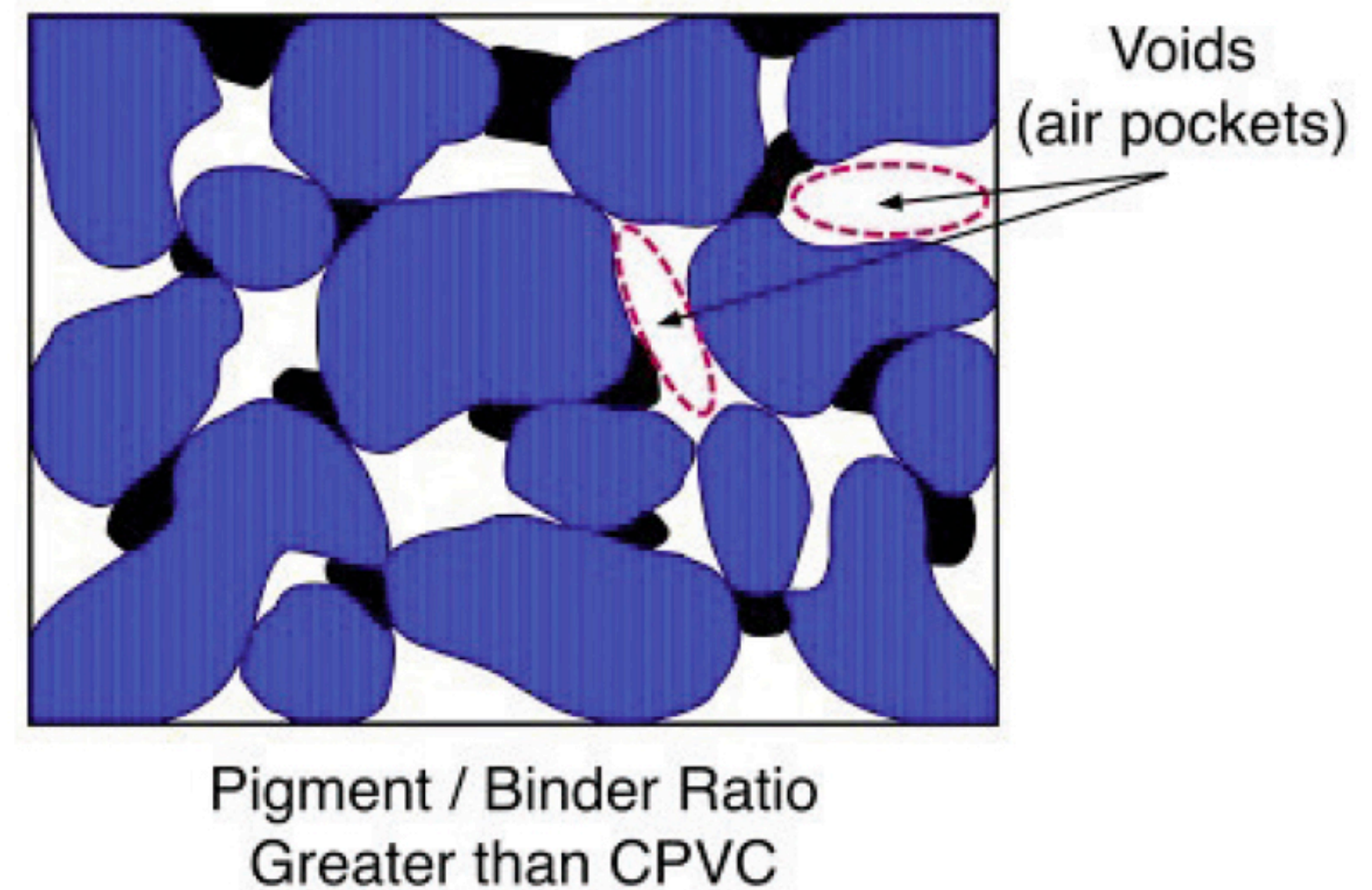
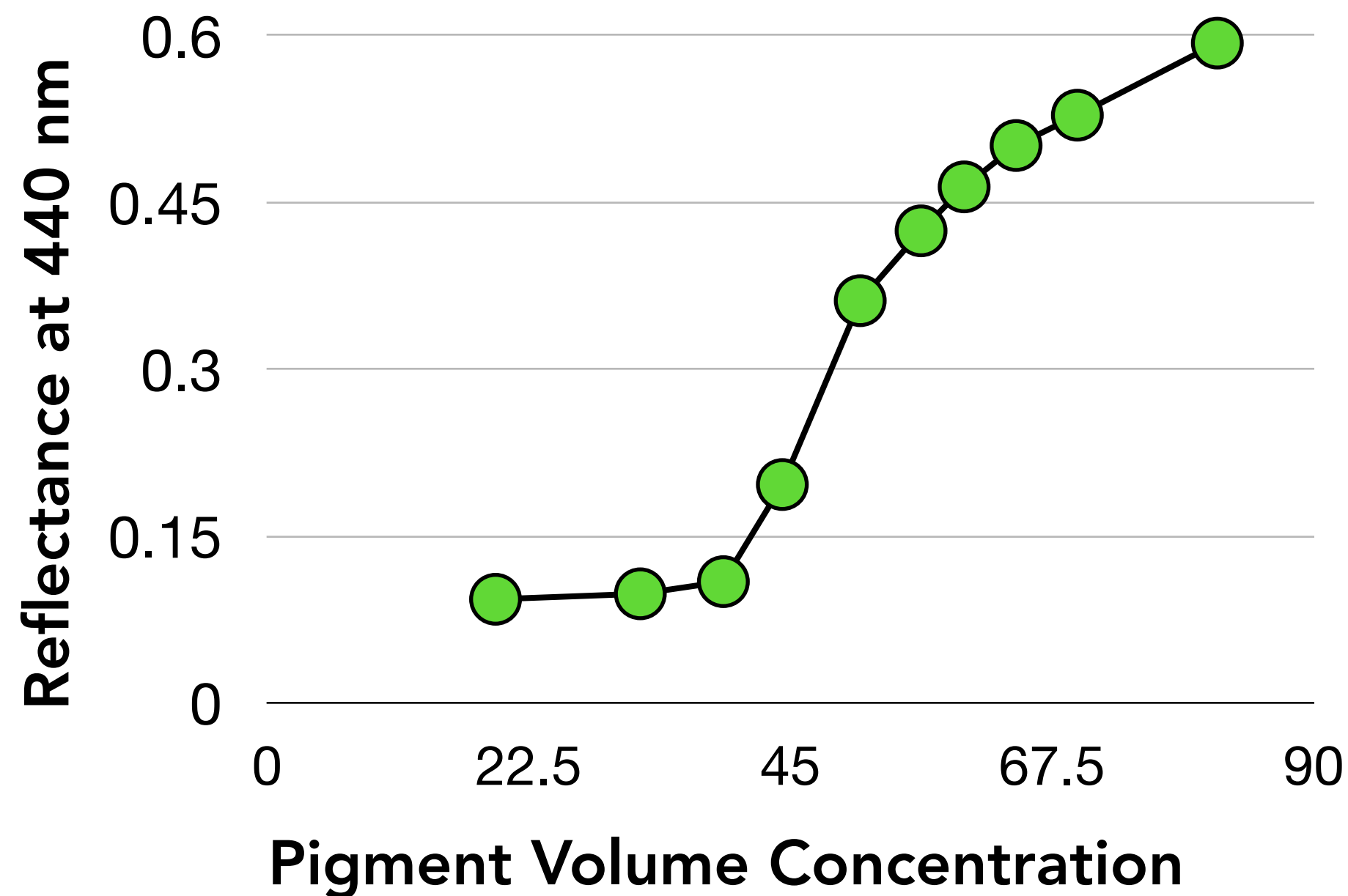


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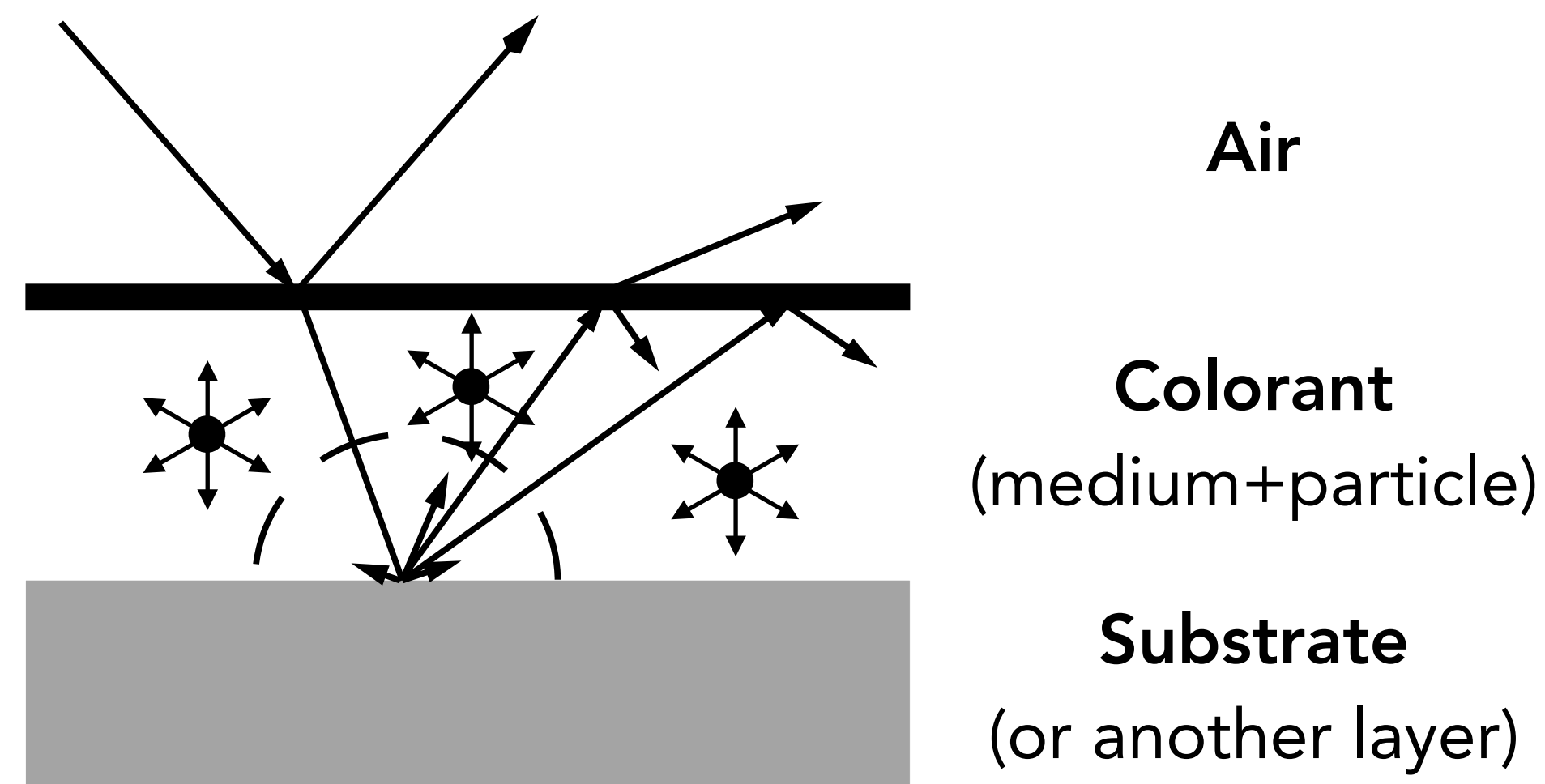
# Surface Reflection



# Considering Surface Reflection

So far we have ignored reflection and refraction at the air/medium interface.

- Note that reflection and refraction are just macroscopic behaviors of light scattering (with interference) off of surface particles. We can reason about it using geometric optics, since a colorant is large enough for that.

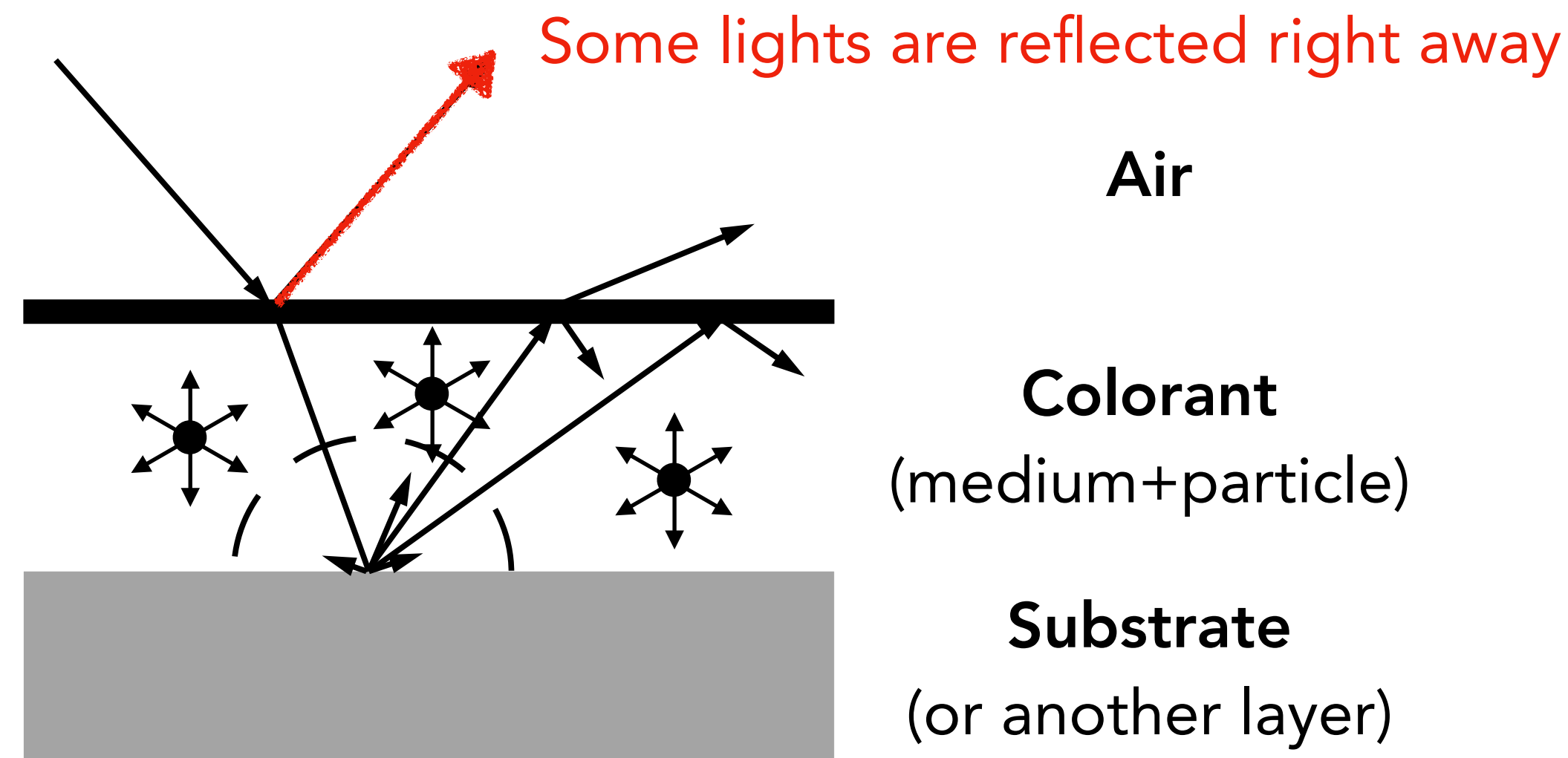




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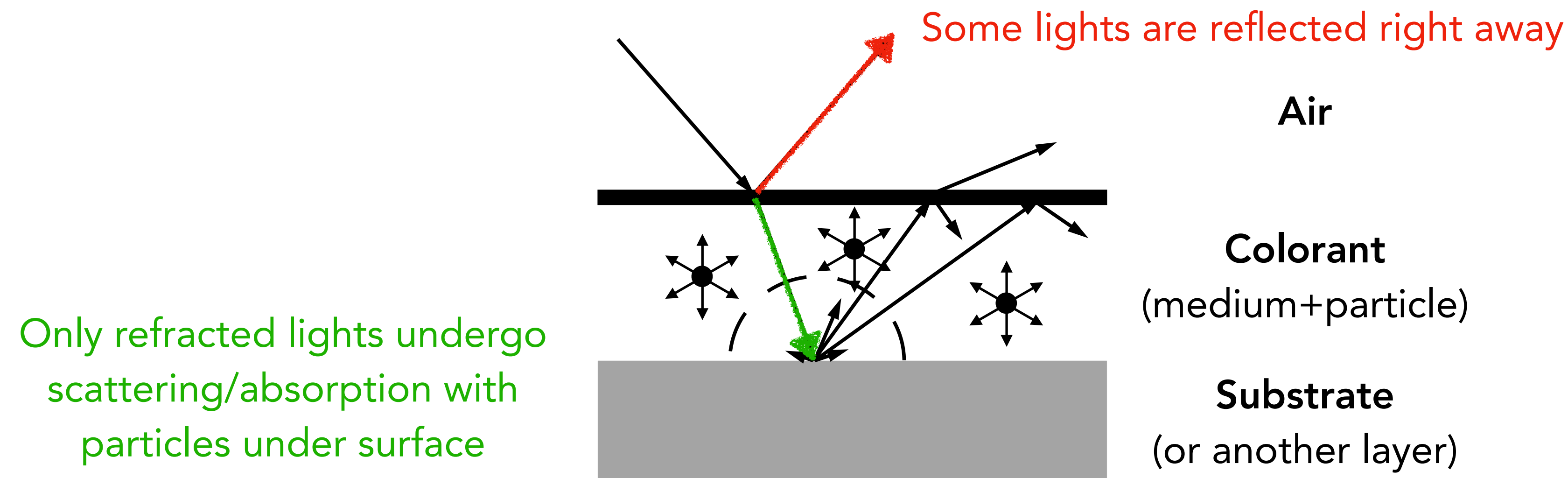




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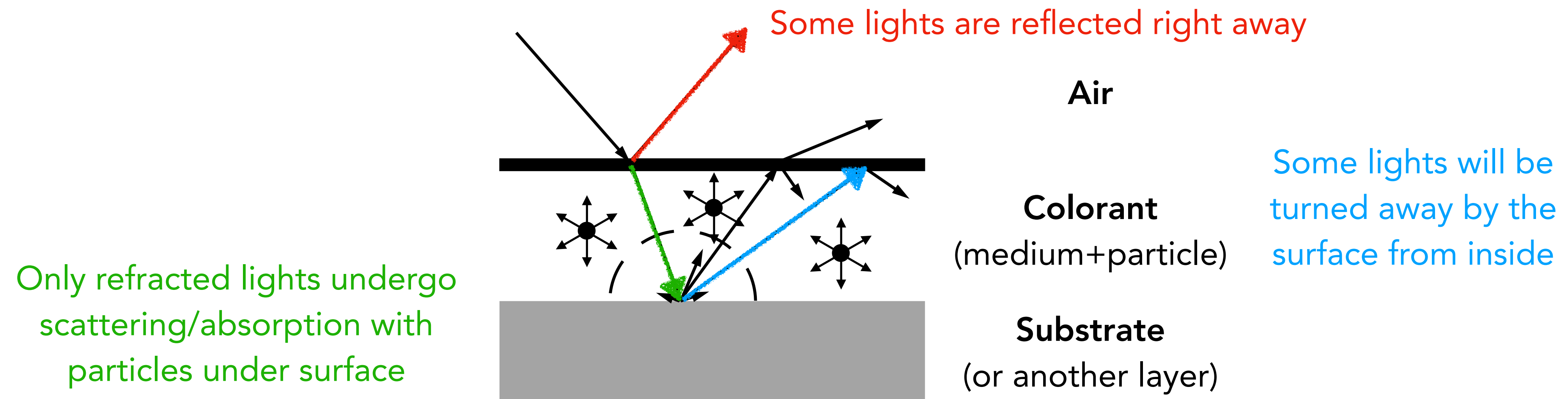




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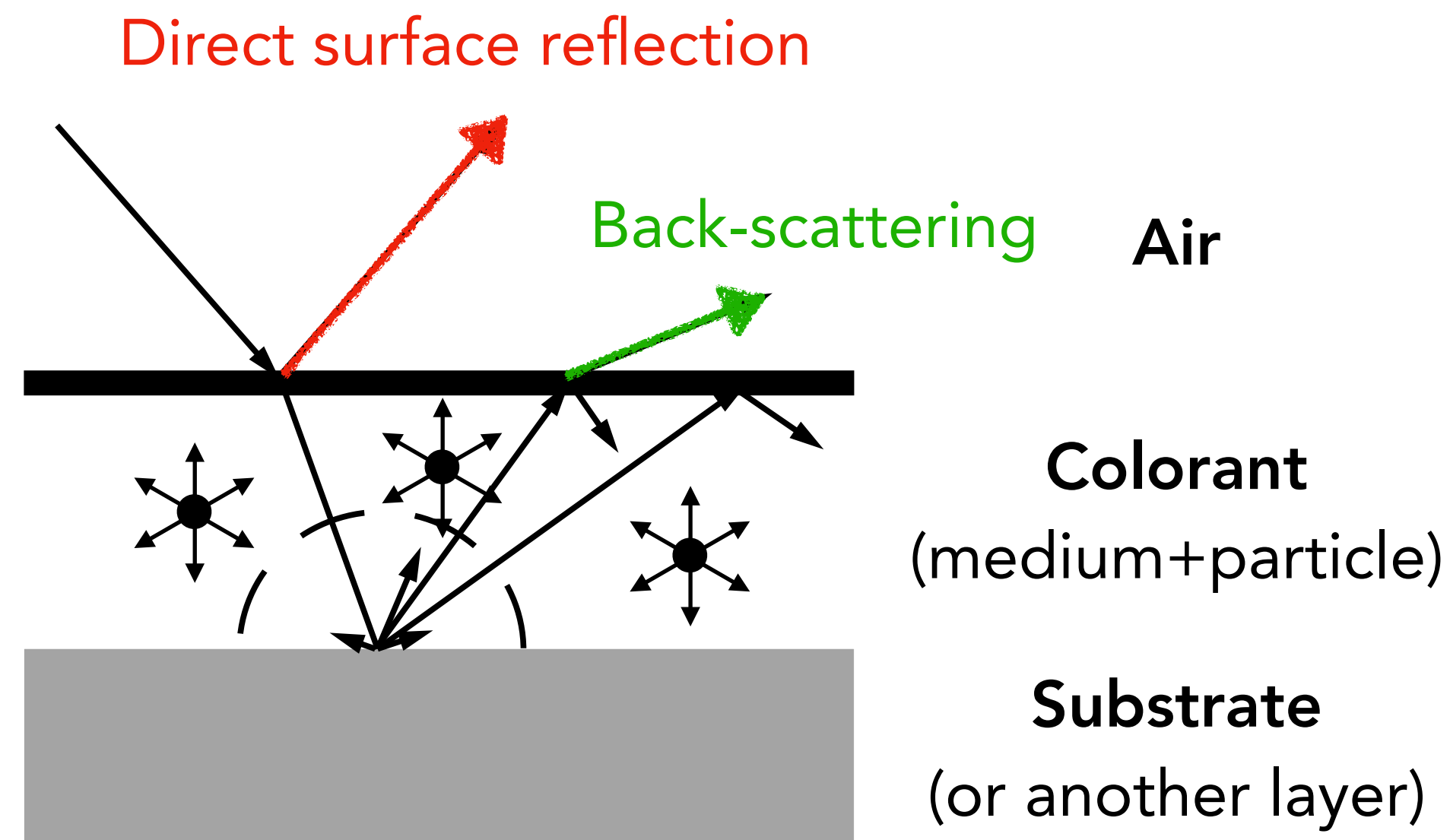
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# Reflectance = Surface Reflection + Back Scattering

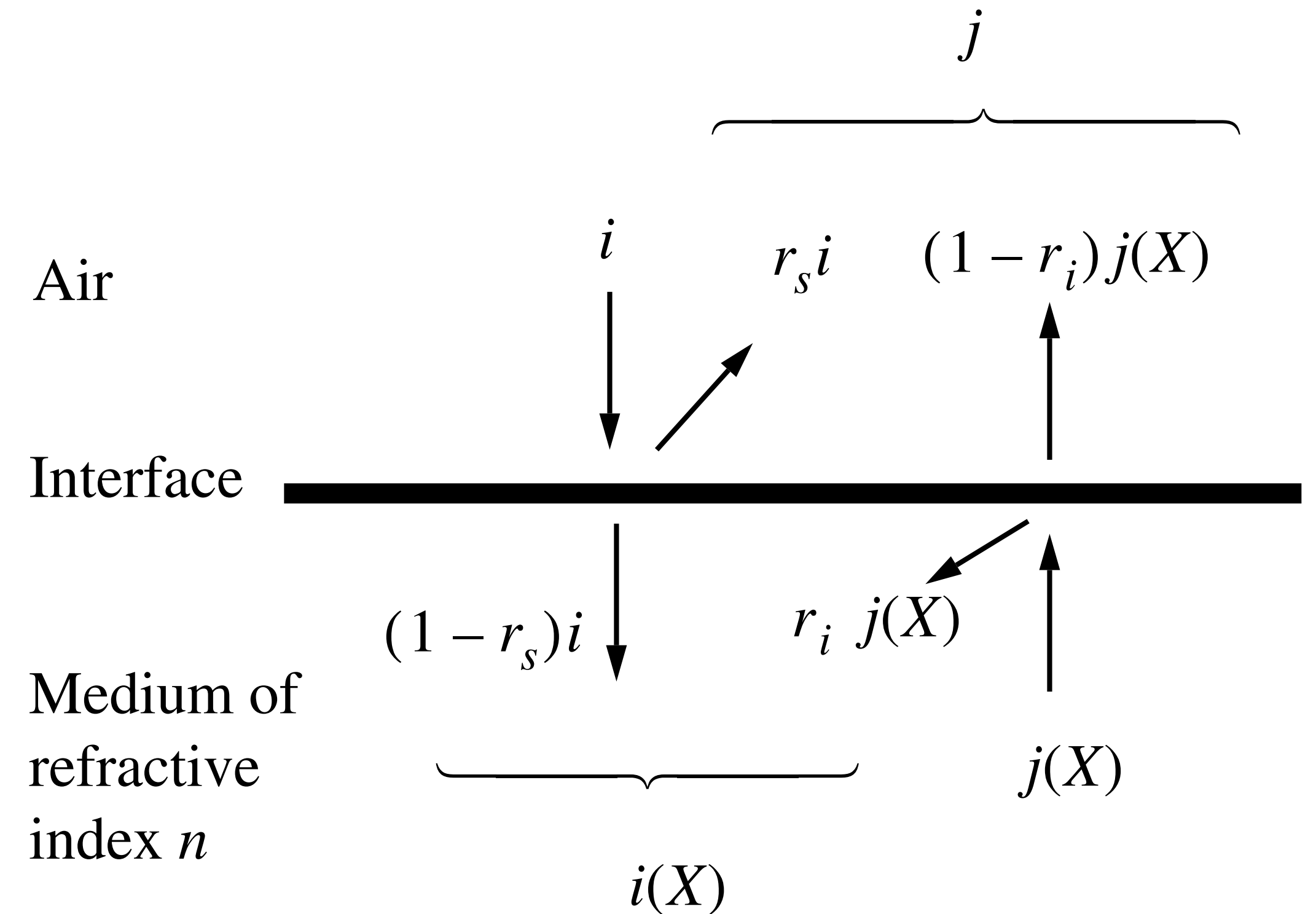
The overall reflection is made up of both surface reflection (photons turned away right at the interface) and back-scattering from photon-particle interactions under the surface (usually called **sub-surface scattering**).





# K-M Model With Surface Reflection Correction

This is called the **Saunderson correction**

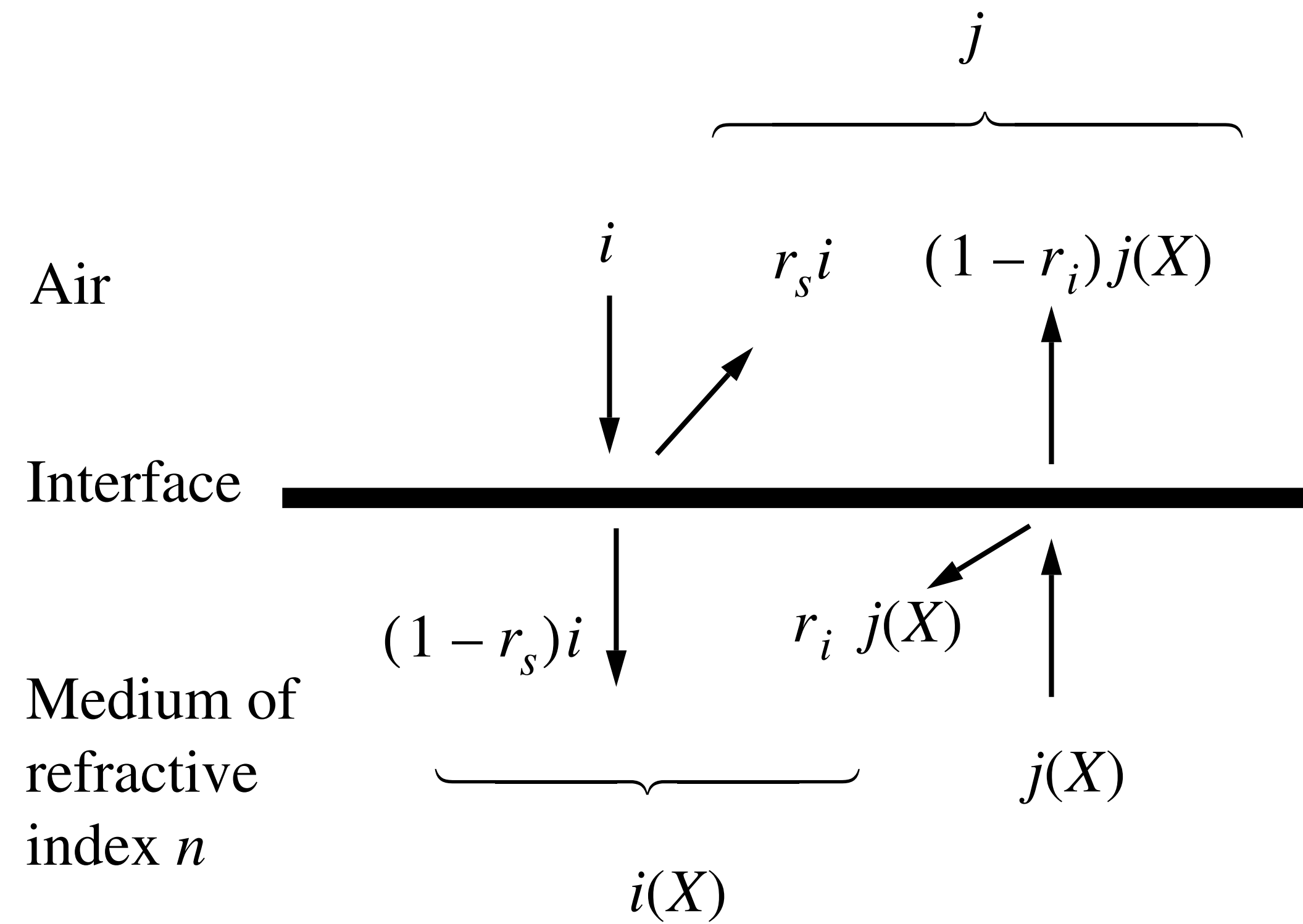




# K-M Model With Surface Reflection Correction

This is called the **Saunderson correction**

$$R = r_s + \frac{(1 - r_s)(1 - r_i) R_X}{1 - r_i R_X}$$



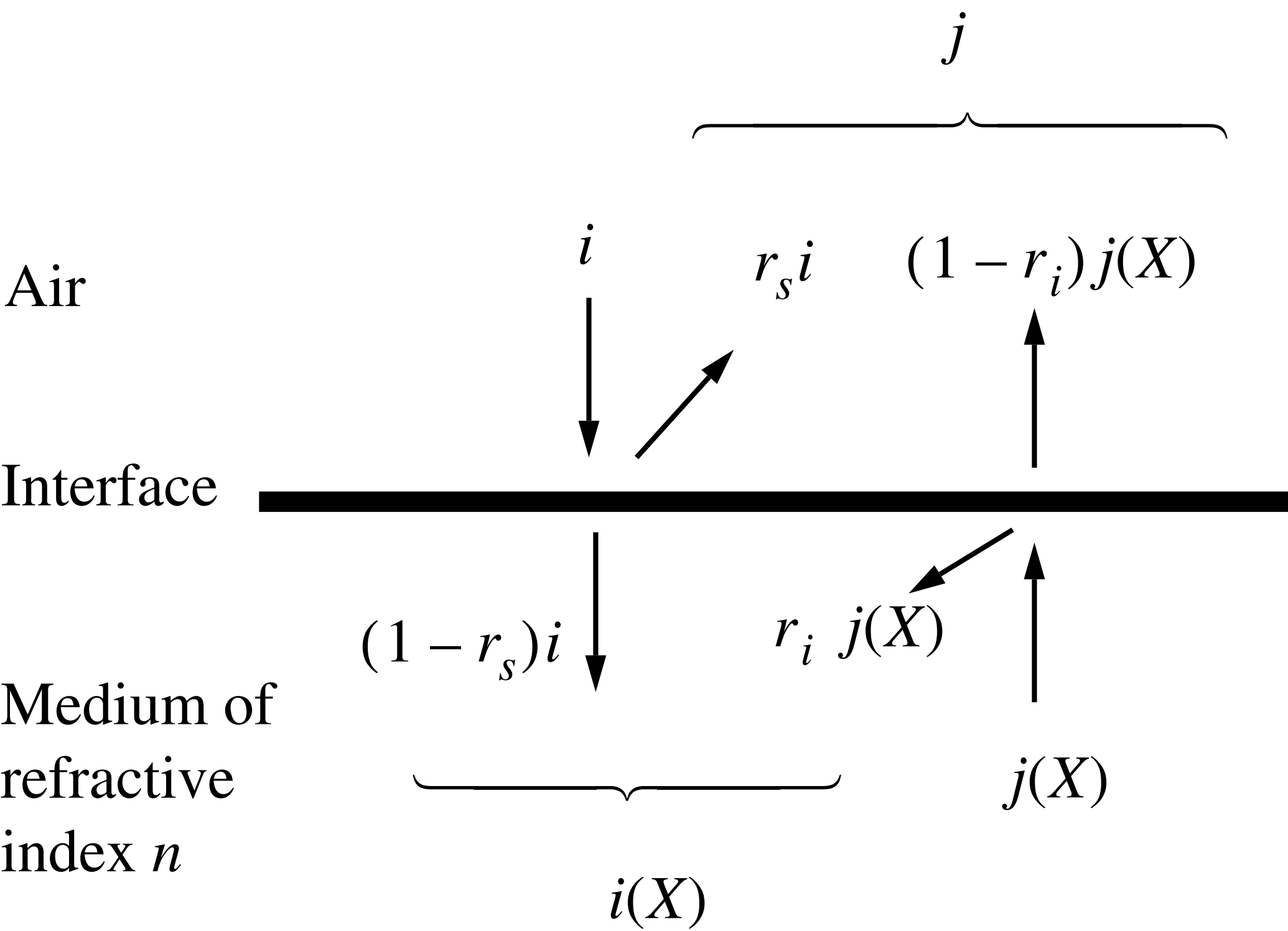


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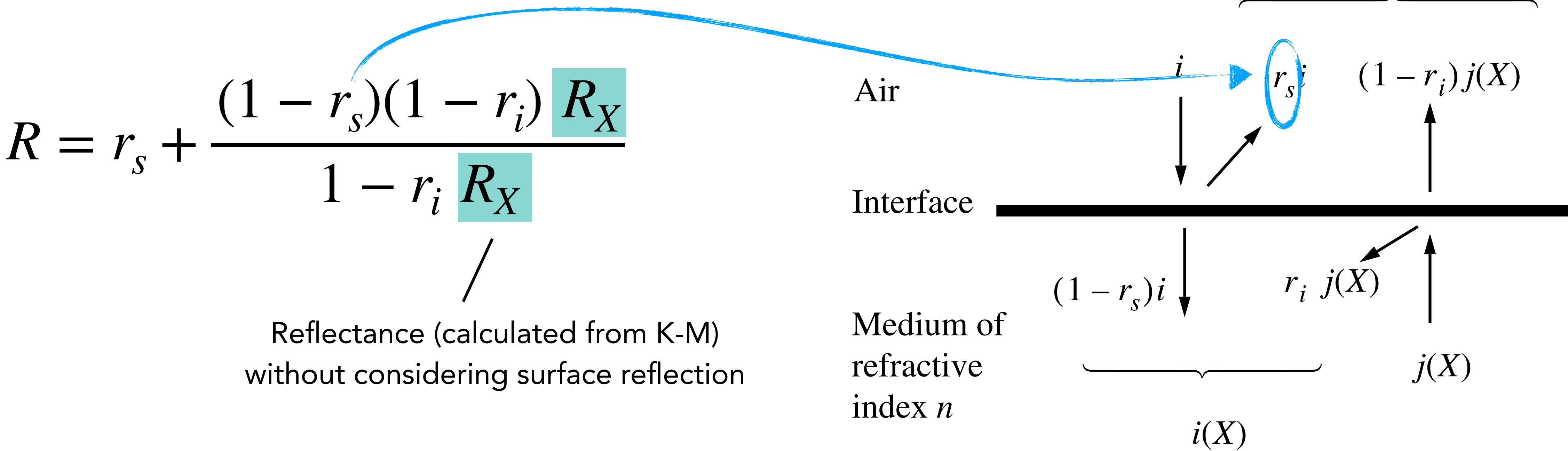
Reflectance (calculated from K-M)  
without considering surface reflection





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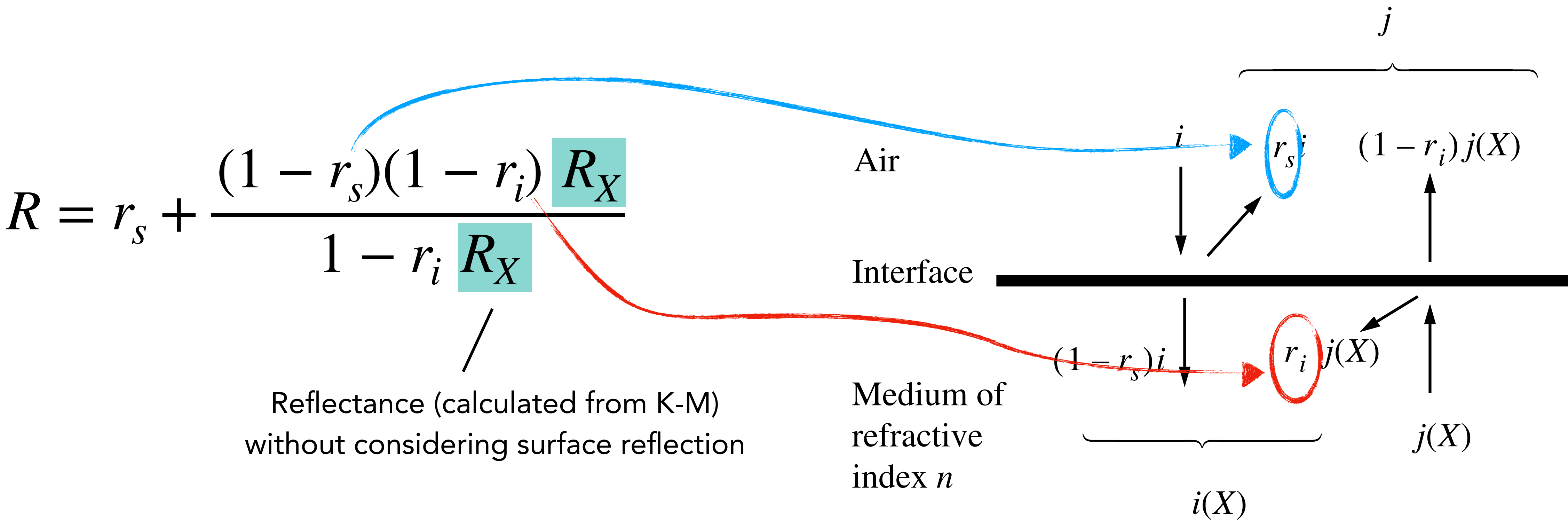
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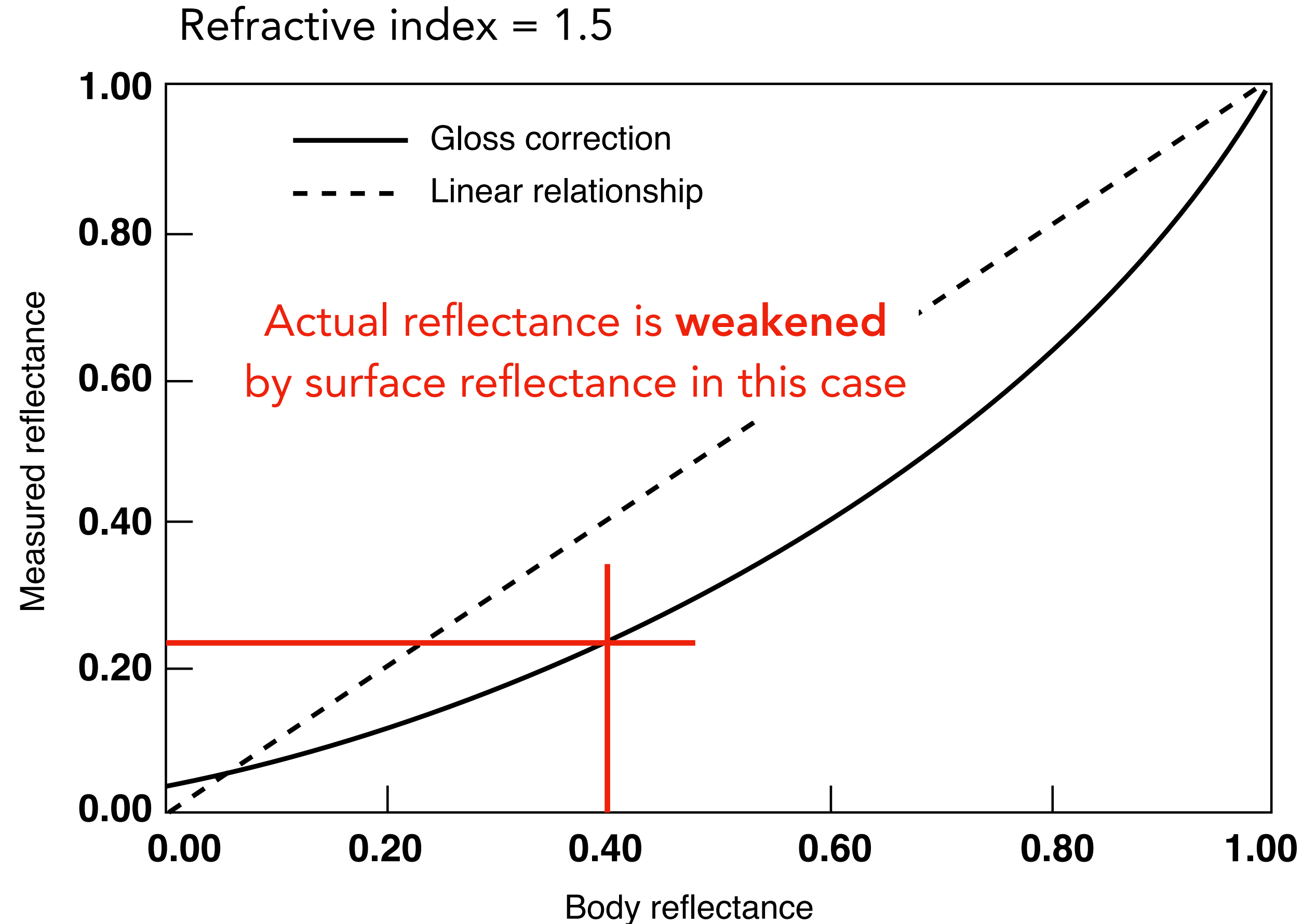
# K-M Model With Surface Reflection Correction

This is called the **Saunderson correction**





# K-M Model With Surface Reflection Correction



Using Fresnel equation

- $r_s$  is 0.092 for diffuse light and 0.04 for when incident lights are perpendicular to the surface.
- $r_i$  is 0.06 for when the internal lights crossing the interface are diffuse.

In practice, parameters are fit with observations, just like other parameters in K-M model.



# Material Classification

In graphics, material is mainly classified into two categories.



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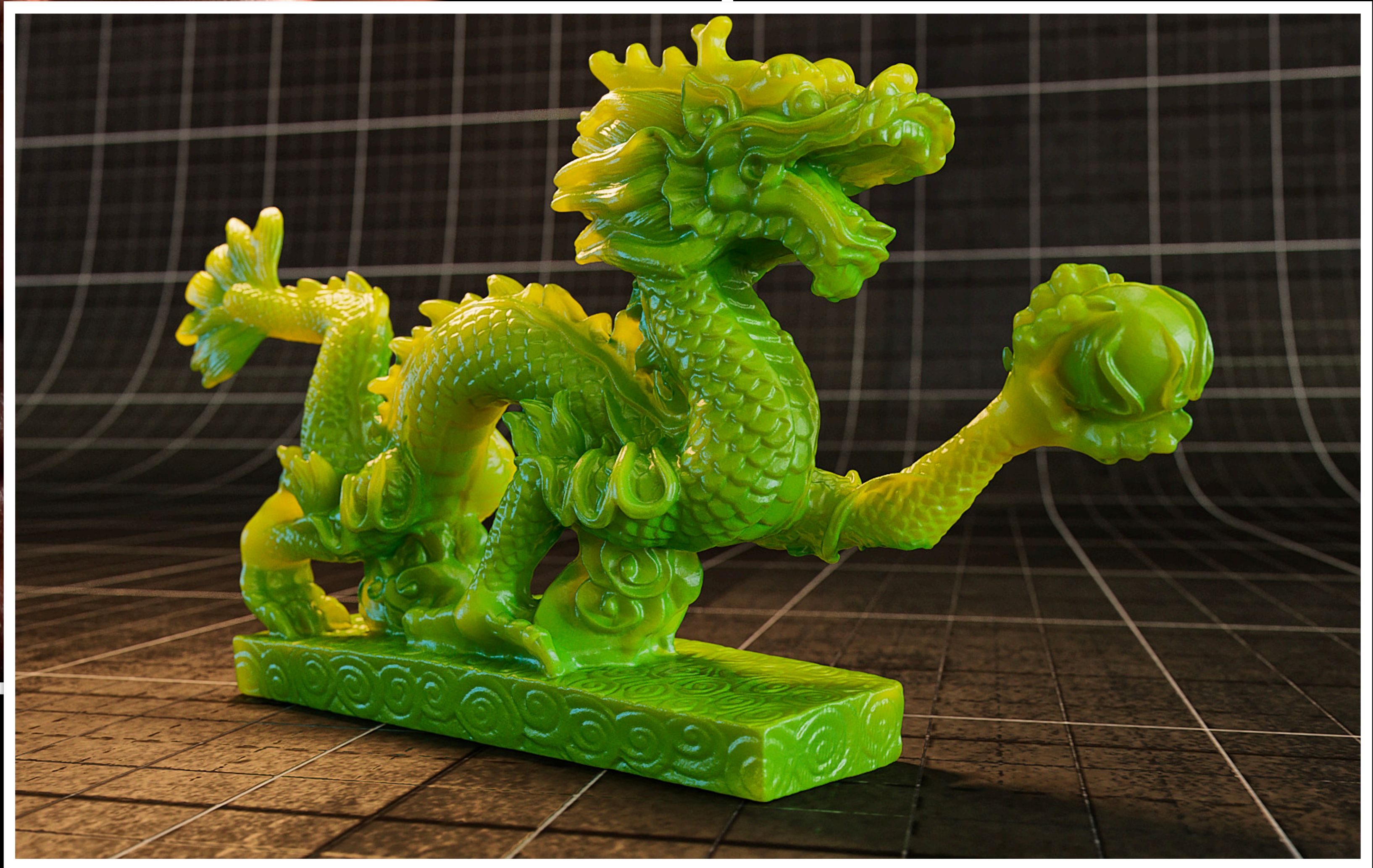
A big part of photorealistic rendering is material modeling. It's always a trade-off between rendering time vs. realism.

- But note that you don't have to be physically correct, just need to be photorealistic.

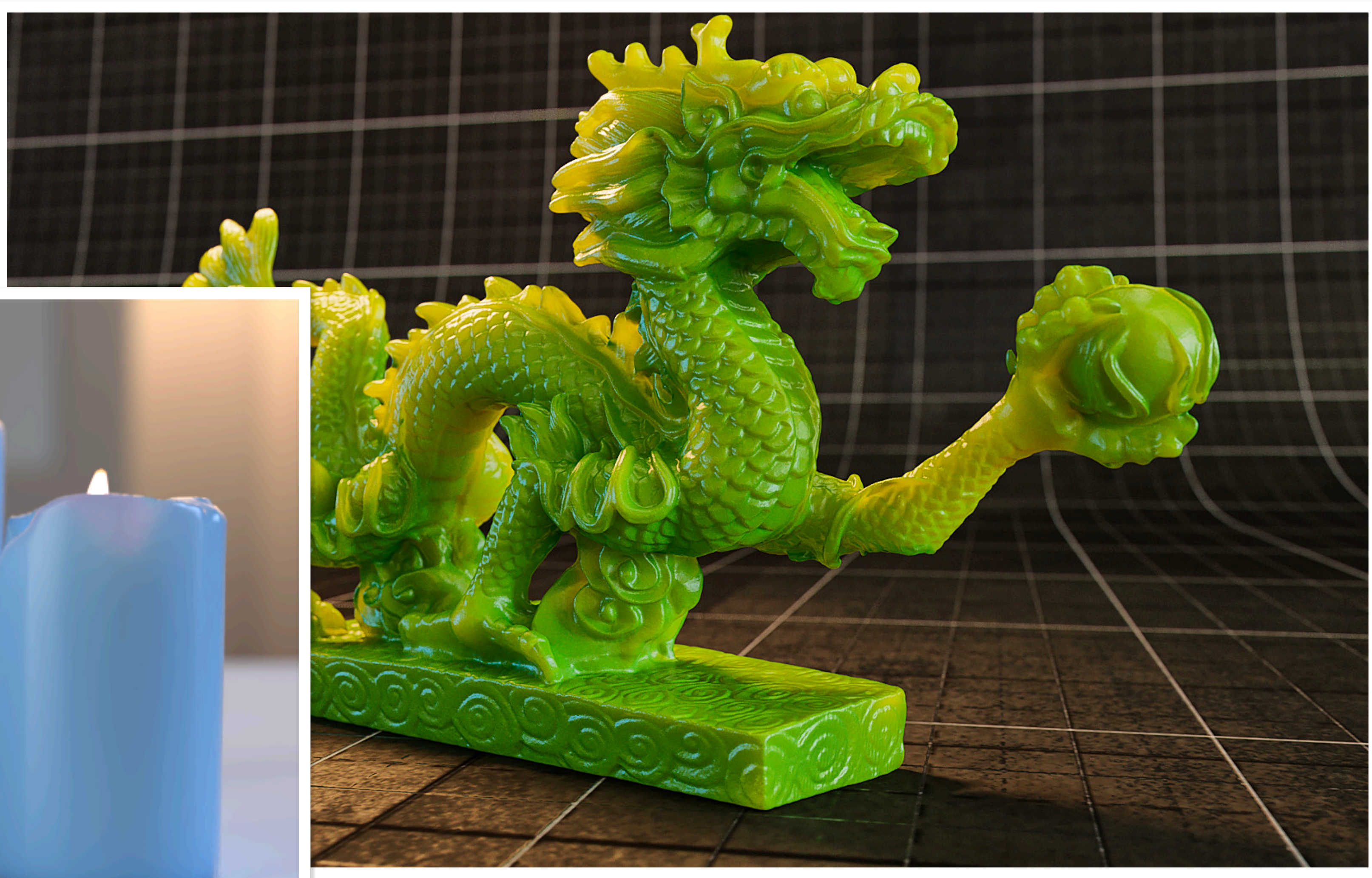










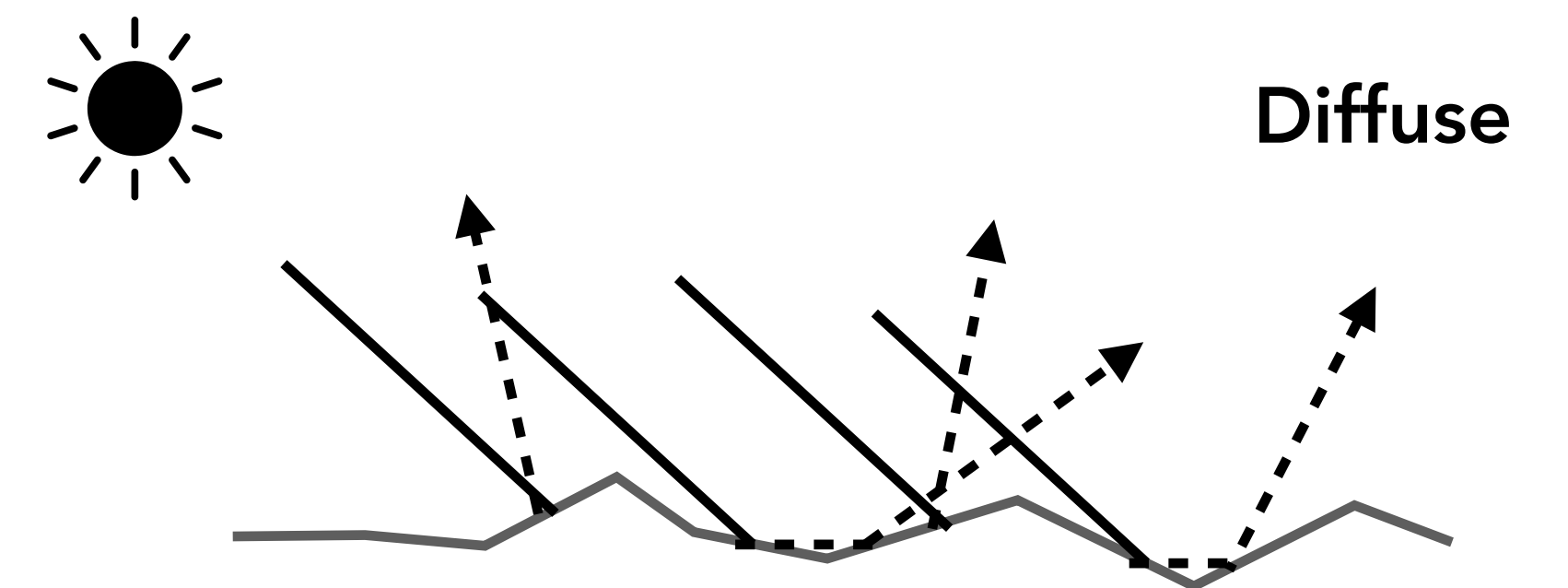
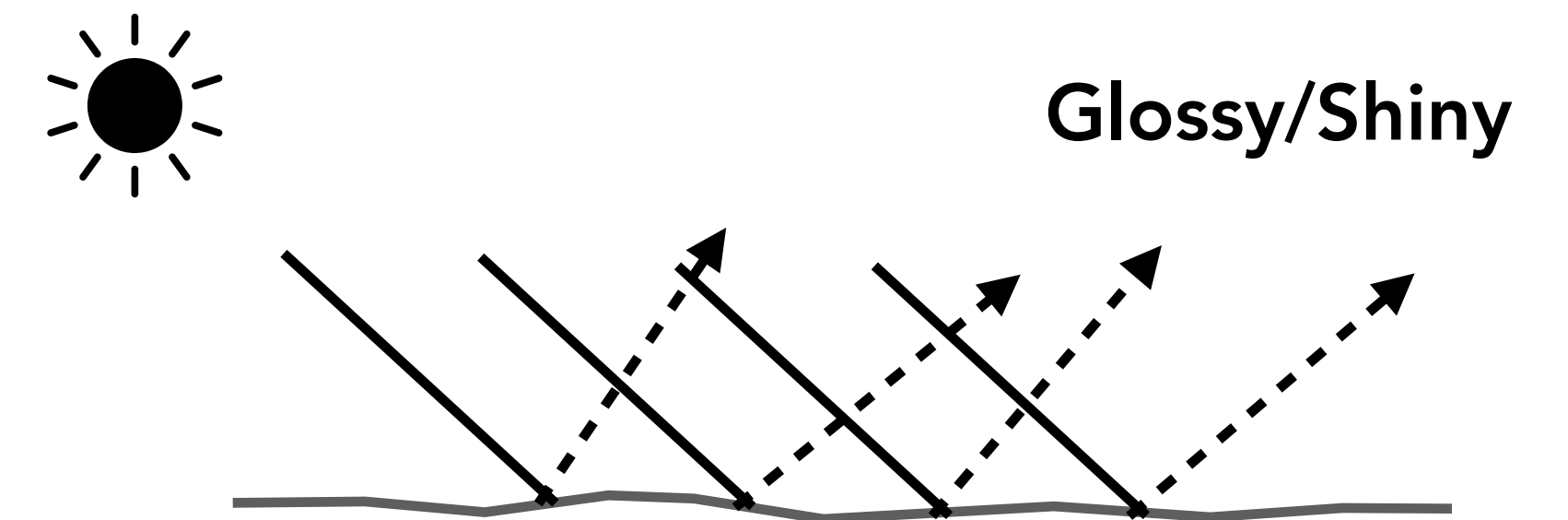
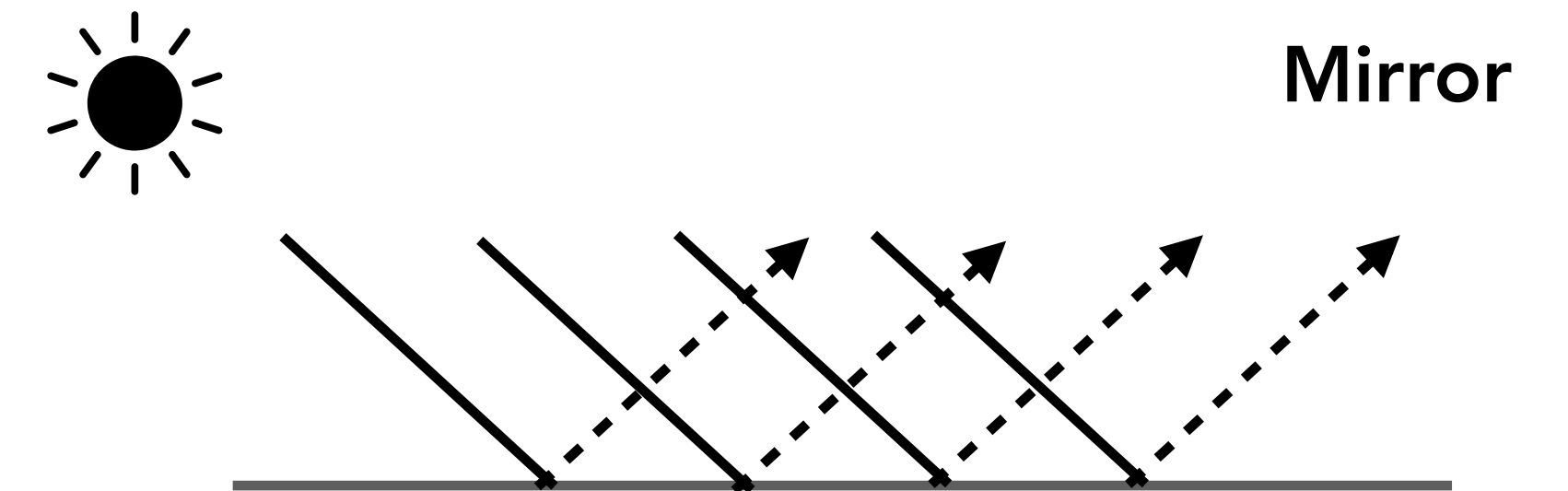




# Modeling Surface

Surface reflection is modeled by the “roughness” of the surface.

- Rough surfaces can be modeled as a collection of small mirrors or “*microfacets*”.





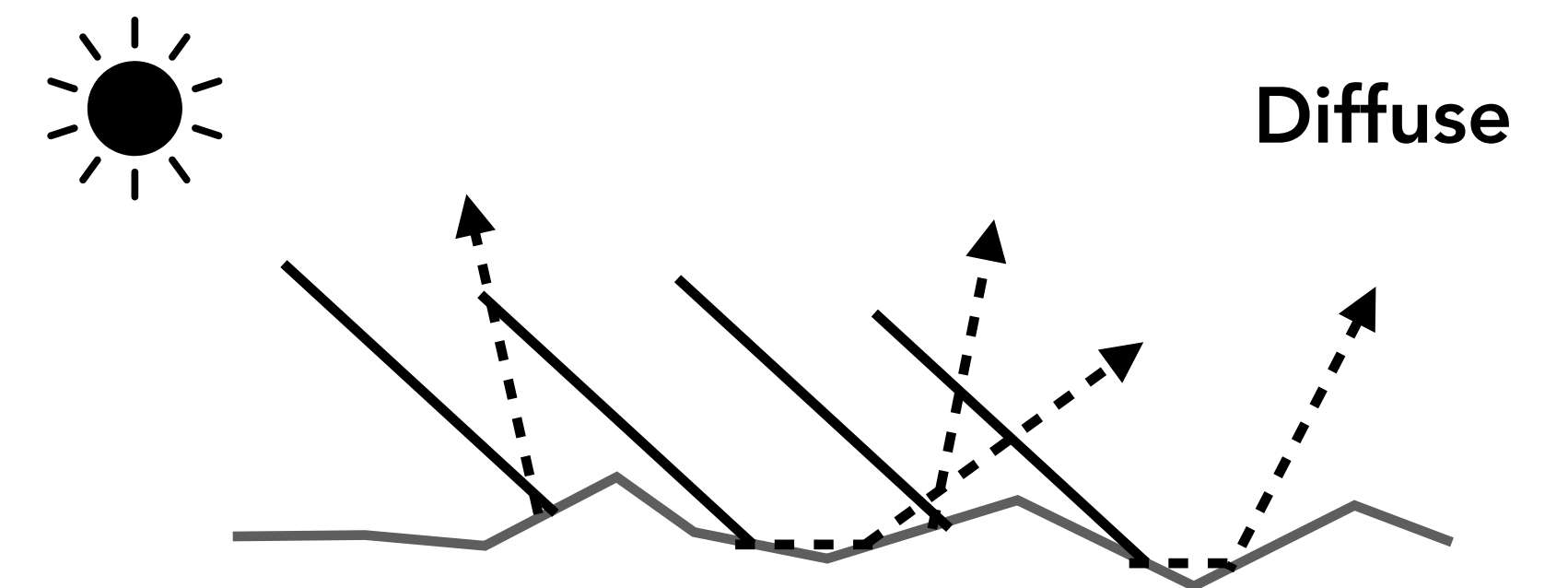
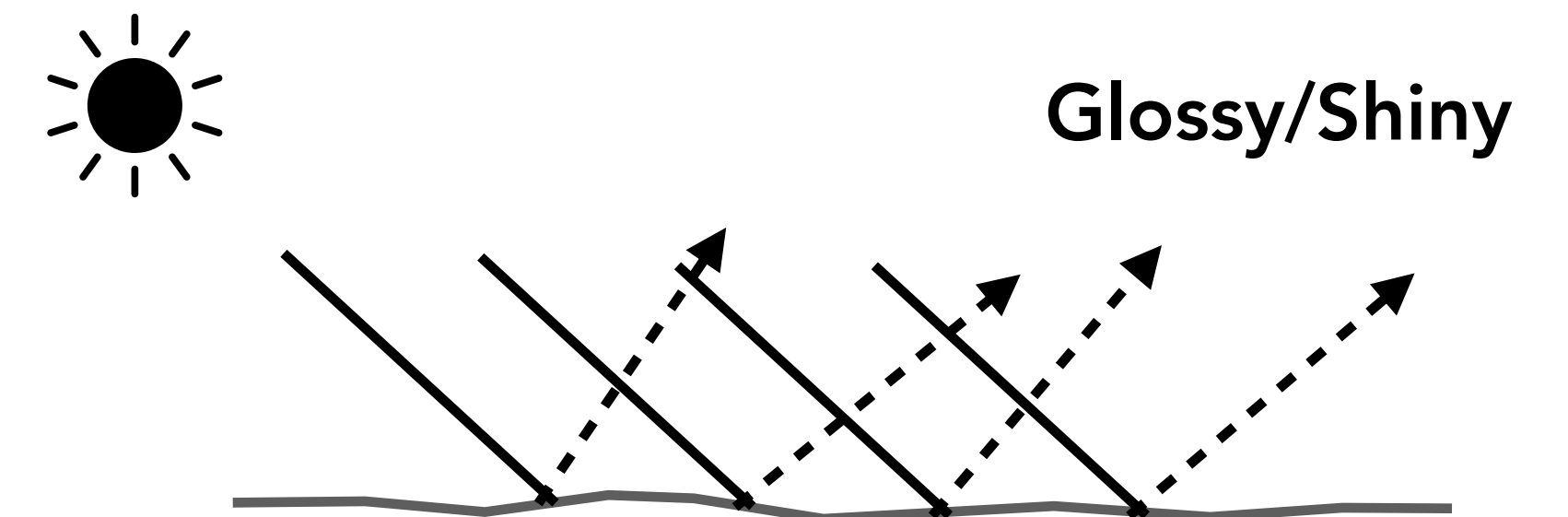
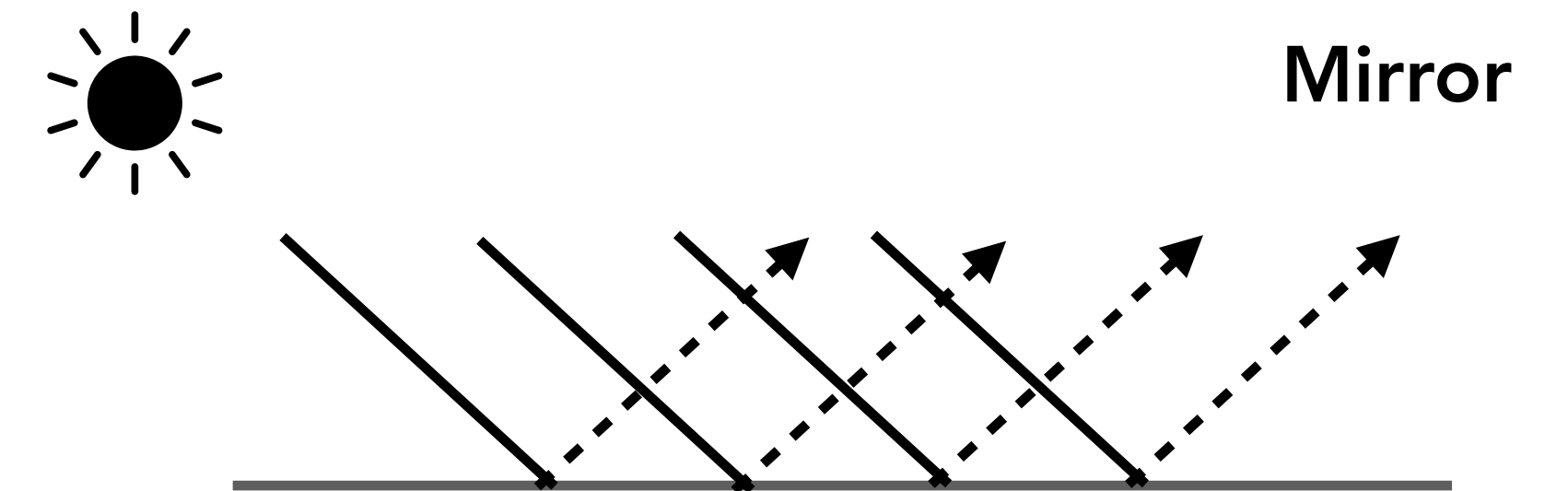
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Surface types:

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- Glossy (shiny). Microfacet orientations are similar but not the same.
- Diffuse (matte). Lights reflected multiple times between “micro-mirrors”. Result: reflecting evenly to all angles regardless incident angle.





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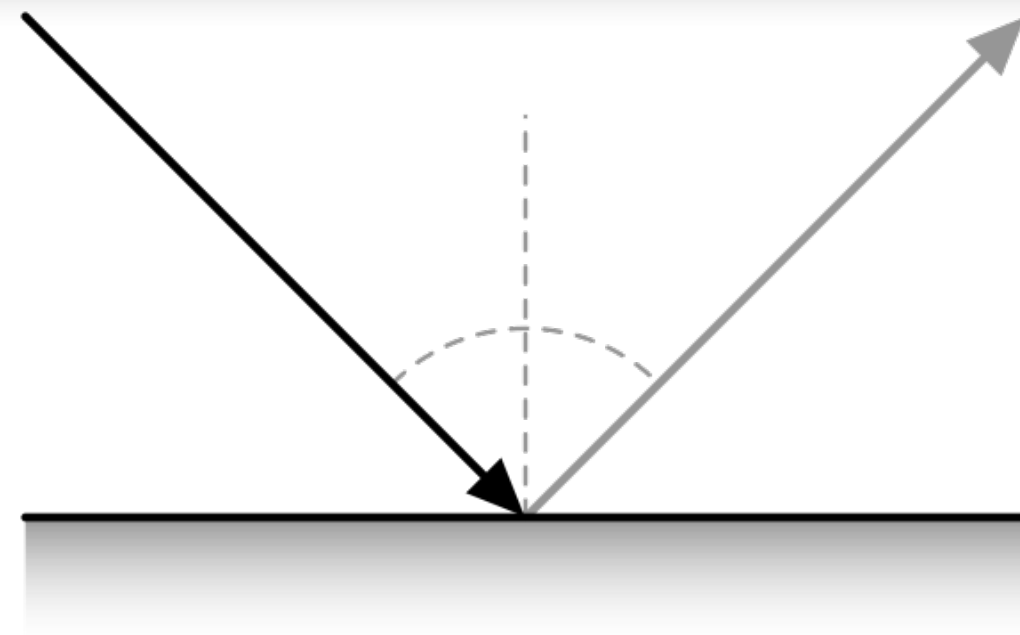
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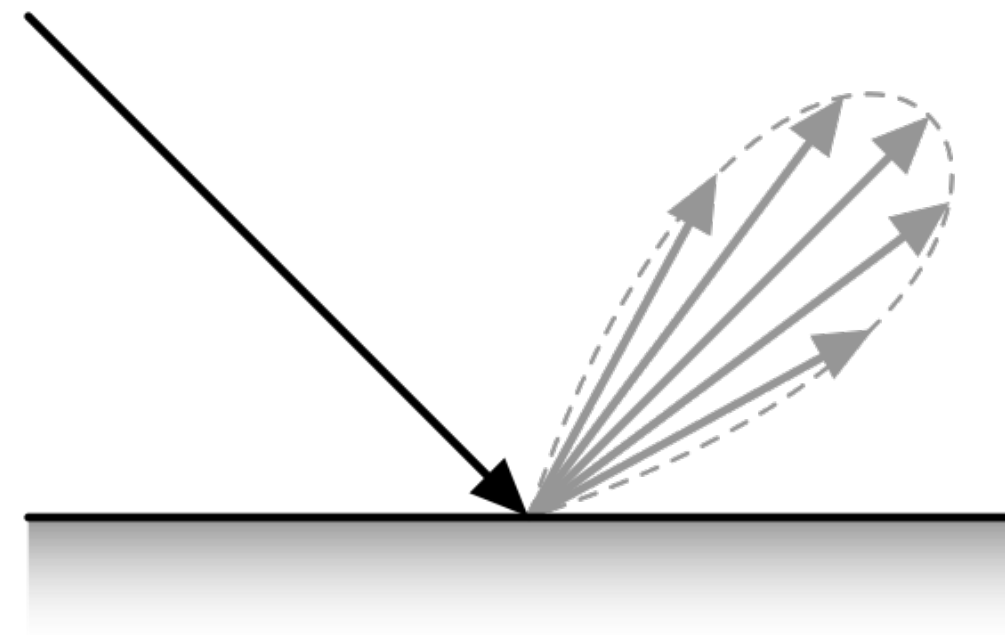


# Surface Types

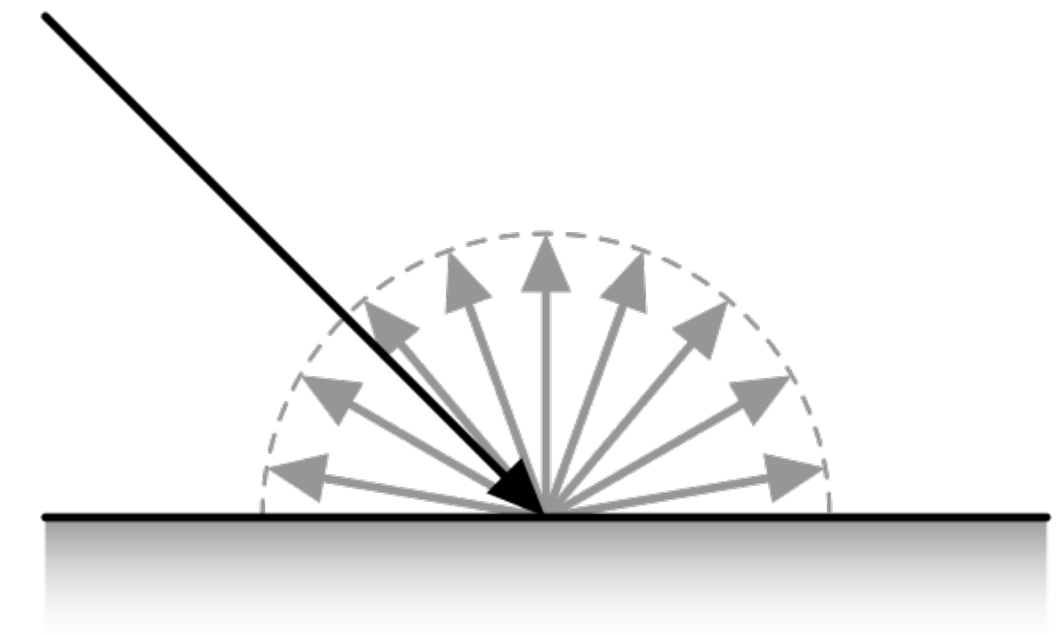
Mirror/Glass (Specular)



Glossy (Specular)

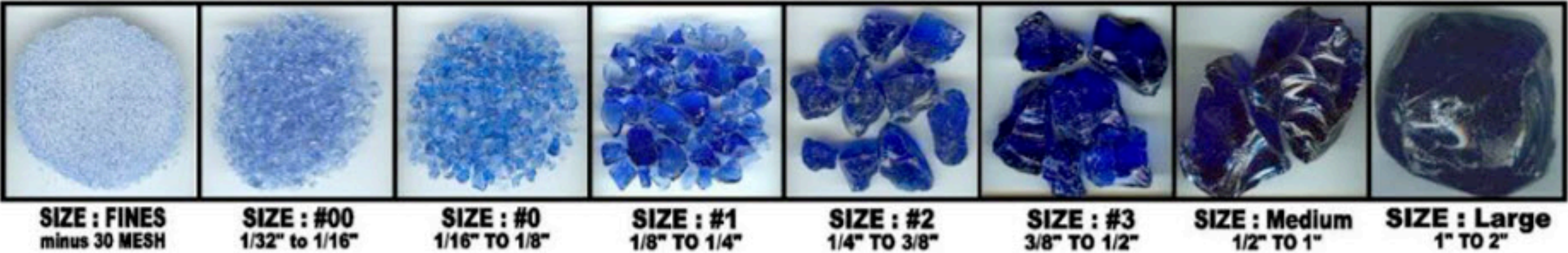


Diffusion (Not Specular)





# Puzzle







Modernist  
Encaustic

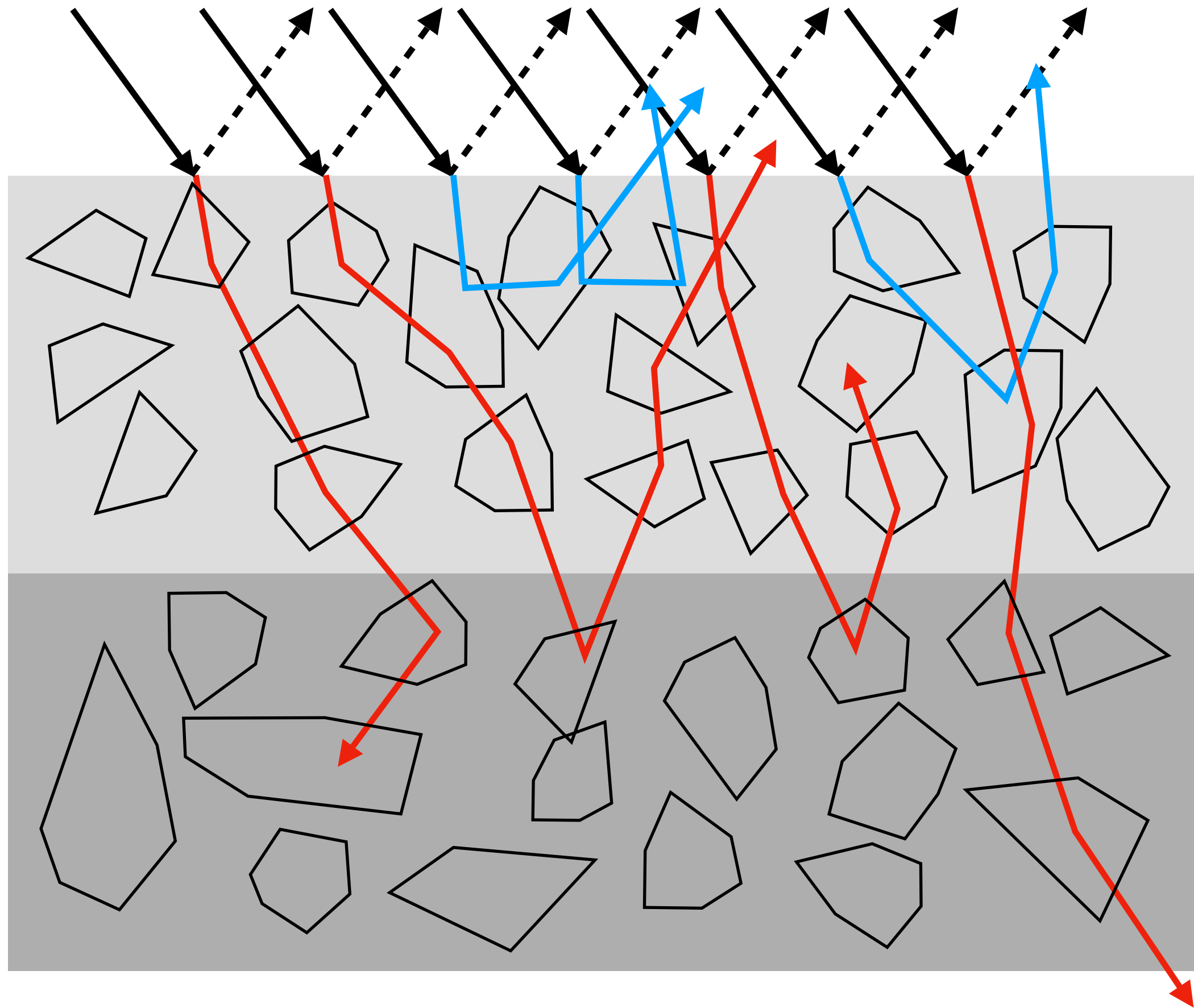
Titanium-rutile crystal & Rutile pigment



# Layering vs. Mixing

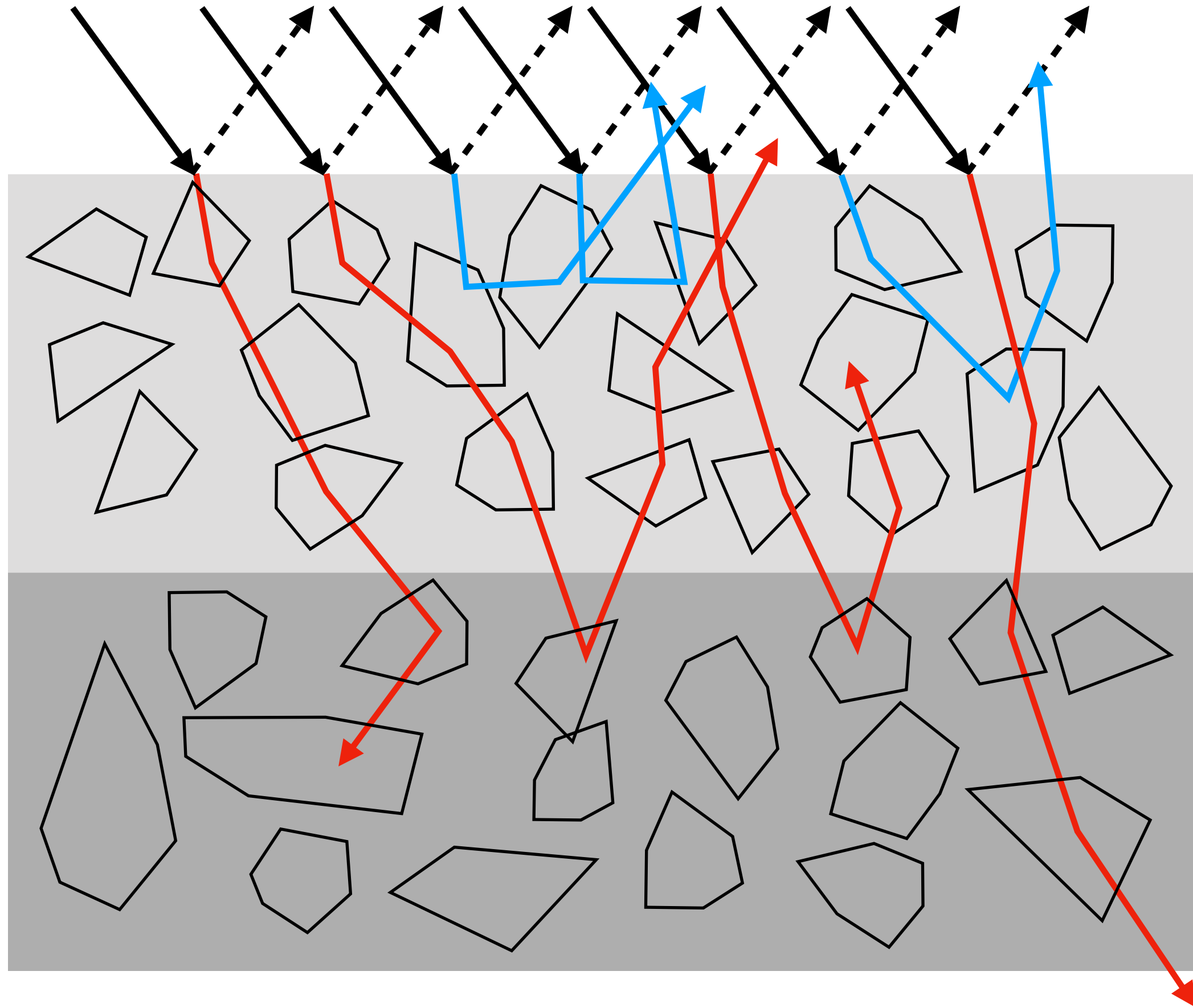


# Layering





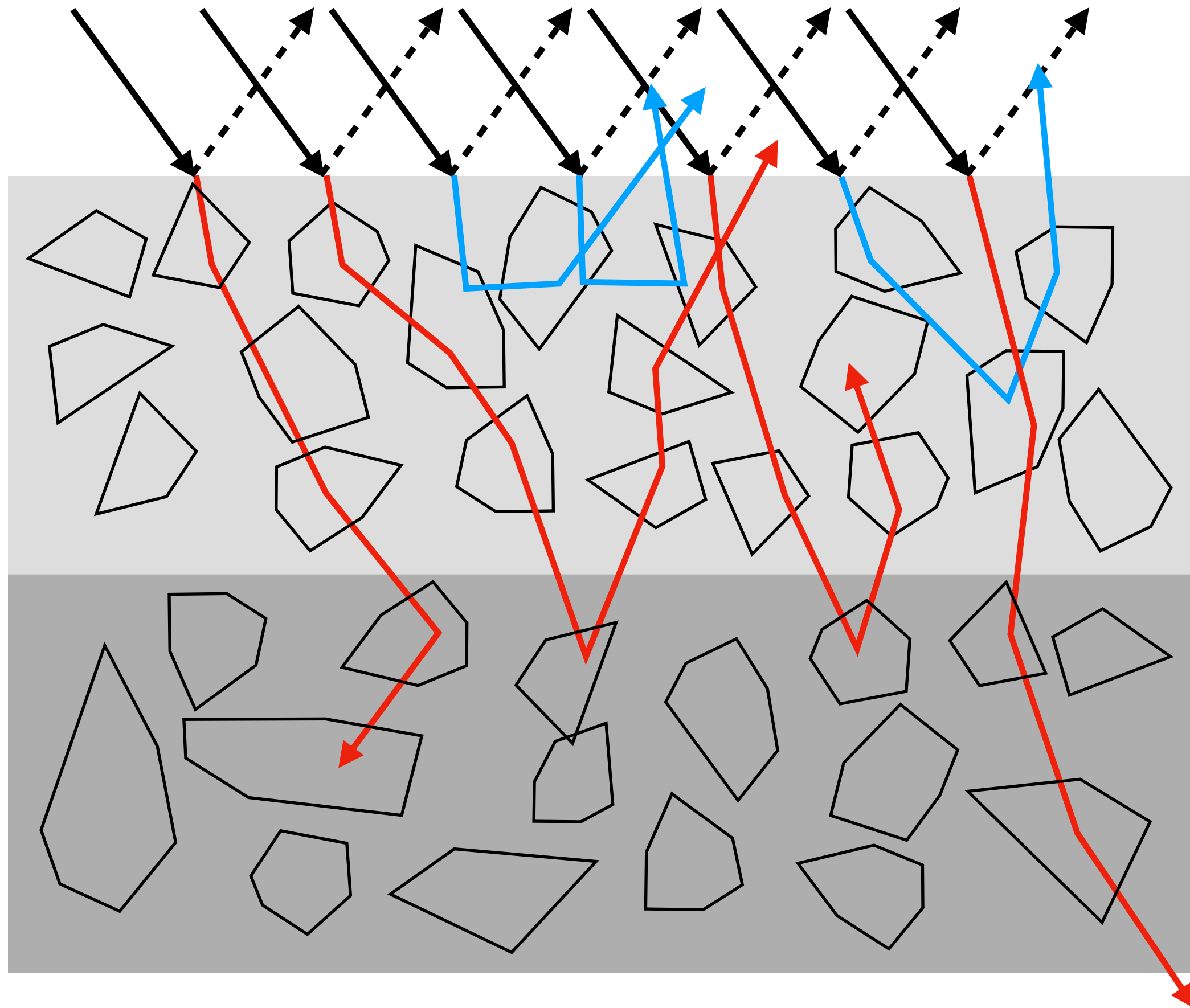
# Layering



$$R_1 = \frac{1 - R_g[a_1 - b_1 \coth(b_1 S_1 X_1)]}{a_1 - R_g + b_1 \coth(b_1 S_1 X_1)}$$



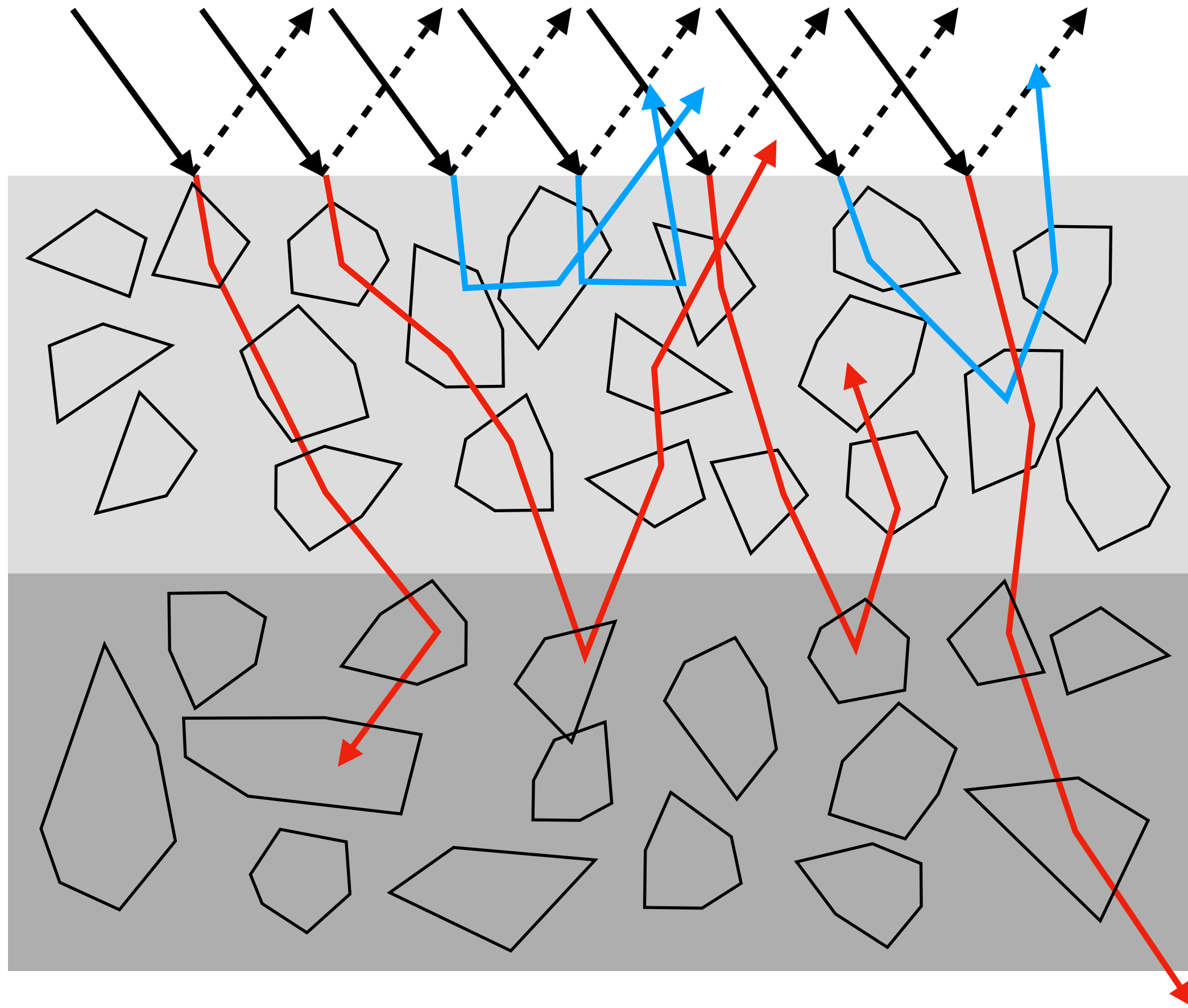
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# Layering



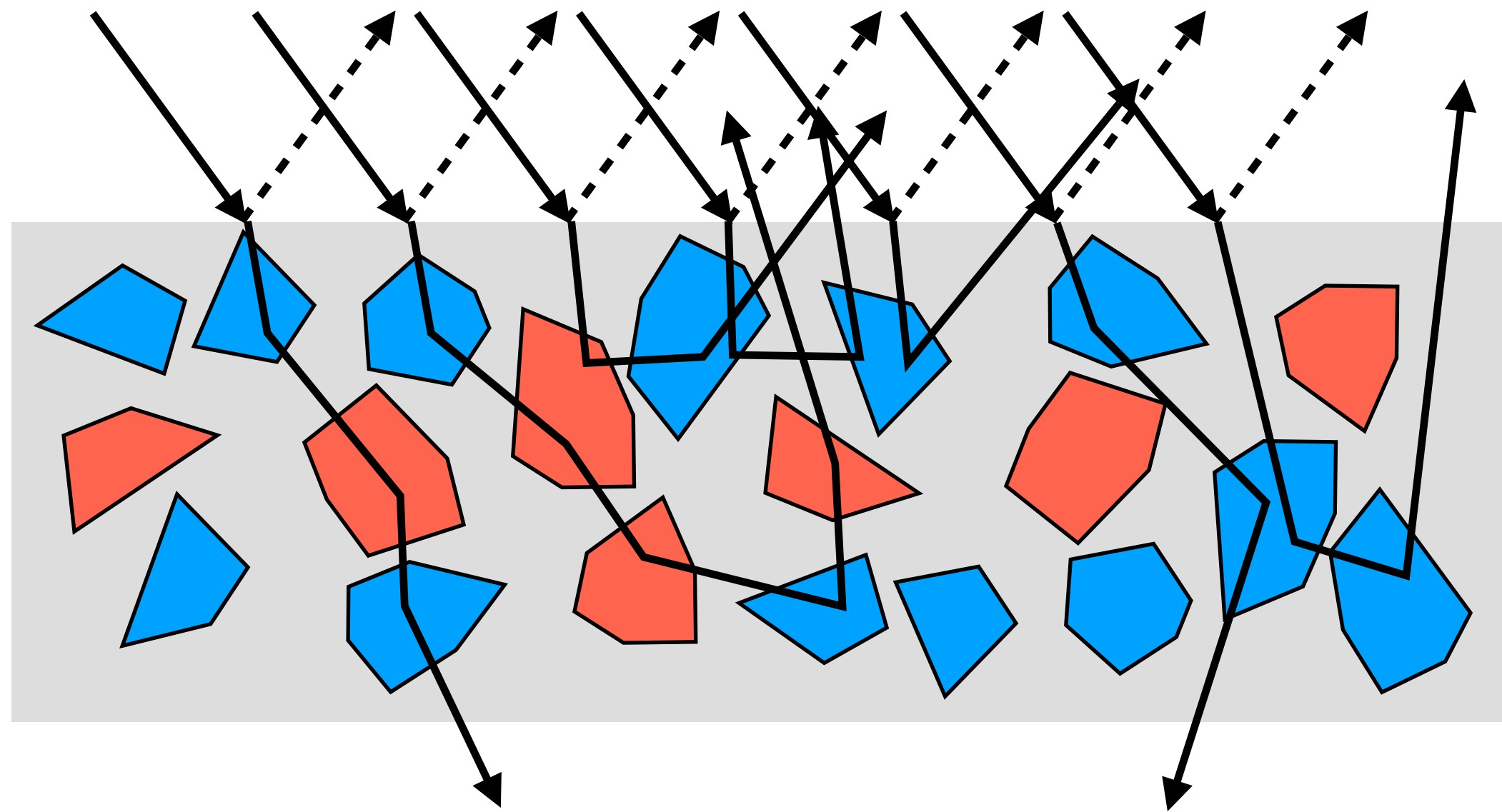
$$R_1 = \frac{1 - R_g[a_1 - b_1 \coth(b_1 S_1 X_1)]}{a_1 - R_g + b_1 \coth(b_1 S_1 X_1)}$$

$$R_2 = \frac{1}{a_2 + b_2 \coth(b_2 S_2 X_2)}$$



# Mixing

$$R_X = \frac{1 - R_g[a - b \coth(bSX)]}{a - R_g + b \coth(bSX)} \quad a = \frac{K + S}{S} \quad b = \sqrt{a^2 - 1} \quad \coth x = \frac{e^{2x} + 1}{e^{2x} - 1}$$



$$K = \sum_i K_i c_i$$

Relative concentration of each pigment

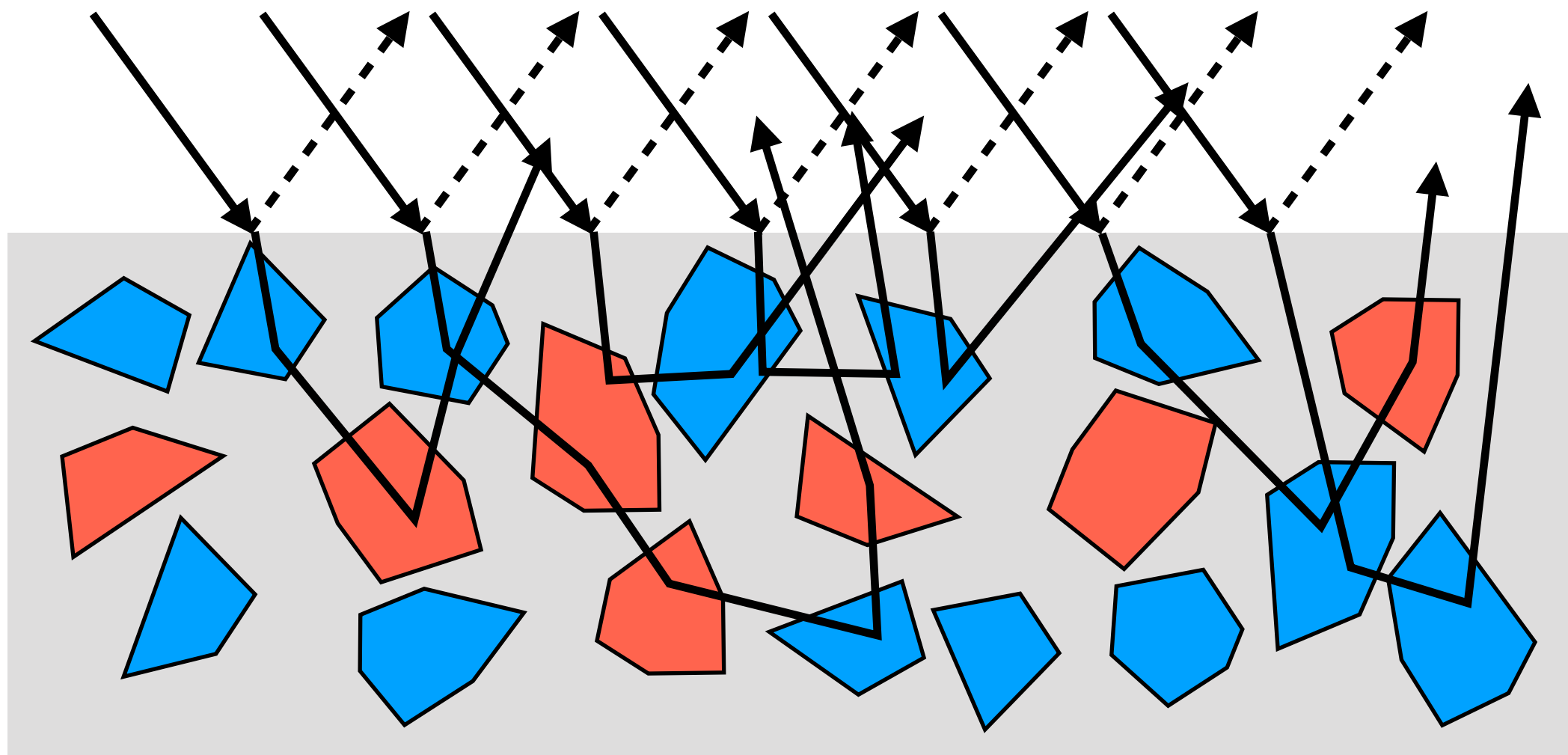
$$S = \sum_i S_i c_i$$

Absorption and scattering coefficients of each pigment



# Mixing When Hiding is Complete

$$\frac{K}{S} = \frac{\sum_i K_i c_i}{\sum_i S_i c_i} = \frac{(1 - R_\infty)^2}{2R_\infty}$$



This set-up is commonly used to determine the K/S coefficients of different pigment.

- Based on reflectance measurement.
- See [this](#) paper.

K/S can also be theoretically derived, but since K-M is an empirical approximation, fitting observed data is better.



# Layering == Mixing For Transparent Materials

Transparent materials do not scatter light and only absorb lights. They only take photons out.

It's not important whether the pigments are mixed or layered, i.e., where the pigment particles are. What matters is just the total amount of particles.

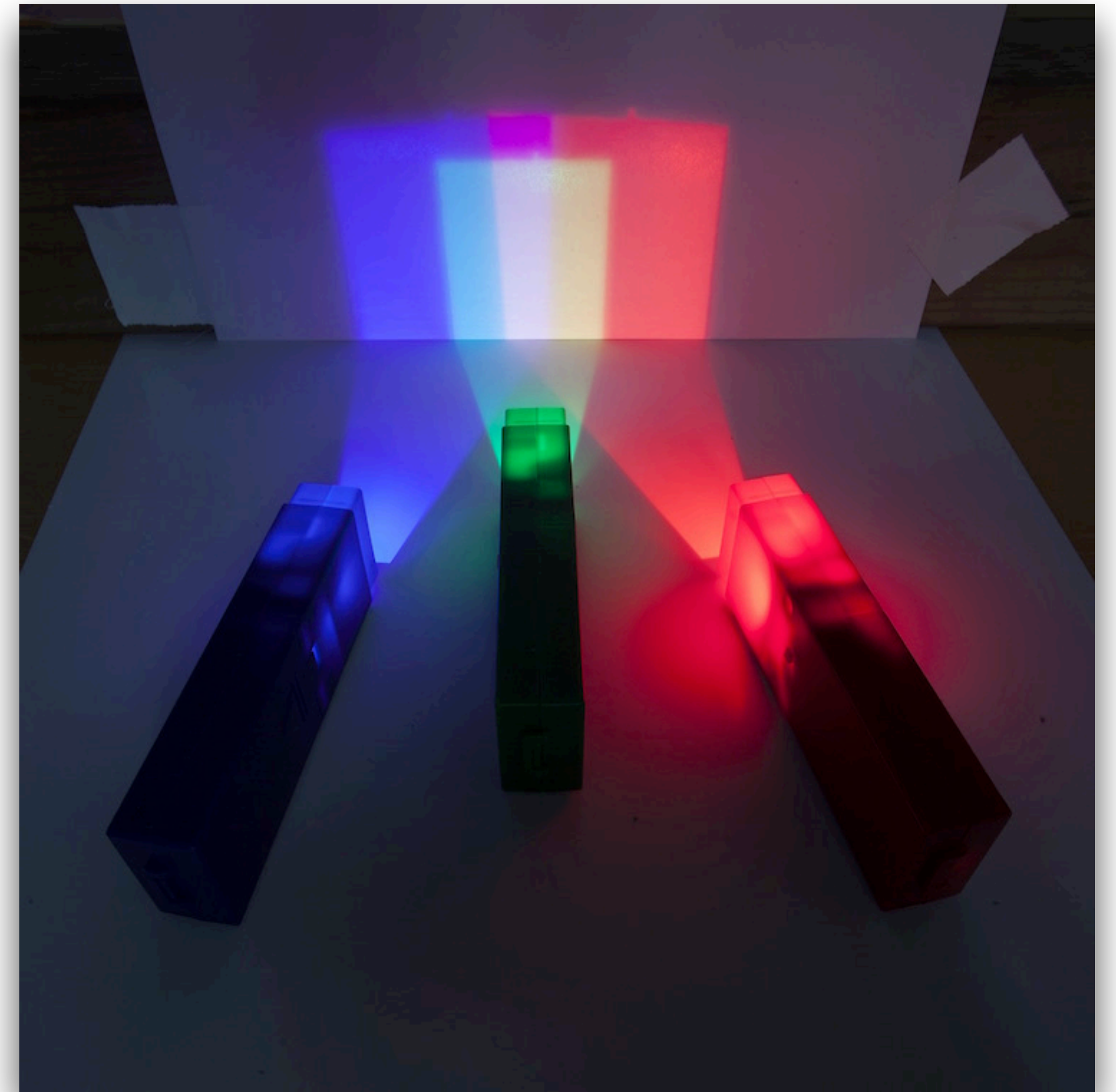
- assuming that the absorption behaviors of individual particles are independent.



# Light Colors are Additive

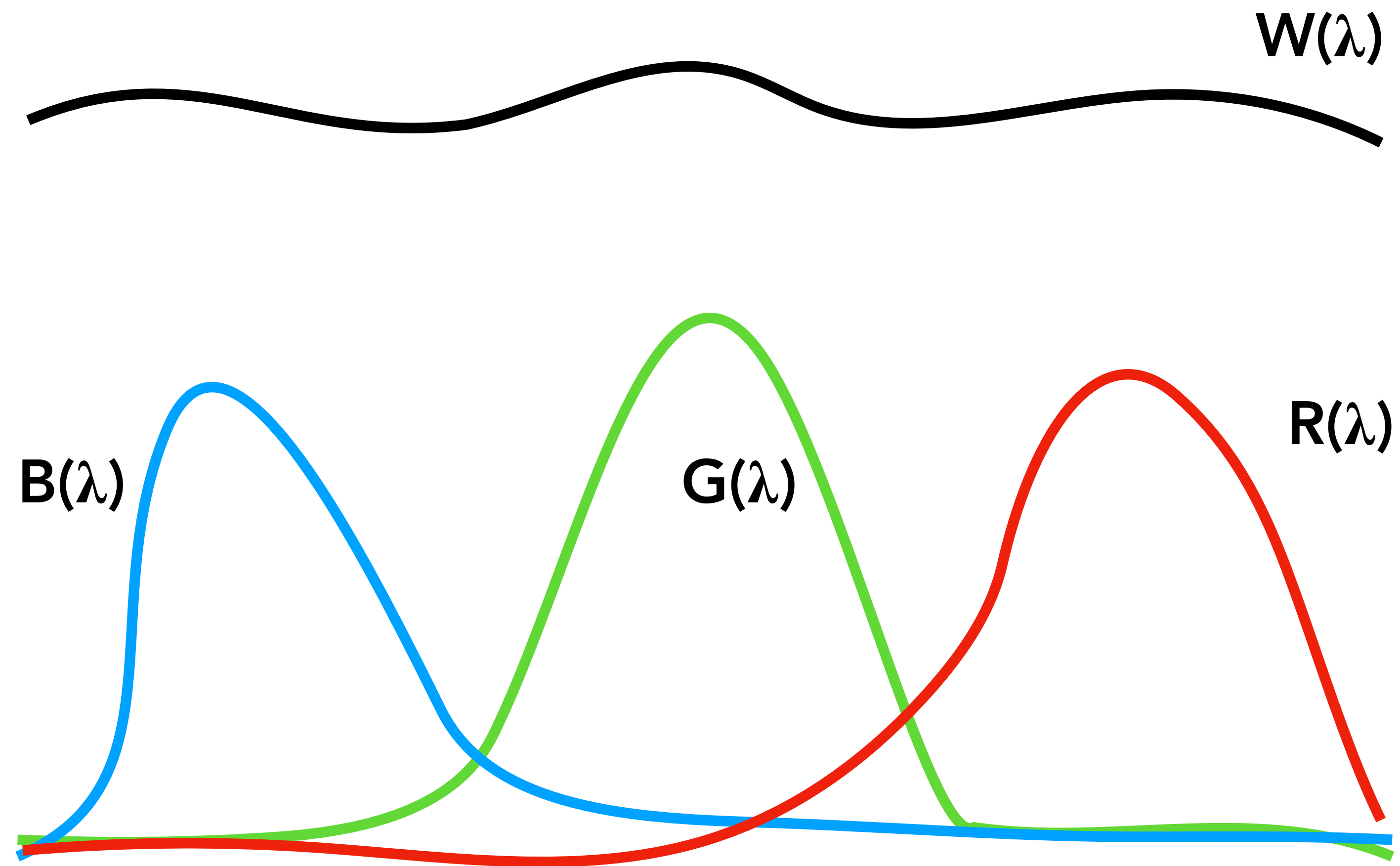
## Mixing lights:

- If we have three lights whose *linear* RGB values are  $[R_1, G_1, B_1]$ ,  $[R_2, G_2, B_2]$ , and  $[R_3, G_3, B_3]$ , when we mix them the mixed light's color is  $[R_1 + R_2 + R_3, G_1 + G_2 + G_3, B_1 + B_2 + B_3]$ .
- Can you prove it?
- The key is that the spectral power distribution of the mixed light is the sum of that of each light. From SPD to RGB color is linear.



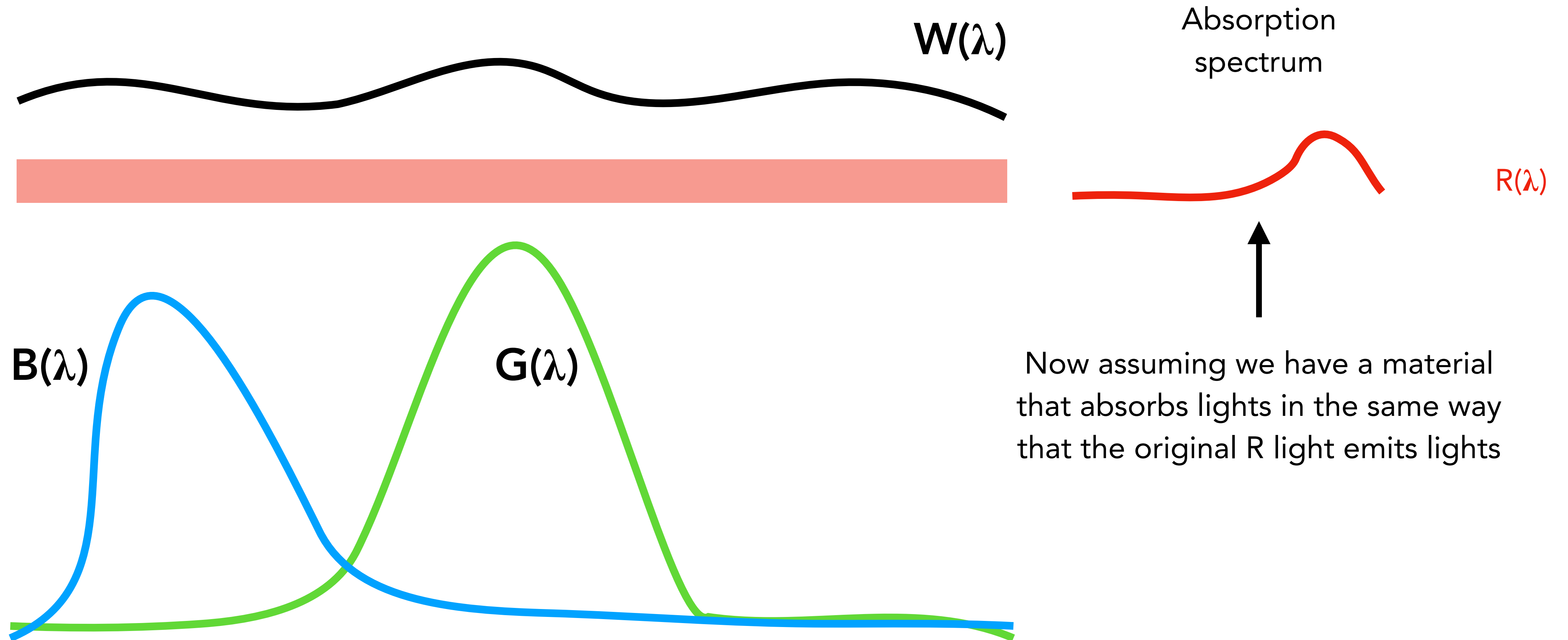


# What is Subtractive Color Space Really?



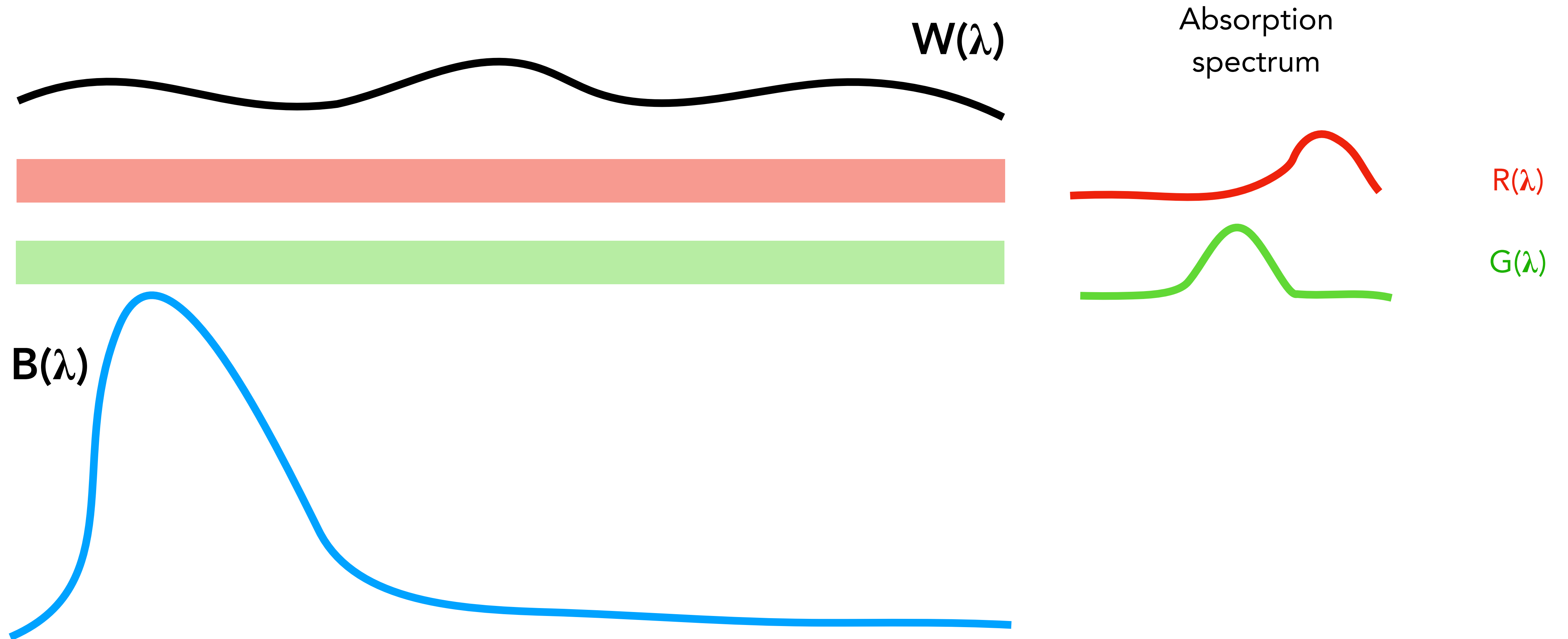


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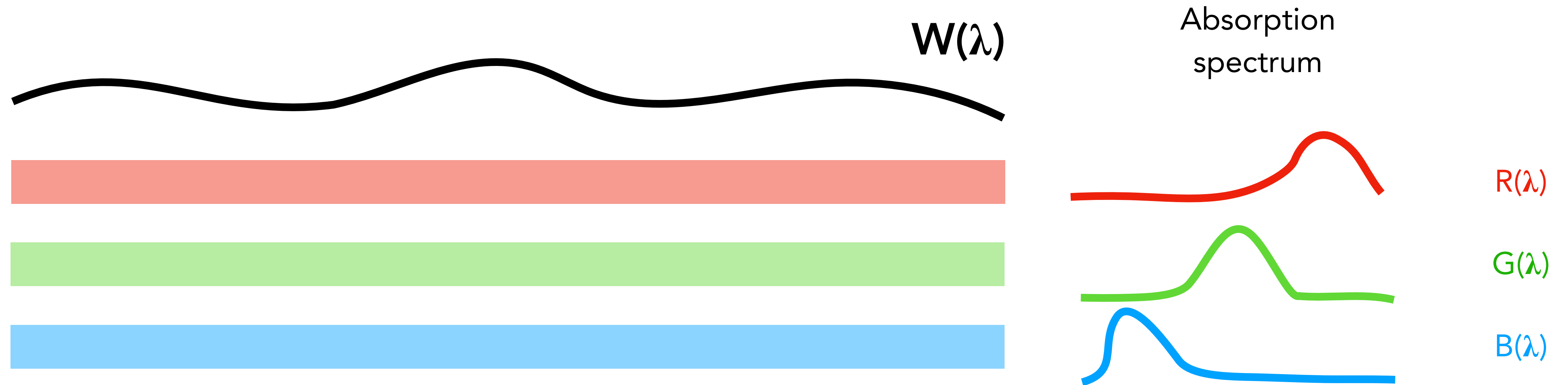


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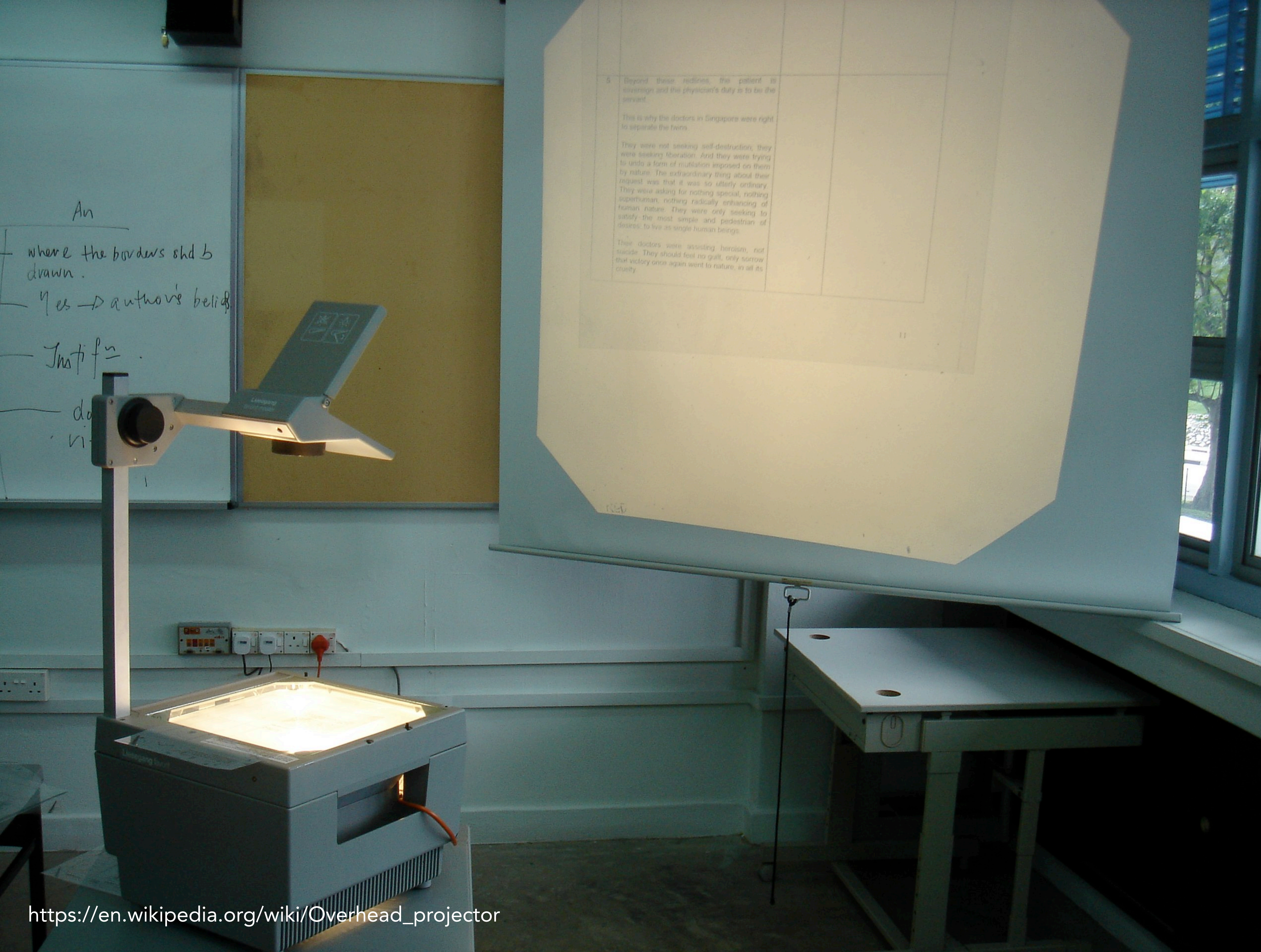
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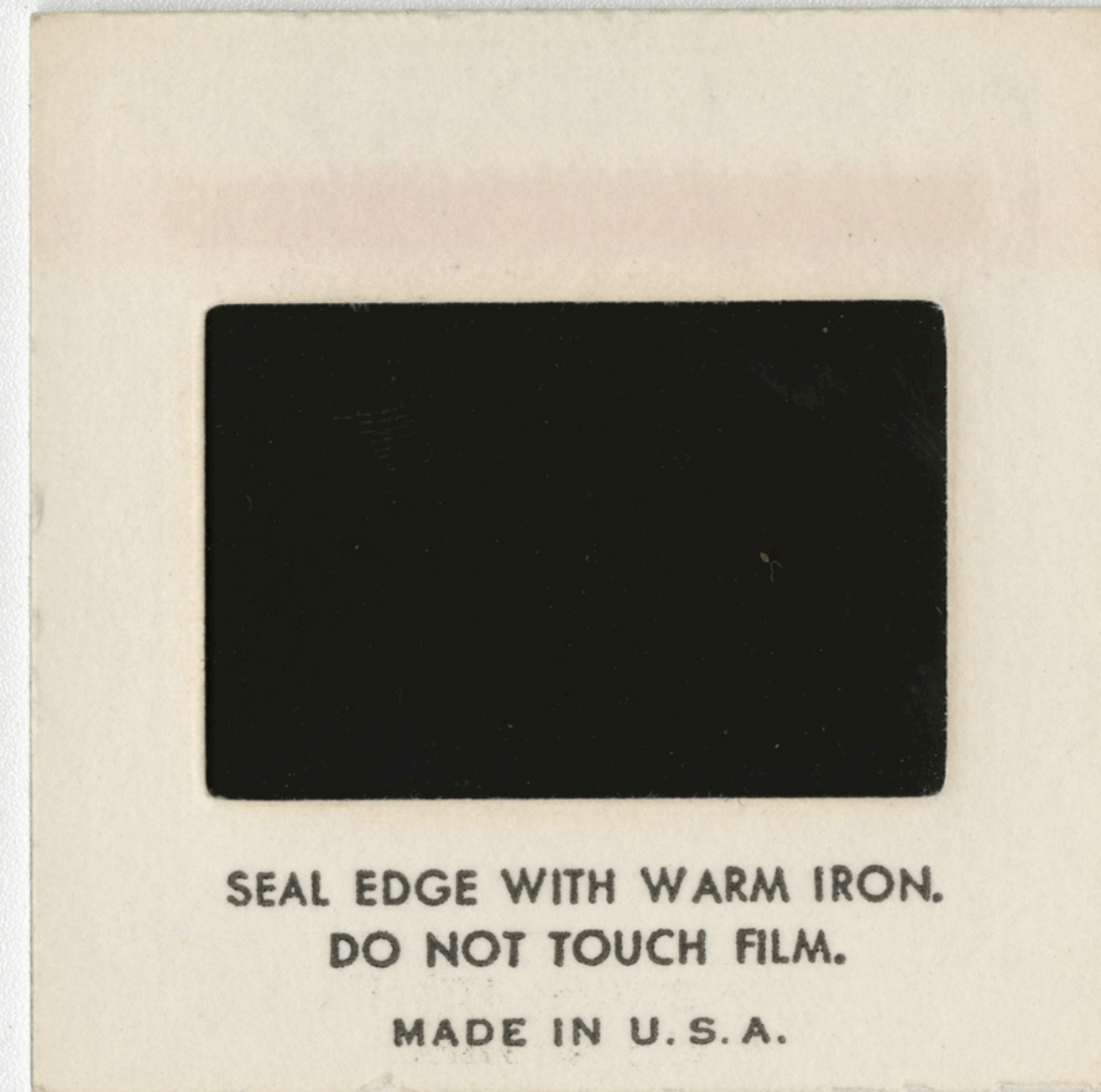
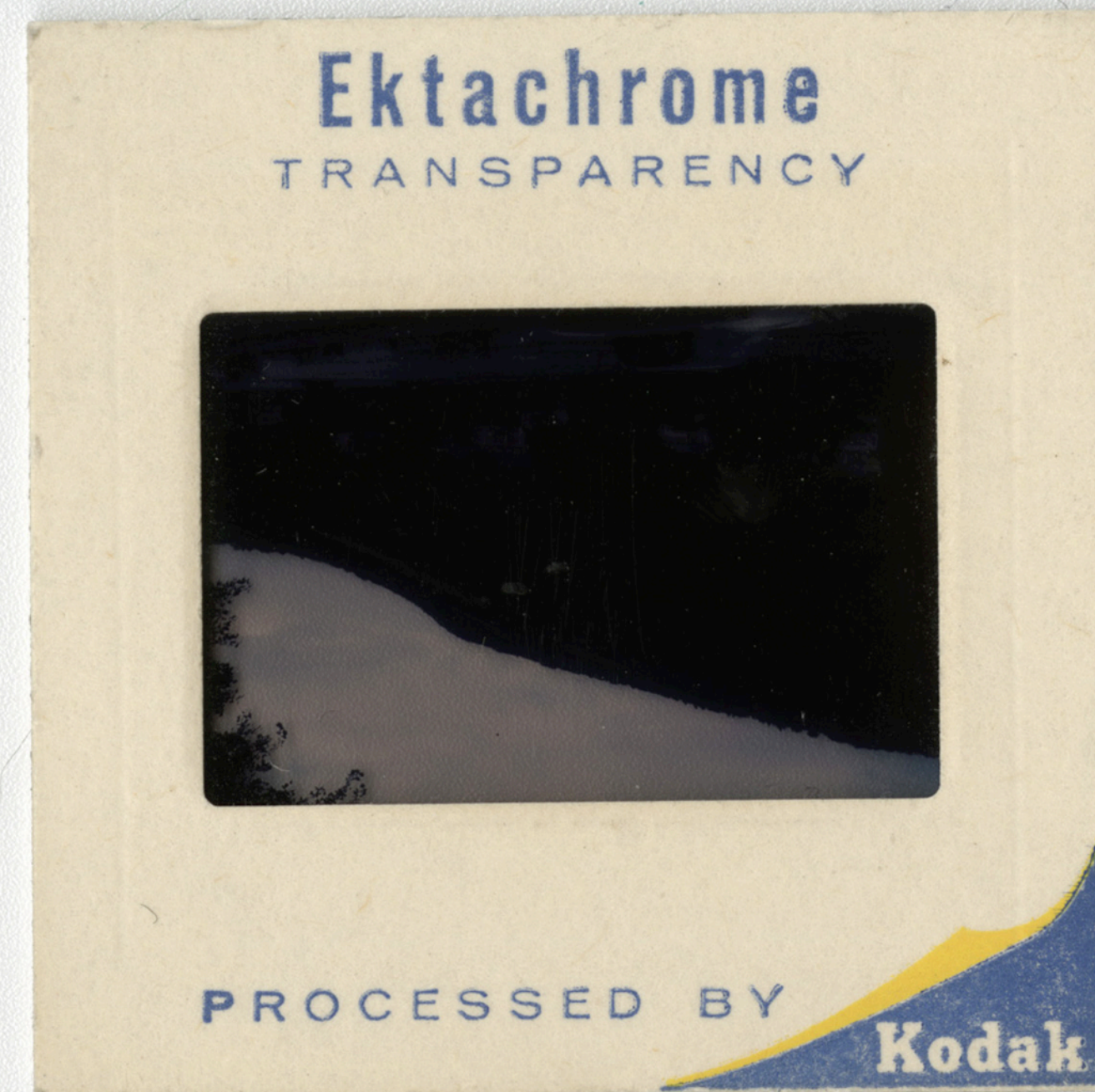
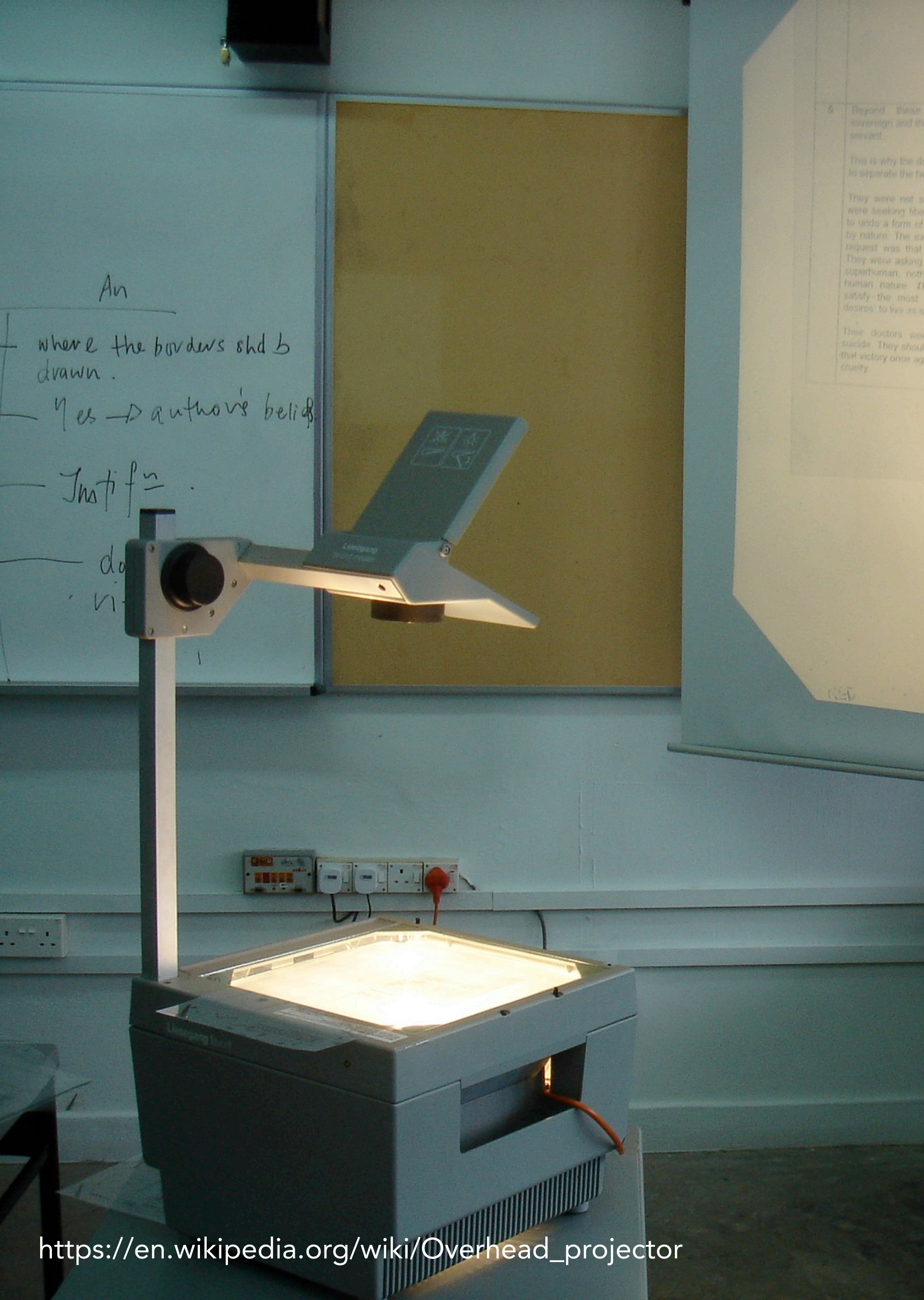
Subtractive color space applies **ONLY** to strictly transparent material.

- Nearly perfect for slides/transparencies printed with dyes, pretty good for traditional photographic prints on photographic media.
- Rather poor for today's digital color printers, because they also **scatter** lights.











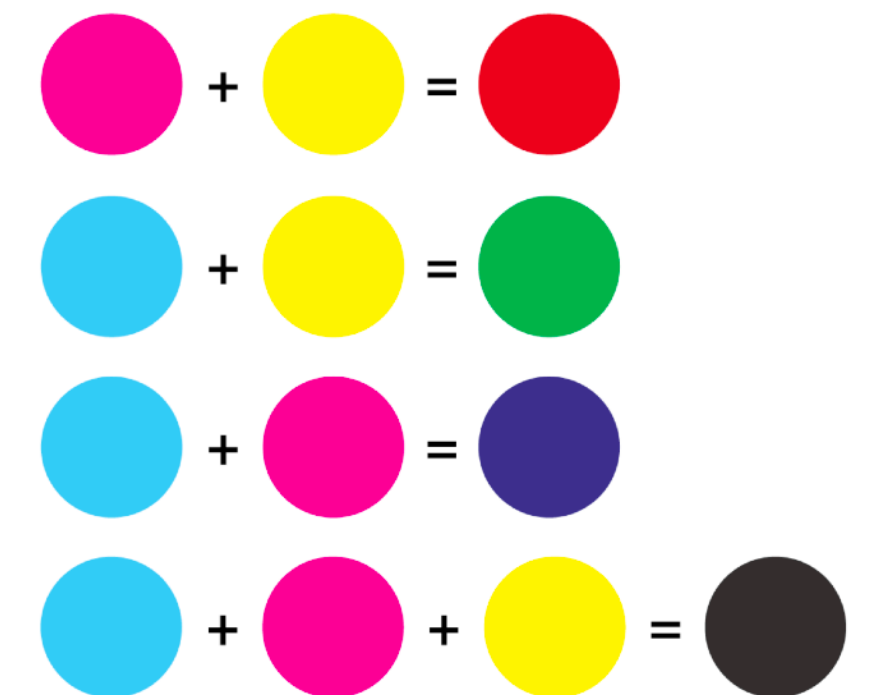
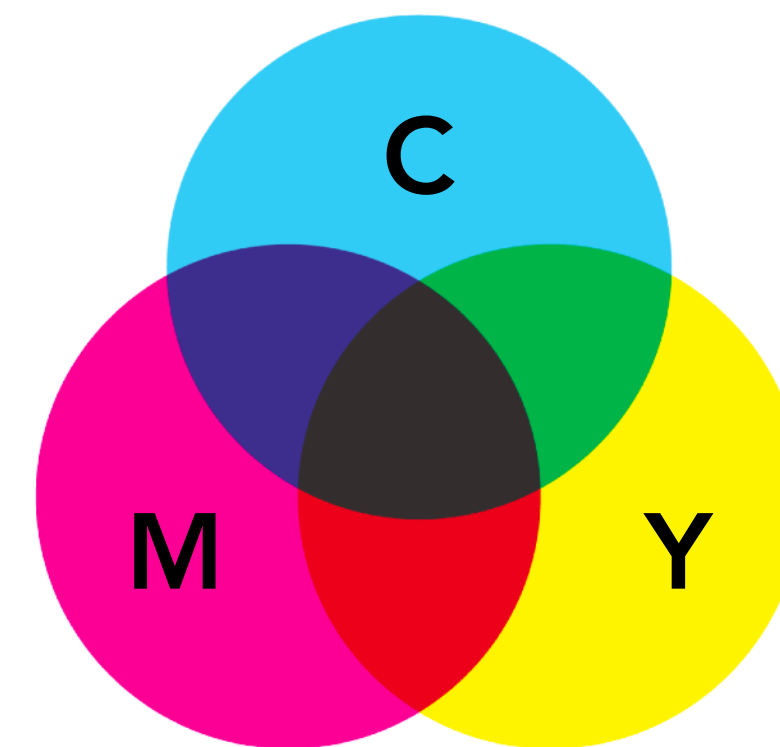
# Is Mixing Materials Subtractive?

Conventional wisdom is that subtractive color model should be used when discussing paint/material mixing. Really?

Cyan + Yellow = Green because:

- Cyan material absorbs Red light; Yellow material absorbs Blue light
- Cyan and Yellow mixture would then absorb both Red and Blue, leaving only Green

By this logic, Blue + Yellow = Black/Gray













# Mixing Materials is (Generally) NOT Subtractive!

Mixing paints: does subtractive color space apply? A few issues.

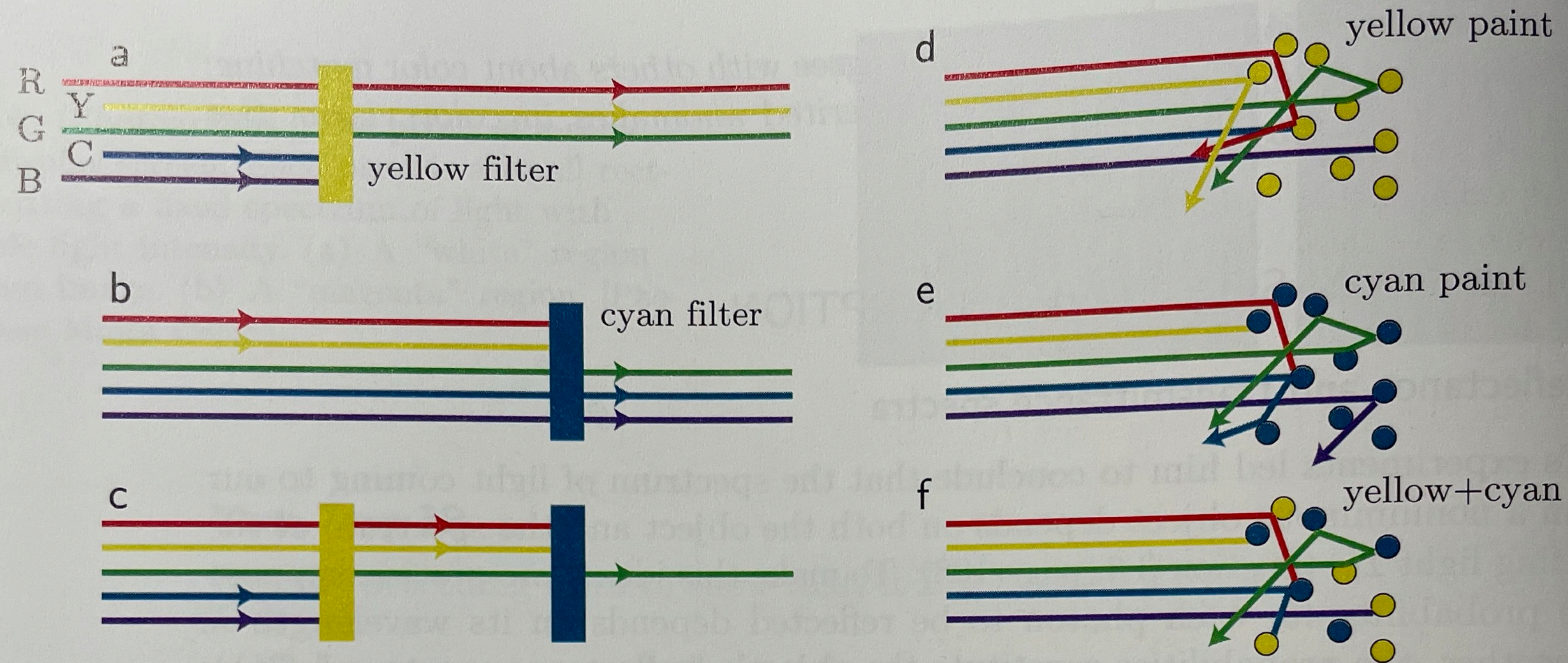
- First of all, a paint's color depends on illumination. So it's meaningless to discuss the color of a paint (or any non-emitting material) without illumination information.
- But more importantly paints also **scatter** lights.

To calculate the color of a paint mixture, we should:

- Get the spectral scattering and absorption coefficient for each pigment
- Calculate the reflectance spectrum of the mixture using the K-M model
- Use the spectrum of the illumination to calculate the R, G, B values of the mixture



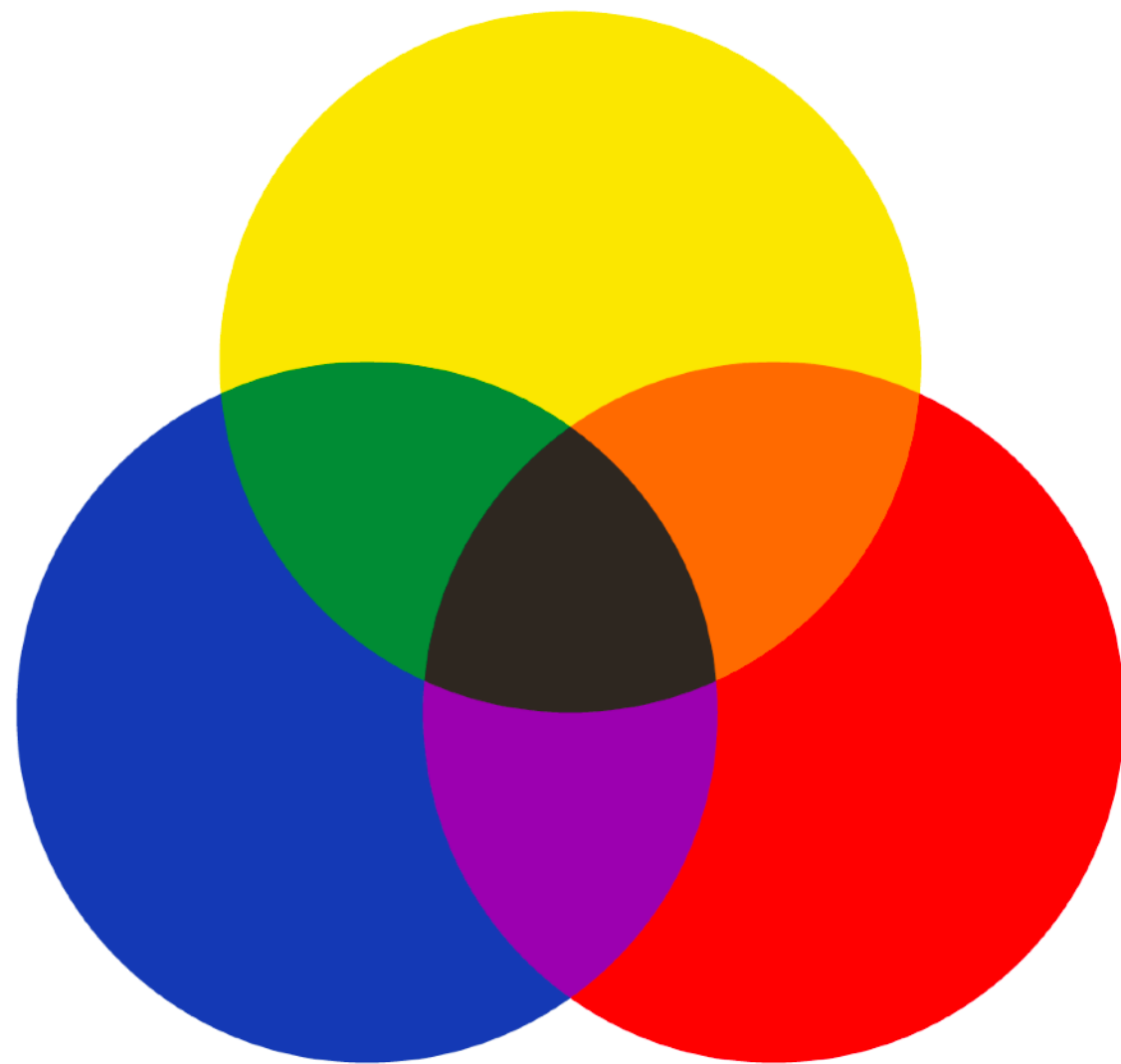
# True or False?



**Figure 3.7:** [Schematics.] **Subtractive color combination.** (a) A “yellow” filter transmits a band of wavelengths including spectral green, yellow, and red. It blocks other wavelengths, including those corresponding to blue and cyan. (b) Similarly, a “cyan” filter instead blocks red and yellow light. (c) Placing these two filters in succession blocks everything except green light. (d,e,f) Yellow and cyan chromophores are distributed within the fibers of a piece of paper. Multiple scattering of light combines their effects, similarly to (c).



# Artists Know Better



The **RYB** model used by artists





# Add White to Adjust Saturation Levels

Skillful painters usually use just three primary paints plus white to create different saturation levels



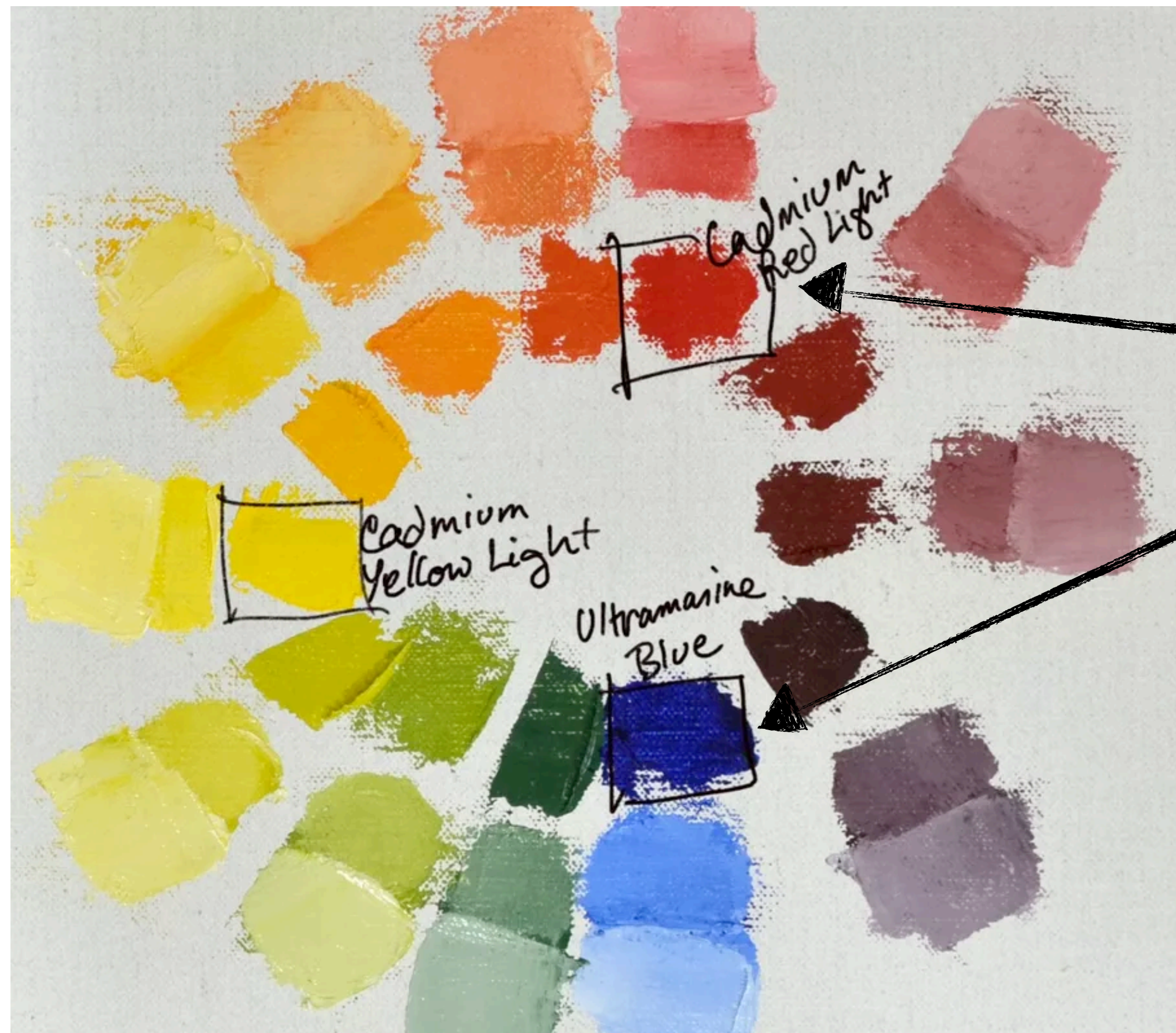


# Masstone vs. Tint





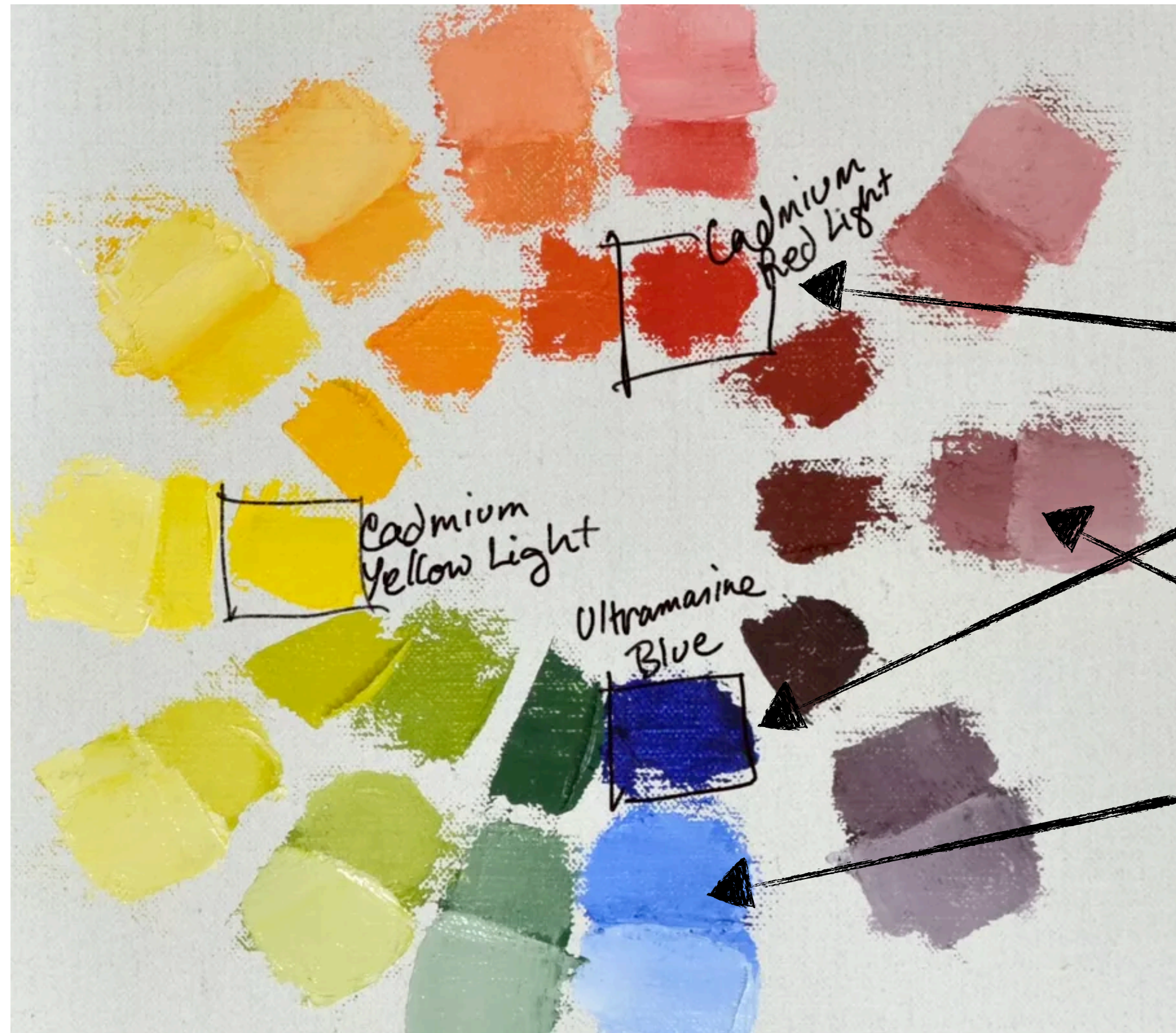
# Masstone vs. Tint



Masstone: pure color



# Masstone vs. Tint

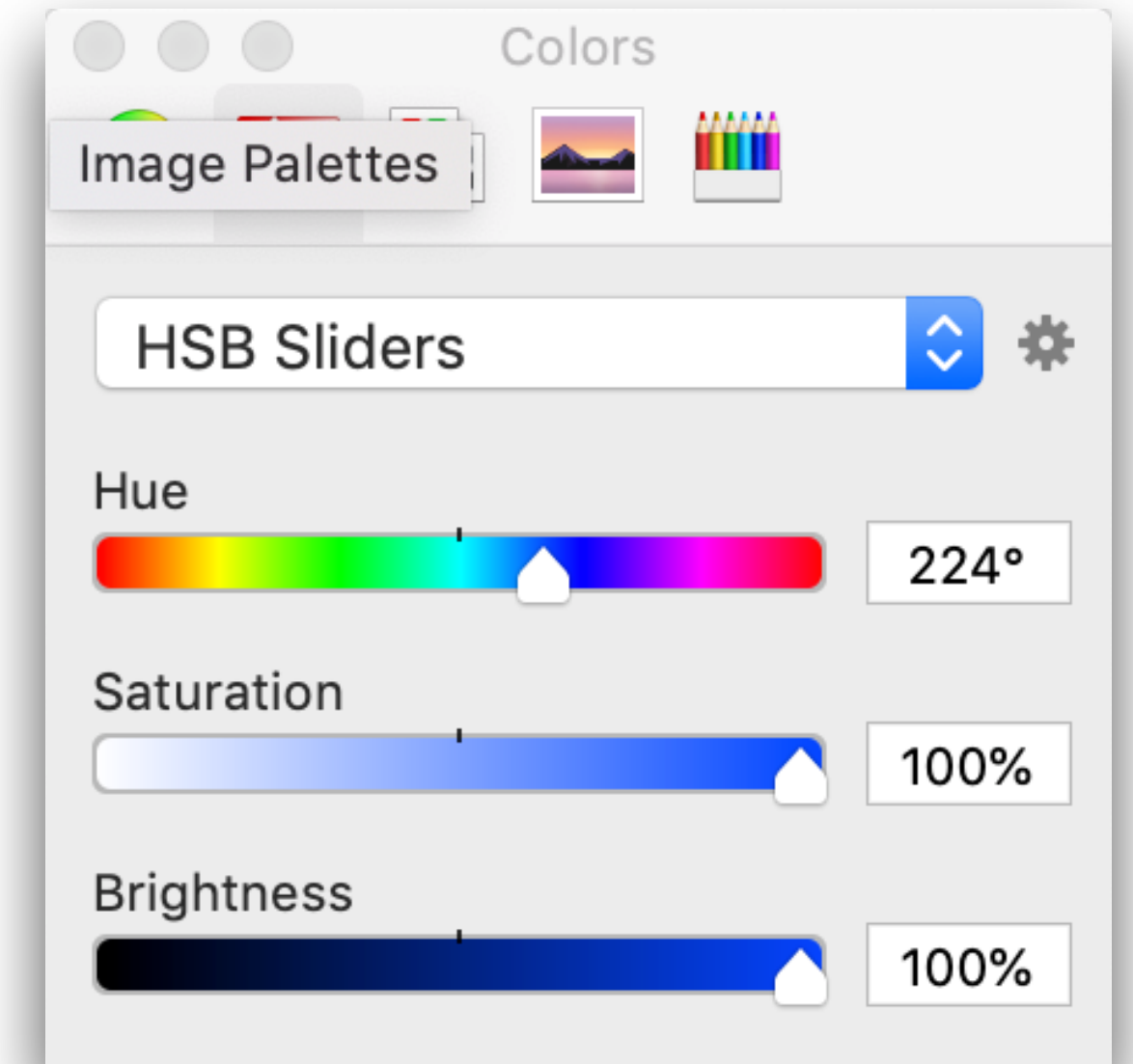
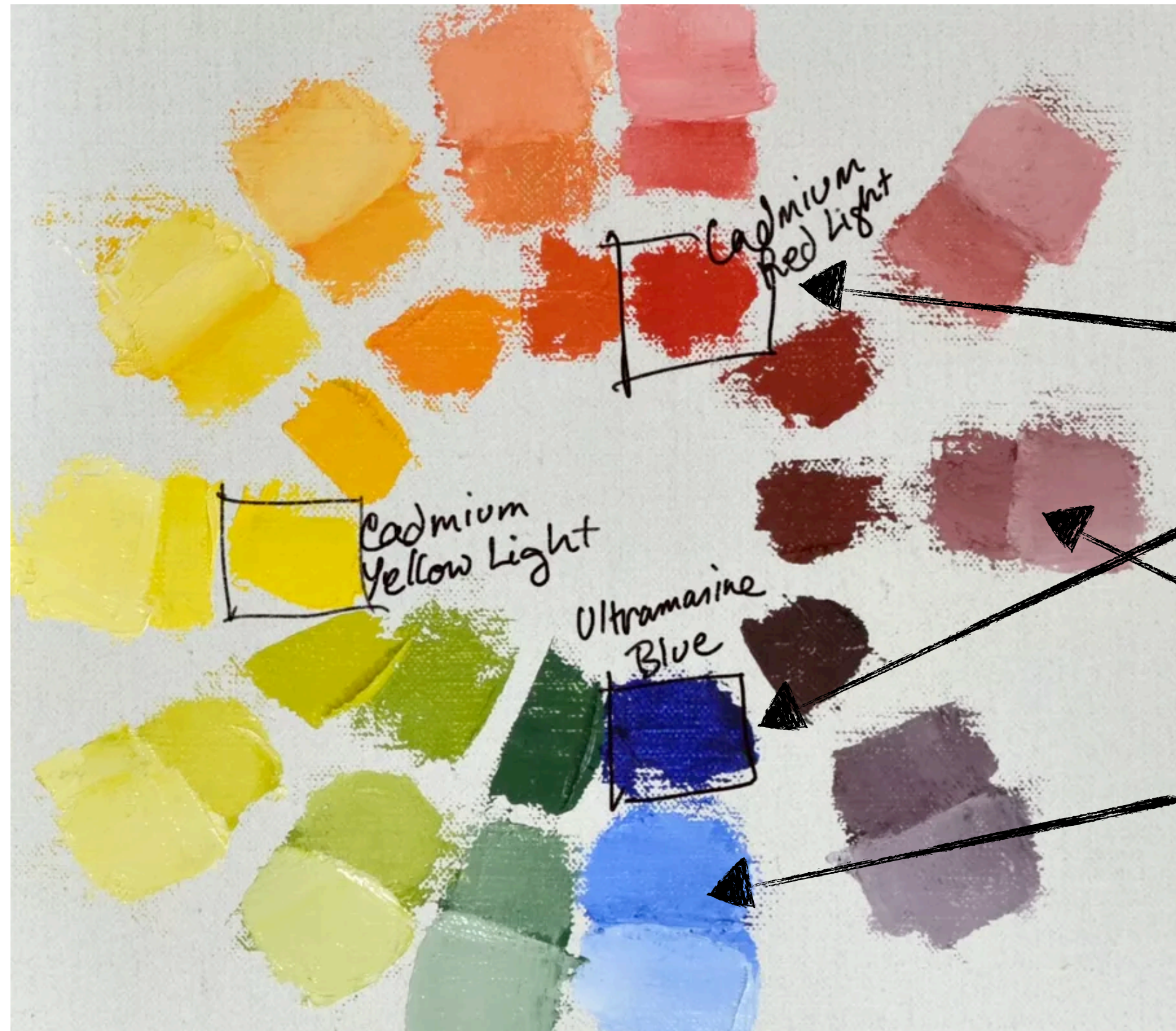


**Masstone:** pure color

**Tint:** mixing pure color with white (desaturation)



# Masstone vs. Tint

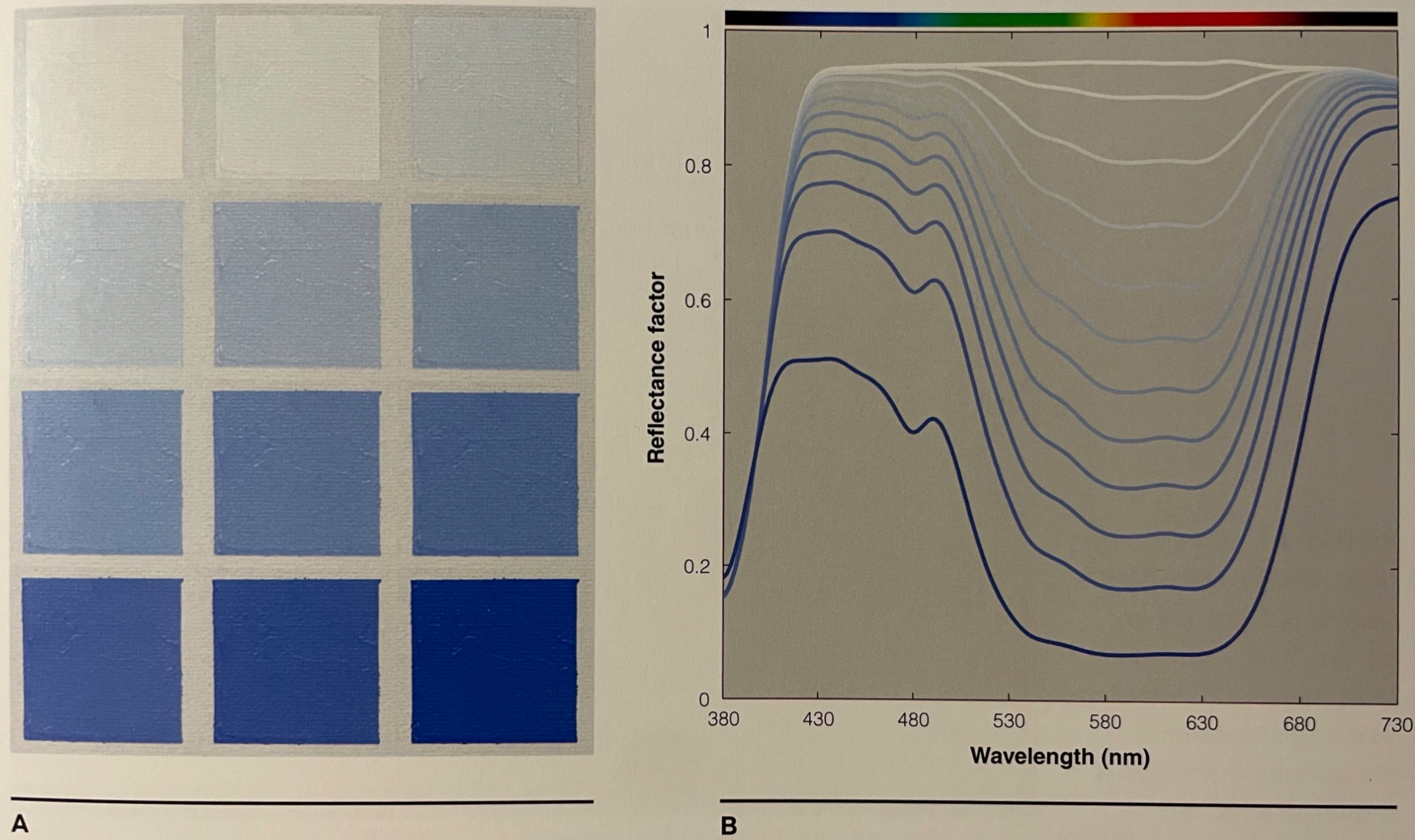


**Masstone:** pure color

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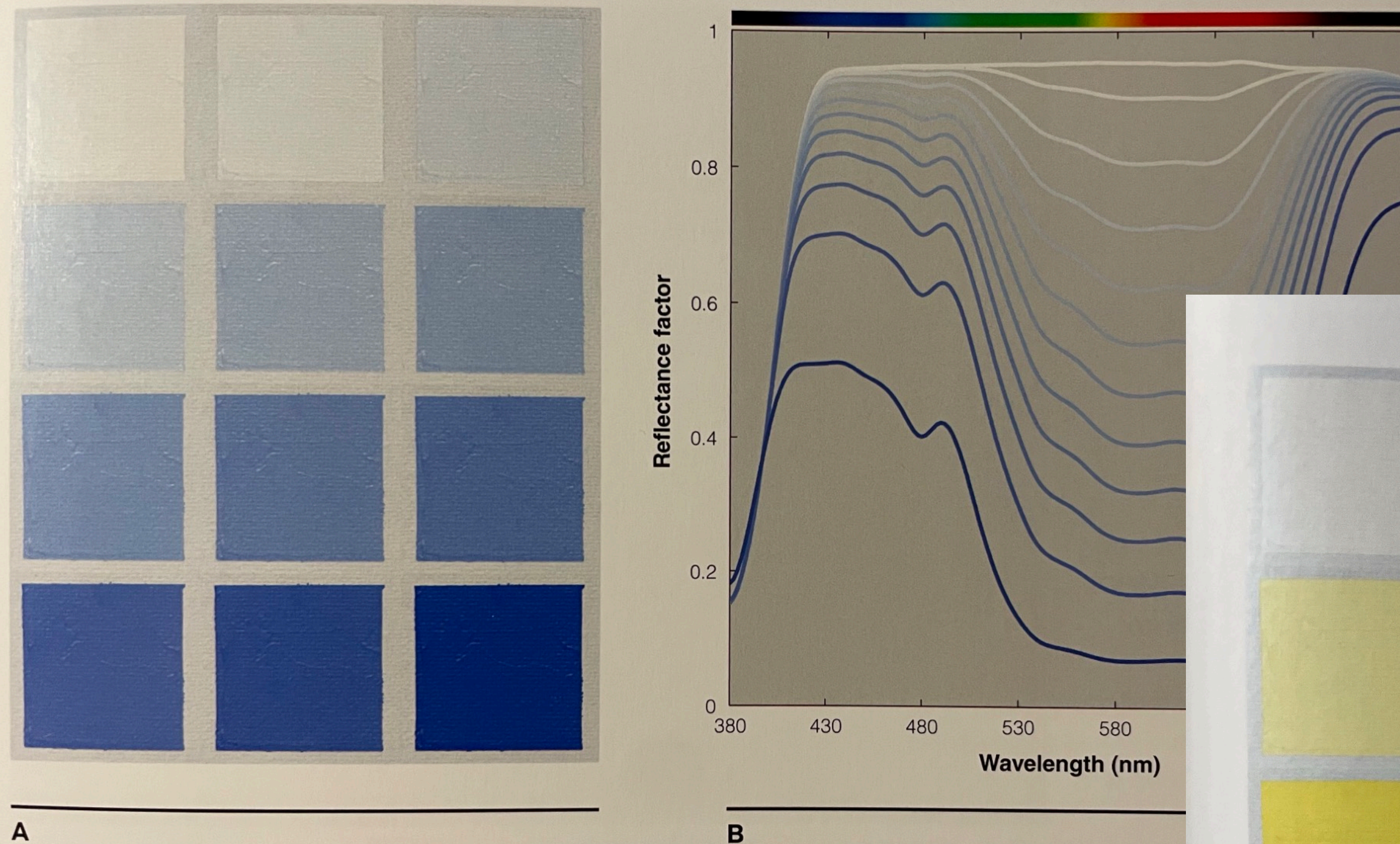
## Spectral reflectance vs. cobalt blue tint



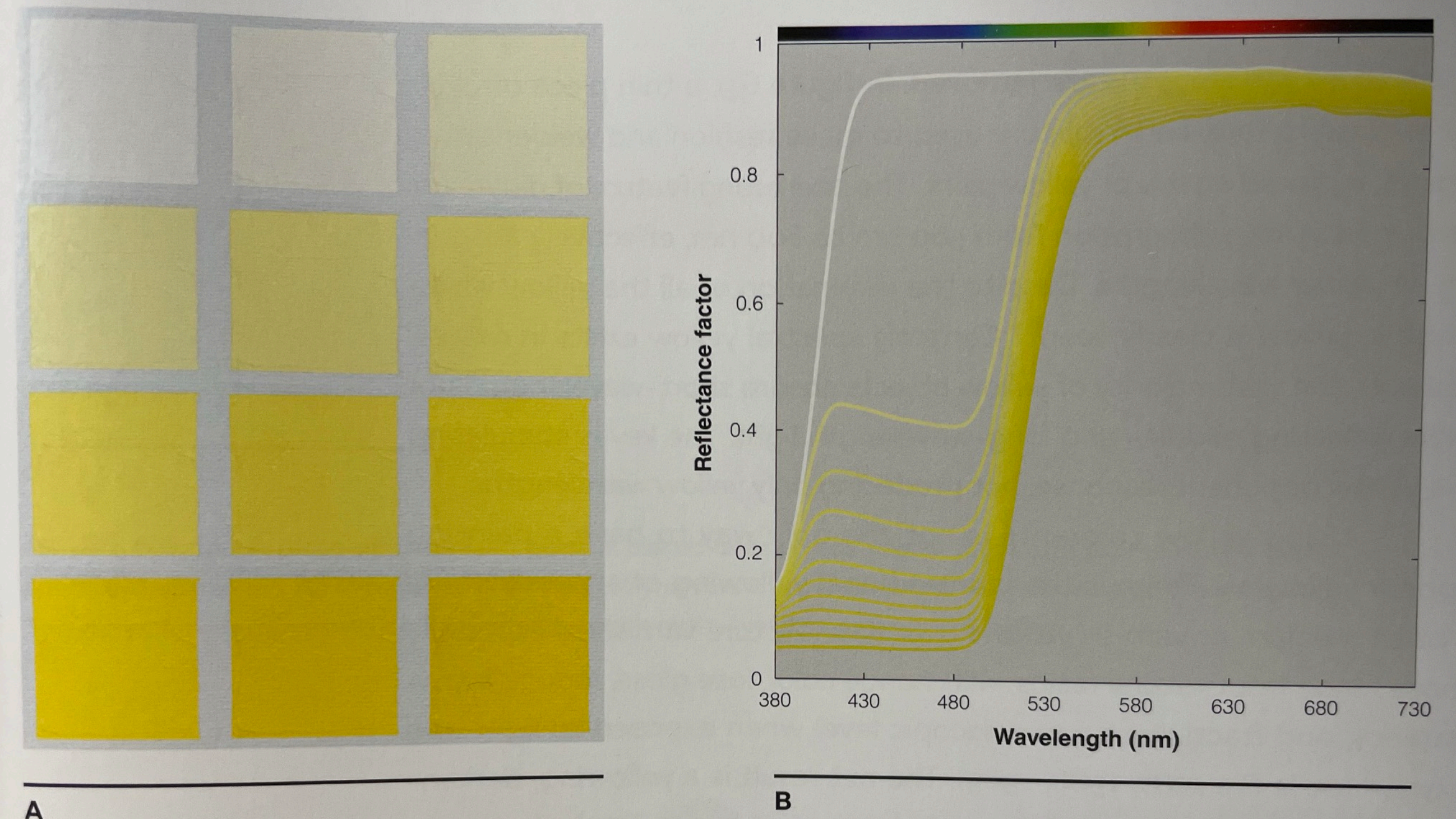
**FIGURE 1.6.** (A) Tint ladder of titanium white and cobalt blue paints, with (B) its reflectance spectra. The high reflectance at very long wavelengths and the mitt-like shape at short wavelengths are characteristic for cobalt blue, irrespective of binding medium.



## Spectral reflectance vs. cobalt blue tint



**FIGURE 1.6.** (A) Tint ladder of titanium white and cobalt blue paint. (B) Reflectance spectra. The high reflectance at very long wavelengths and the dip-like shape at short wavelengths are characteristic for cobalt blue, in any binding medium.



**FIGURE 6.7.** (A) Tint ladder of arylide yellow and titanium white, with (B) its reflectance spectra. With an increase in the concentration of yellow, absorption of short-wavelength light increases, reducing reflectance.

## Spectral reflectance vs. arylide yellow tint



# Applicability of K-M Model

A precise theoretical material model would require modeling of both the 3D microstructure and the non-isotropic nature of the flux. However, we will almost never have the data required for such modeling and therefore have to rely on simplifying assumptions and often empirical modifications, which is what K-M does.

The accuracy is determined by the validity of the simplifications and how well the empirical modifications are able to fit the data. At some point, the empirical modifications are more like parametric functions to **fit observations** rather than rigorously justified physical models.



# Painting vs. Dyeing vs. Printing



# Painting vs. Dyeing vs. Printing

Similarities:

- Mixing colorants to produce color
- All have a substrate (e.g., canvas vs. textile vs. paper) on which colorants are applied

The Kubelka-Munk model has been applied to all three areas for predicting colors, reverse-engineering colorants given a sample, etc.











# Colorant Differences

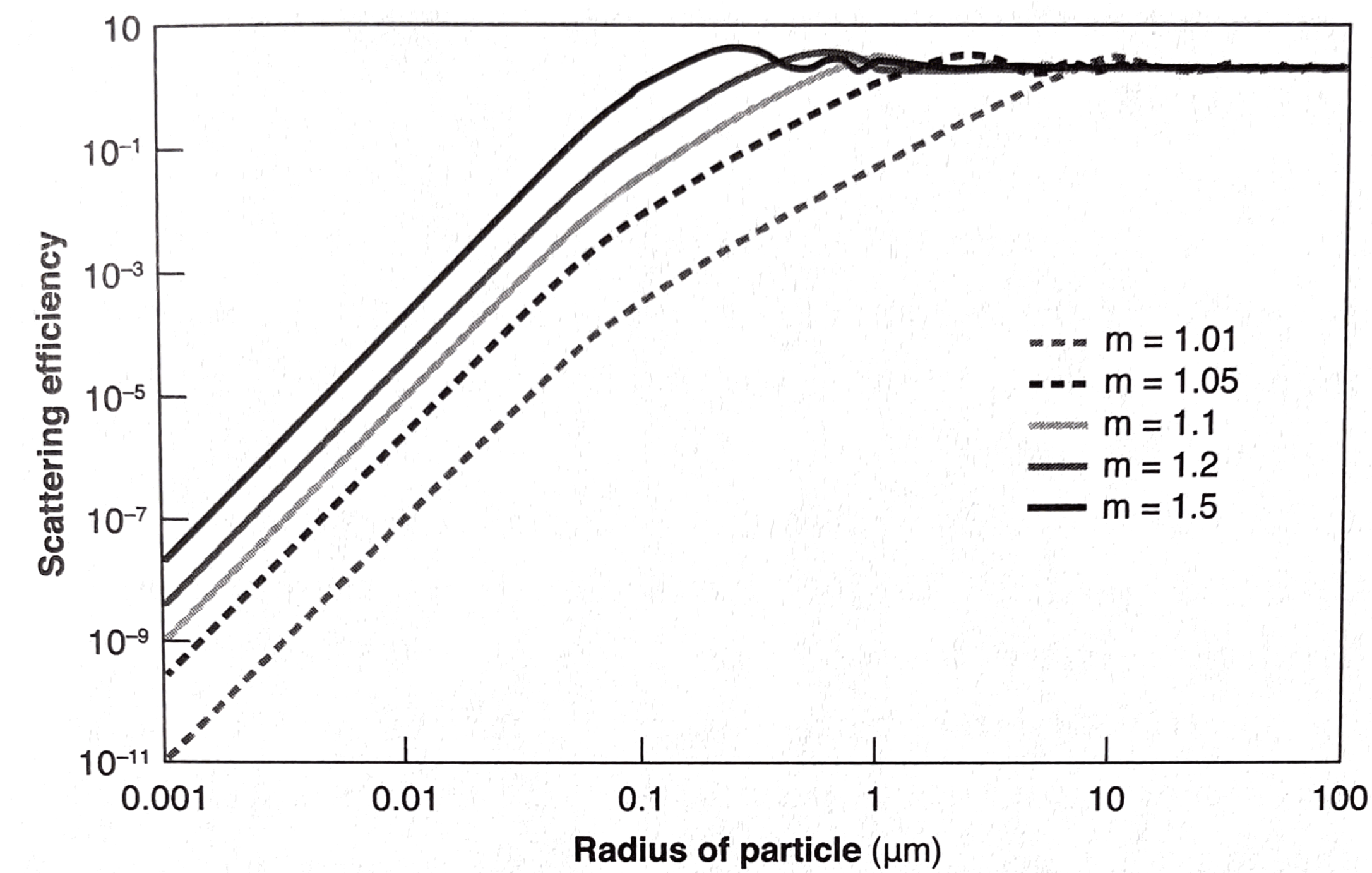
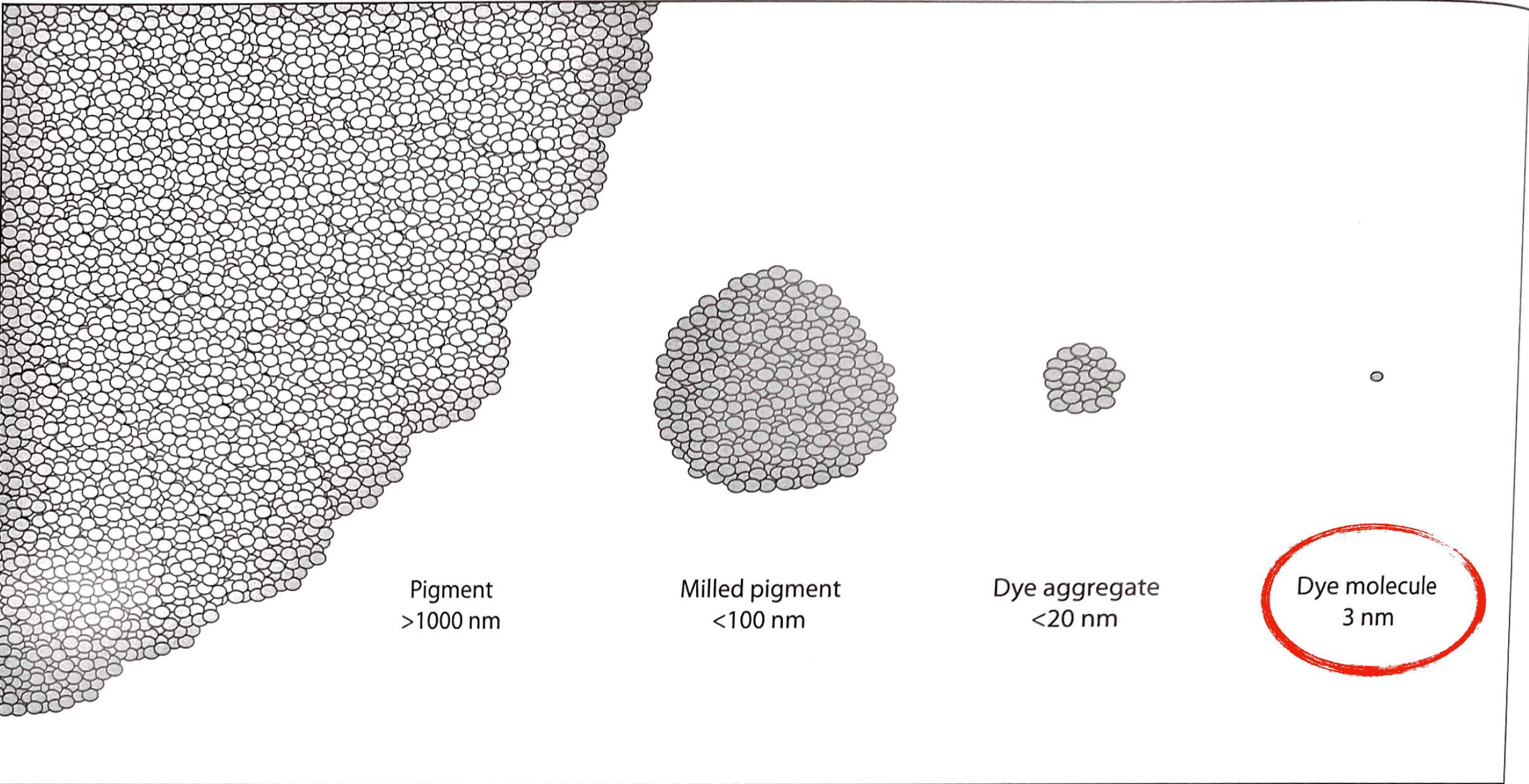
Painting uses **pigments**, and dyeing uses **dyes**.

- Dye molecules are orders of magnitude smaller than pigment particles.
- Dyes are water soluble (chemically bond with the water molecules).
- So dyes scattering efficiency is quite low, mostly transparent/translucent.

Printing inks can either be pigments or dyes.

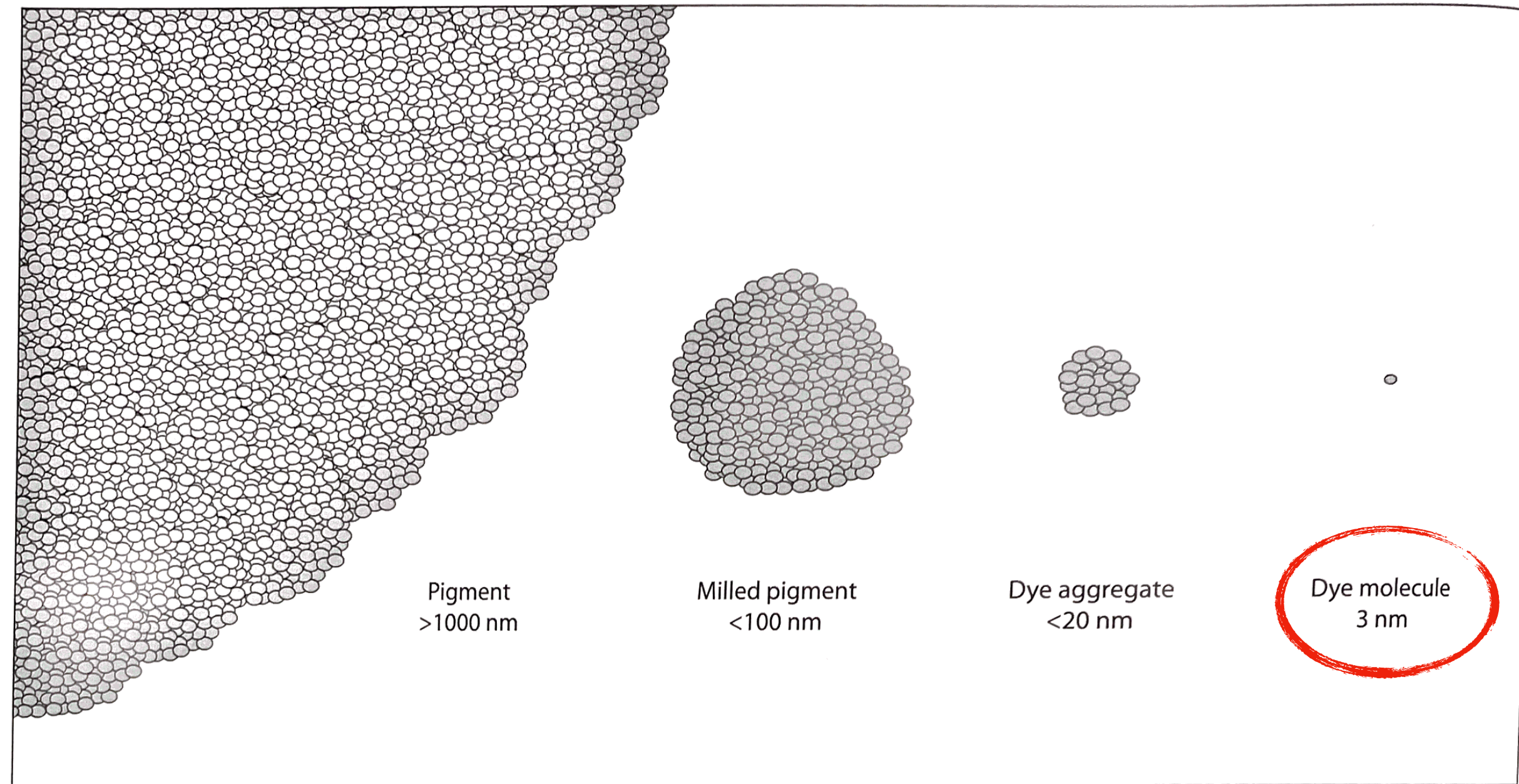


# Colorant Differences





# Colorant Differences





# Layering vs. Mixing

**Painting** can be layering or mixing depending on what artists decide to do.

- Glazing/vanishing are translucent layers.
- Pentimento is intentionally painting over/hiding under-layers.

**Dyeing** colorants are usually (pre-)mixed.

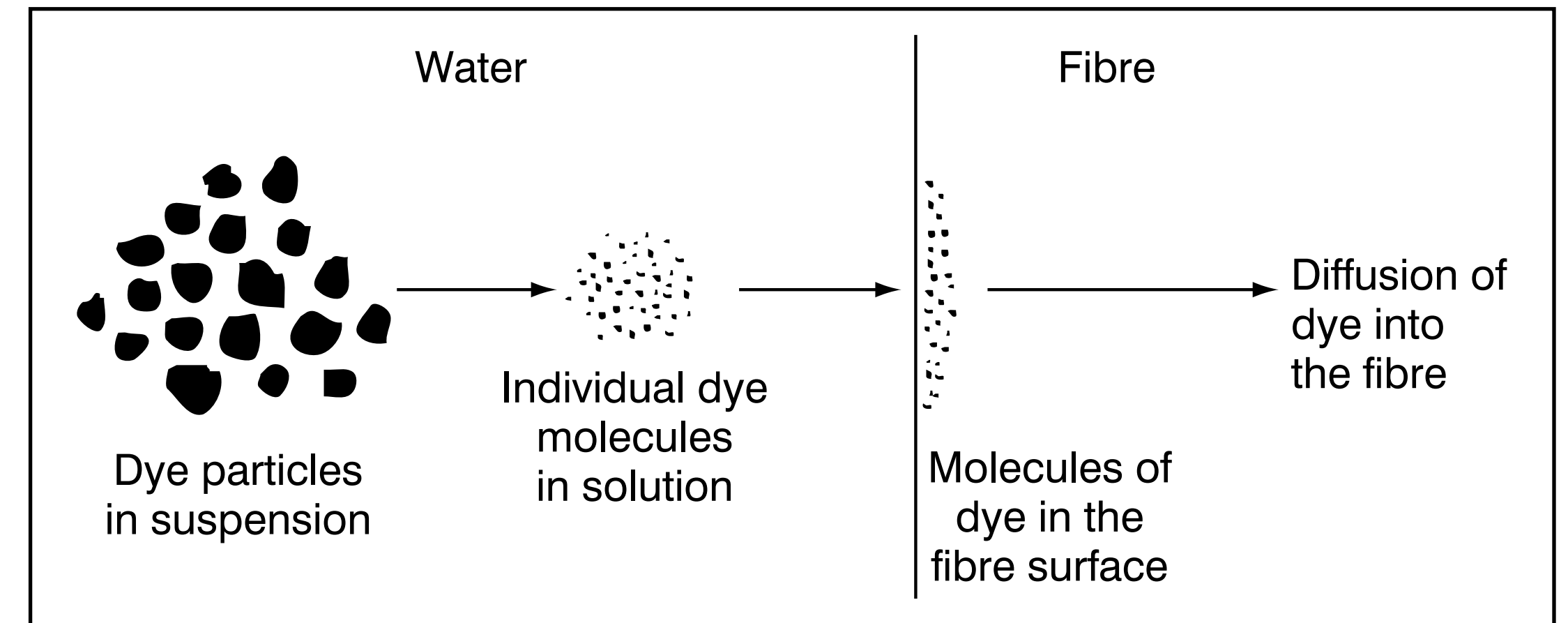
**Printing** inks are complicated; they are neither completely layered nor completely mixed.



# Substrate Interaction

**Painting:** paint usually completely hides the substrate so the color usually doesn't depend on the substrate.

**Dyeing:** dyes are molecularly dispersed into the substrate (fibers). Dyes, while absorb lights like crazy, do not scatter nearly as much as the fibers, which contribute to most of the scattering and participate in color formation.







Paints are usually thick enough to  
hide the substrate (e.g., canvas)





Dyes usually penetrate directly into the fibers.  
The scattering efficiency of dyes is low.



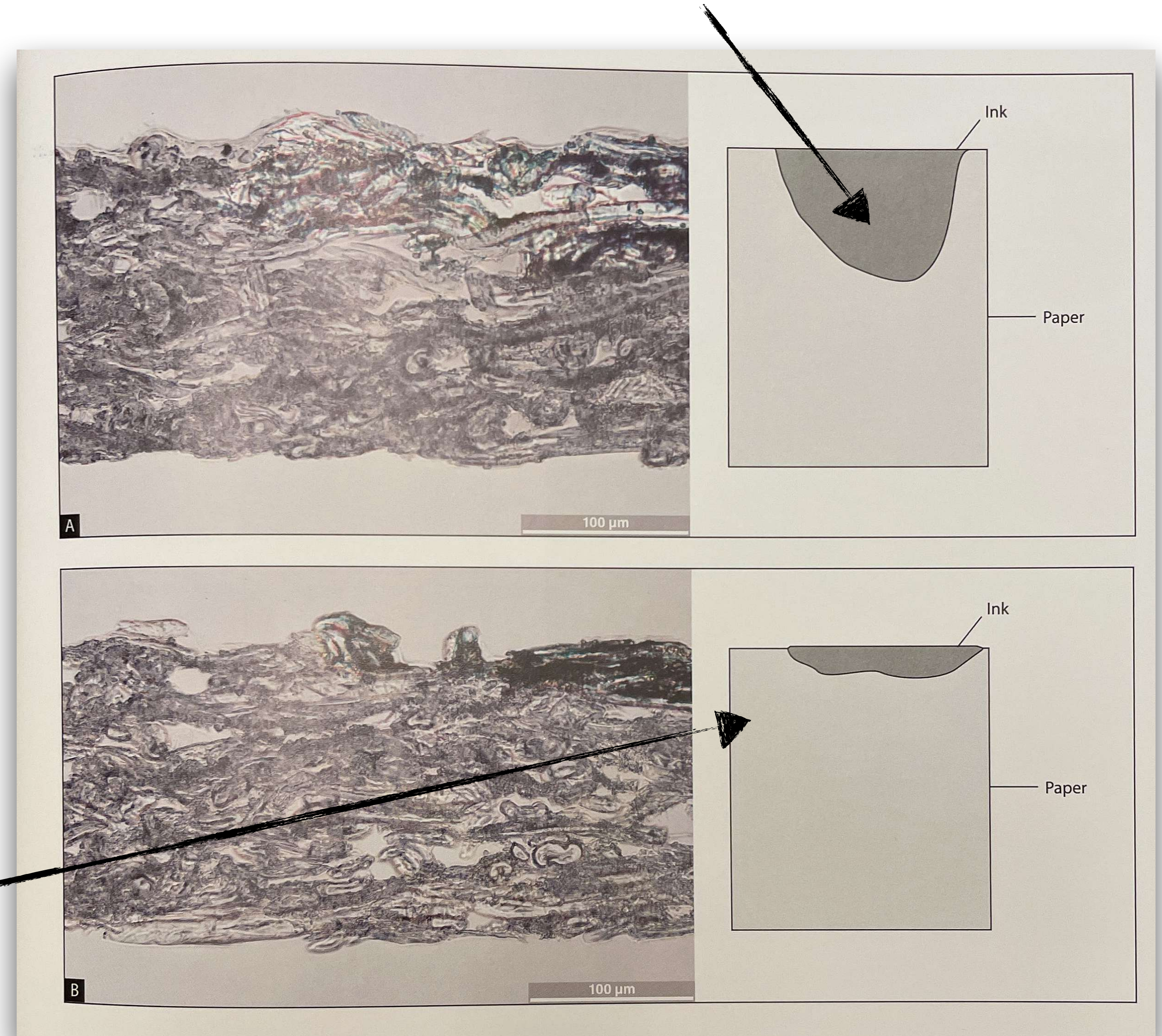
# Substrate Interaction

## Printing: somewhat in-between.

- The ink-mixture penetrates into the substrate (paper), but doesn't completely absorb lights (you can still see light from the other side!)
- So the paper contributes to color formation and is effectively just another colorant in the K-M model.

Cross sections of Epson Ultrachrome 7 pigment-based ink printed on paper. Pigment particles are larger. They are not fully penetrated.

Cross sections of Epson R340 dye-based ink printed on paper. Dye particles are smaller; they penetrate into paper well.





# Pointillism and Half-Tone Printing

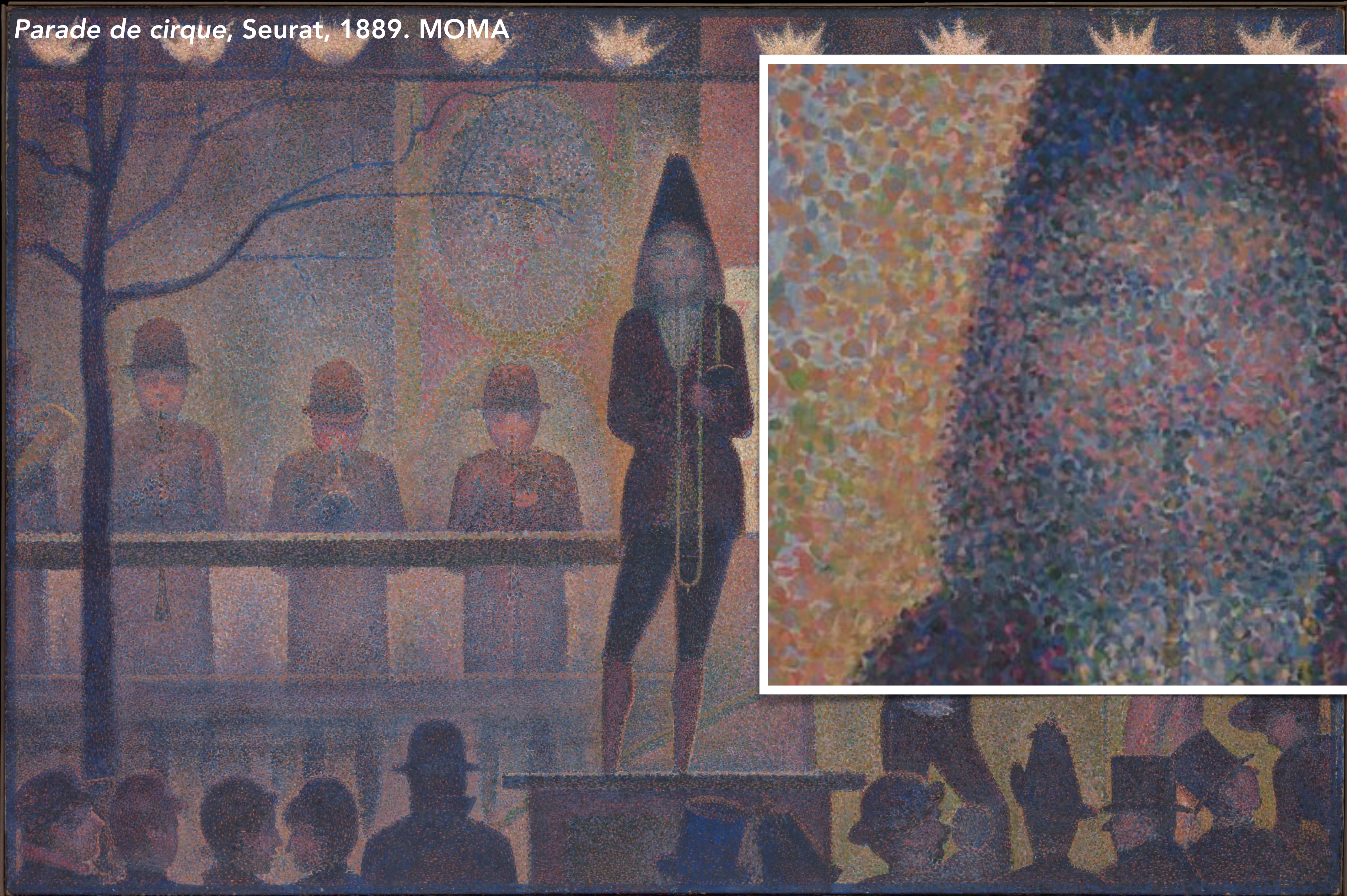


Parade de cirque, Seurat, 1889. MOMA





Parade de cirque, Seurat, 1889. MOMA







**Self-Portrait, Vincent van Gogh,  
1887. Art Institute of Chicago.**



**Regatta in Venice, 1903-04, Henri-Edmond  
Cross. Museum of Fine Arts, Houston.**



# Pointillism

A style of Neo-impressionism. A variant of Divisionism one might say.

*"By requiring the viewer to combine the colors **optically** instead of **physically** mixing pigments, Divisionists believed they were achieving the maximum luminosity scientifically possible."*

In principle, they are no different from the sub-pixels of modern displays — spatial integration on the retina — but at a larger scale. Pointillists paints must be viewed far away for colors to spatially mix.



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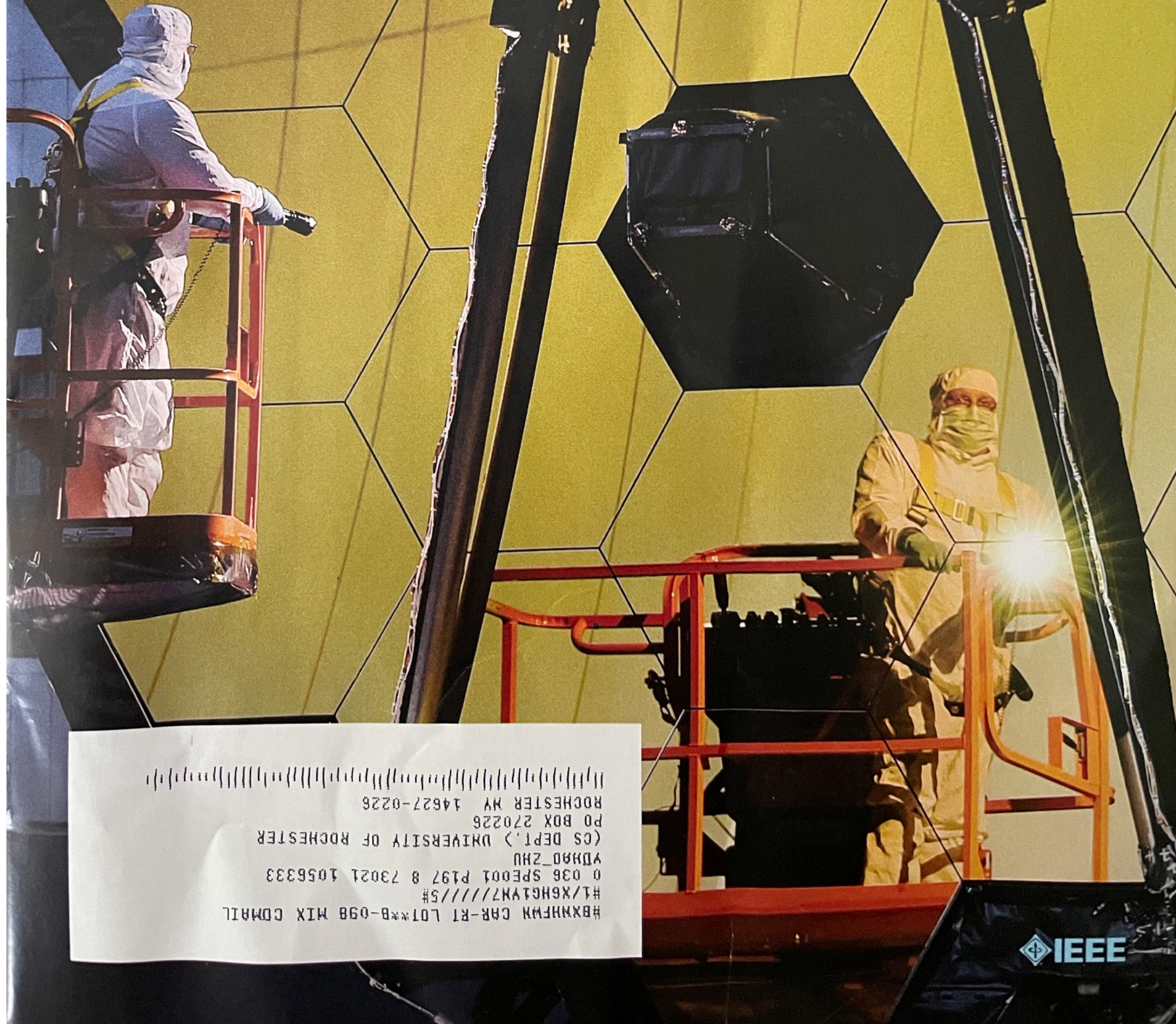
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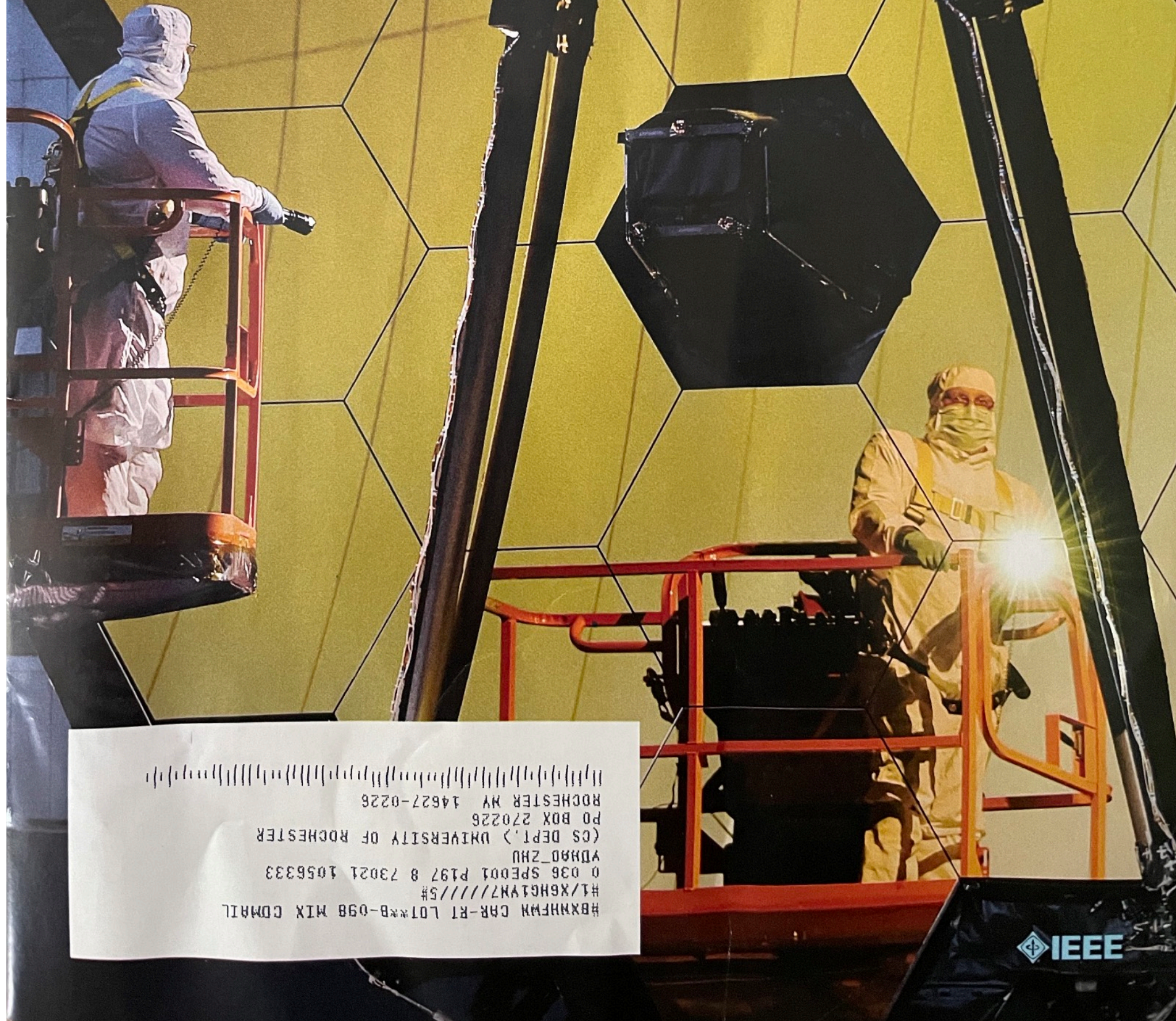
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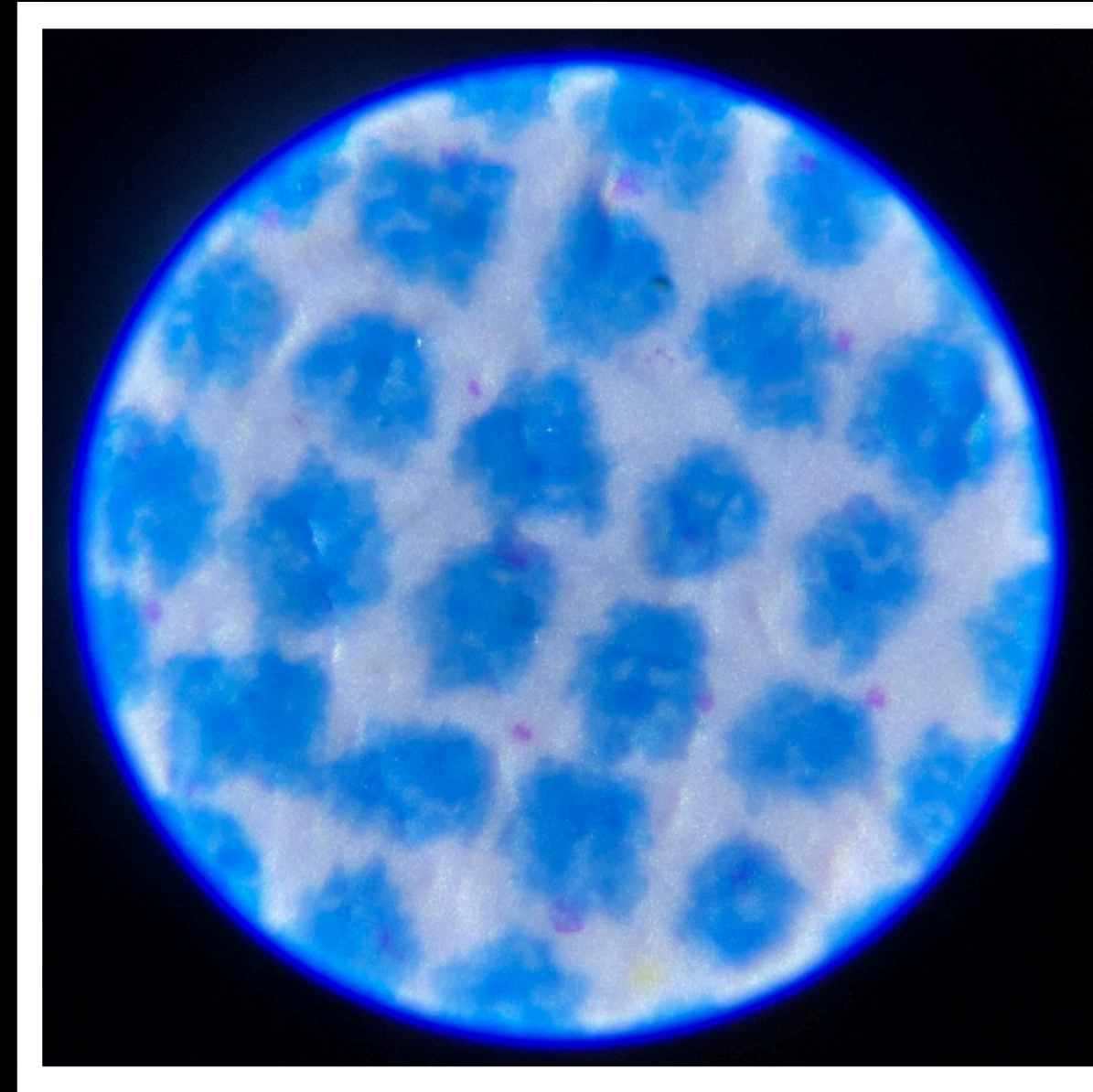
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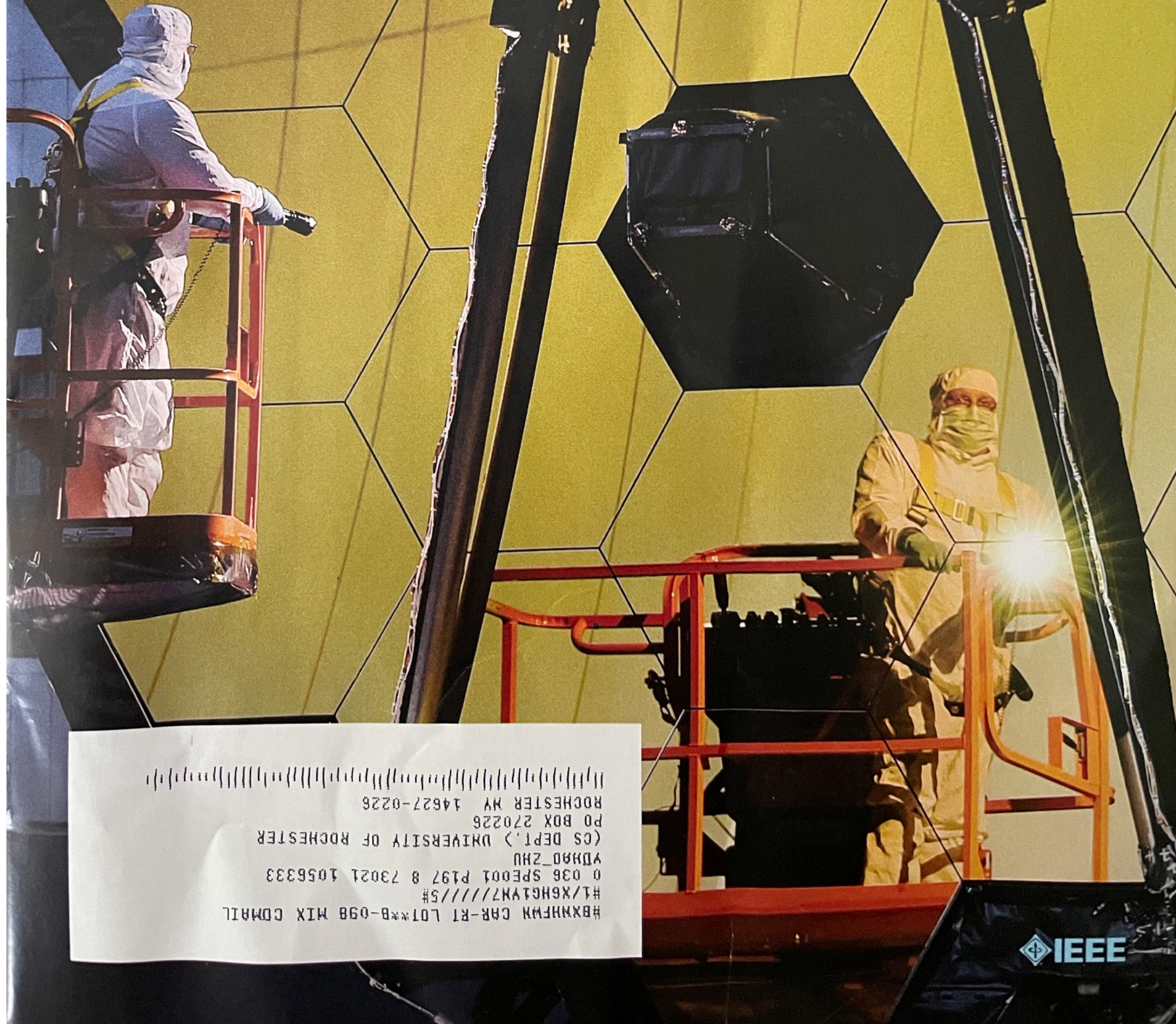
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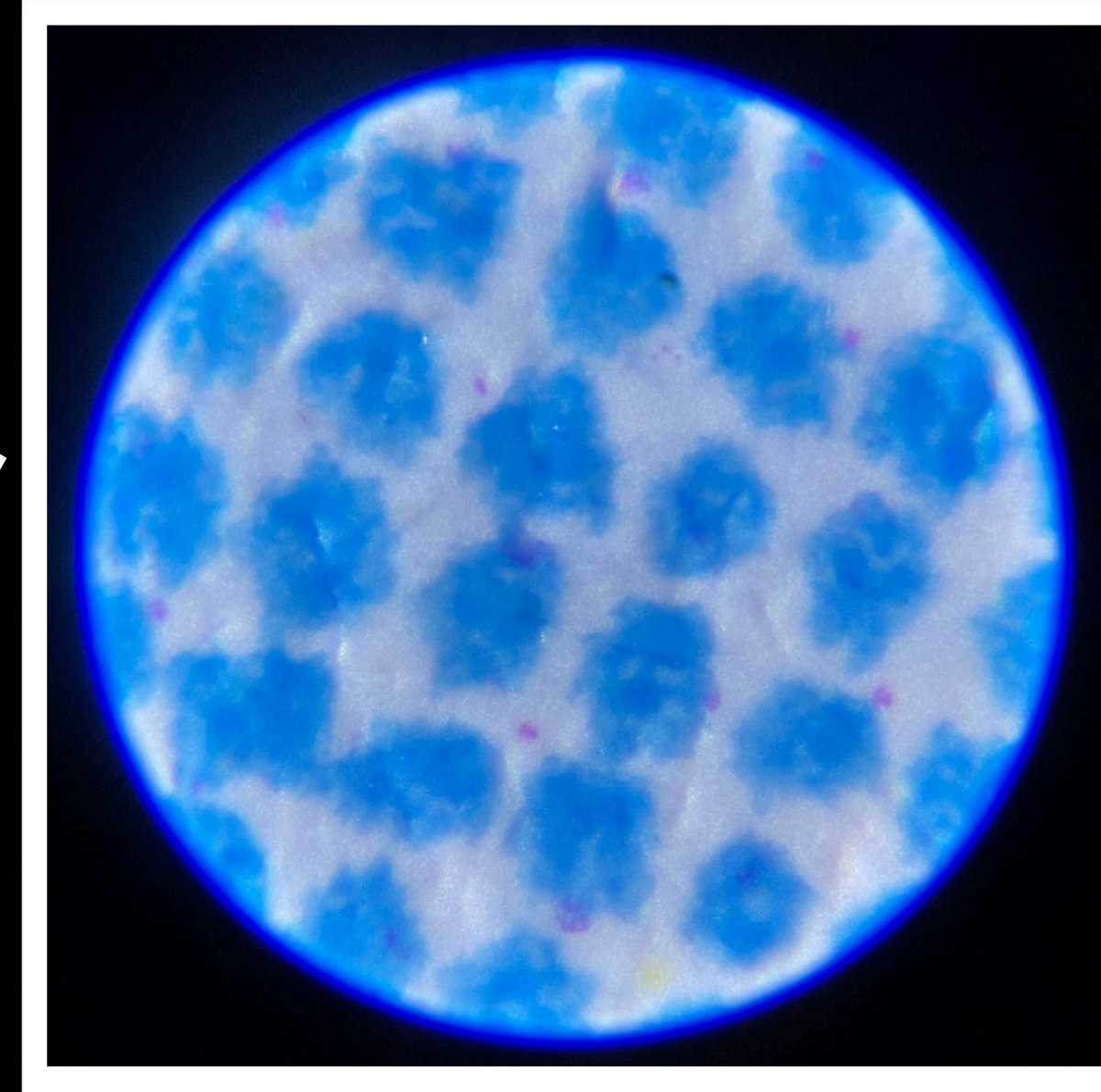
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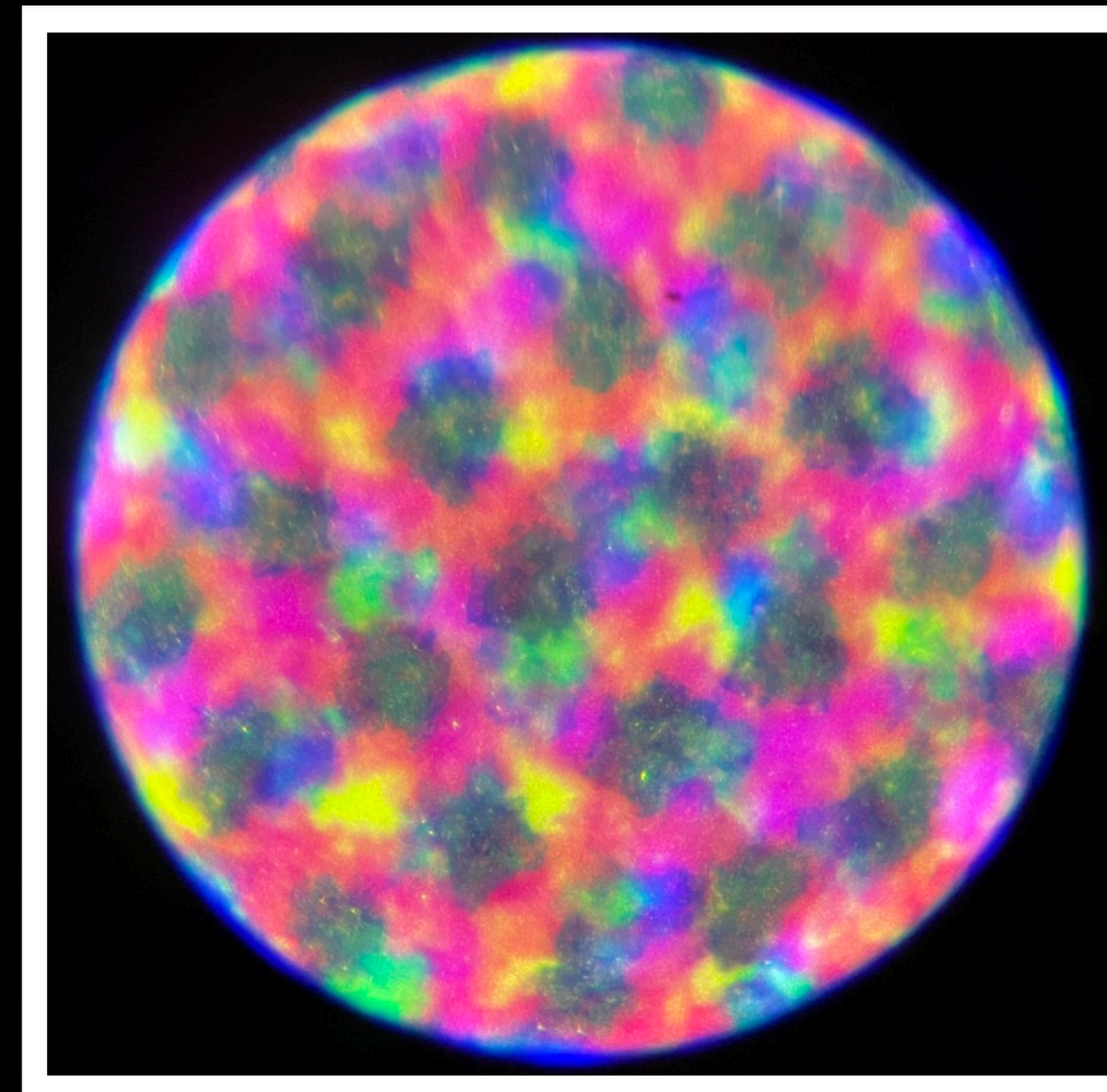
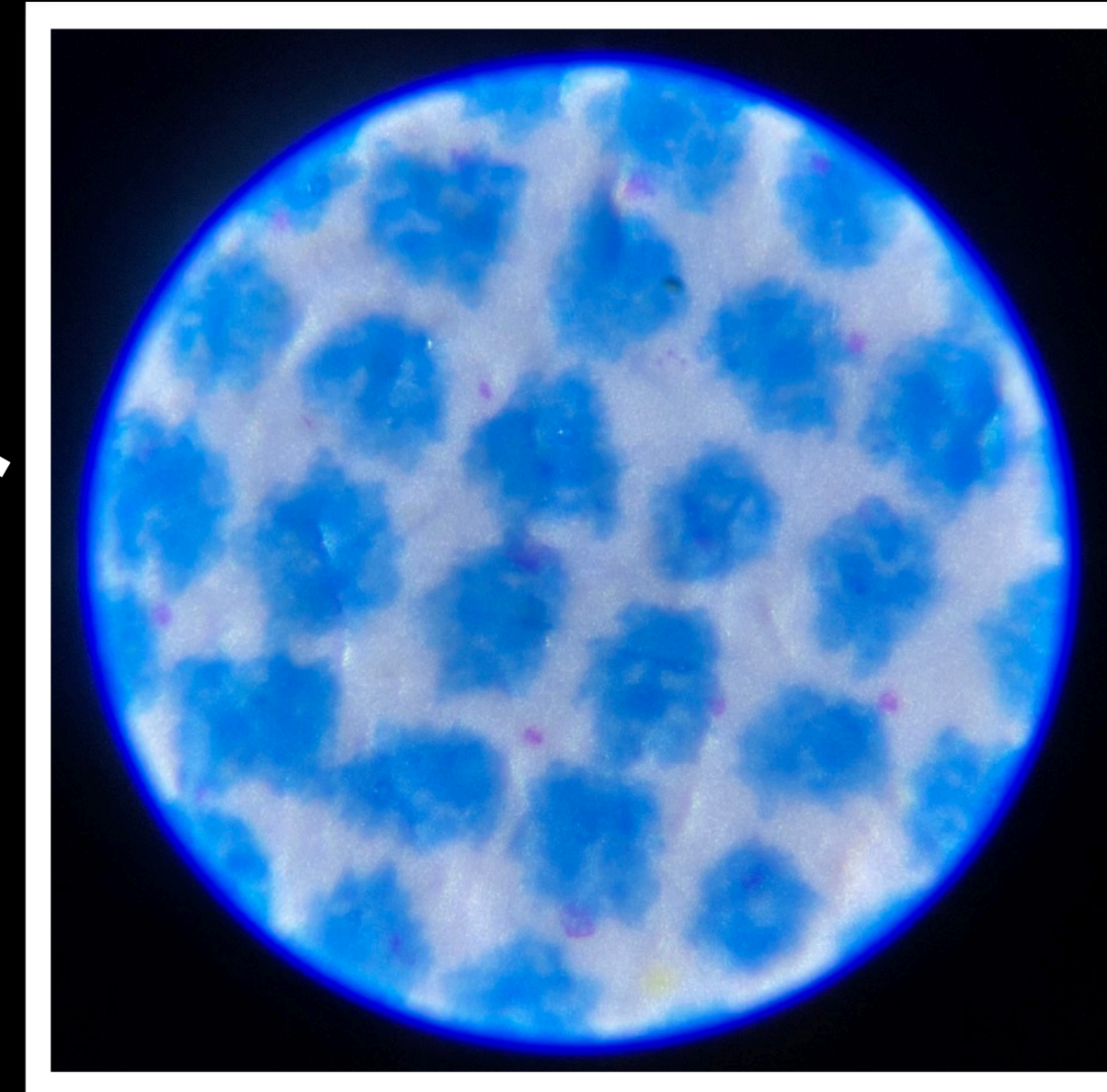


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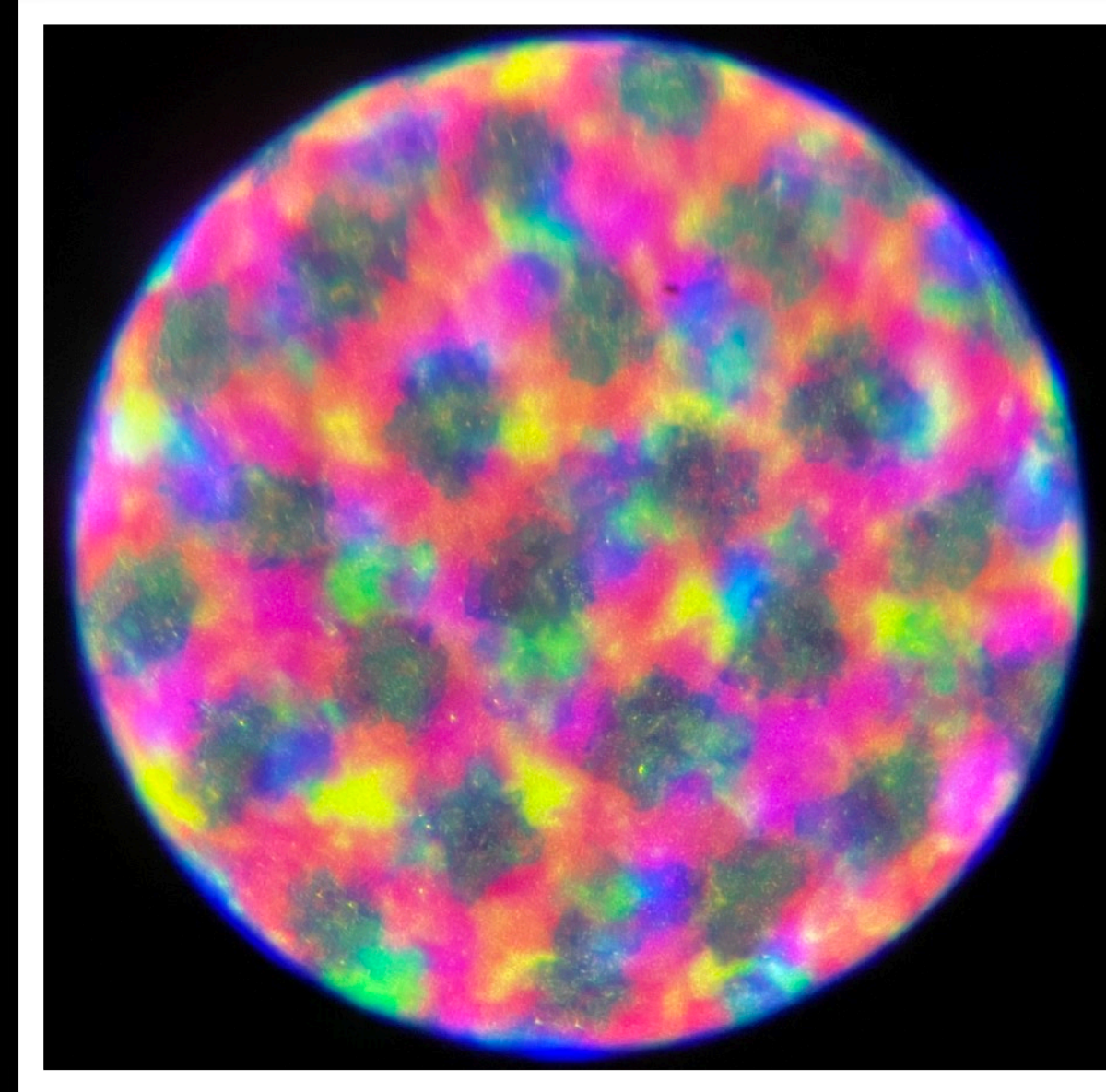
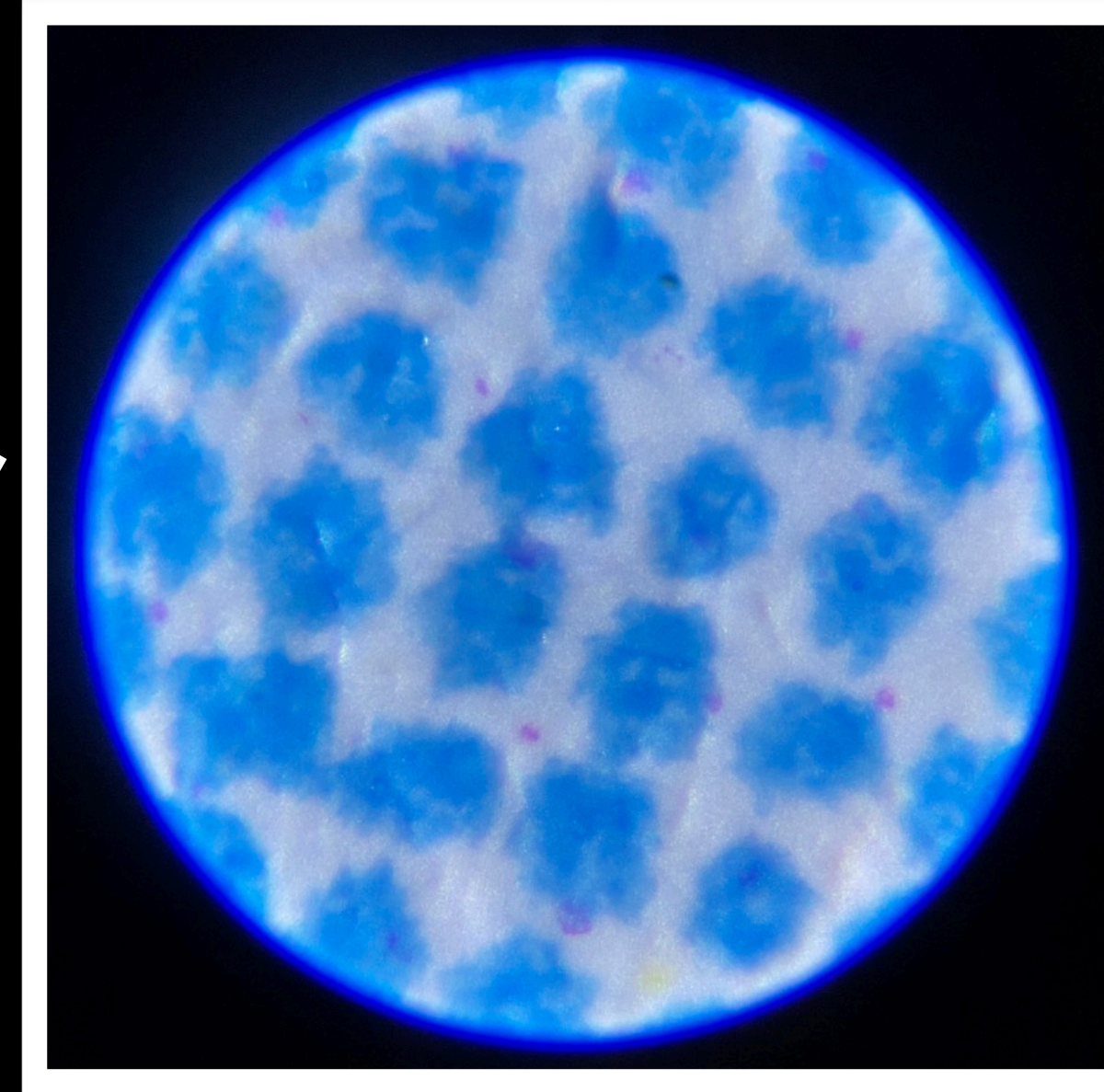
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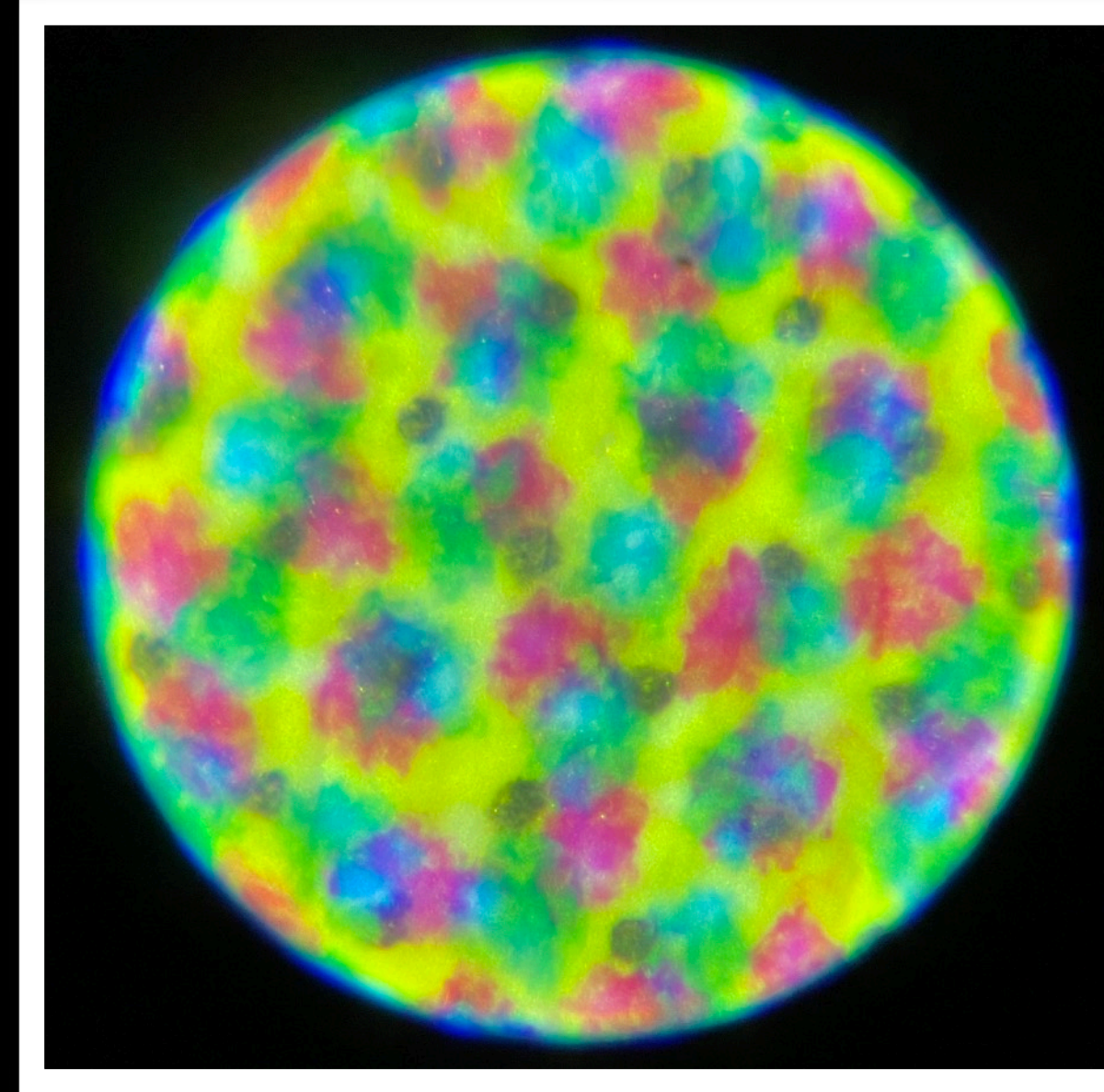
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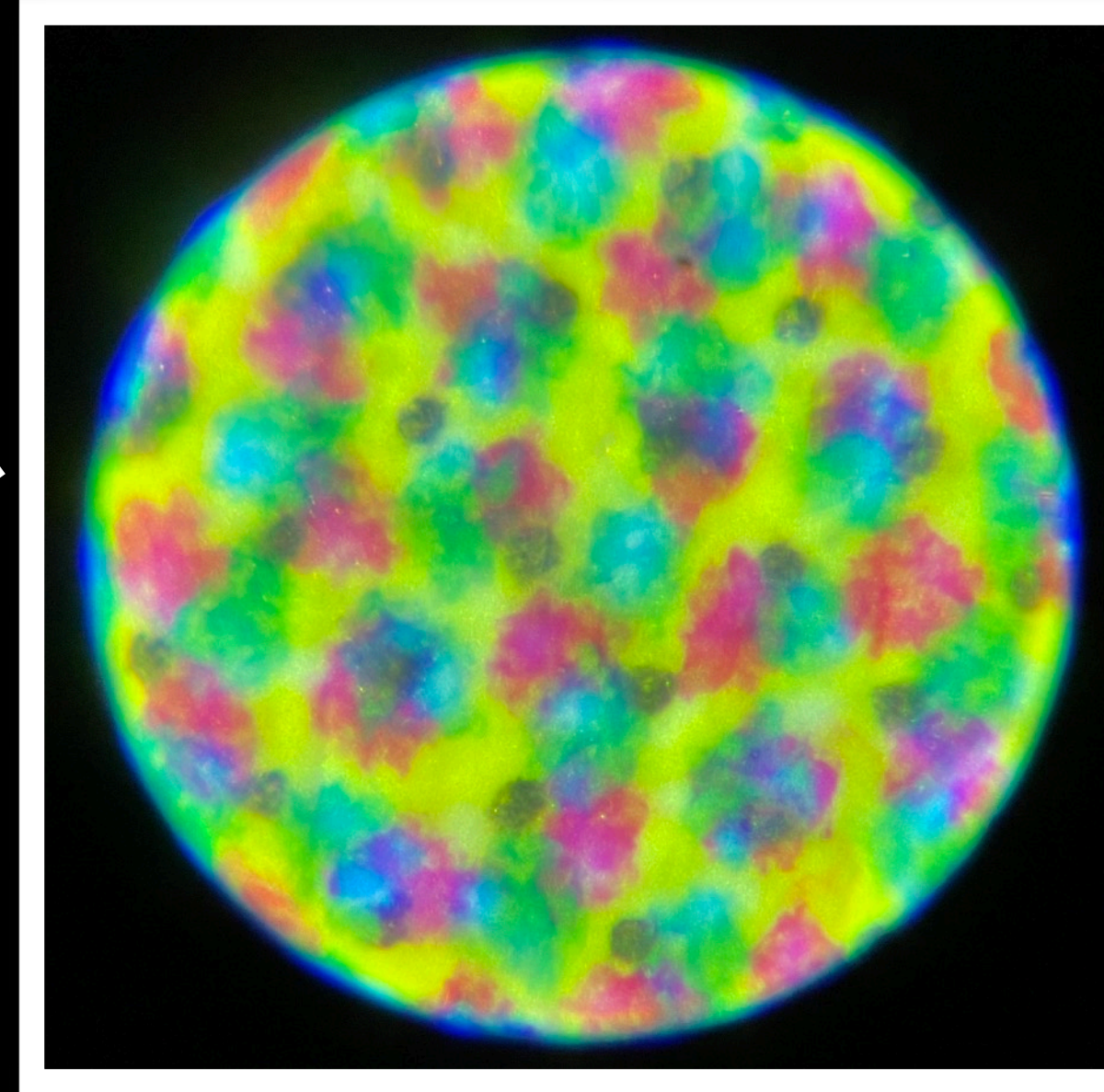
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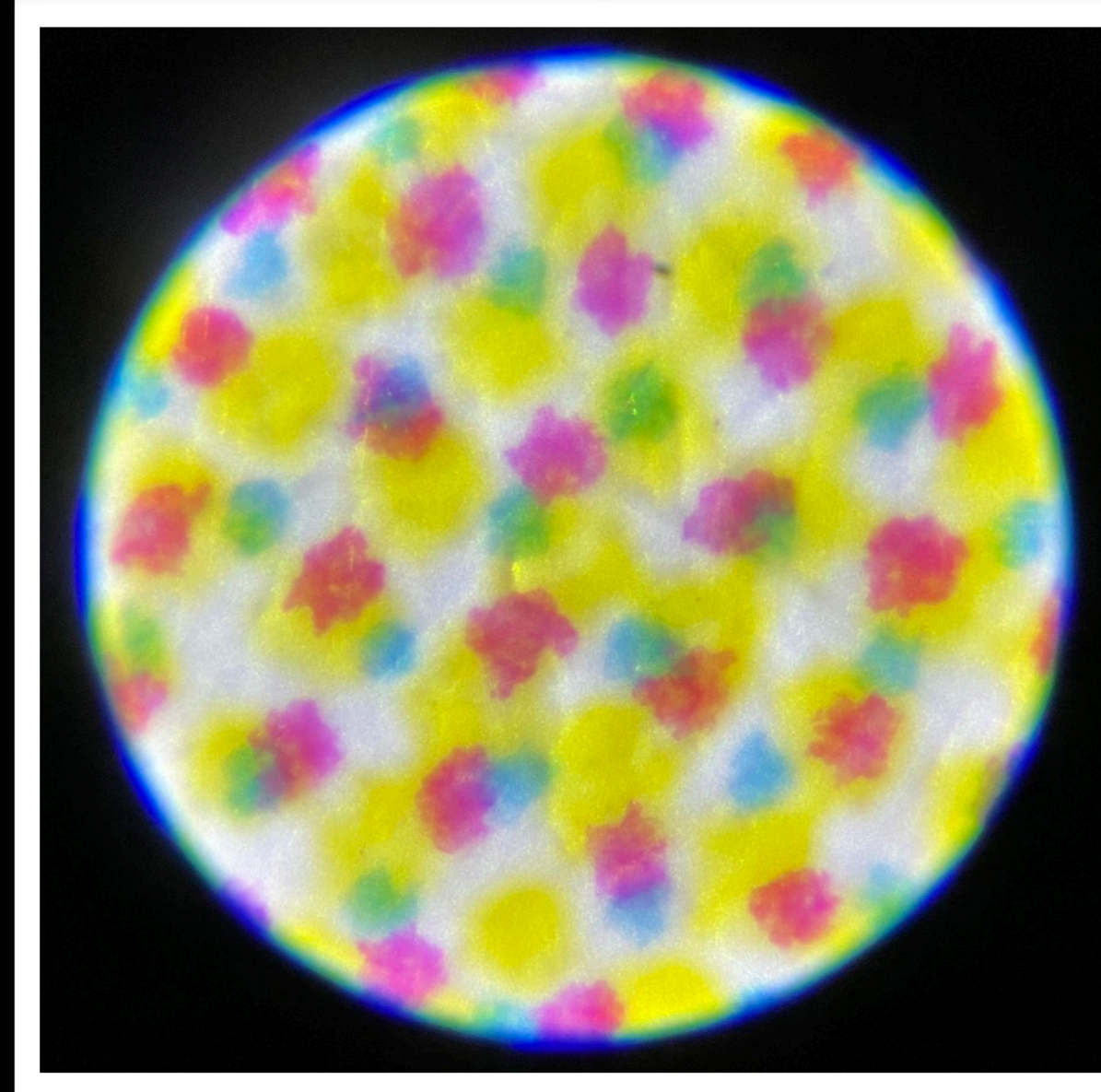
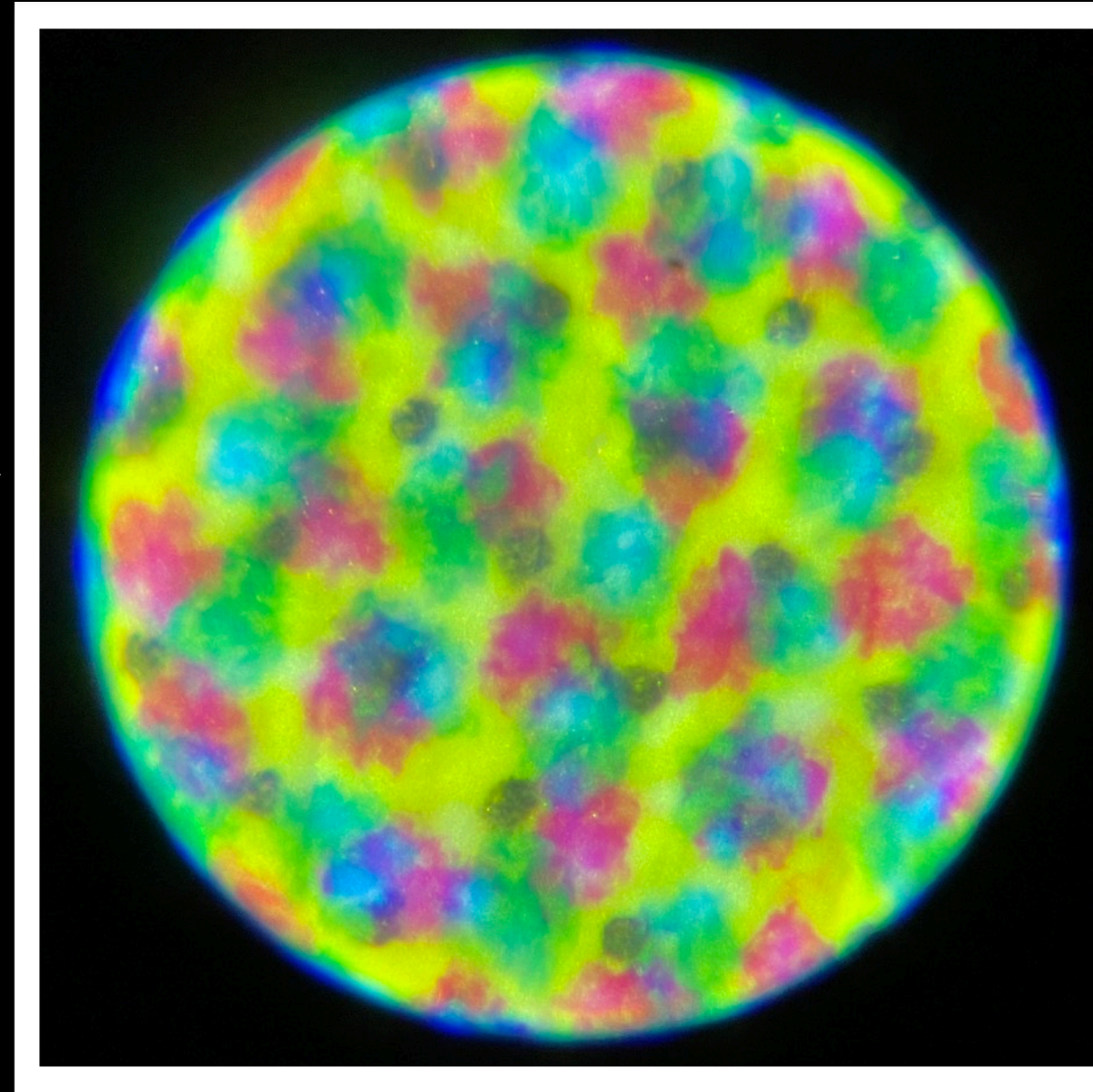
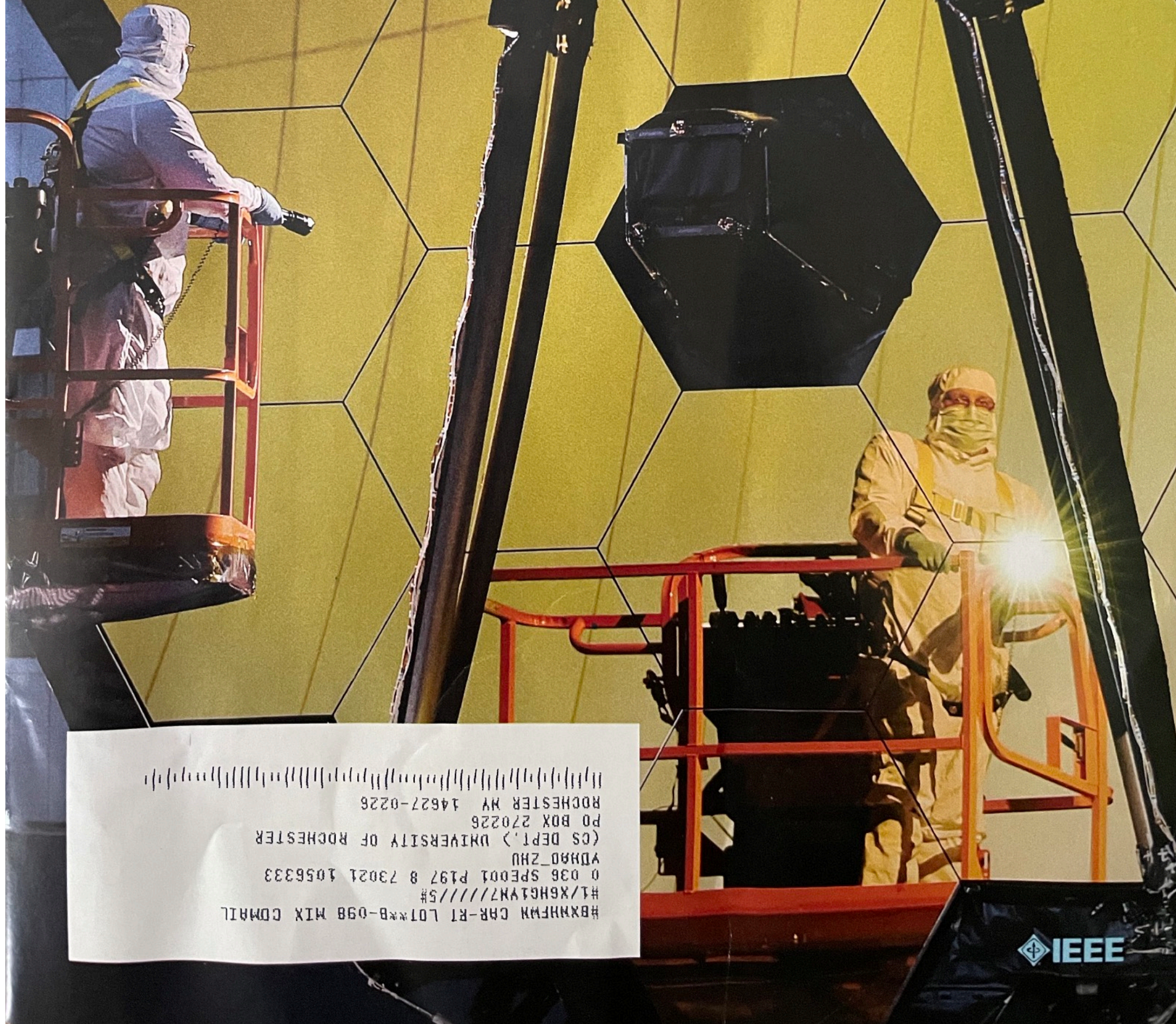
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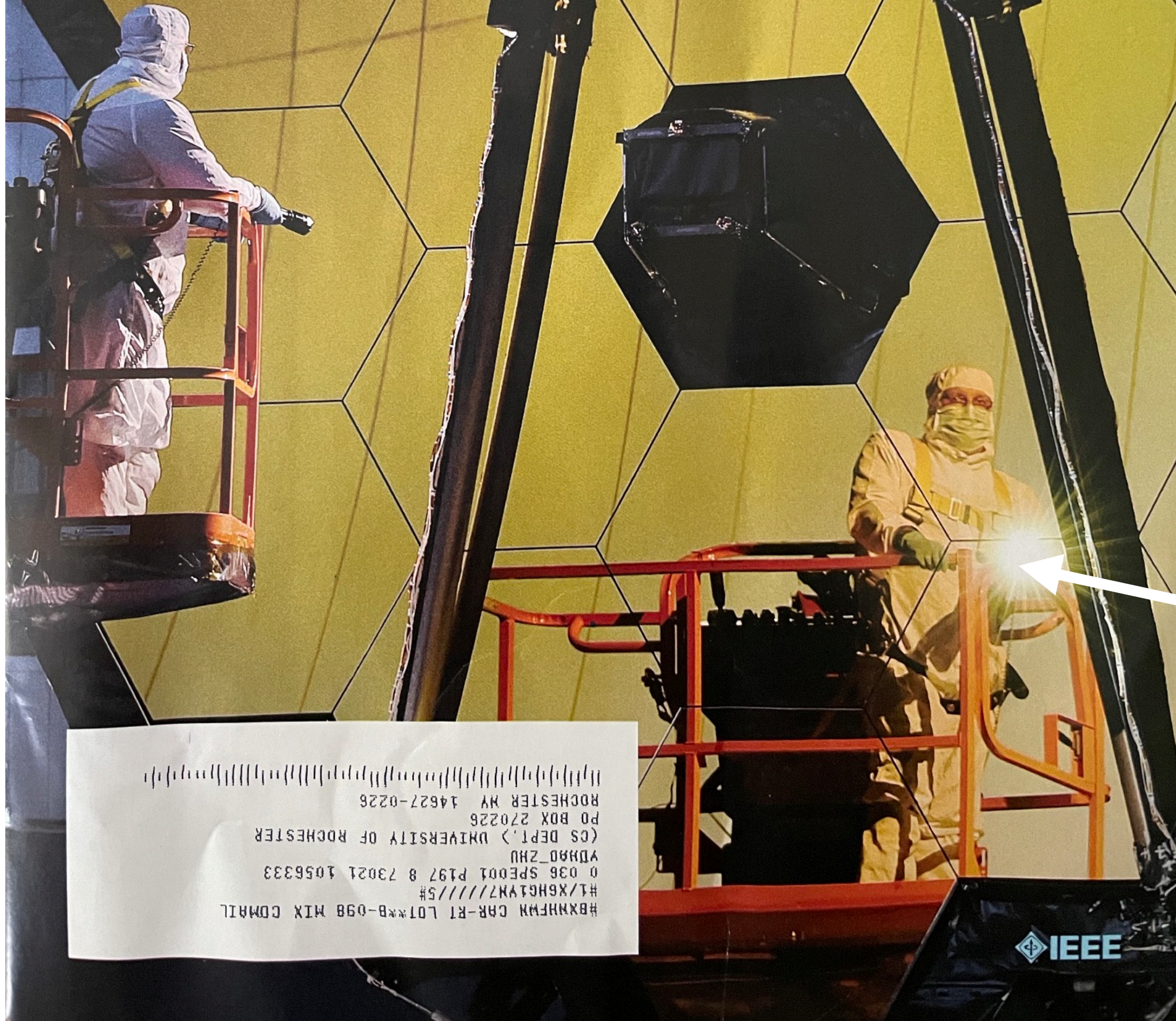
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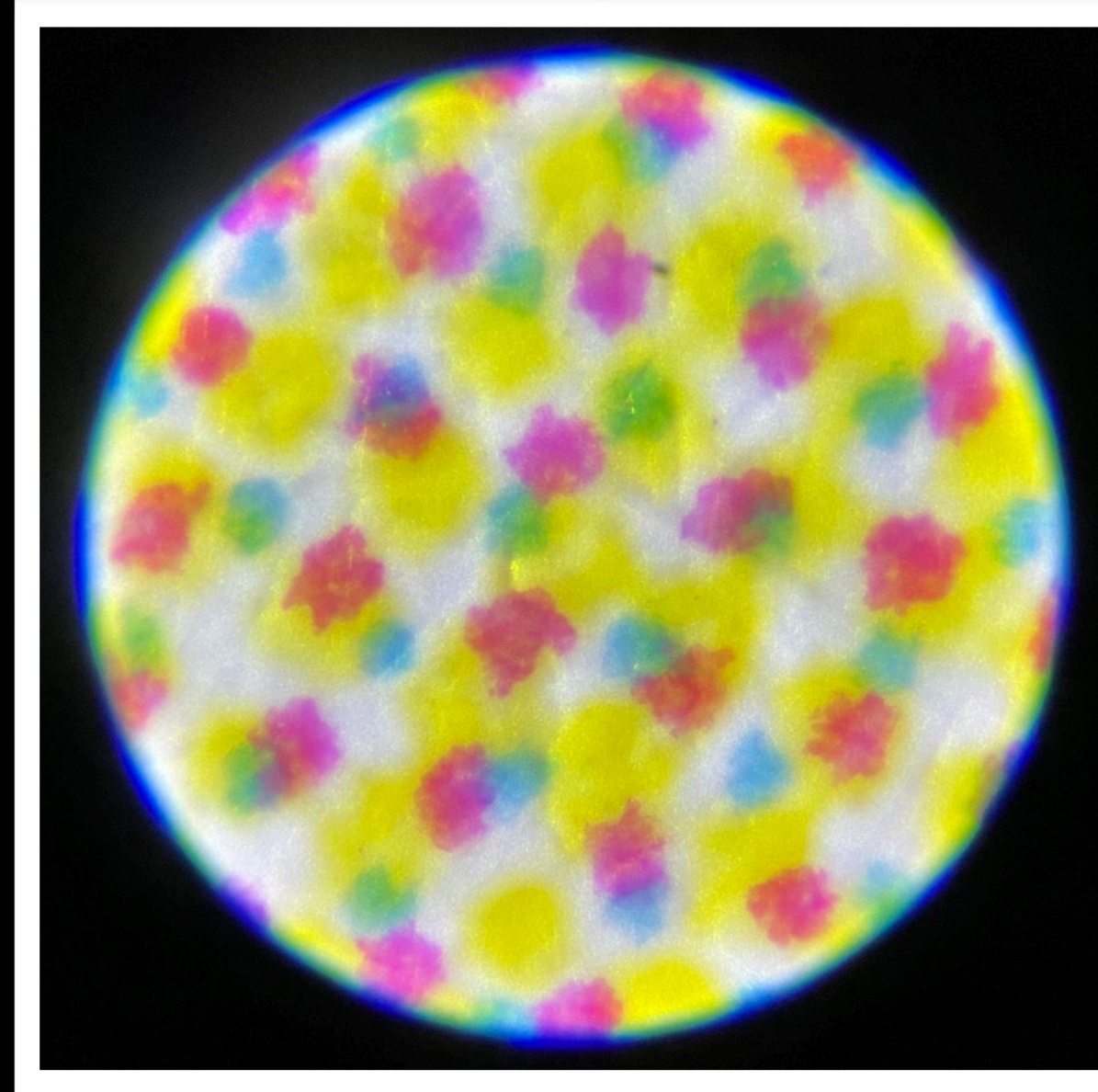
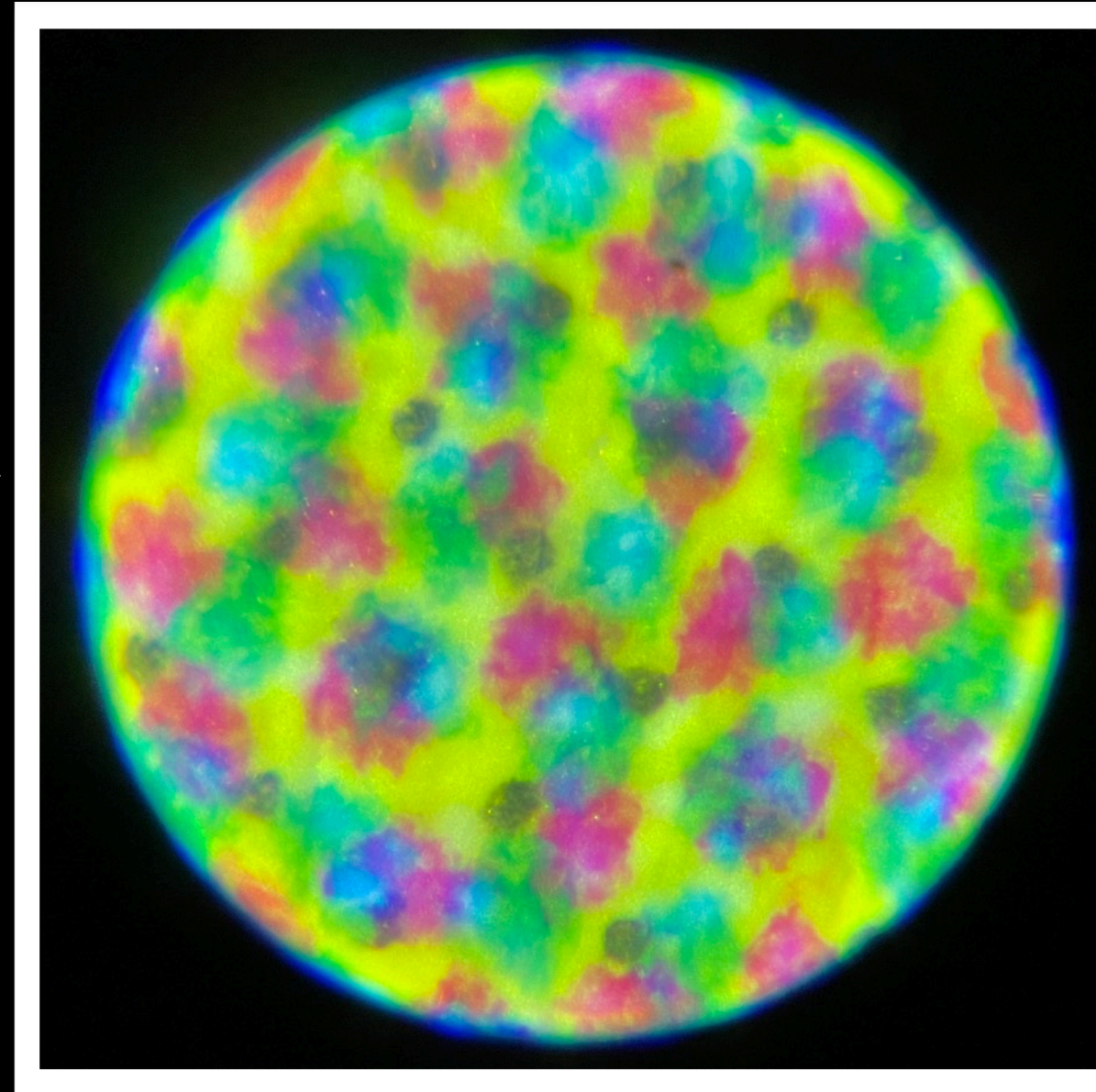
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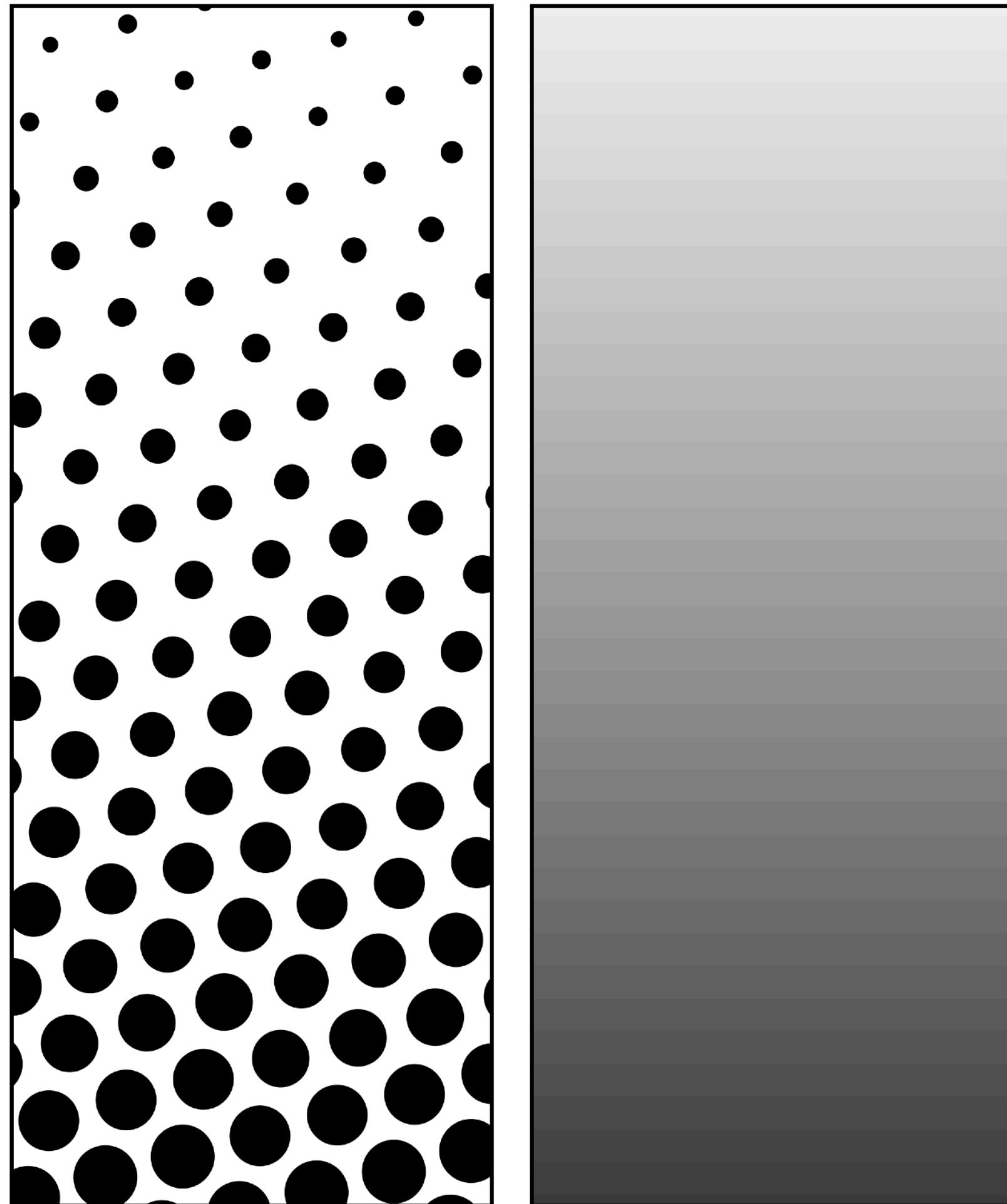


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# (Gray-Scale) Halftoning Printing



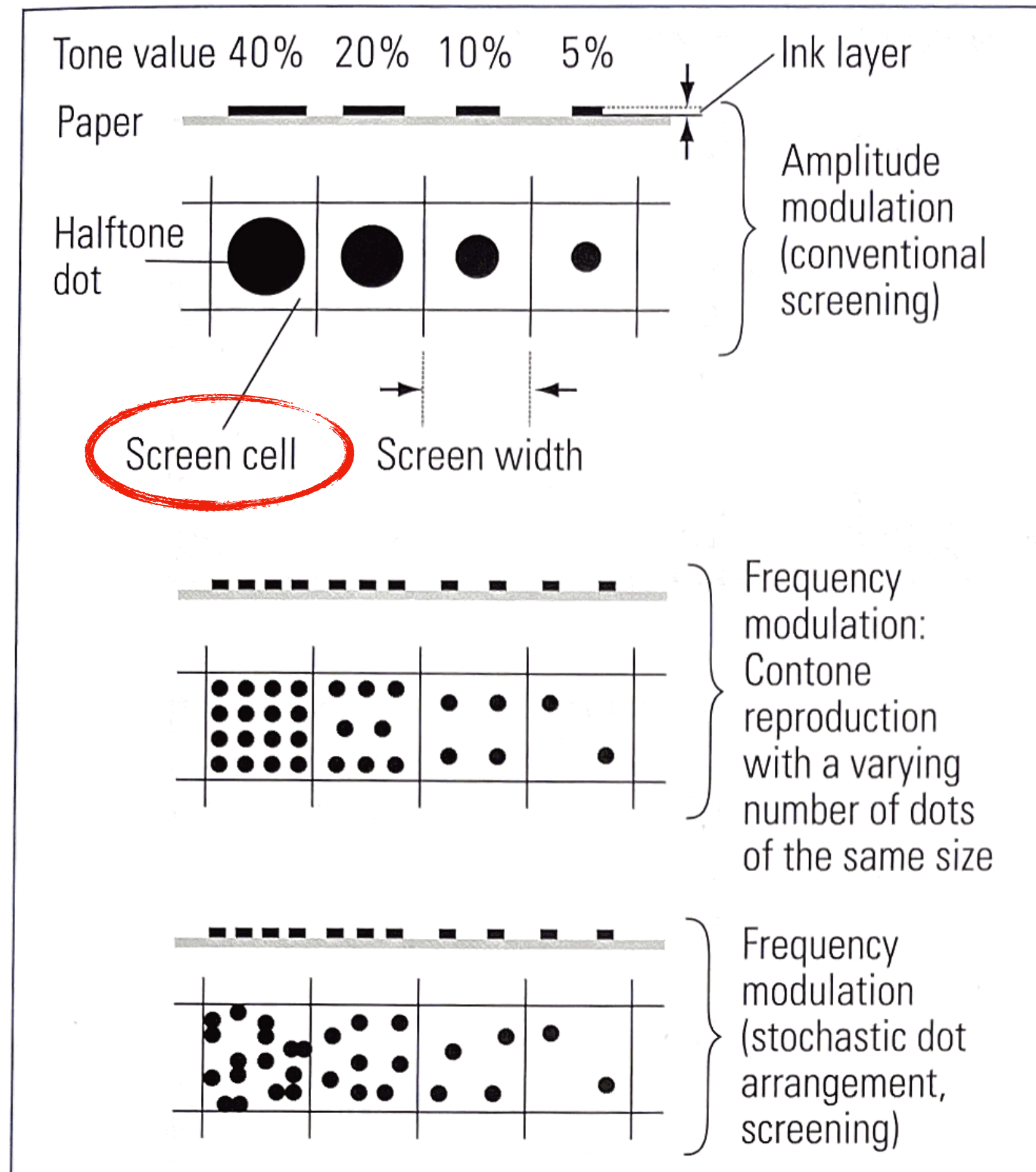
Operates on the principle of spatial integration, just like displays and pointillism.

Use dot sizes and/or frequency to mimic continuous intensity change.

- Controlling dot sizes is called amplitude modulation (**AM**).
- Controlling dot spacing is called frequency modulation (**FM**); requires more dots per inch (DPI) from the printer.



# (Gray-Scale) Halftoning Printing



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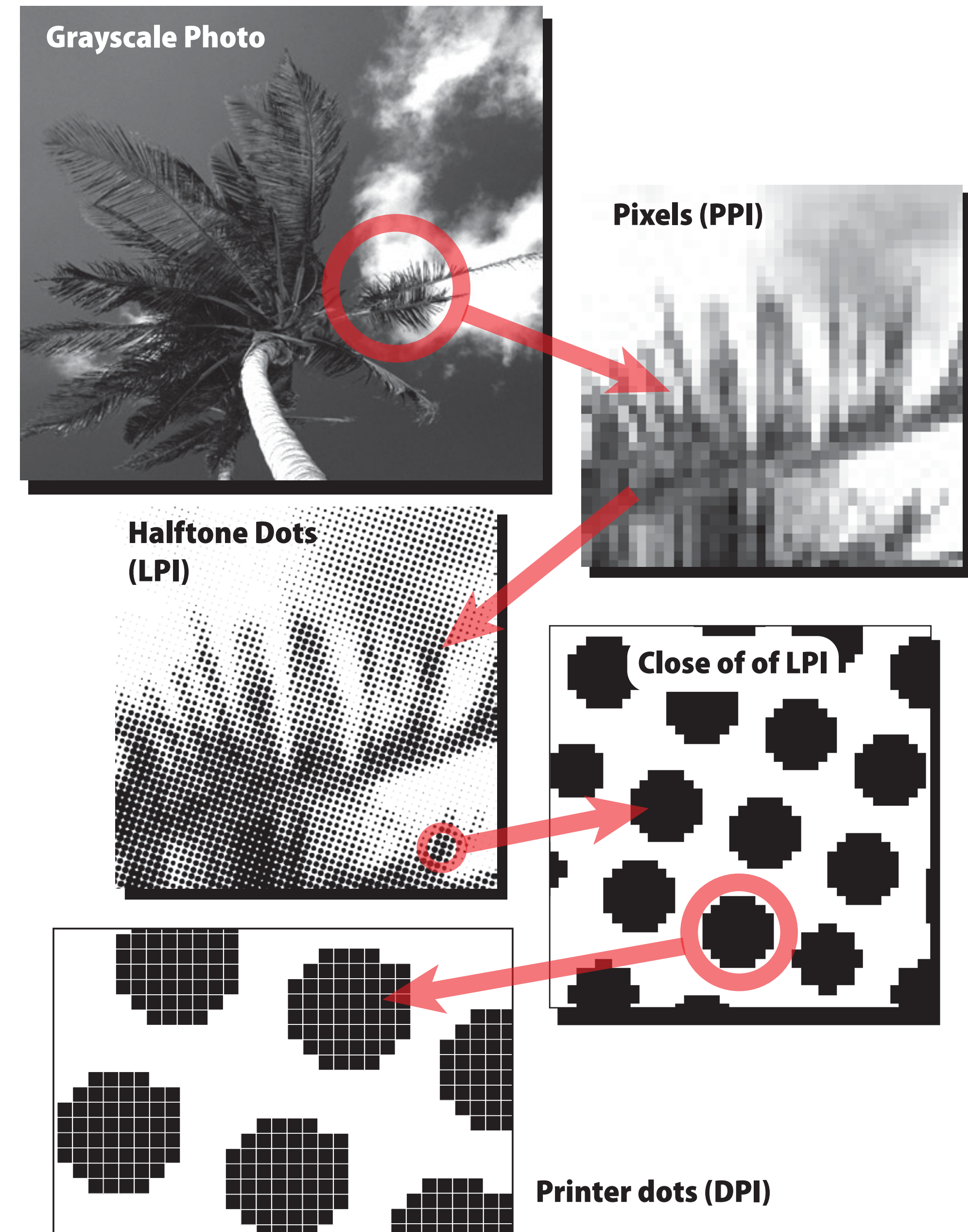


# Digital Halftoning

Each screen cell is divided into a tile of dots. By controlling how many dots are filled in a tile we get to approximate different dot sizes, i.e., gray scales.

- This is how digital halftoning implements amplitude modulation.

# of gray scales that can be mimicked = number of dots in a screen cell + 1.

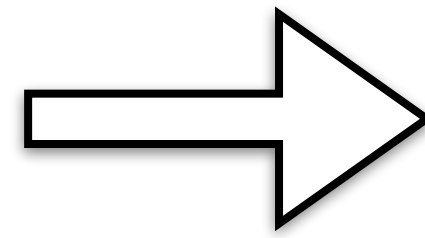




# Color Halftoning

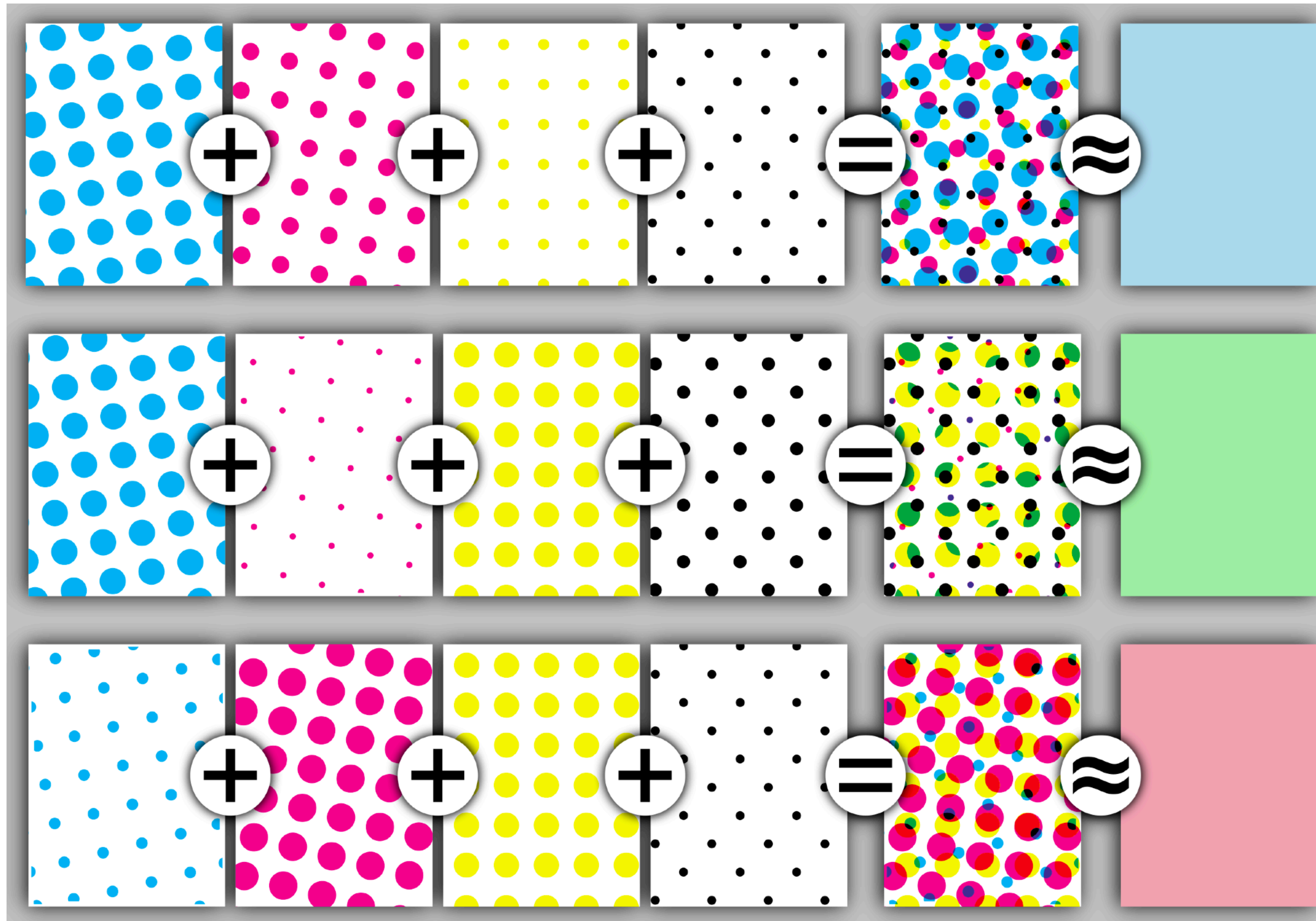


Color  
Separation





# Color Halftoning

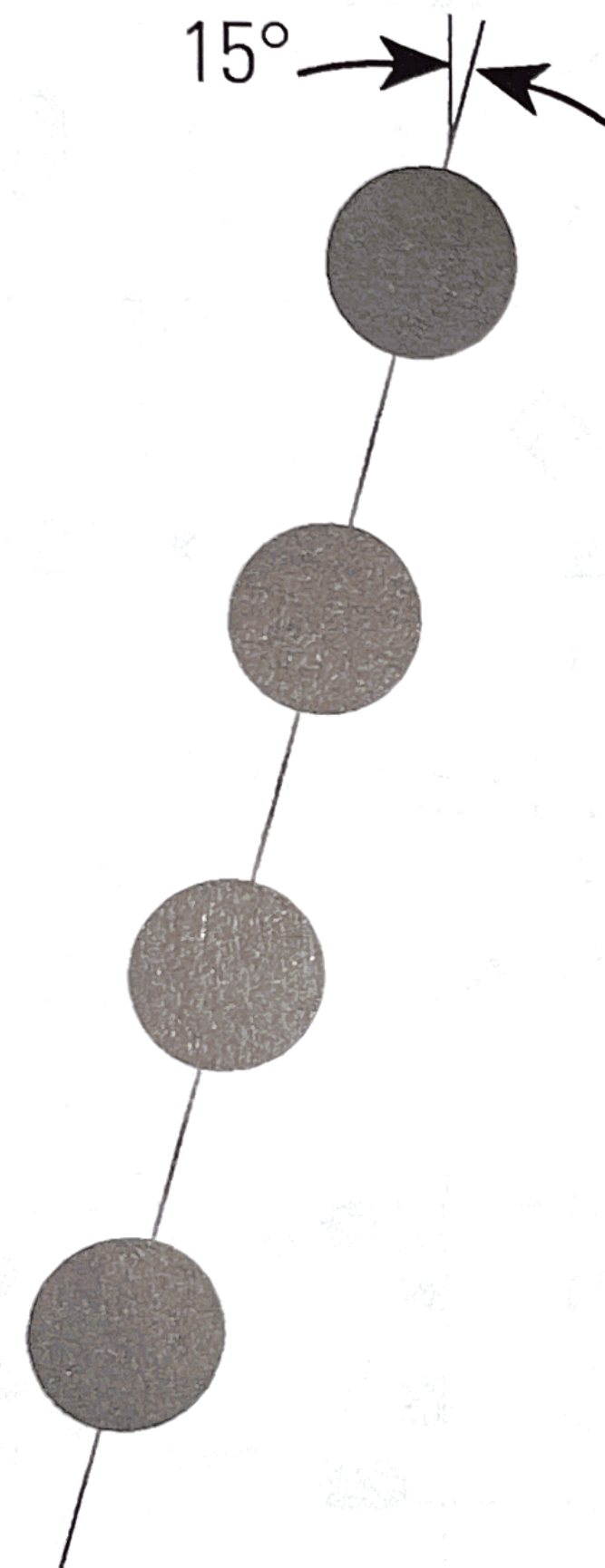


Each color separation is individually half-toned just like a gray-scale image.

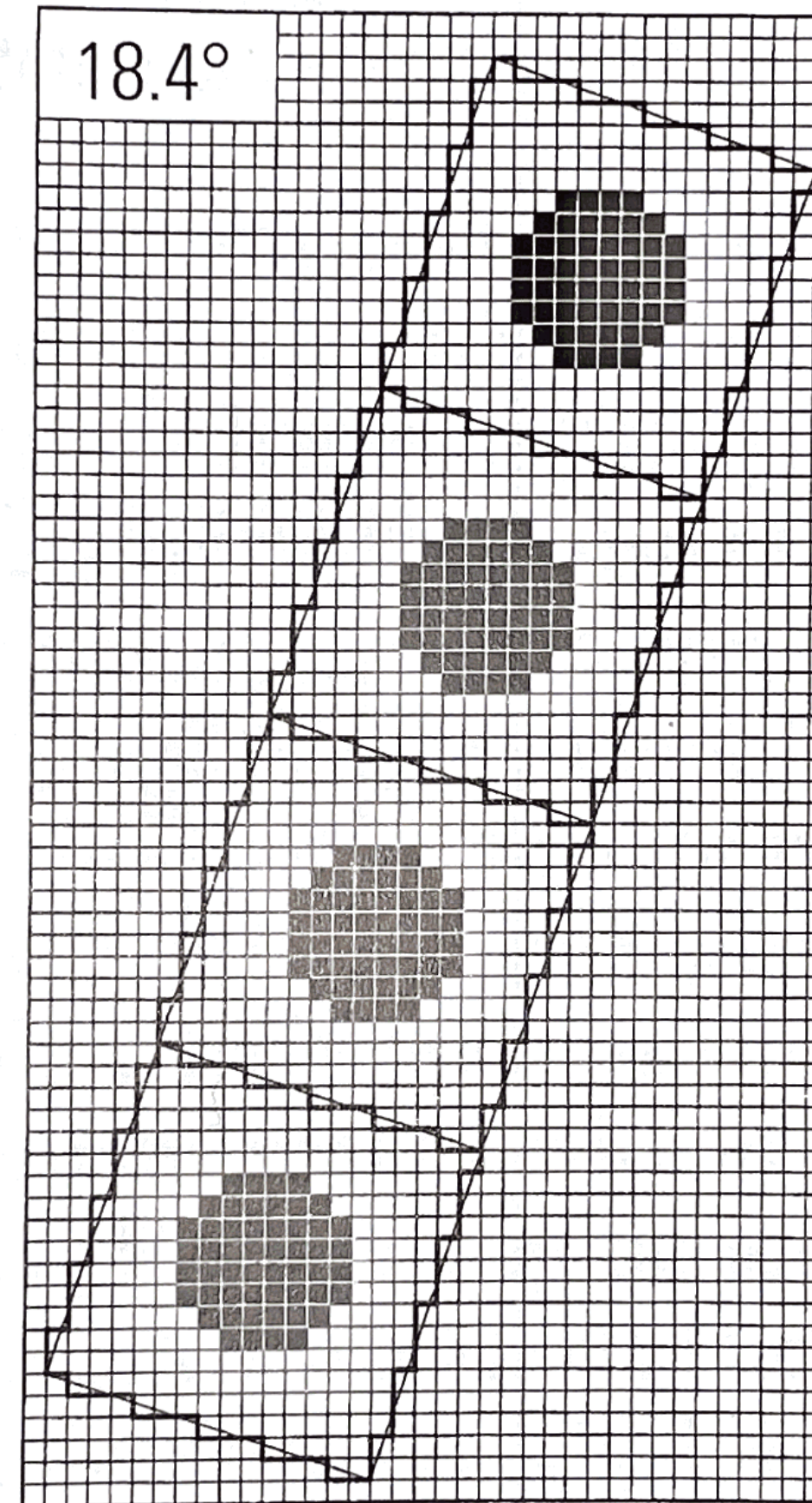
Each color separation's halftone pattern has a unique angle to reduce aliasing.



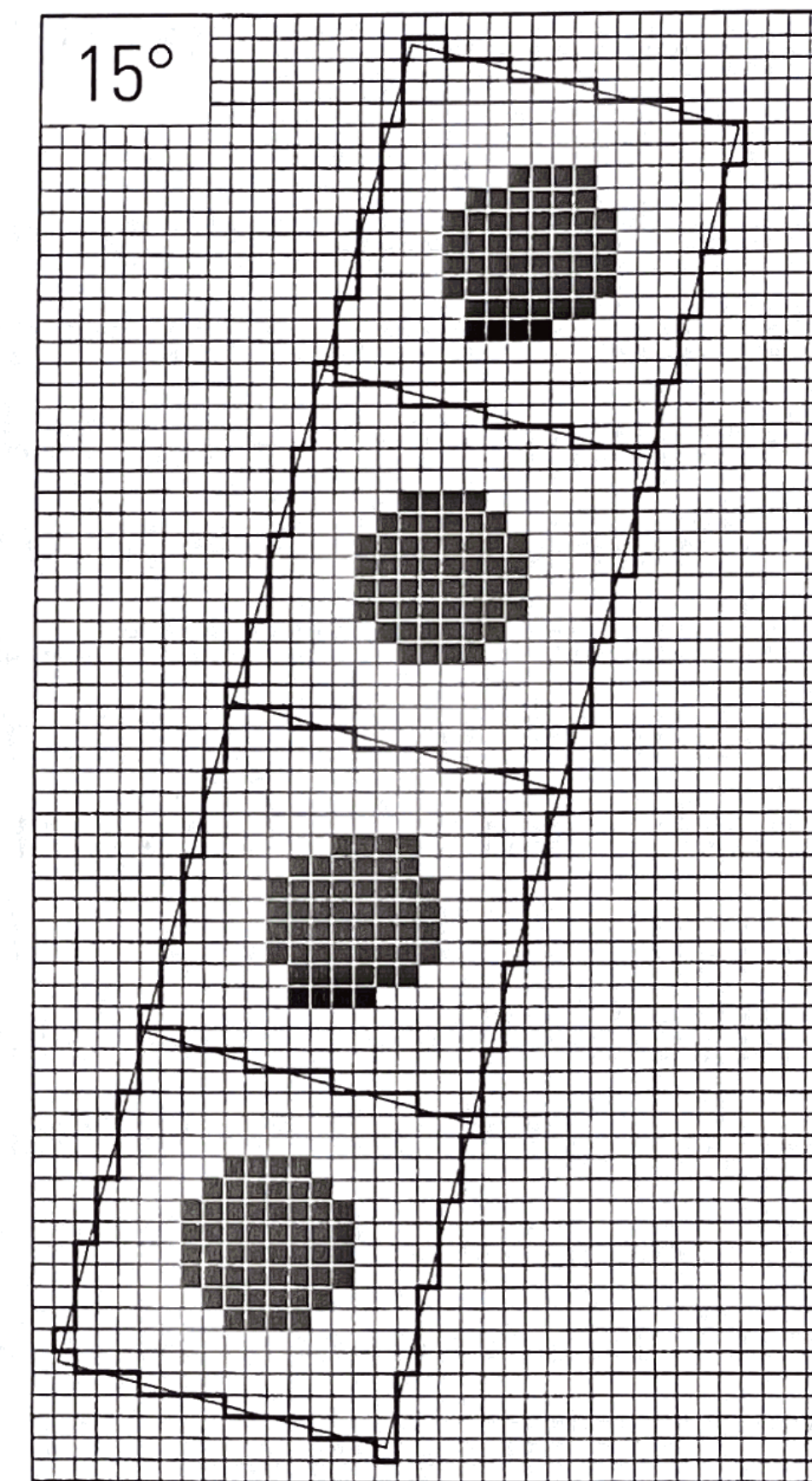
# Screen Orientation in Digital Halftoning



analog



"rational"

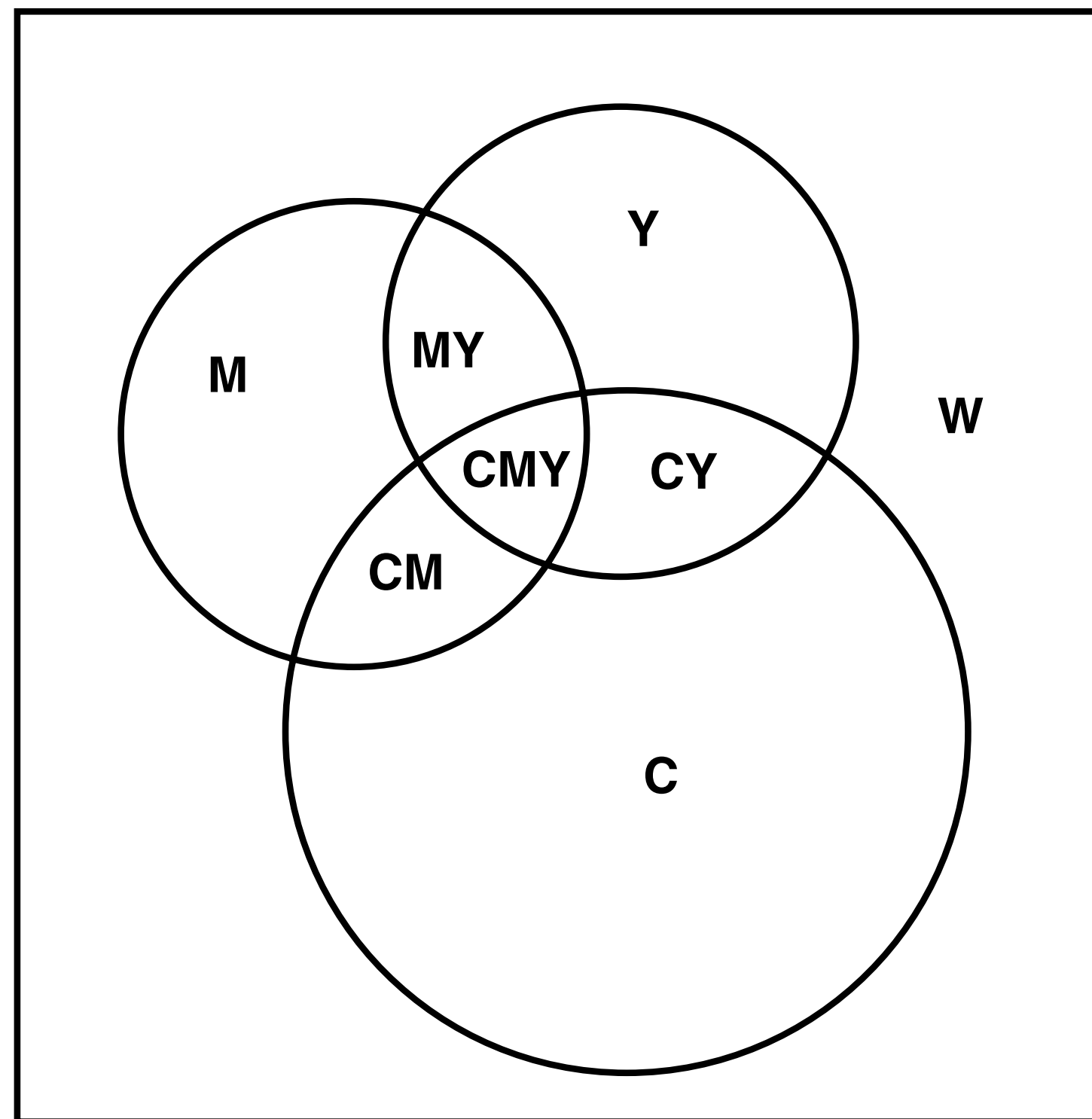


"irrational"

digital



# Modeling Color Halftoning



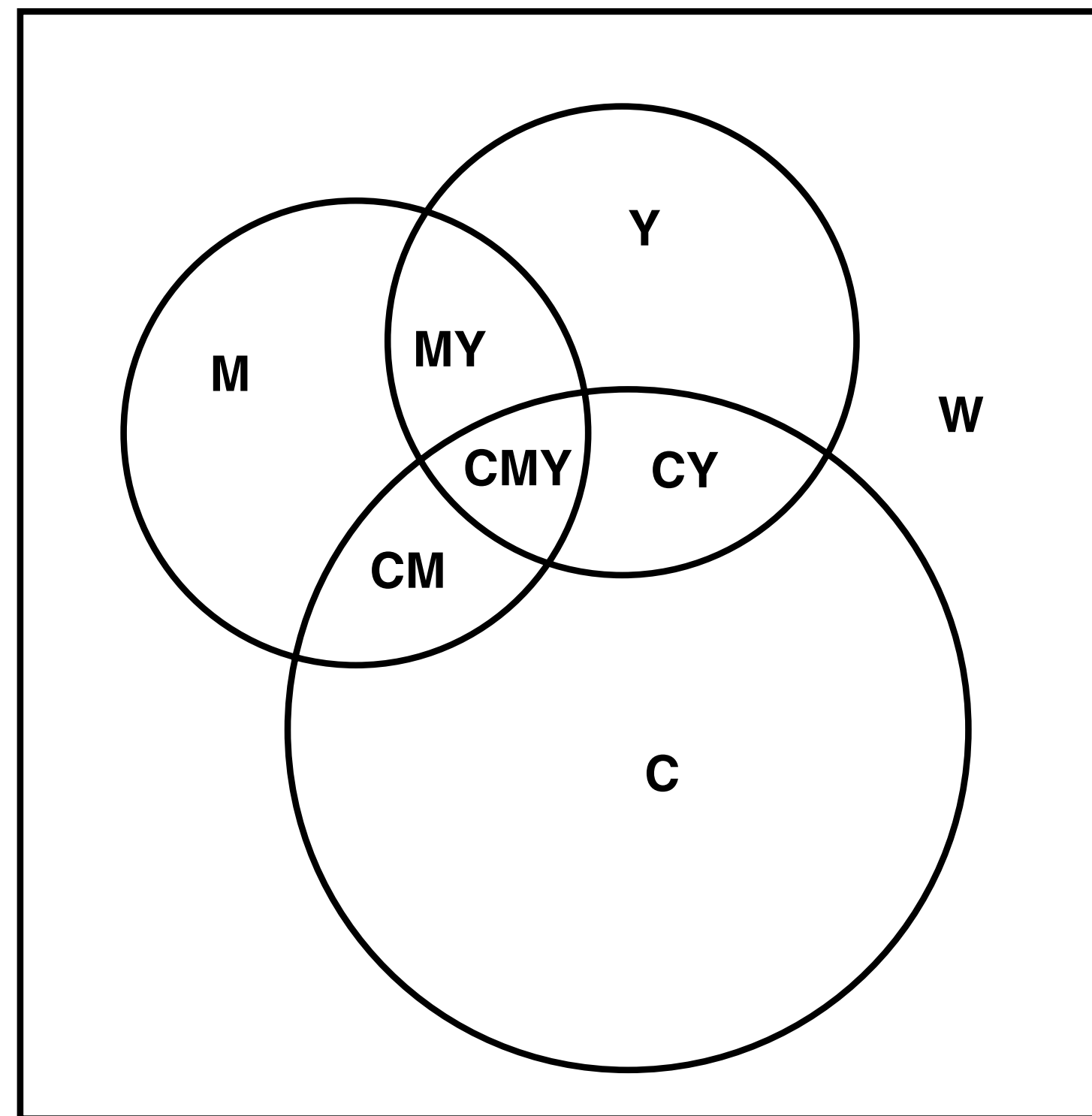
Mixing C, M, Y colorants to create in total 8 different colorants, each has a unique reflectance spectrum.

- The reflectance spectra of secondary colorants are dictated by both absorption and scattering, like discussed before; can use K-M model to approximate them but are usually measured.
- Because of scattering, mixing C, M, and Y doesn't exactly produce black, so usually a dedicated black (K) colorant is used.



# Modeling Color Halftoning

$$R = A_C R_C + A_M R_M + A_Y R_Y + A_{CM} R_{CM} + A_{MY} R_{MY} + A_{CY} R_{CY} + A_{CMY} R_{CMY} + A_W R_W$$



Area of the CMY colorant

Reflectance of the CMY colorant

The total reflectance is area-weighted reflectance of the eight colorants.

When printing, what we can control is the dot sizes of the C, M, Y colorants.

- Assuming the fractional areas of the three colorants are  $c$ ,  $m$ , and  $y$ , what are the areas of the others?



# Modeling Colorant Areas

$$R = \underbrace{A_C}_{c(1-m)(1-y)} R_C + \underbrace{A_M}_{(1-c)m(1-y)} R_M + \underbrace{A_Y}_{(1-c)(1-m)y} R_Y + \underbrace{A_{CM}}_{cm(1-y)} R_{CM} + \underbrace{A_{MY}}_{(1-c)my} R_{MY} + \underbrace{A_{CY}}_{c(1-m)y} R_{CY} + \underbrace{A_{CMY}}_{cmy} R_{CMY} + \underbrace{A_W}_{(1-c)(1-m)(1-y)} R_W$$

The assumption is that dot placements of the colorants are statically independent. Given this assumption:

- The fractional area of the C colorant can be seen as the **probability** that a point is hit by the C colorant
- The fractional area of the CMY colorant is the **joint probability** that a point is simultaneously hit by the C, M, and Y colorants



# Inverse Design

$$R = \underbrace{A_C}_{c(1-m)(1-y)} R_C + \underbrace{A_M}_{(1-c)m(1-y)} R_M + \underbrace{A_Y}_{(1-c)(1-m)y} R_Y + \underbrace{A_{CM}}_{cm(1-y)} R_{CM} + \underbrace{A_{MY}}_{(1-c)my} R_{MY} + \underbrace{A_{CY}}_{c(1-m)y} R_{CY} + \underbrace{A_{CMY}}_{cmy} R_{CMY} + \underbrace{A_W}_{(1-c)(1-m)(1-y)} R_W$$

Given an RGB value of an image pixel, we need to calculate the fractional areas of  $c$ ,  $m$ , and  $y$ , i.e., the dot sizes of the three colorants, so that the resulting color matches.

- Usually this is done over a tile of pixels over which our eyes spatially integrate. So it depends on the print size and the viewing distance.

The reverse process can either be analytical derived or use a look-up table.