

Lecture 5: Human Visual System: Spatial and Color Vision

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CSC 292/572, Fall 2022
Mobile Visual Computing

Logistics

Written assignment 2 is up and is due Sept. 23 11:30 AM.

- Can work in groups of 2.

Grades of WA1/WA0 will be posted soon.

The Roadmap

Theoretical Preliminaries

Human Visual Systems

Color in Nature, Arts, & Tech

(a.k.a., the birth, life, and death of light)

Digital Camera Imaging

Modeling and Rendering

Applications



Eye Anatomy

Spatial Vision

Color Vision

Colorimetry

Color science is a vastly interesting and broad discipline. We can only covering the basics. Dig (slightly) deeper using our interactive tutorials: <https://www.cs.rochester.edu/courses/572/colorvis/>.

Red Without Any Red Pixel



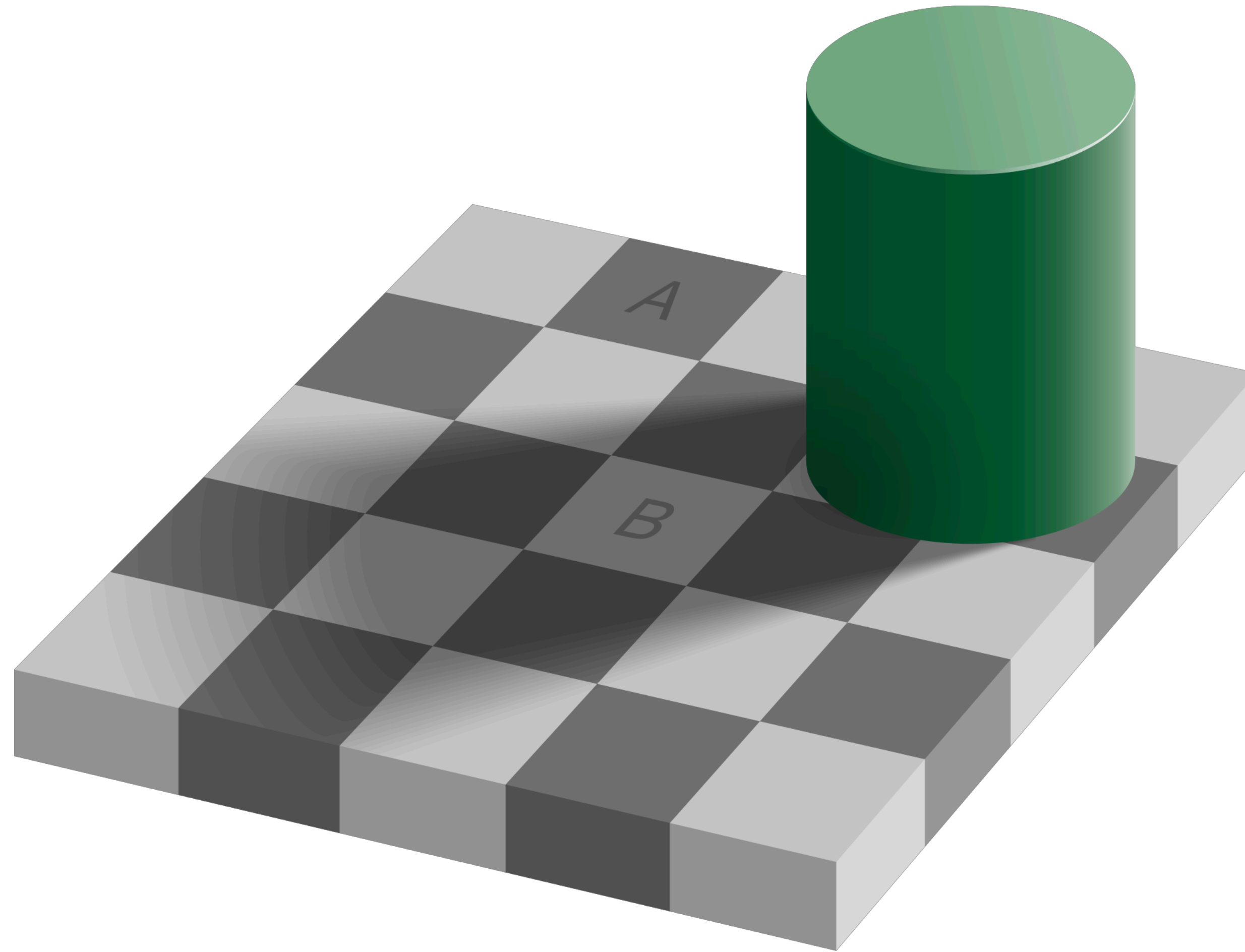
The Dress



The Dress



Checker Shadow Illusion

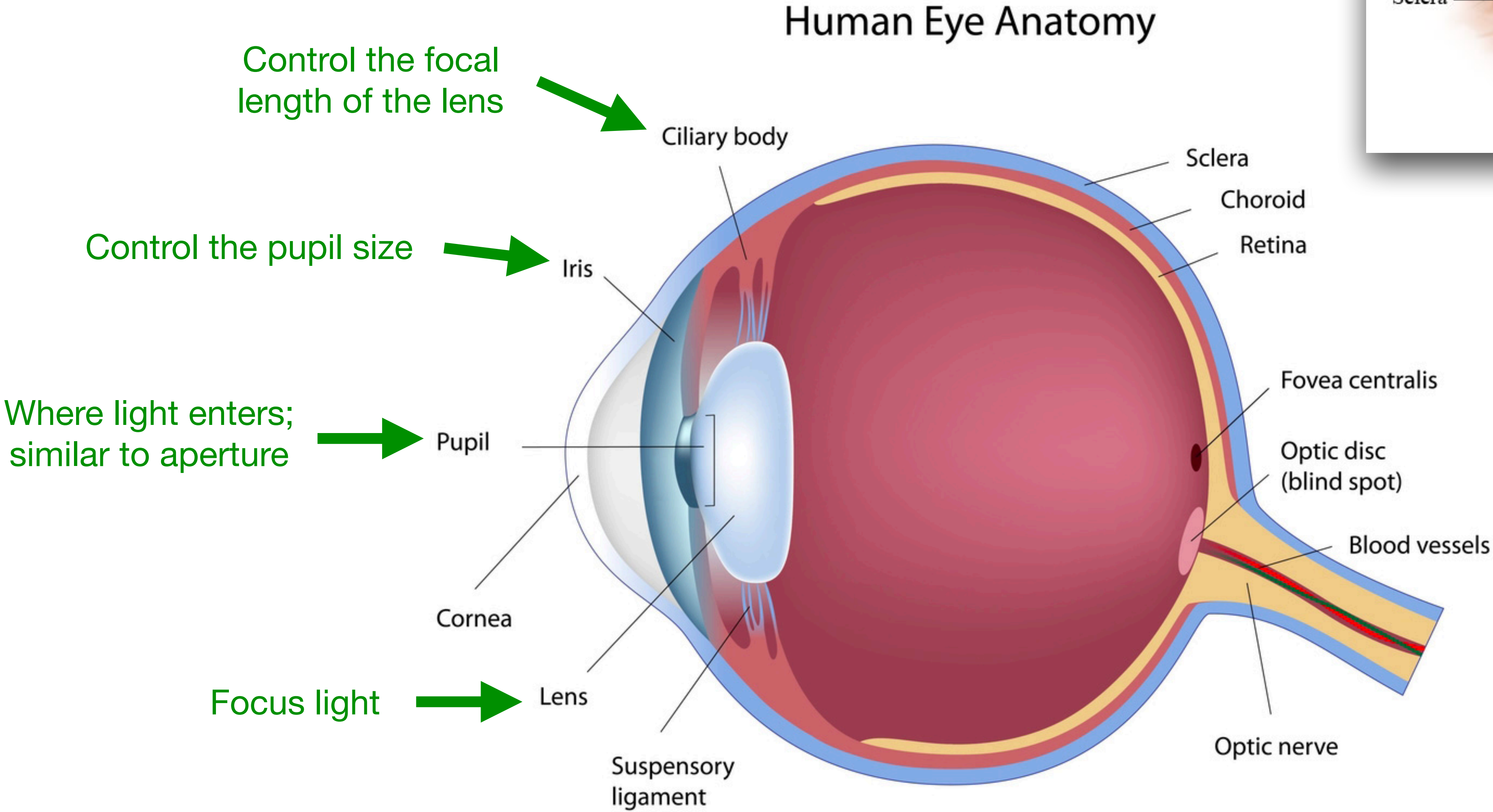
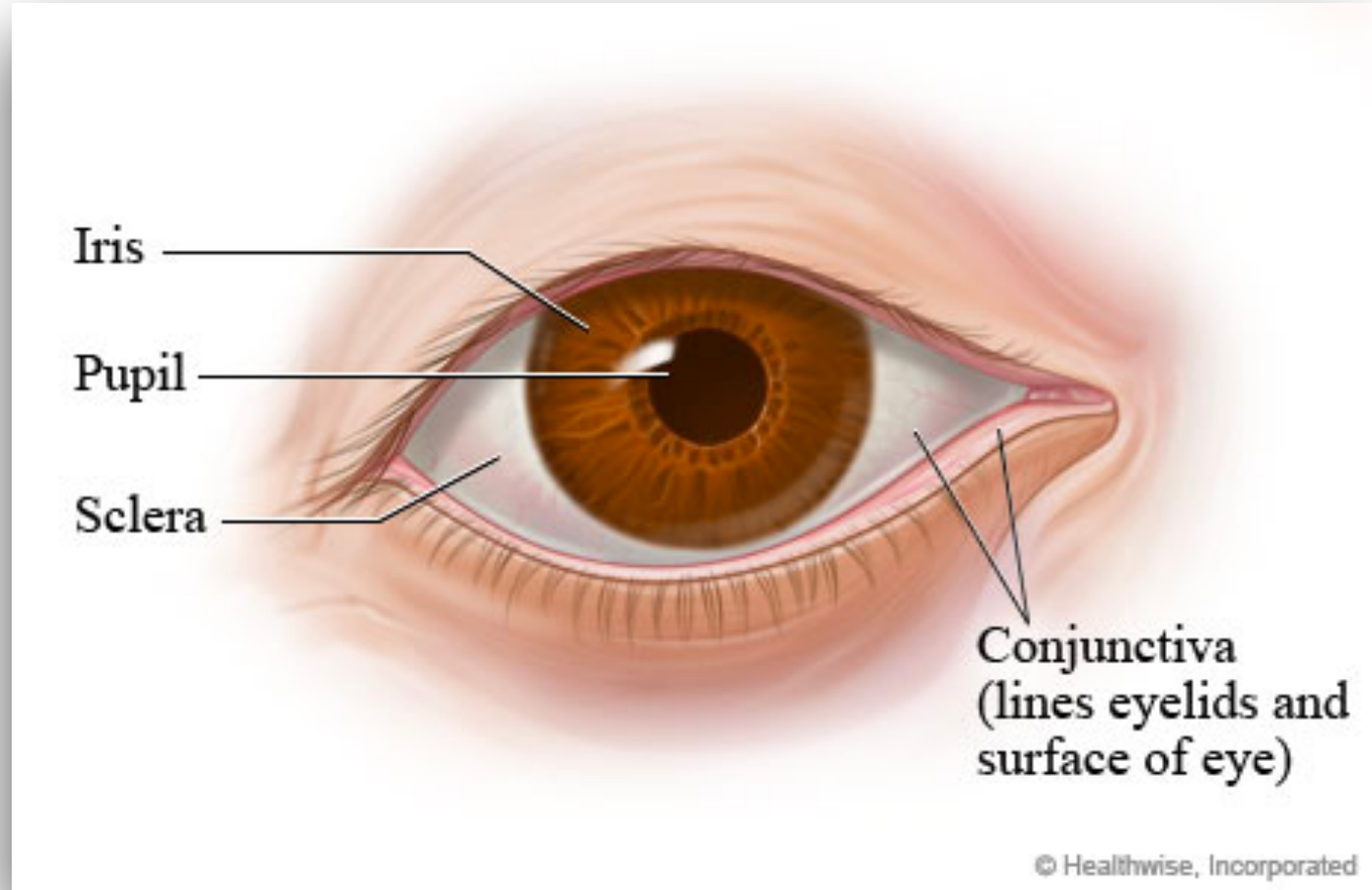


Checker Shadow Illusion



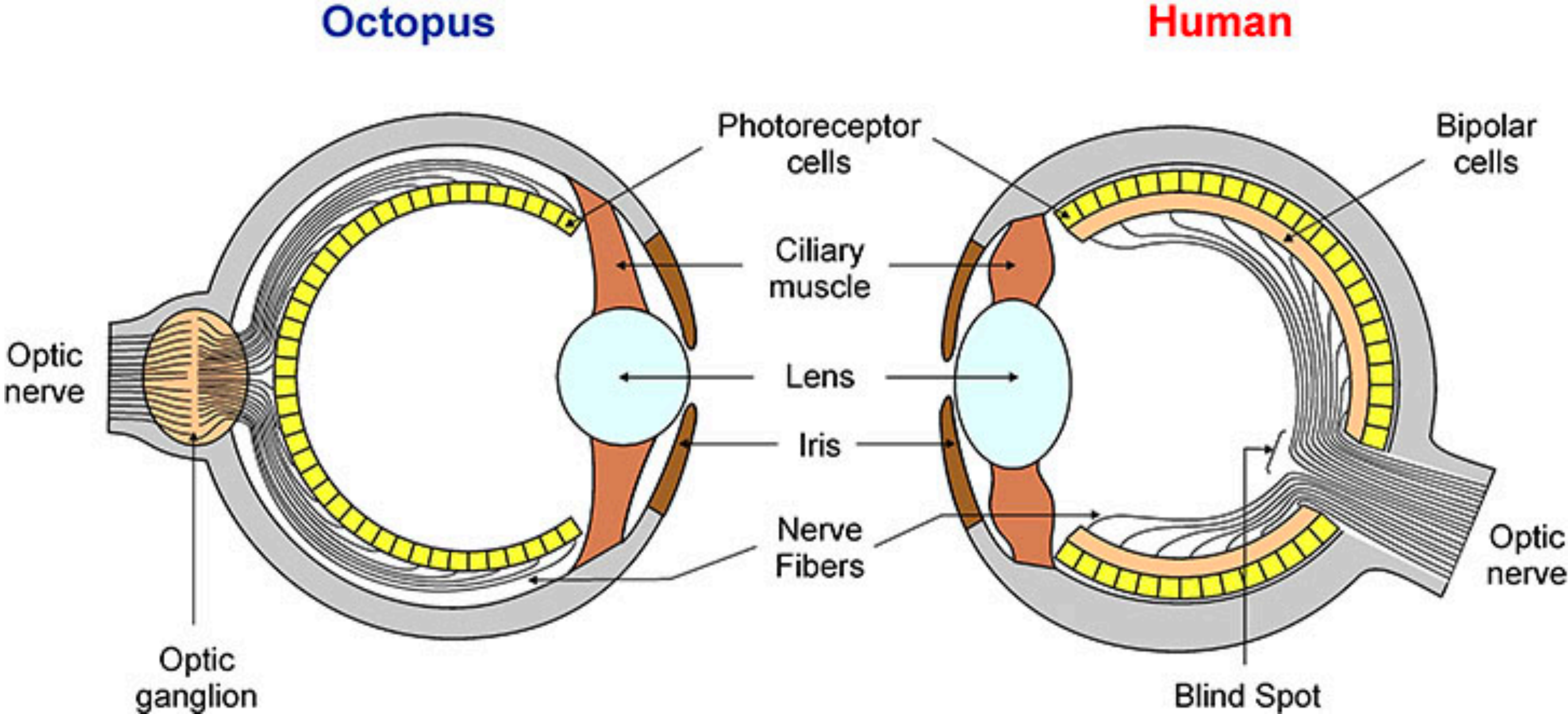
Eye Anatomy

Human Eye Anatomy



Blind Spot

Nerve fibers route before the retina in human eyes



Blind Spot: "See" It For Yourself

Test:

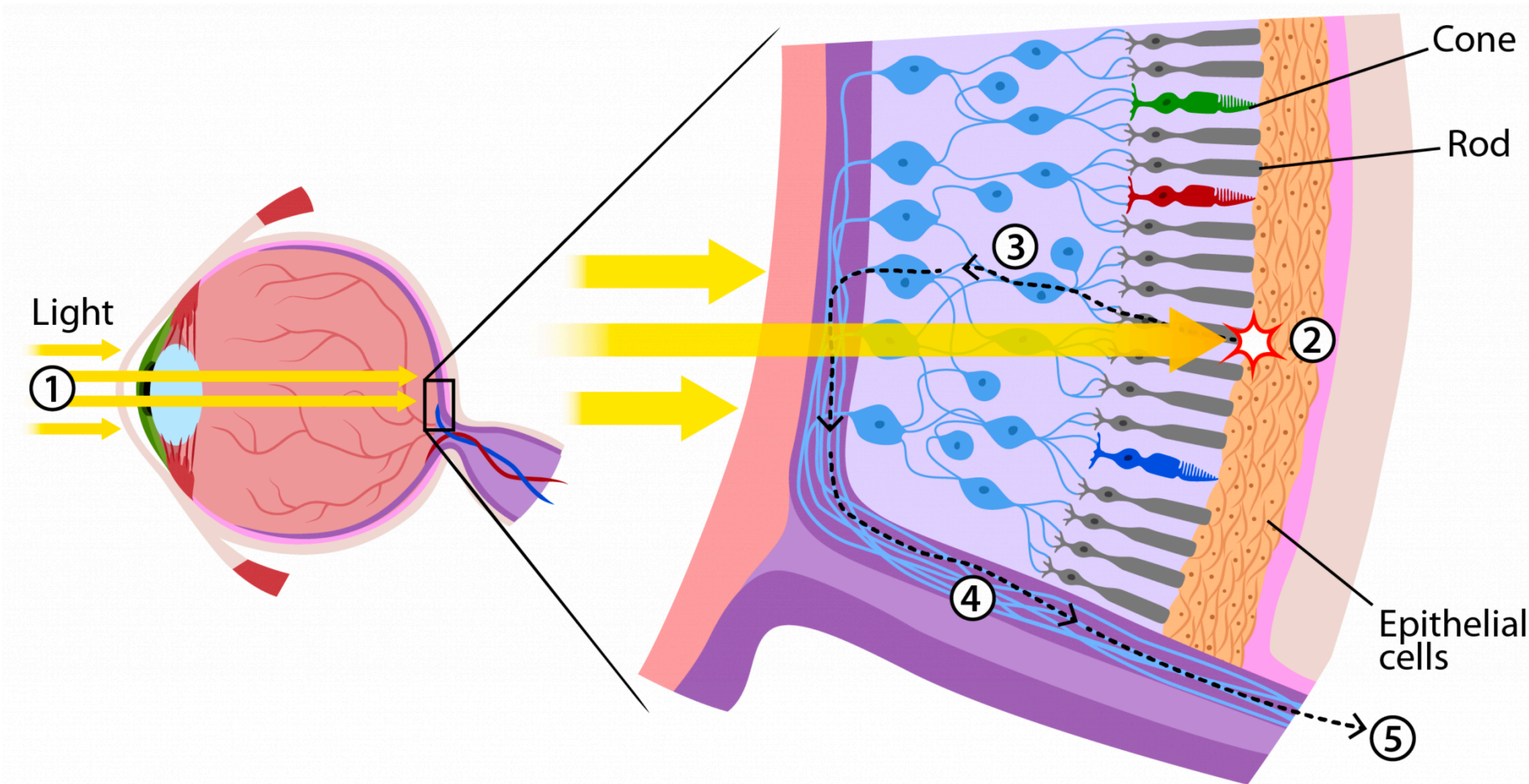
- Cover your LEFT eye and stare at the cross with your RIGHT eye.
- Slowly move towards the computer screen while still staring at the cross with your RIGHT eye.
- At some point the black circle will disappear.

Blind spot doesn't appear dark because your brain "makes stuff up"



Photoreceptors: Cones and Rods

Cells that **convert optical signals (lights) to electrical signals** (voltage potentials), which then stimulate biological processes in your brain.



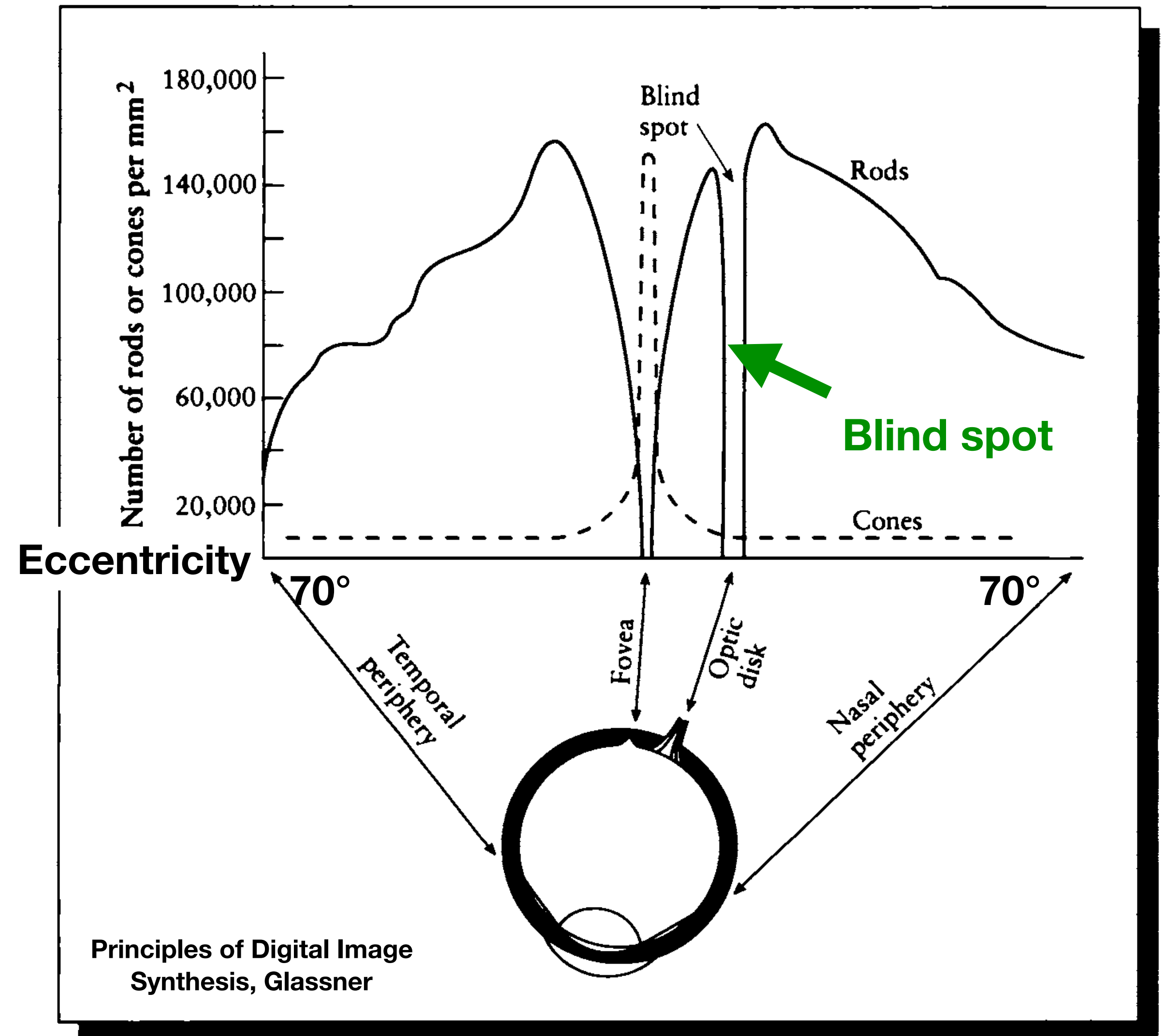
Photopic vision: activated at typical light levels; ~6 million cones. Provides color sensation.

Scotopic vision: activated at low light levels; ~120 million rods. Provides no color sensation.

Photoreceptor Density Distribution

Uneven. The retina area with the highest cone density is called *fovea*.

Fovea angle is about 2° . Peripheral vision has very low acuity.



Foveated Rendering



From Light to Visual Perception

Light

For the purpose of understanding color, we consider light as photons.

Lights emit photons. Materials emit, absorb, and/or scatter photons.

Each photon has energy, dictated by its frequency/wavelength.

Energy of a $\lambda = 500$ nm
photon: $\sim 4 \times 10^{-34}$ J

Energy of a $\lambda = 750$ nm
photon: $\sim 2.7 \times 10^{-34}$ J

Energy of a photon — $E = hf = \frac{hc}{\lambda}$

Planck constant. $\sim 6.63 \times 10^{-34}$ J·s

Light frequency

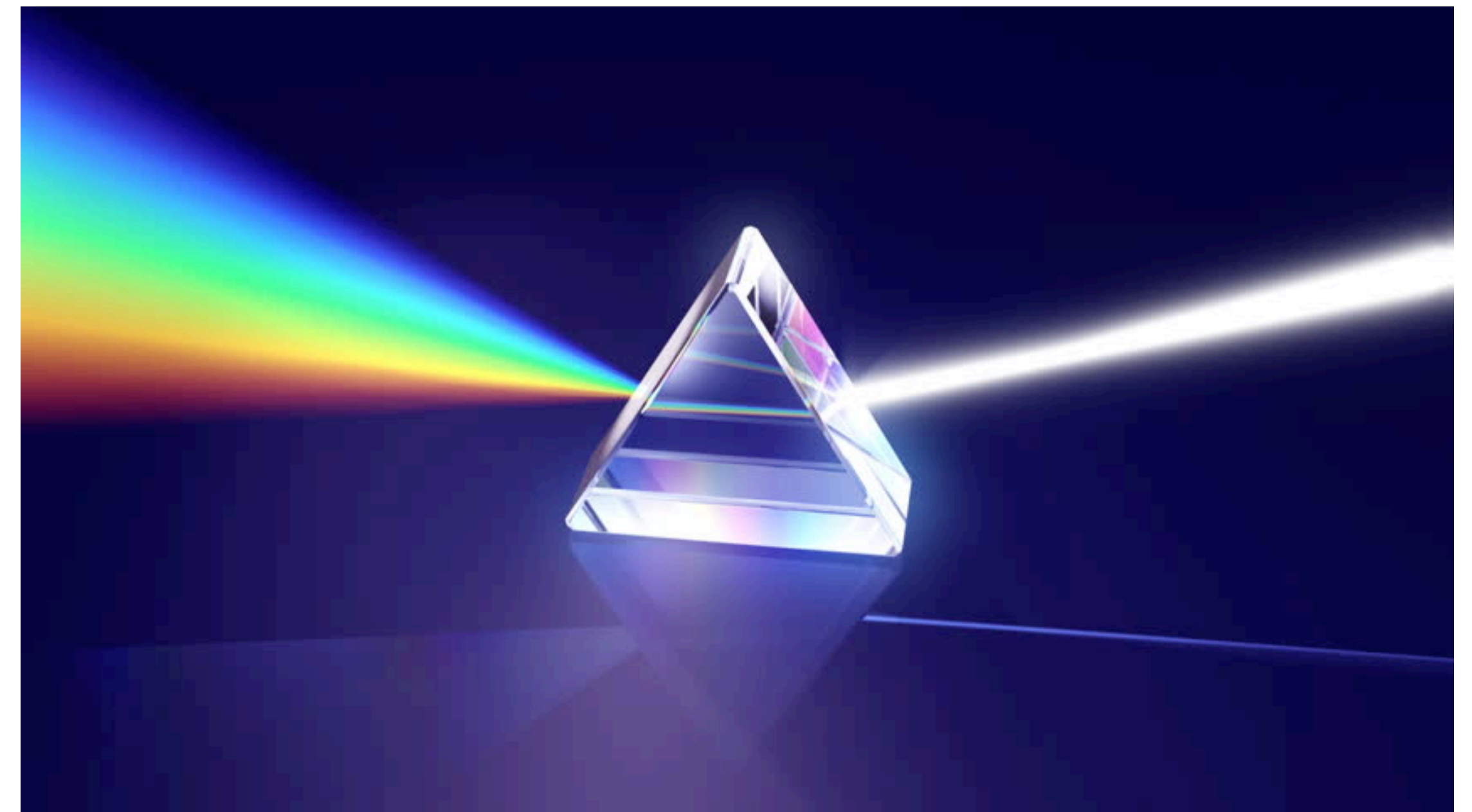
Speed of light (m/s)

Light wavelength

The diagram shows the equation $E = hf = \frac{hc}{\lambda}$ in green. Lines connect the variables to their definitions: E to 'Energy of a photon', h to 'Planck constant. $\sim 6.63 \times 10^{-34}$ J·s', f to 'Light frequency', c to 'Speed of light (m/s)', and λ to 'Light wavelength'.

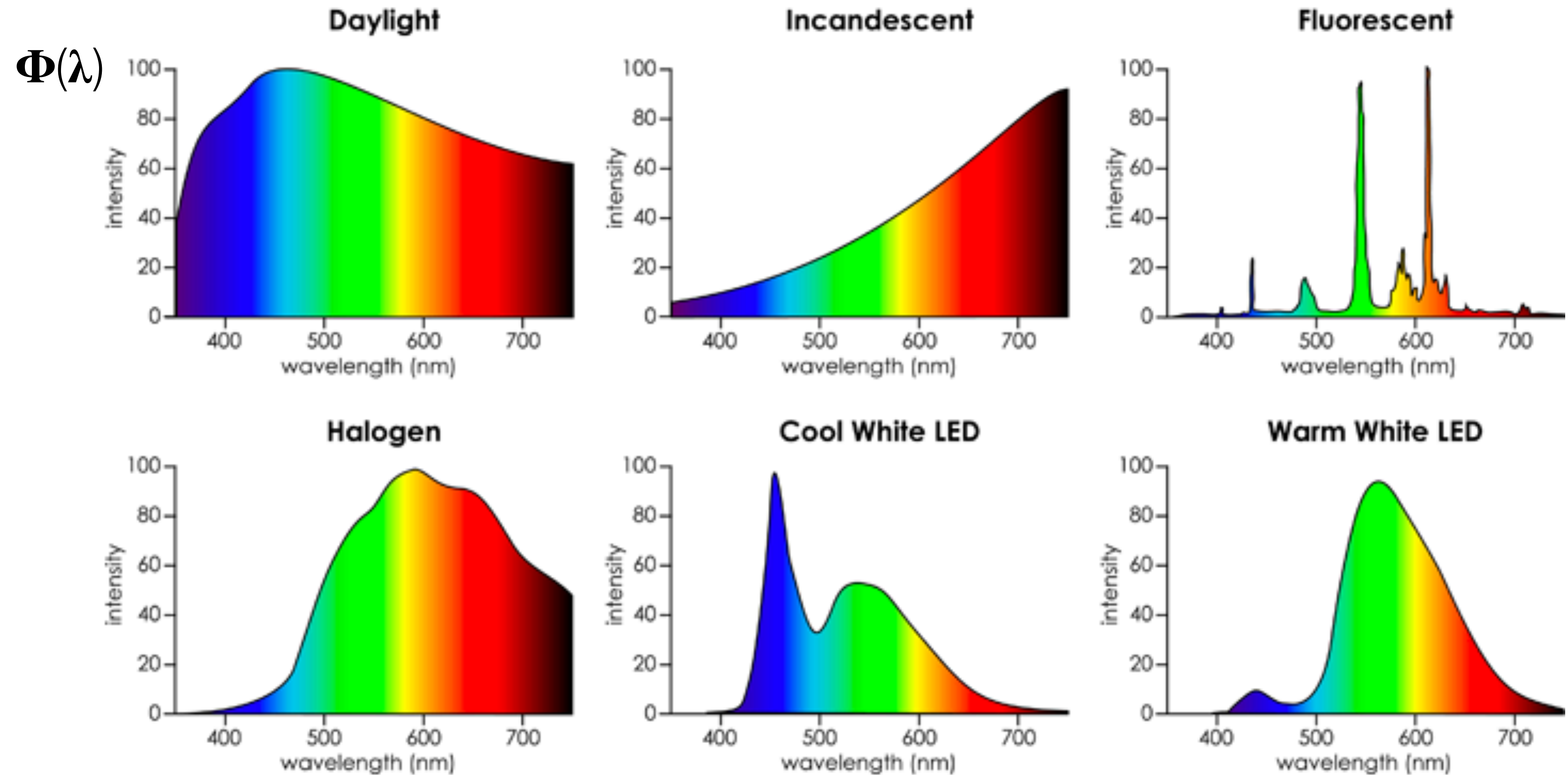
Light Spectrum

Most light sources mix photons of different wavelengths.



Spectral Power Distribution

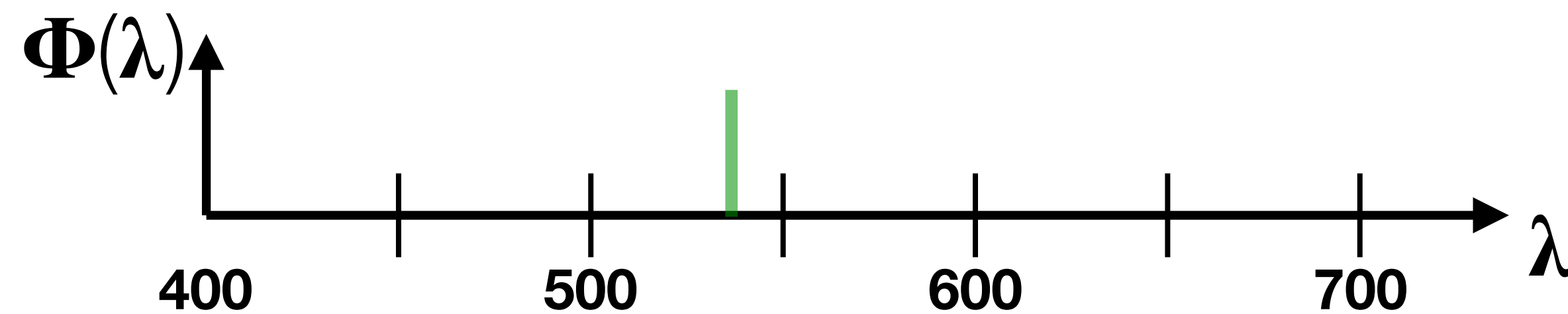
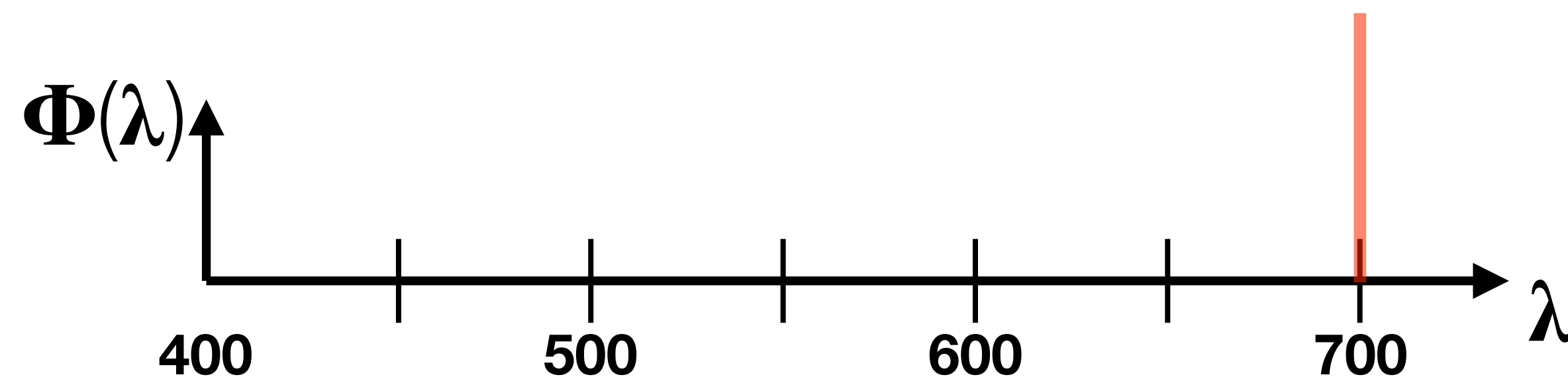
SPD $\Phi(\lambda)$: the power distribution by wavelength.
Unit is W/nm.



Spectral Light

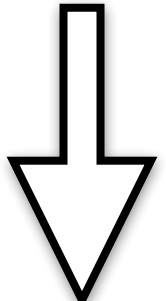
Single-wavelength (monochromatic) light are called **spectral light**.

- SPDs of spectral lights are very narrow spikes.
- Few natural lights are spectral lights. Lasers can generate spectral lights.

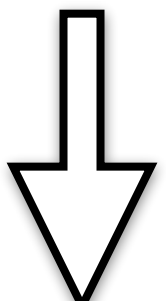


Cone Responses

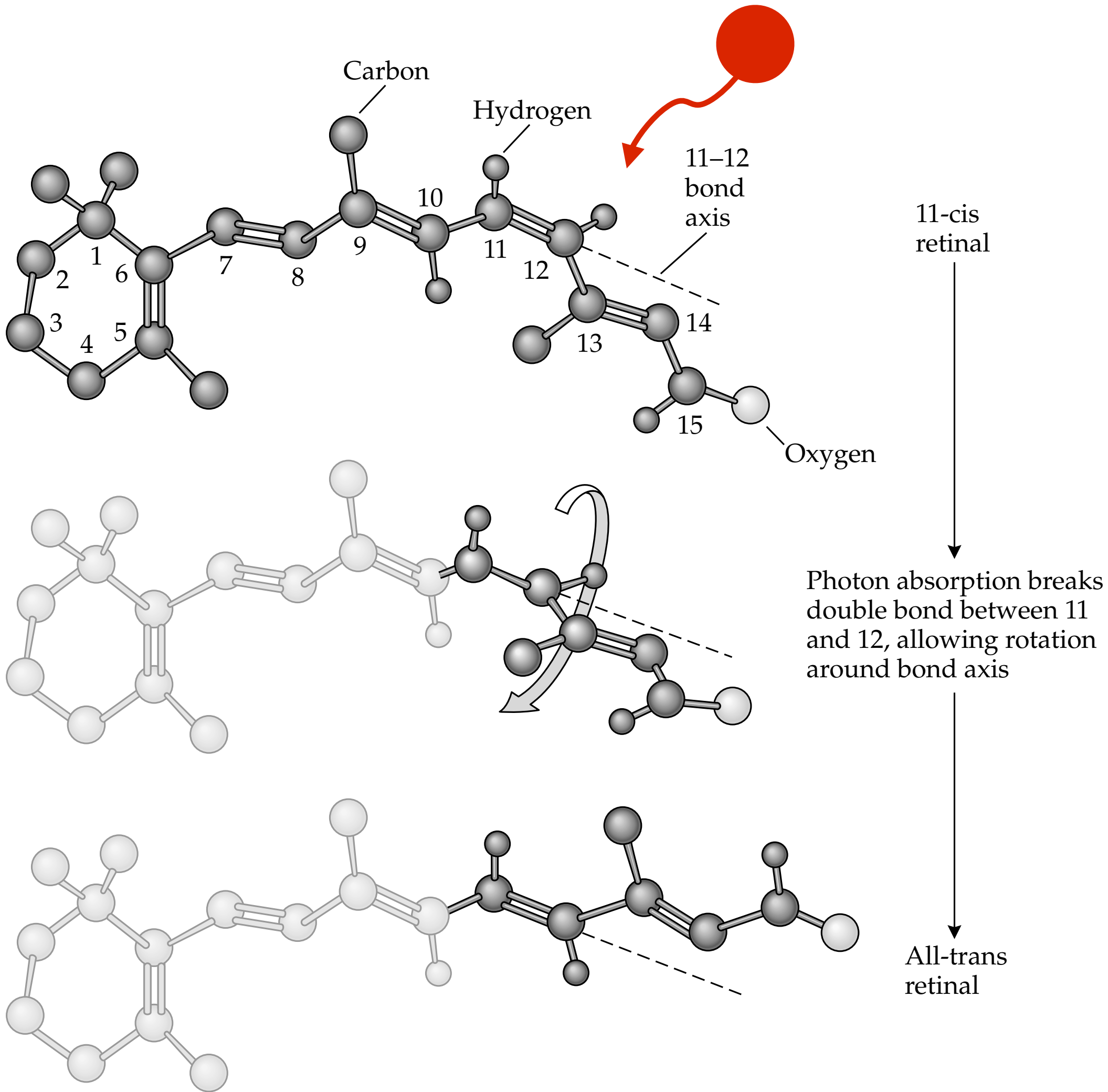
Optical Signal (λ)



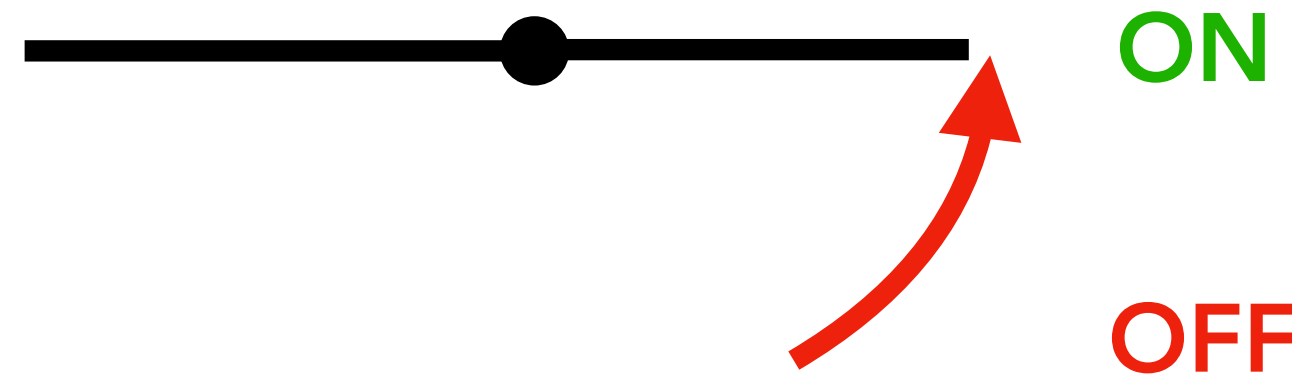
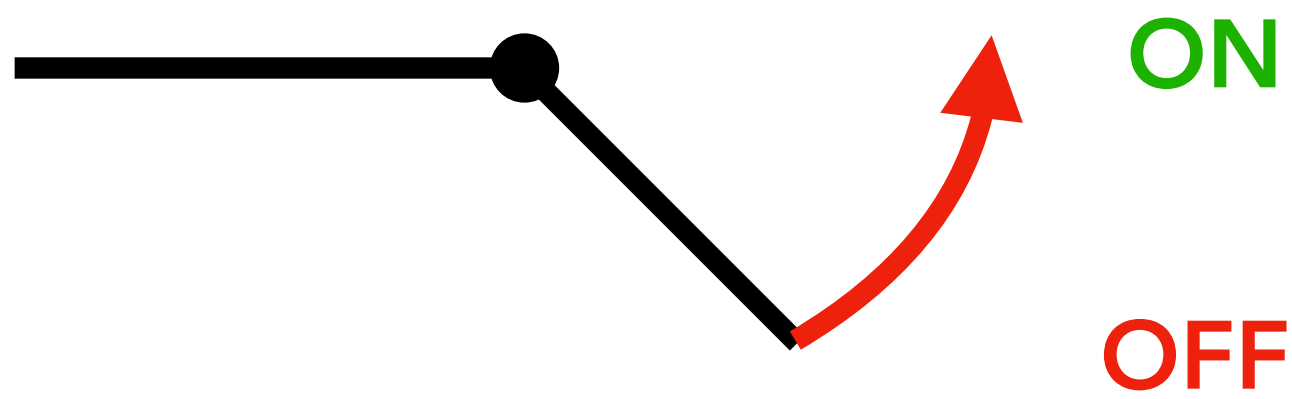
Cones



Electrical Signal (λ)



Think of this molecule as a photon-sensitive switch

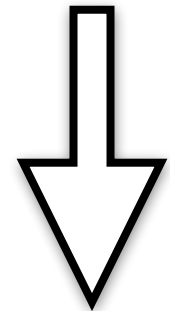


"The Human Eye: Structure and Function", by Clyde W. Oyster (1999)

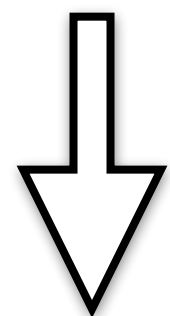
What Dictates Photon Absorption?

Two factors that decide whether a photon is absorbed.

Optical
Signal (λ)



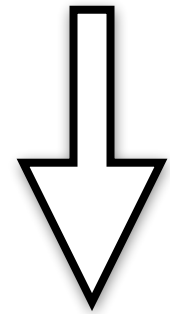
Cones



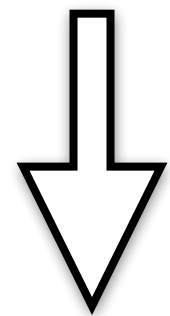
Electrical
Signal (λ)

What Dictates Photon Absorption?

Optical
Signal (λ)



Cones



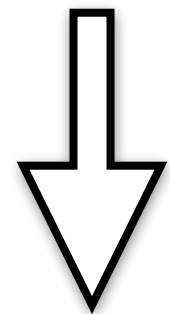
Electrical
Signal (λ)

Two factors that decide whether a photon is absorbed.

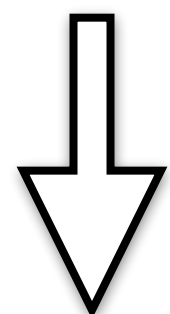
1. Which type of cone a photon hits?

What Dictates Photon Absorption?

Optical
Signal (λ)



Cones



Electrical
Signal (λ)

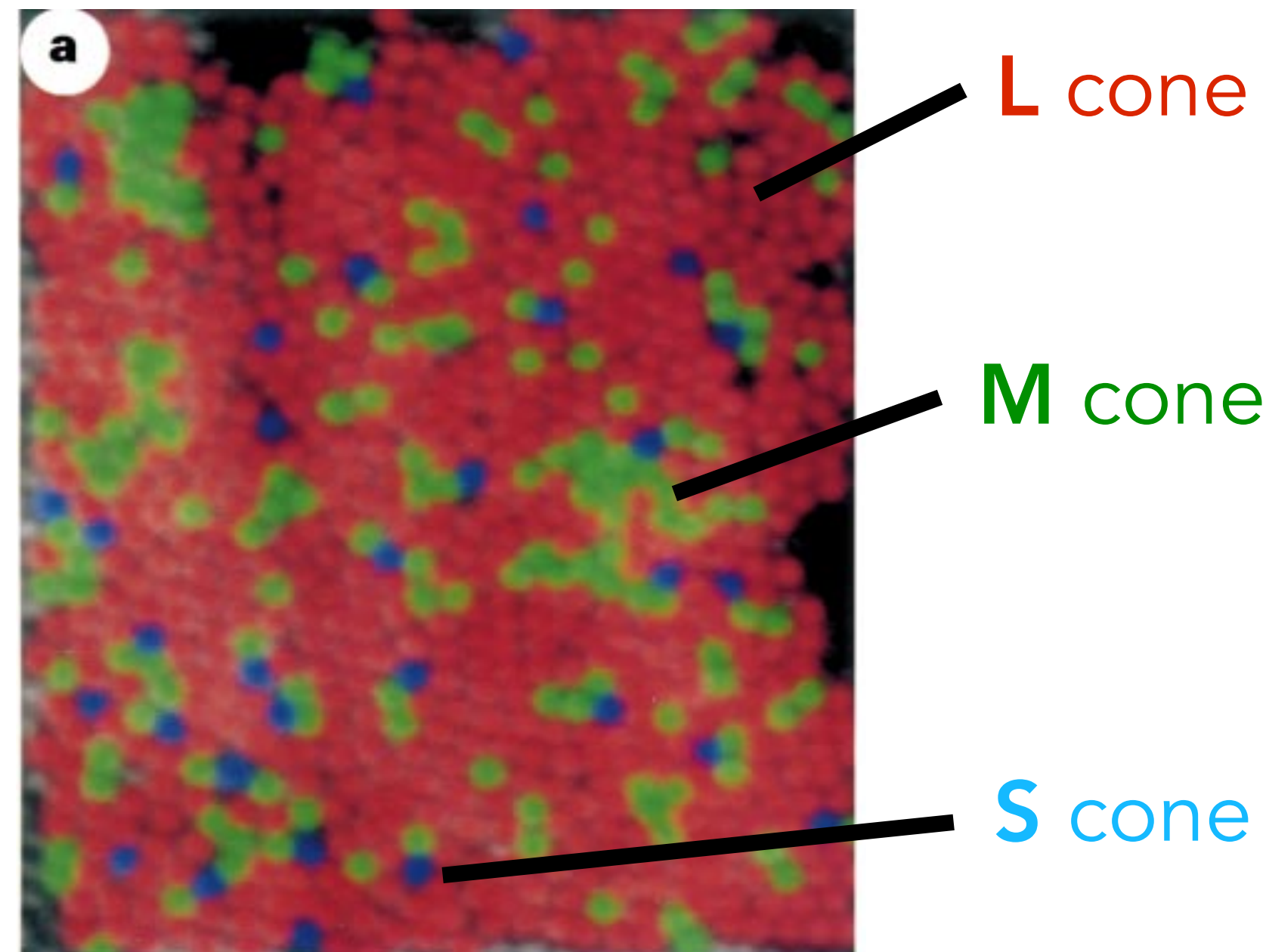
Two factors that decide whether a photon is absorbed.

1. Which type of cone a photon hits?
2. What's the wavelength of the photon?

Three Cone Types

Three types of cone exists

- Long (L), Medium (M), Short (S)
- At 10° fovea: ~63% L, ~31% M, ~6% S



David Williams, UR



The arrangement of the three cone classes in the living human eye

Austin Roorda^{††} & David R. Williams^{*}

^{*}Center for Visual Science, University of Rochester, Rochester, New York 14627, USA

Human colour vision depends on three classes of receptor, the short- (S), medium- (M), and long- (L) wavelength-sensitive cones. These cone classes are interleaved in a single mosaic so that, at each point in the retina, only a single class of cone samples the retinal image. As a consequence, observers with normal trichromatic colour vision are necessarily colour blind on a local spatial scale¹. The limits this places on vision depend on the relative numbers and arrangement of cones. Although the topography of human S cones is known^{2,3}, the human L- and M-cone submosaics have resisted analysis. Adaptive optics, a technique used to overcome blur in ground-based telescopes⁴, can also overcome blur in the eye, allowing the sharpest images ever taken of the living retina⁵. Here we combine adaptive optics and retinal densitometry⁶ to obtain what are, to our knowledge, the first images of the arrangement of S, M and L cones in the living human eye. The proportion of L to M cones is strikingly different in two male subjects, each of whom has normal colour vision. The

^{††}Present address: University of Houston, College of Optometry, Houston, Texas 77204-6052, USA

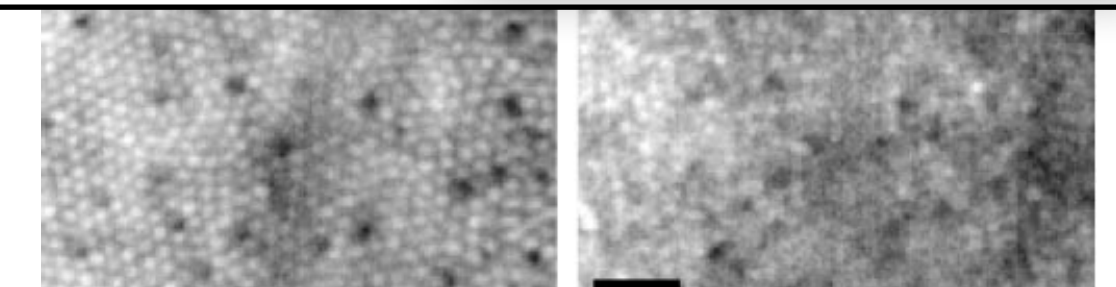


Figure 1 Images of the right eye of subject JW. The mosaic was illuminated with a 4-ms flash (1 degree diameter, ~0.3 μ J) through a 2-mm entrance pupil. Light of wavelength 550 nm was used to maximize absorbance by L- and M-cone photopigment. Images were obtained with a 6-mm exit pupil at a retinal eccentricity of one degree nasal from the foveal centre, which is located to the left of the image. For each retinal location, about 50 images taken over 5 days were averaged to increase the signal-to-noise ratio. Fixational eye movements translate the image from flash to flash, requiring registration with cross-correlation before averaging. **a**, A registered sum of 61 images taken after a full bleach. **b-d**, Absorbance images of the patch of cones shown in **a**, defined as 1 minus the ratio of the absorbance of a dark-adapted or selectively bleached image to the absorbance of the corresponding fully bleached image. Images of fully bleached retina were obtained following exposure to 550-nm light (70-nm bandwidth, 37×10^6 troland-seconds). Images of dark-adapted retina were taken following 5 min of dark adaptation. **b**, The absorbance image of dark-adapted cones, revealing a sparse array of S cones which appear dark because of their low absorbance at 550 nm. **c, d**, Absorbance images following a 470-nm (**c**) and 650-nm (**d**) selective bleach. Bleaching energies were set by calculation and then modified empirically to achieve the maximum possible difference between M- and L-photopigment concentration. The absorbance images for both bleaching conditions have higher pigment density toward the fovea; this is caused by the increases in the length of the outer segment and in macular pigment, which reduce bleaching in the 470-nm condition. Scale bar represents 5 arcmin of visual angle.

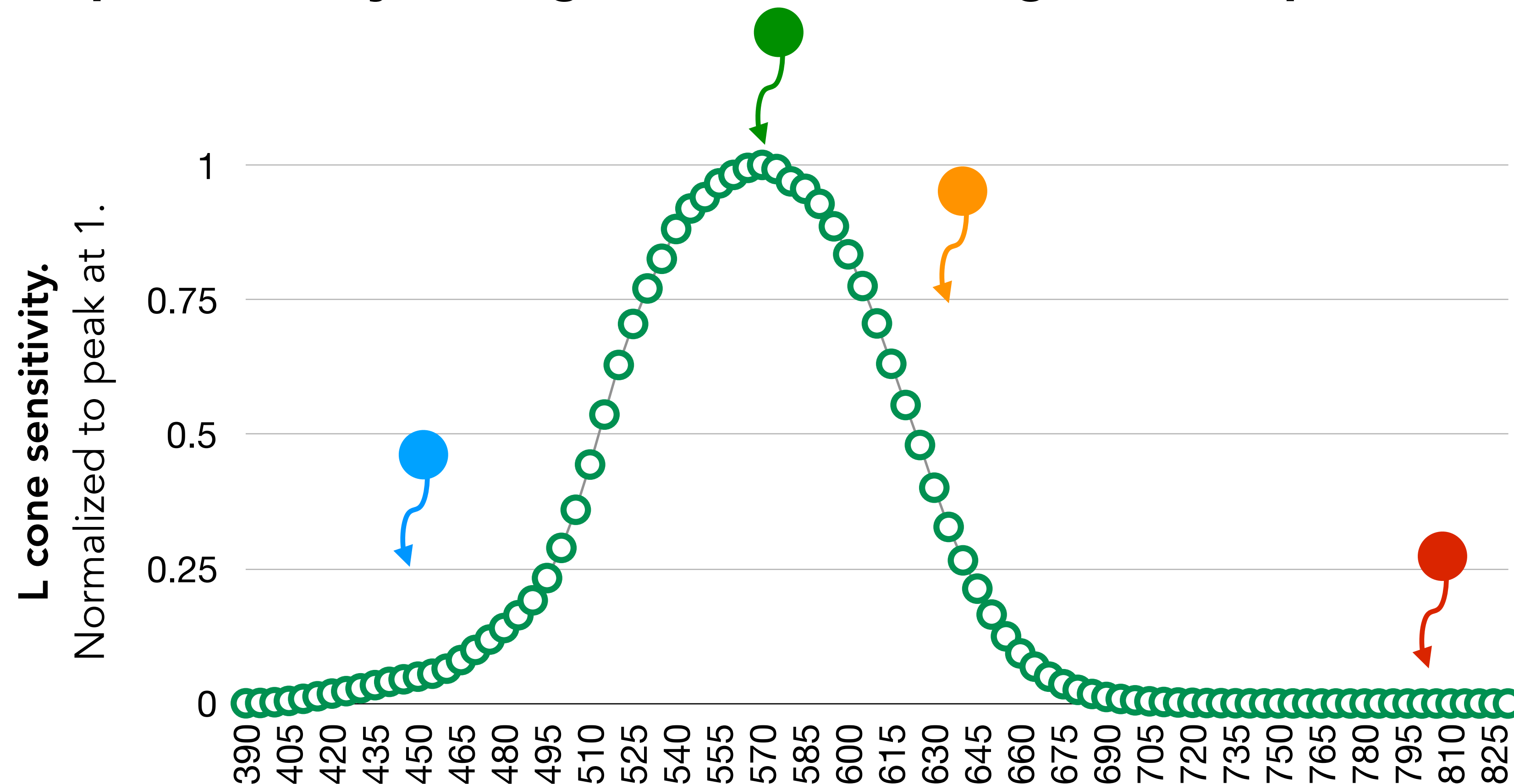
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Spectral Response/Sensitivity of Cones

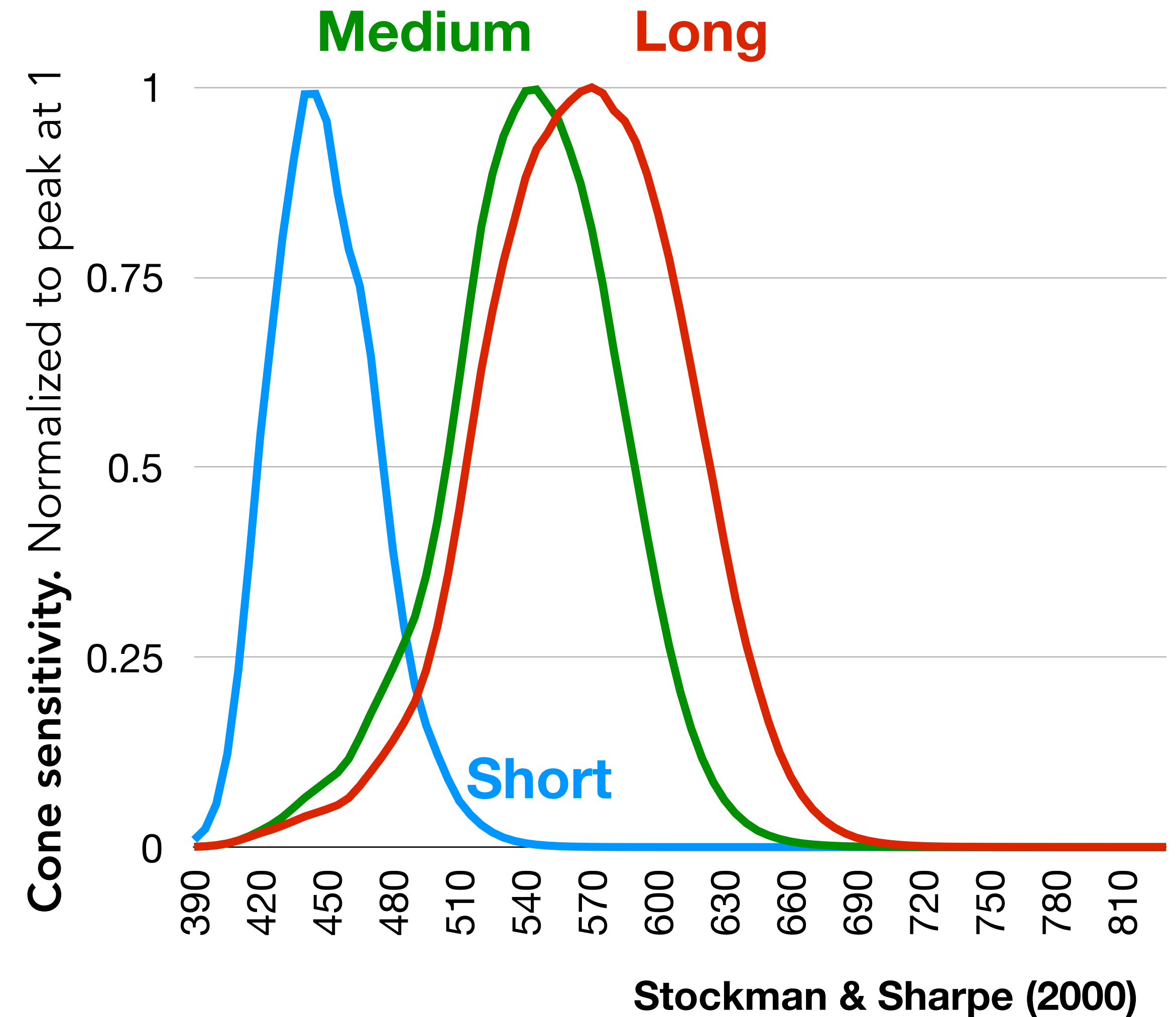
Absorption **probability** changes with wavelength, i.e., **spectral** sensitivity.



* What's shown here is not raw absorption spectra, but measured cone sensitivity, which is resulted from but not equivalent to absorption.

Spectral Response/Sensitivity of Cones

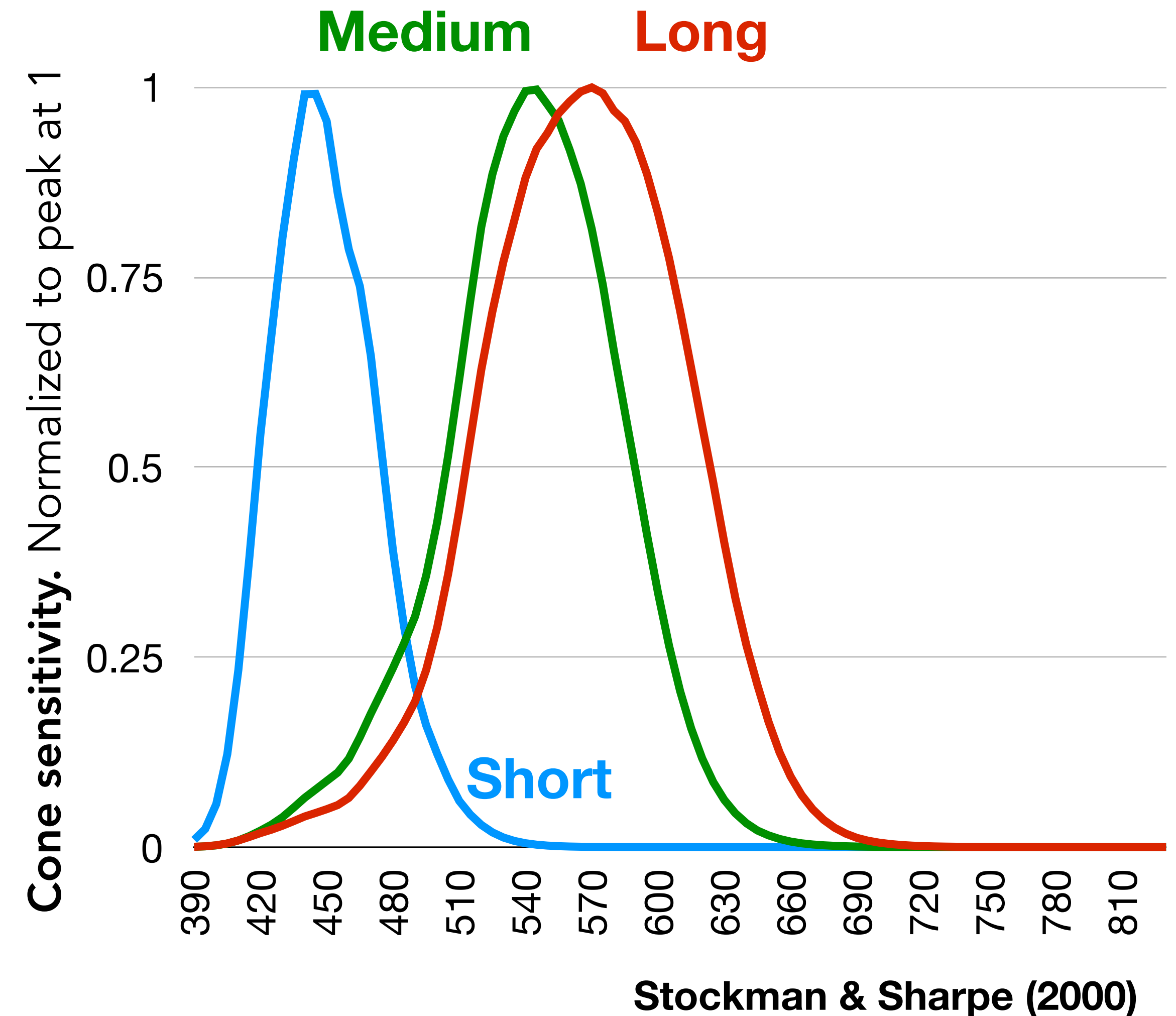
Different cone types have different spectral sensitivity peaks.



Spectral Response/Sensitivity of Cones

Different cone types have different spectral sensitivity peaks.

- L/M/S cones have peak sensitivities at, informally, long, medium, and short wavelength, respectively.

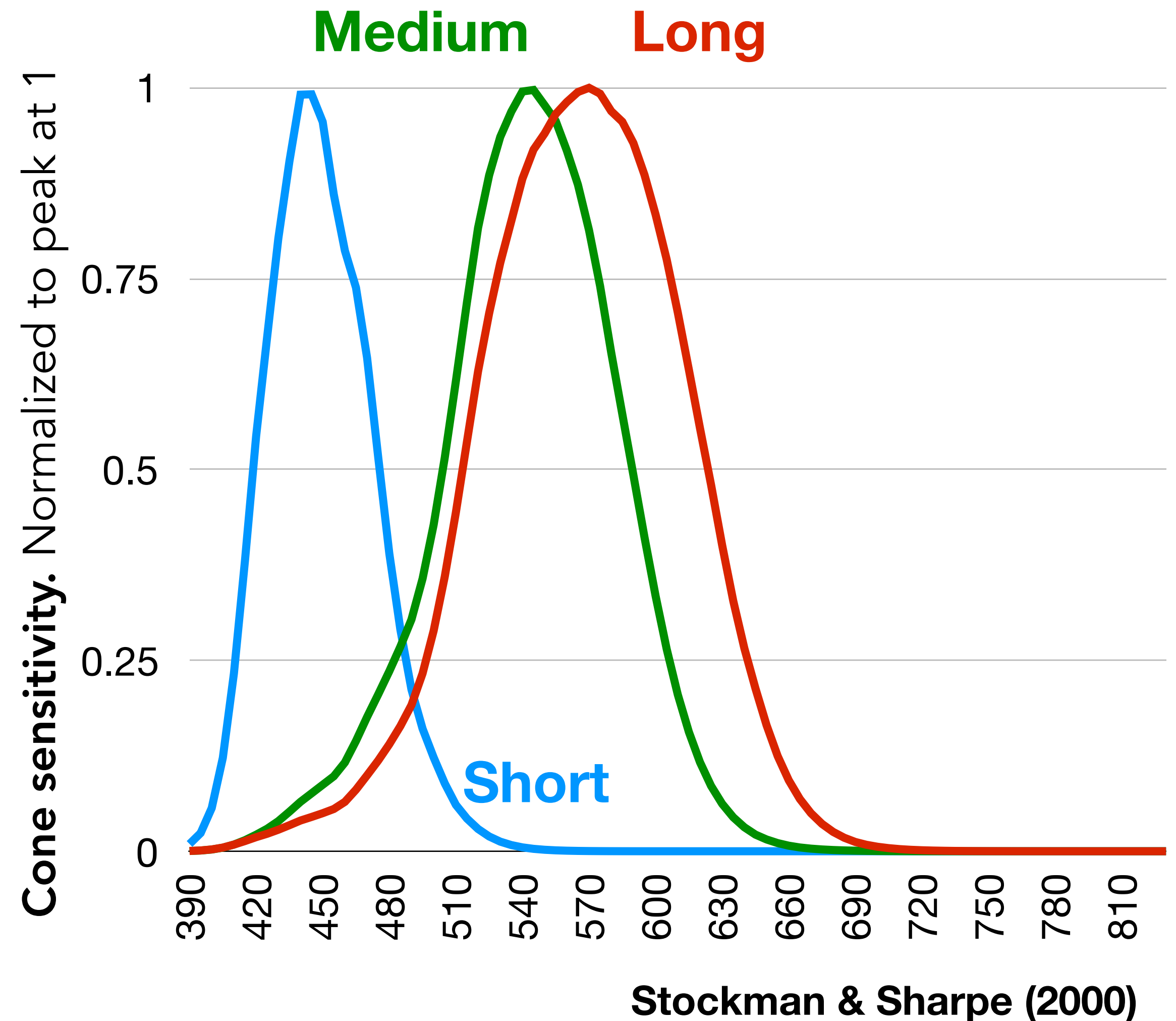


Spectral Response/Sensitivity of Cones

Different cone types have different spectral sensitivity peaks.

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Informally, it tell us for 1 unit power of spectral light (single-wavelength light) at wavelength \mathbf{x} , how much cone responses (\mathbf{y}) is generated.



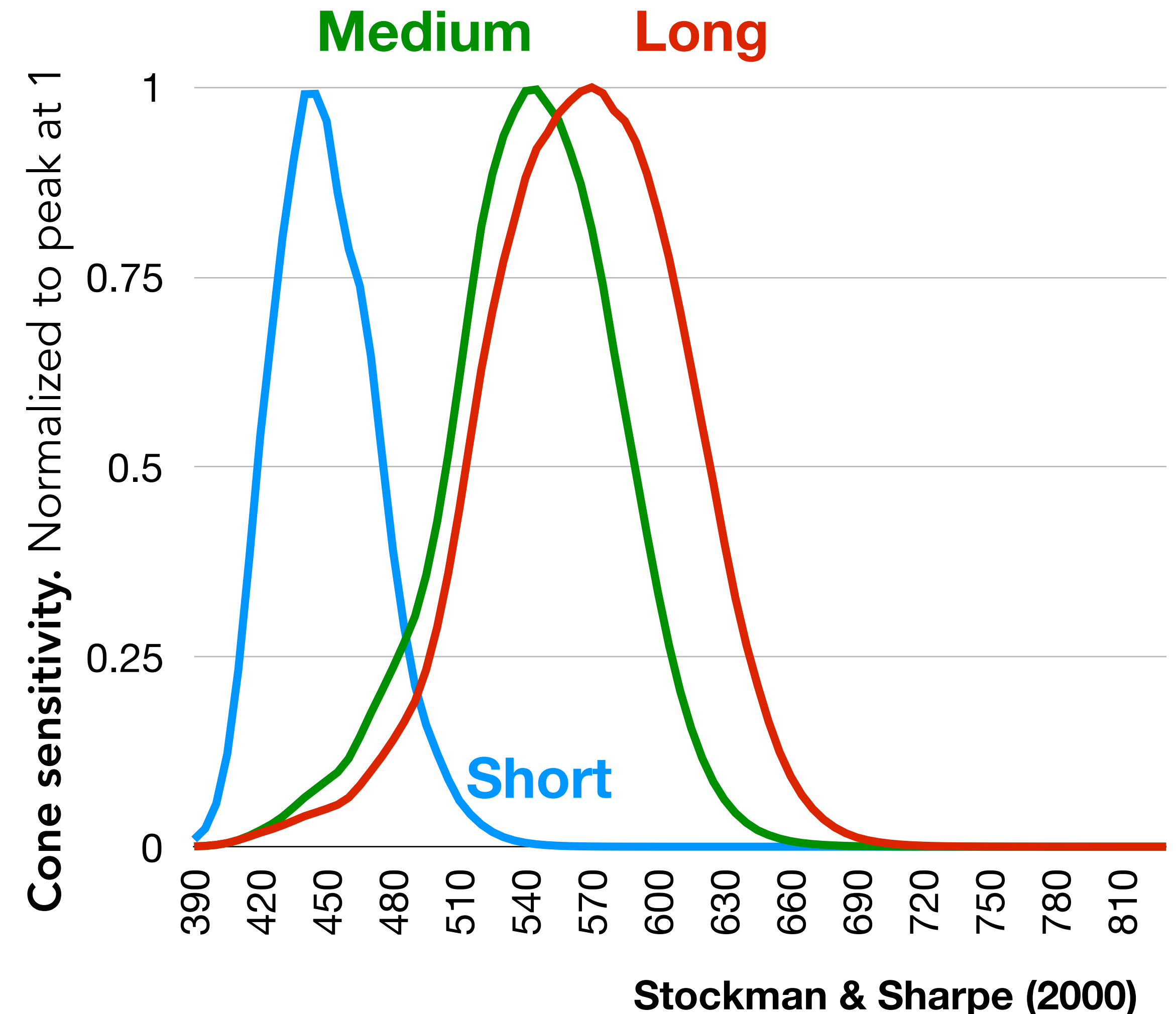
Spectral Response/Sensitivity of Cones

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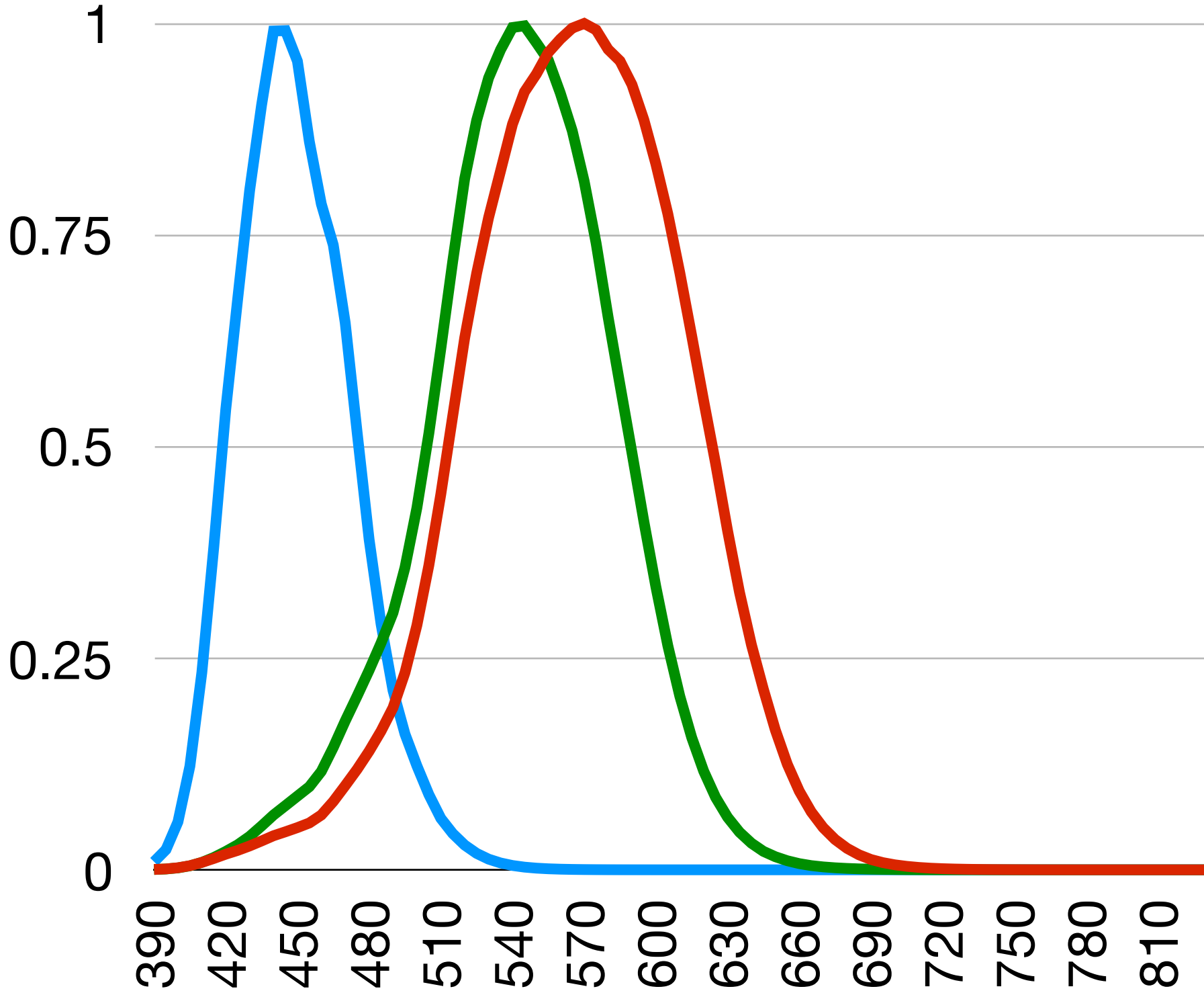
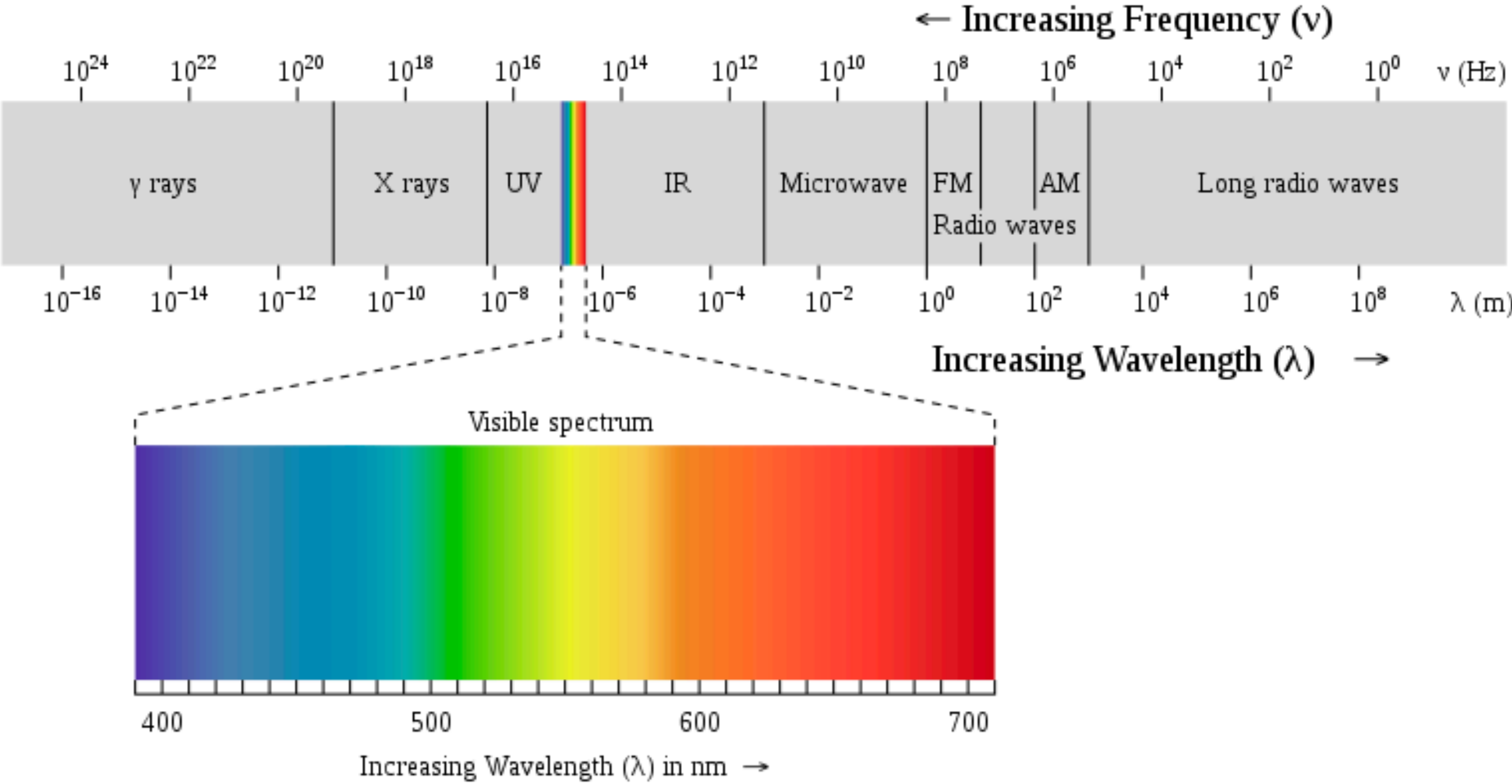
Informally, it tell us for 1 unit power of spectral light (single-wavelength light) at wavelength x , how much cone responses (y) is generated.

- Sometimes called "Cone Fundamentals"



Invisible Light

“Invisible”: Lights (electromagnetic waves) not interacting with cones cells.



Stockman & Sharpe (2000)

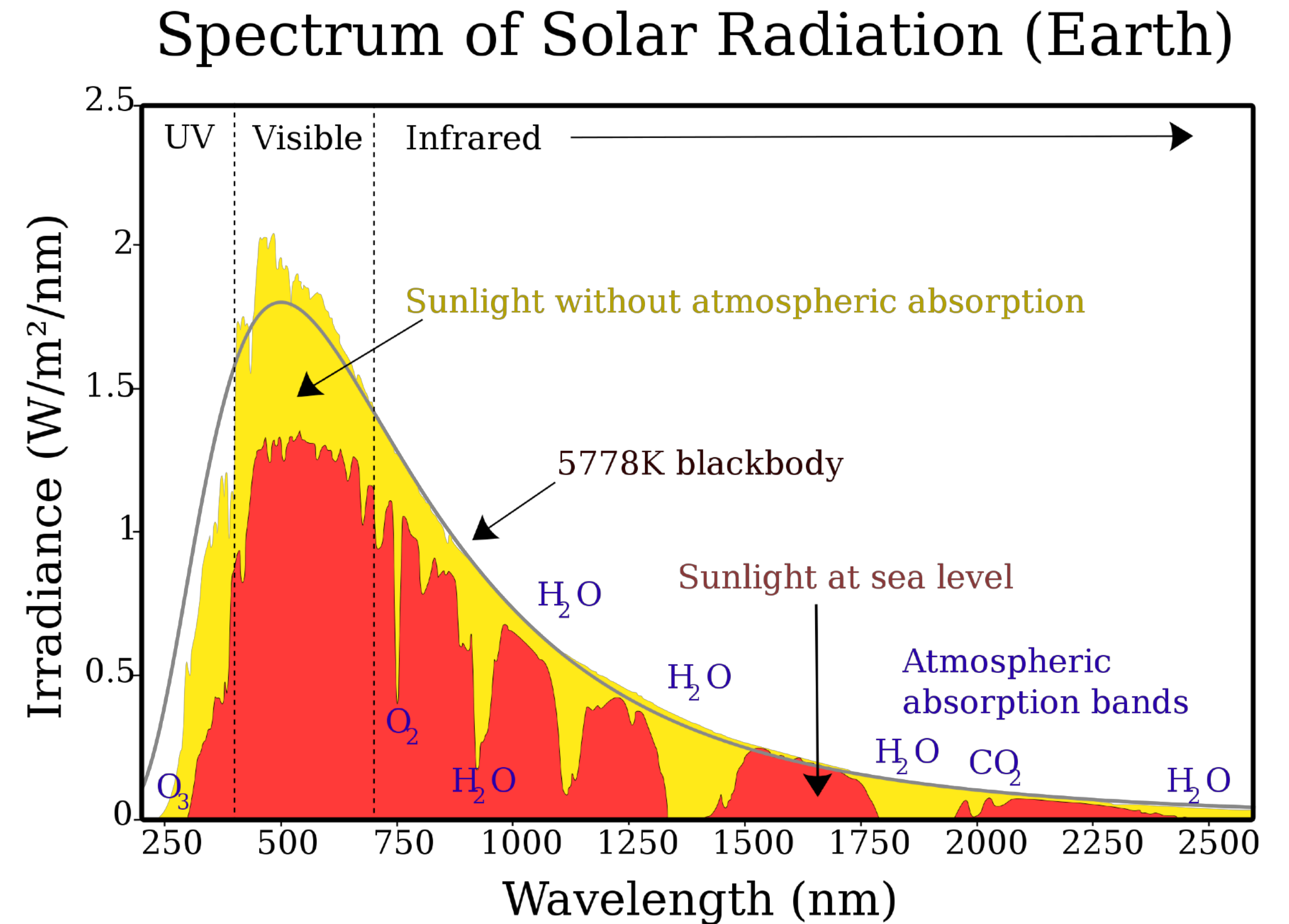
Invisible Light

Sunlight has invisible components, which don't interact with retina photoreceptors, but interact with other kinds of cells.

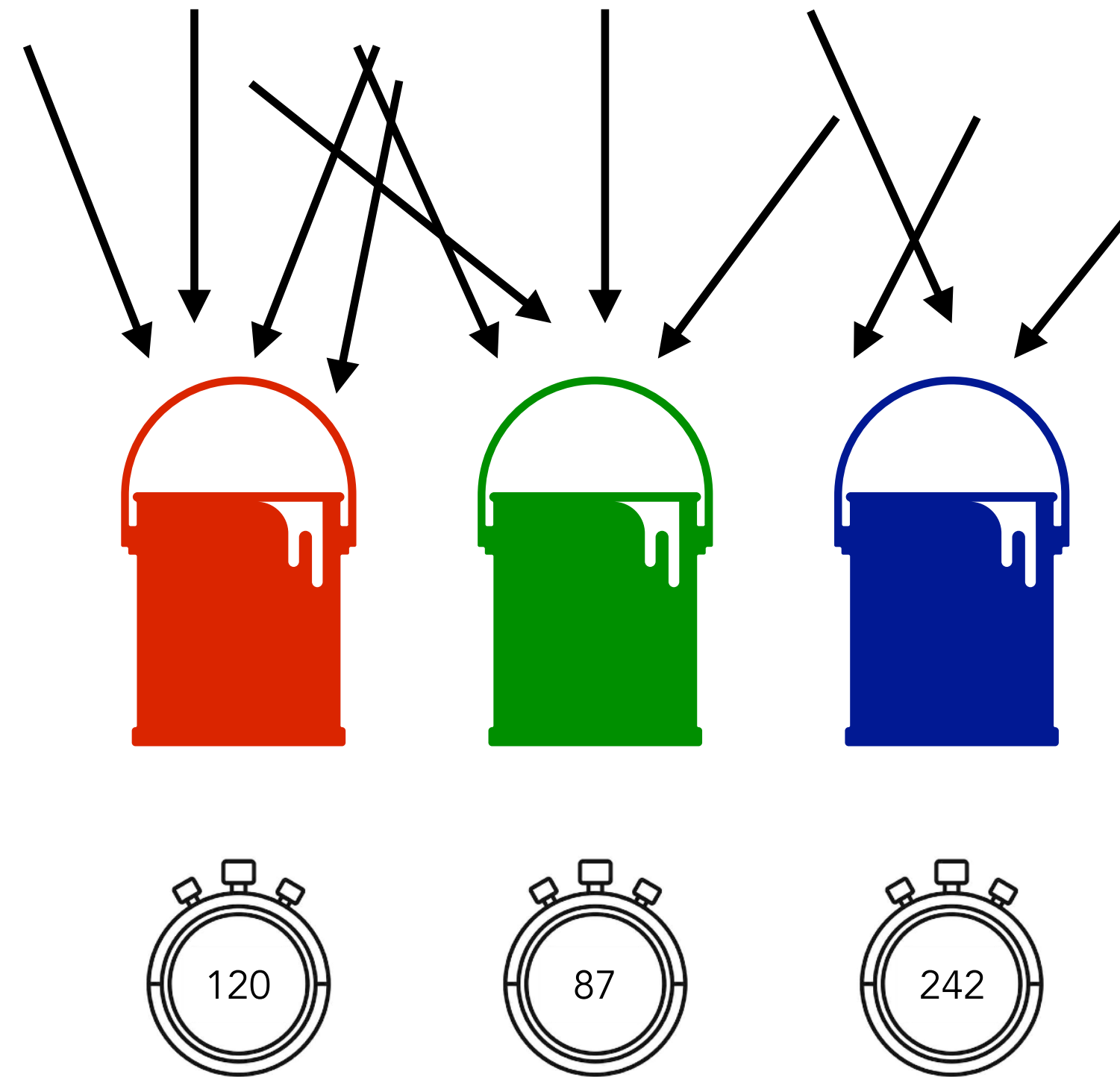
- Case in point: skin cancer, where Ultra-Violet (UV) lights interact with skin cells.

Another example: Infra-Red (IR) cameras can "see" things we can't.

- They use materials that do interact with IR lights. More in the camera module.



A Mental Model of Photoreceptors



Each photoreceptor is a bucket with a small area. It collects (absorbs) photons that fall within the area with **probability** that varies with wavelengths.

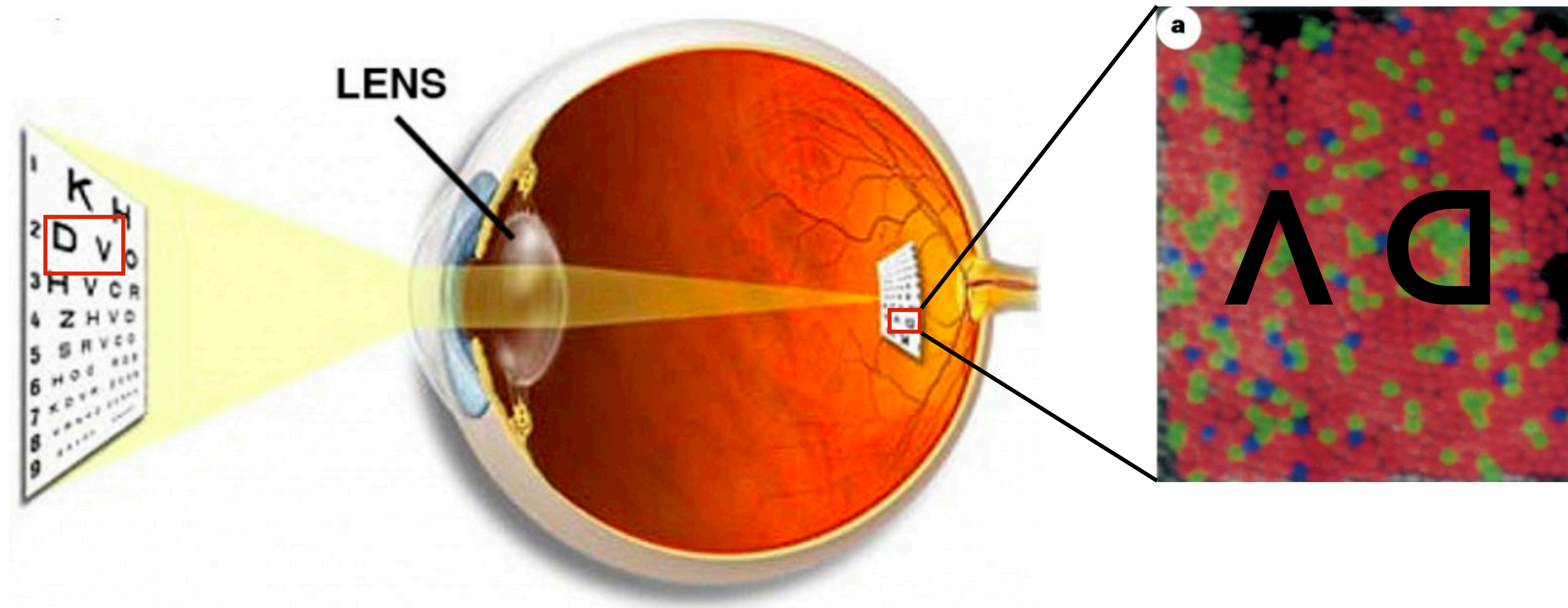
Univariance: once a photon is absorbed, the counter increments by 1, i.e., no difference between photons after absorption.

When a bucket is full (i.e., **saturated**), no photons are collected, and no neural signals are generated. The photoreceptor becomes insensitive to light.

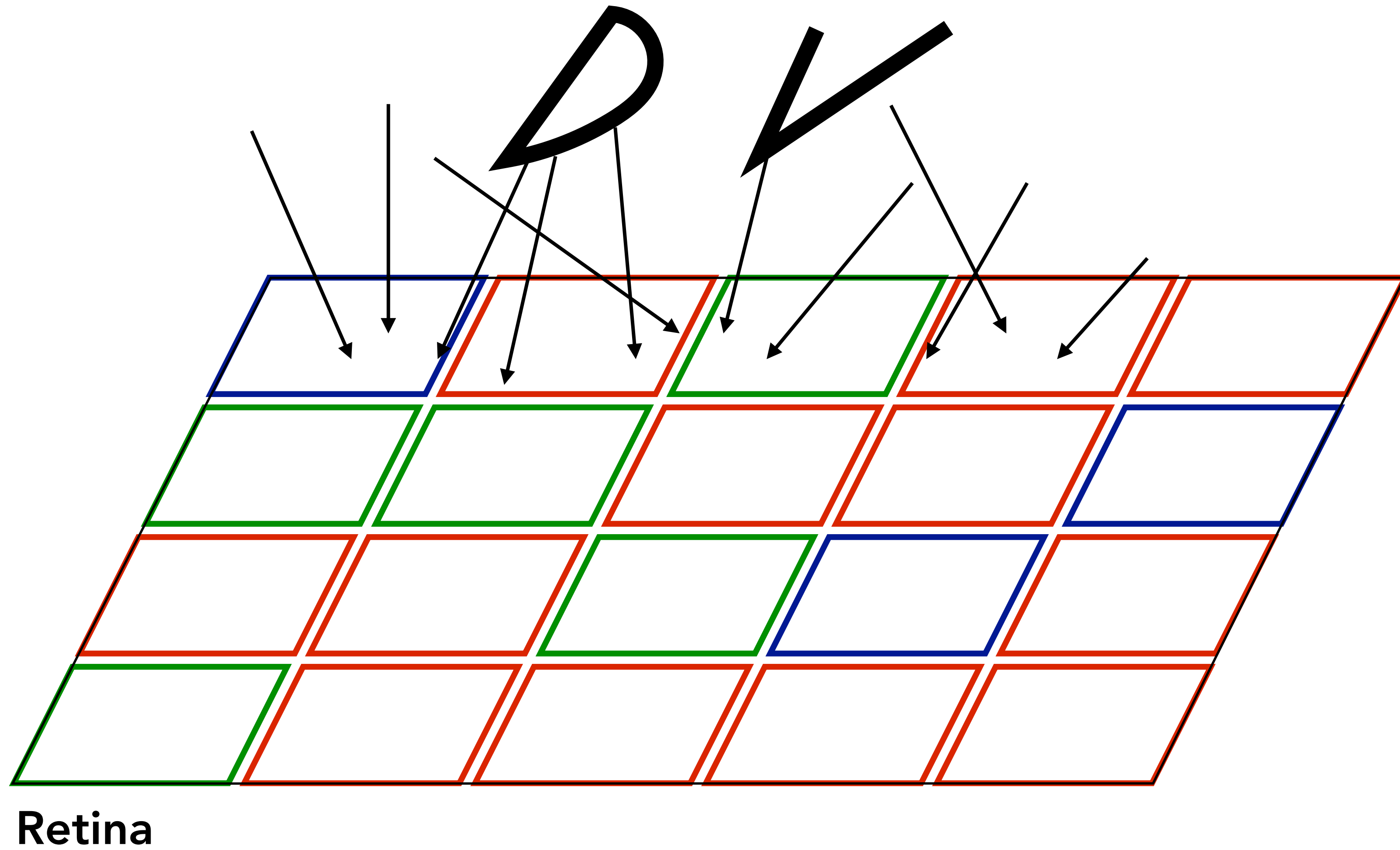
- Rods are saturated during daylight.

Spatial Vision

Retina = Low-Pass Filtering + Spatial Sampling

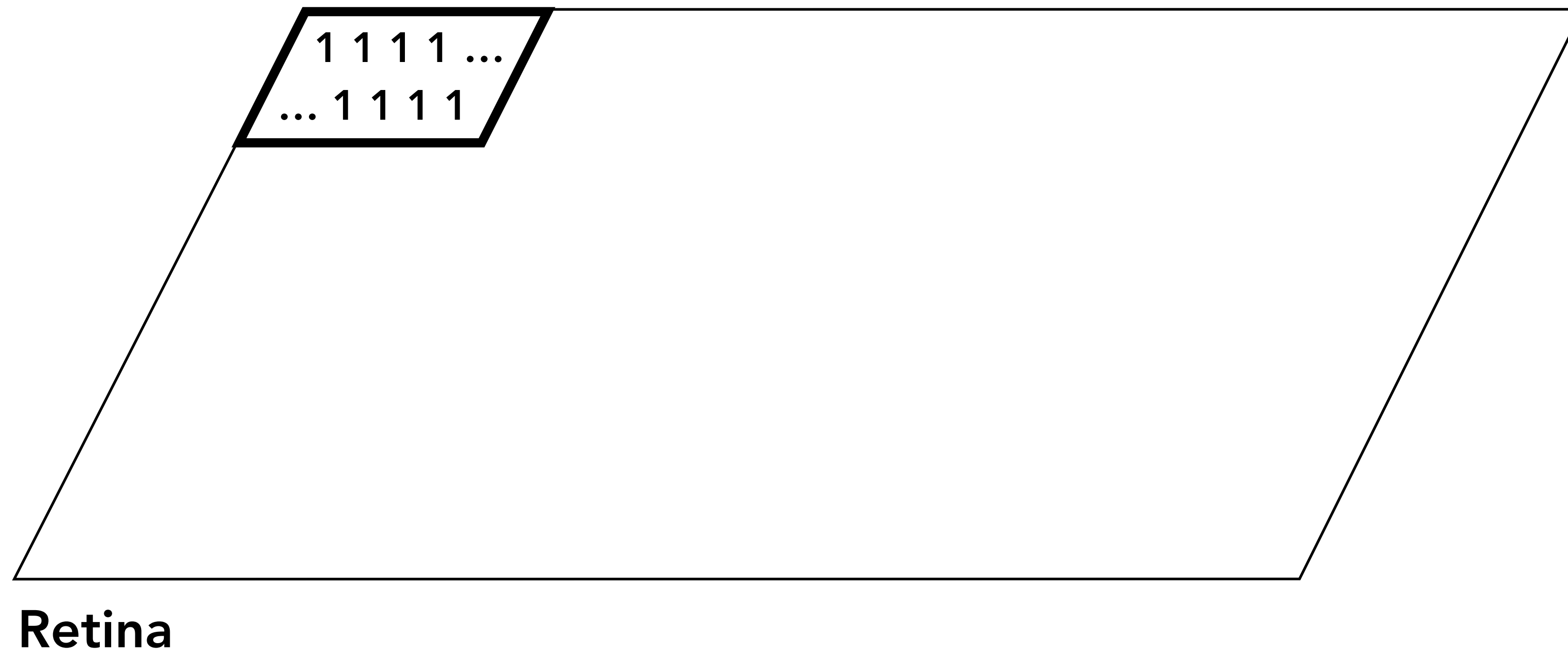


Low-Pass Filtering on Retina



Low-Pass Filtering on Retina

Compare: convolving with a box filter, whose size is the size of a cone cell.



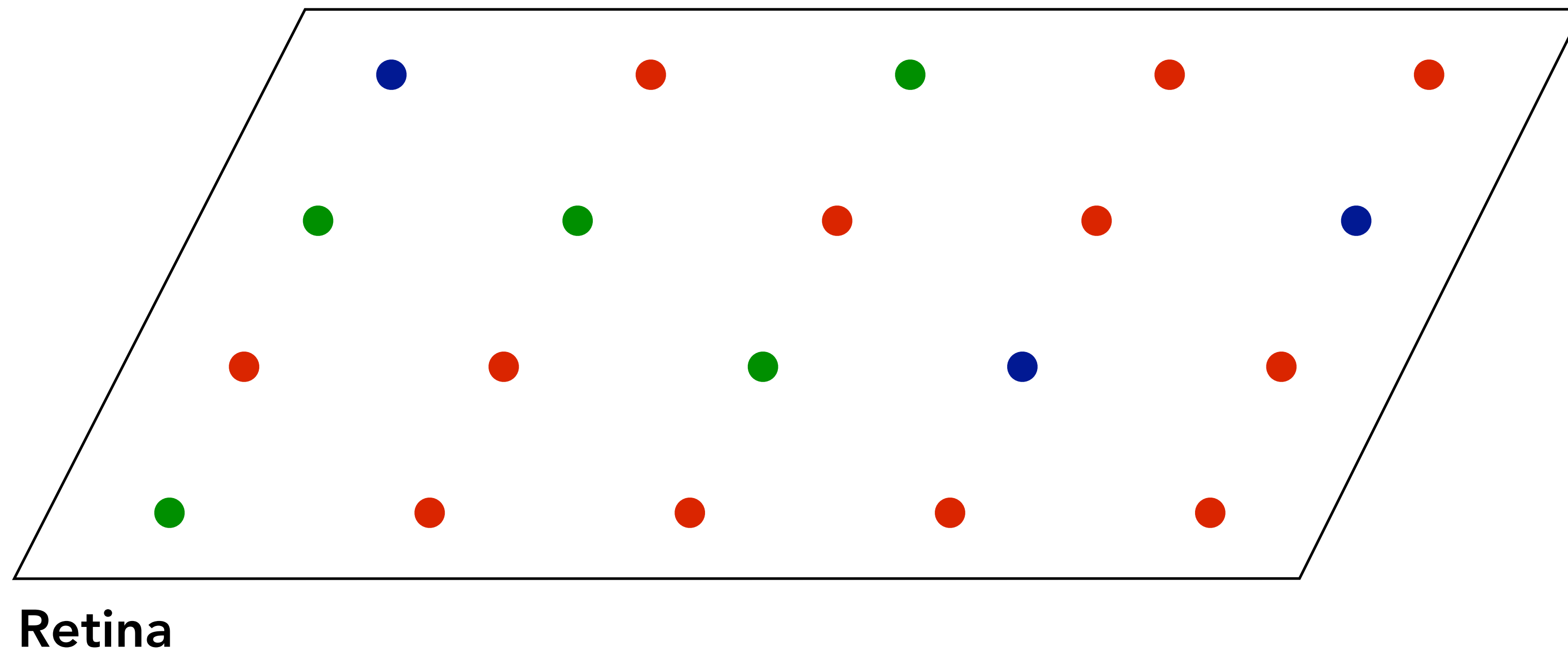
Low-Pass Filtering on Retina

Compare: convolving with a box filter, whose size is the size of a cone cell.



...Followed by 2D Sampling

Sample at each photoreceptor center



Retina is a Low-Pass Filter

Each photoreceptor has an (small) area, which integrates photons.

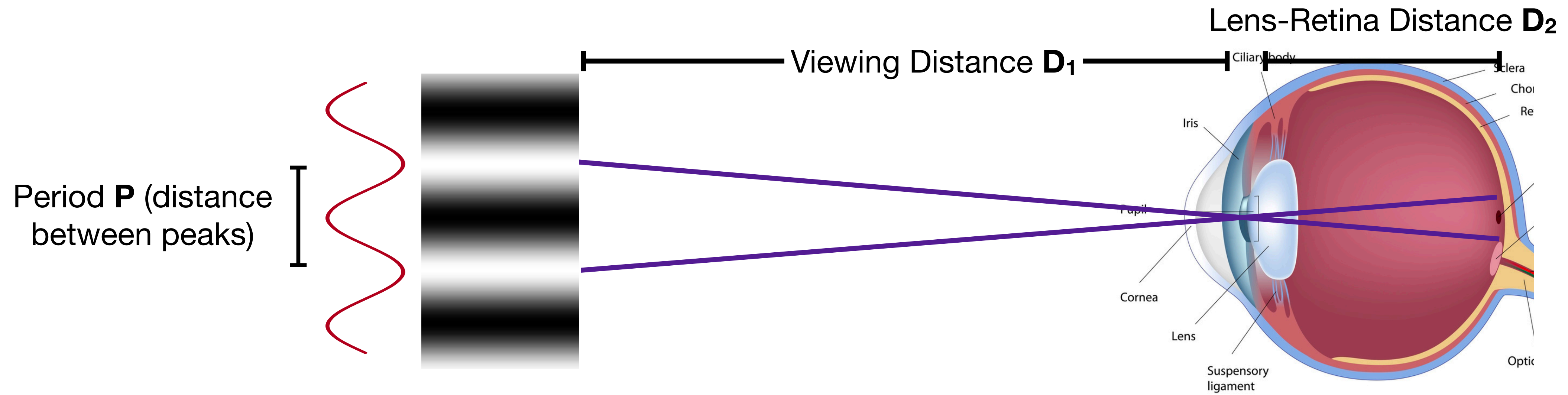
This is equivalent to a 2D box filter, whose width is the photoreceptor size.

Photon detection is thus a two-step process:

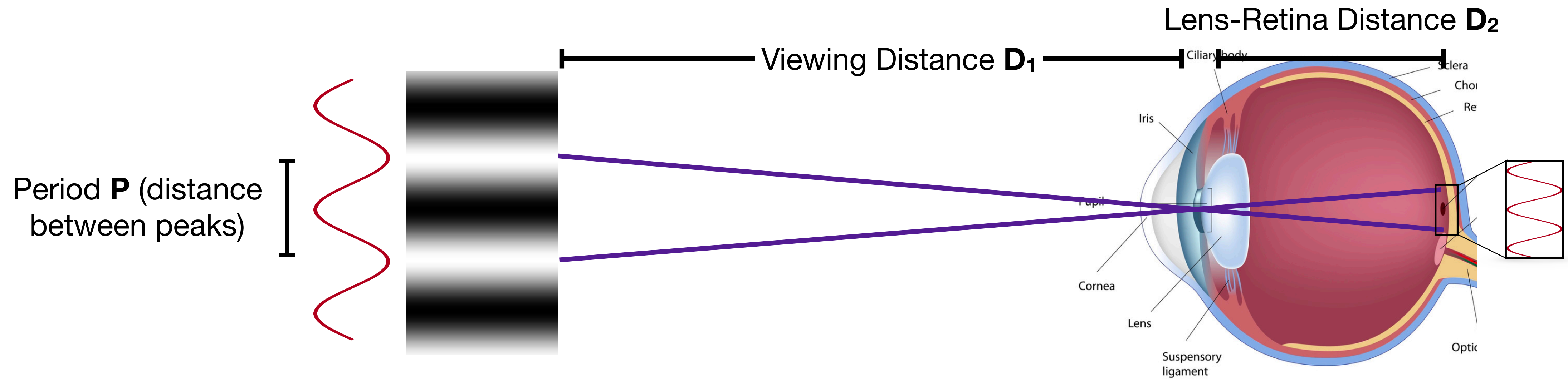
- Filtering the continuous 2D signal on the retina with a 2D box filter
- Followed by a 2D sampling, one sample at each photoreceptor center

Integrating photons is equivalent because convolution followed by sampling is equivalent to calculating the convolution only at the sampled points.

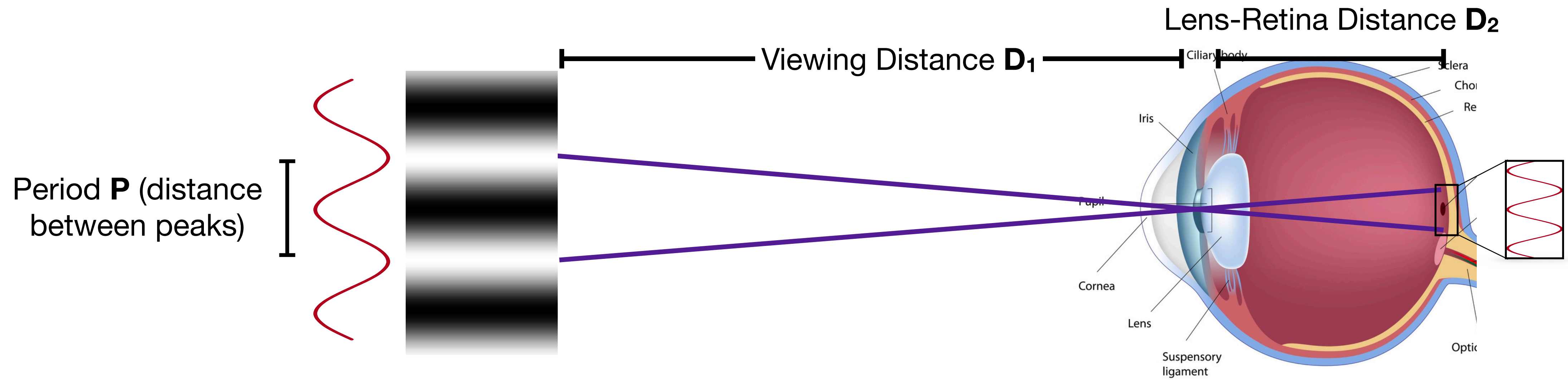
2D Sampling on Retina



2D Sampling on Retina

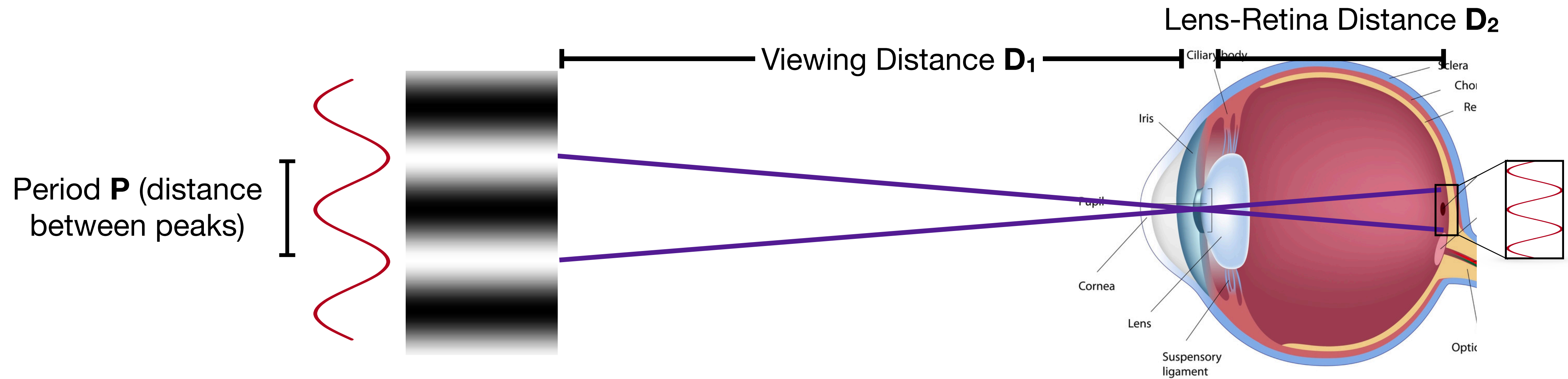


2D Sampling on Retina



Frequency of signal in the scene: $1/P$

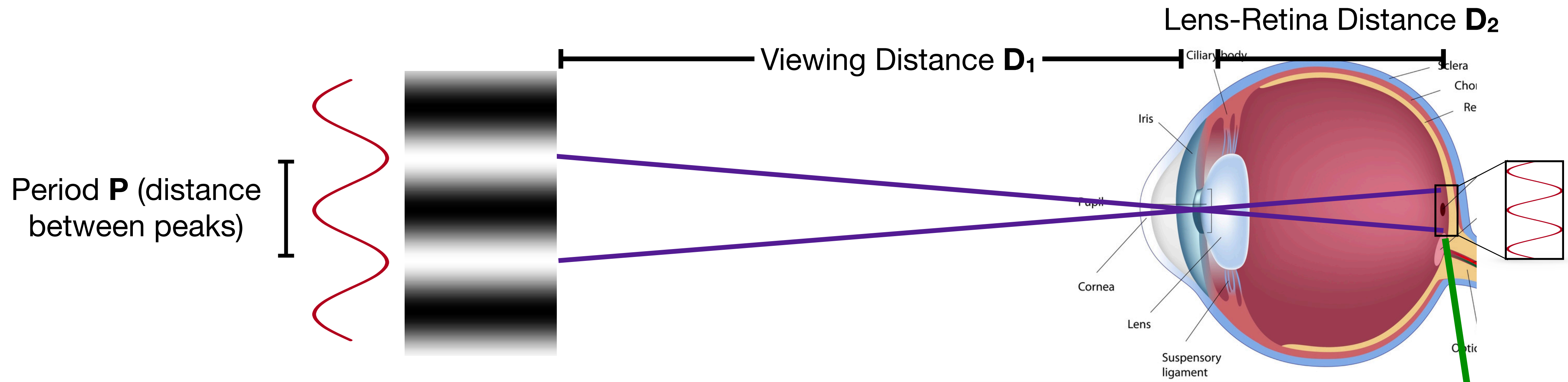
2D Sampling on Retina



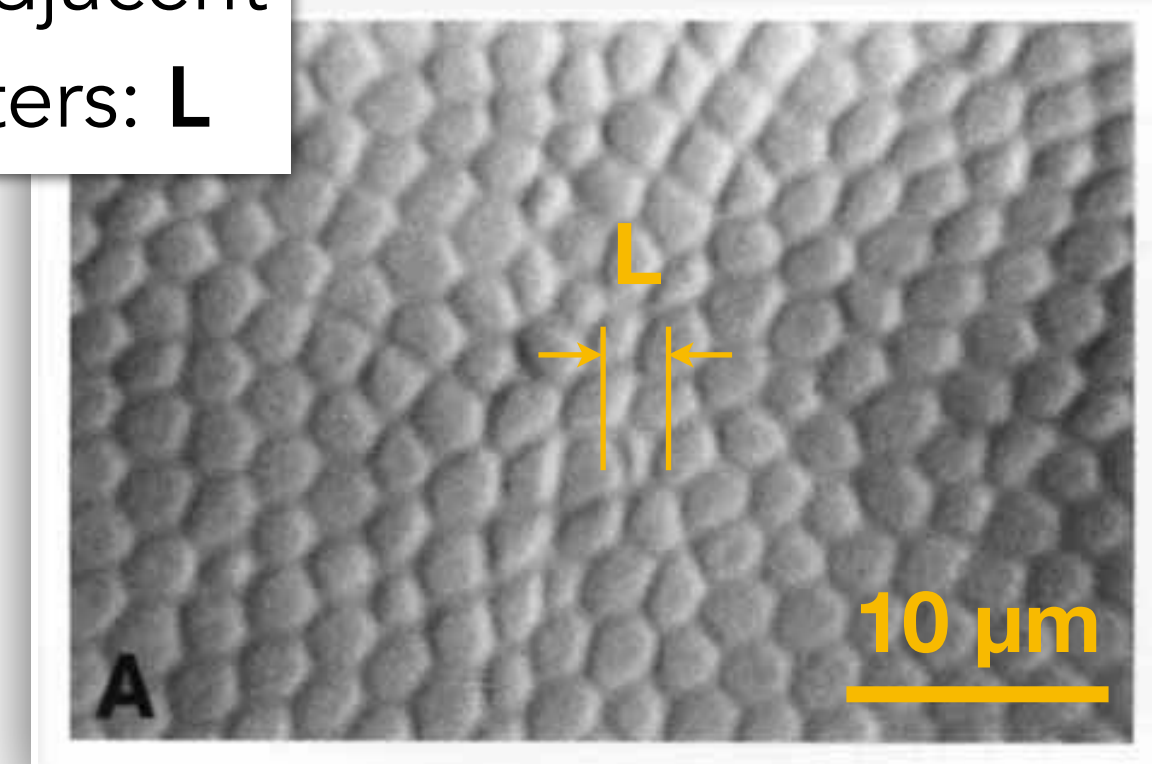
Frequency of signal in the scene: $1/P$

Frequency of signal on the retina
(before sampling): $D_1/(D_2 P)$

2D Sampling on Retina



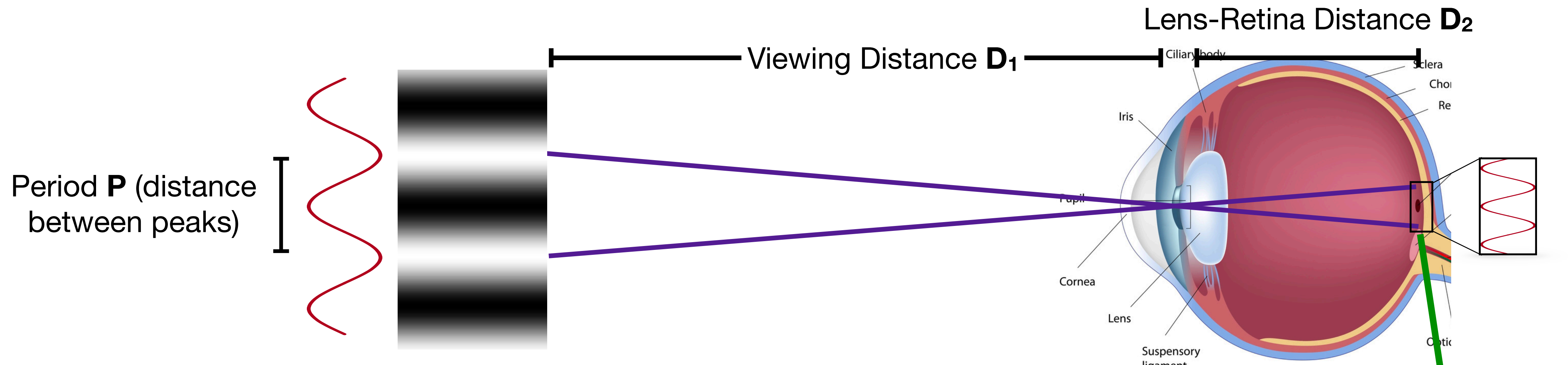
Avg. distance between two adjacent cone cell centers: L



Frequency of signal in the scene: $1/P$

Frequency of signal on the retina (before sampling): $D_1/(D_2 P)$

2D Sampling on Retina



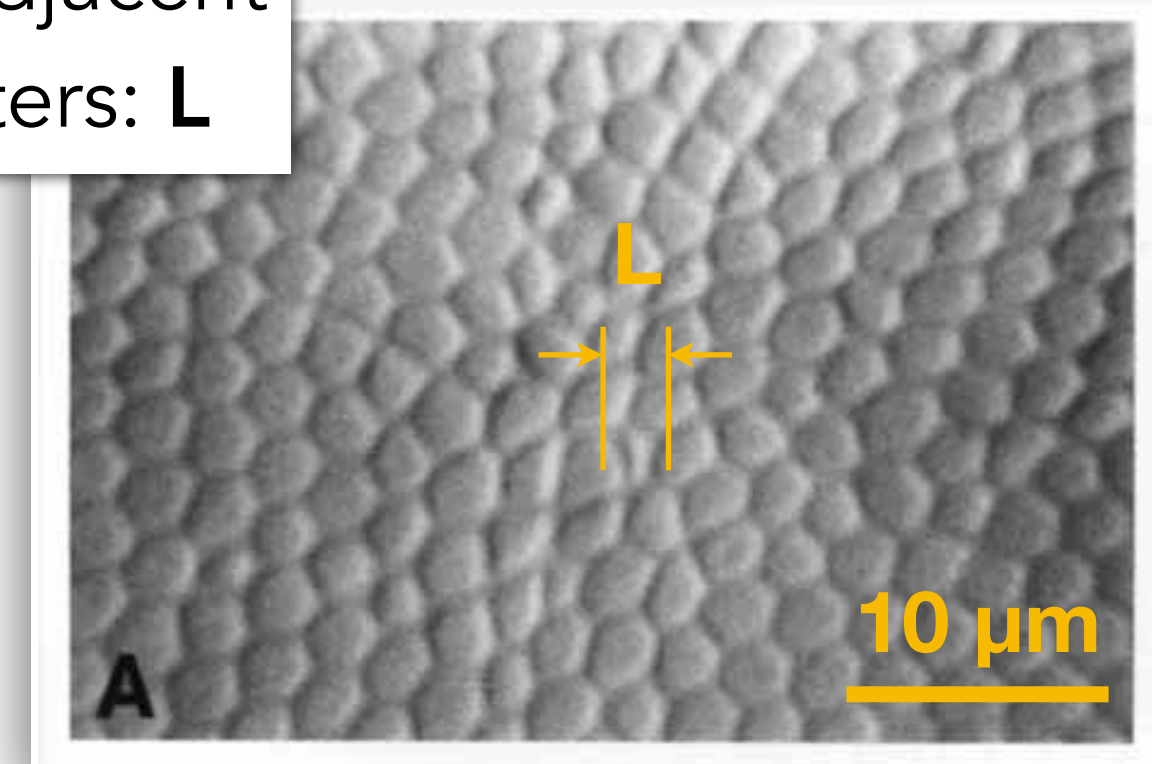
Period P (distance between peaks)

Avg. distance between two adjacent cone cell centers: L

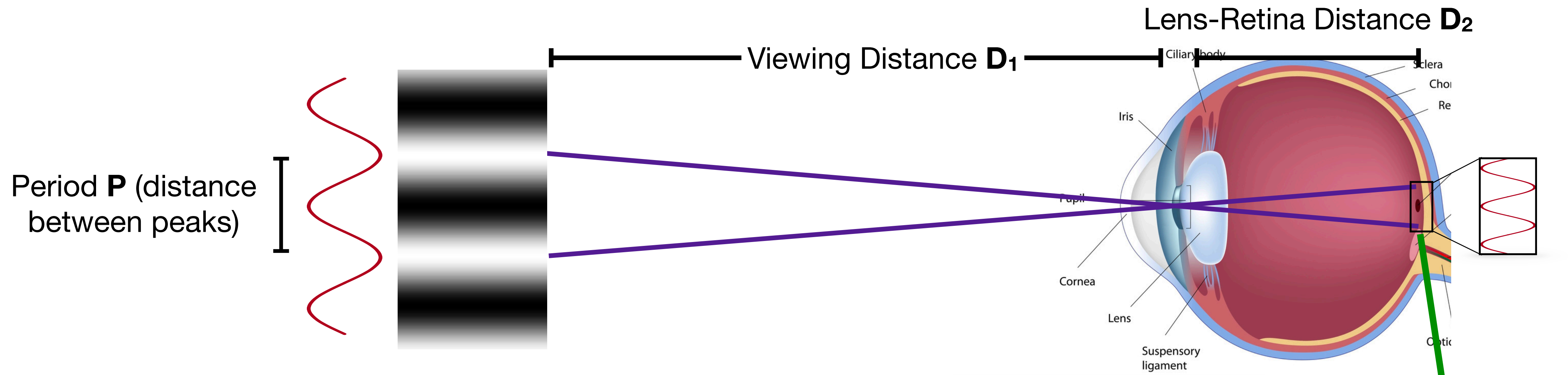
Frequency of signal in the scene: $1/P$

Frequency of signal on the retina (before sampling): $D_1/(D_2 P)$

Sampling frequency of the retina: $1/L$



2D Sampling on Retina



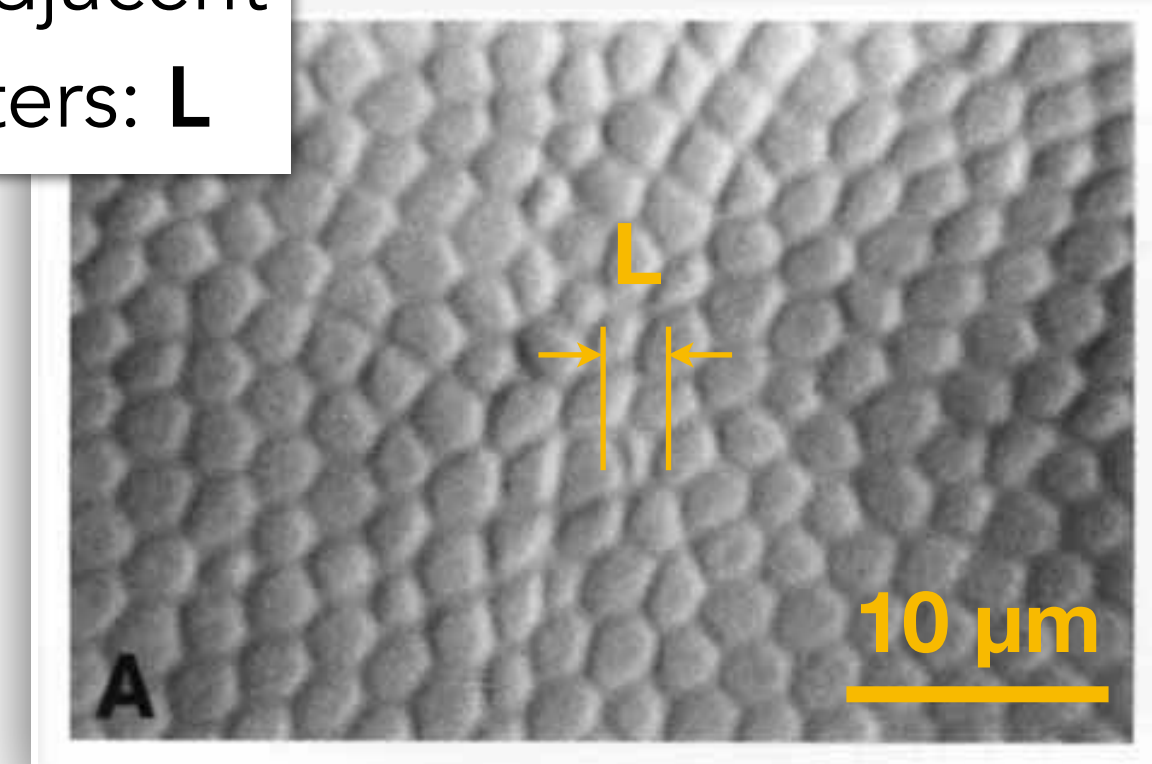
Avg. distance between two adjacent cone cell centers: L

Frequency of signal in the scene: $1/P$

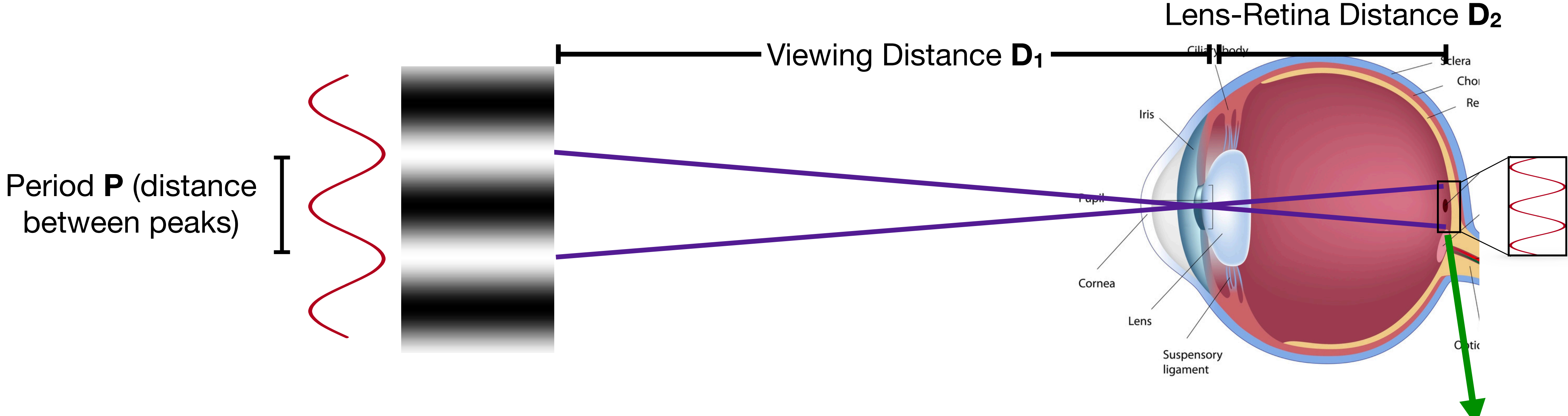
Frequency of signal on the retina (before sampling): $D_1/(D_2 P)$

Sampling frequency of the retina: $1/L$

$1/L > 2 D_1/(D_2 P)$
for the sine wave to be perceivable



Retinal Sampling Rate

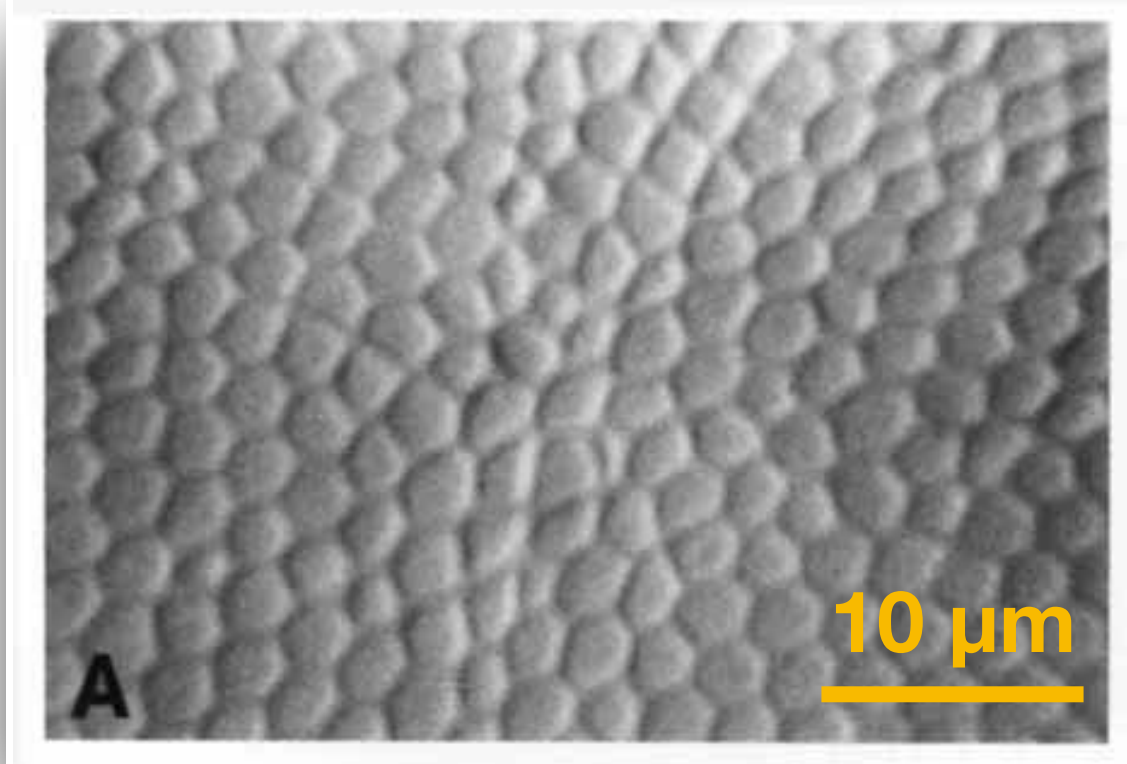


$L \approx 2.5 \mu\text{m}$ (at fovea) and
 $D_2 \approx 16.7 \text{ mm}$ (data from
 Visual Ecology Ch 6, 2014).

$$\frac{P}{D_1} > \frac{2L}{D_2}$$

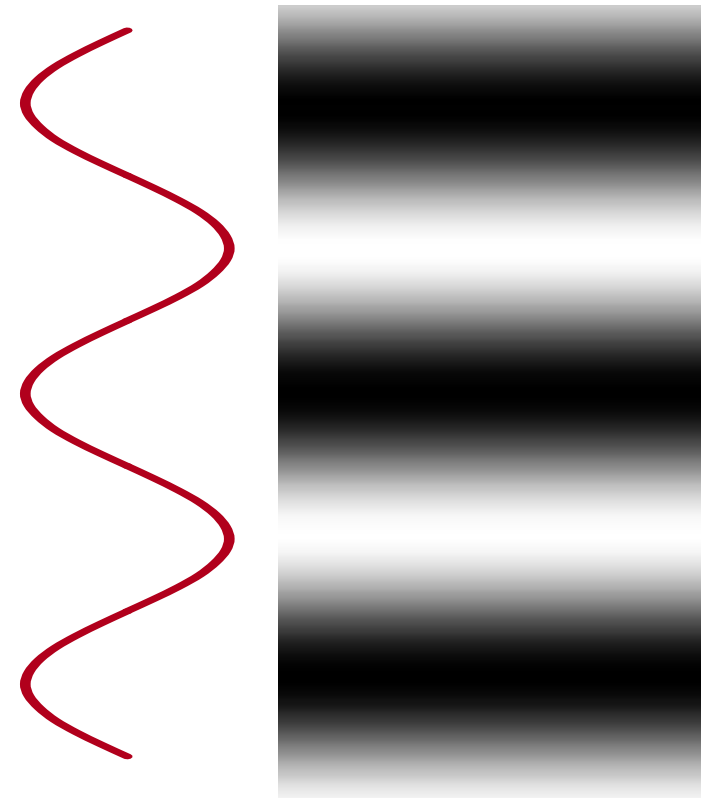


$$2L/D_2 \approx 0.0003$$



iPhone 12 Pro Max Example

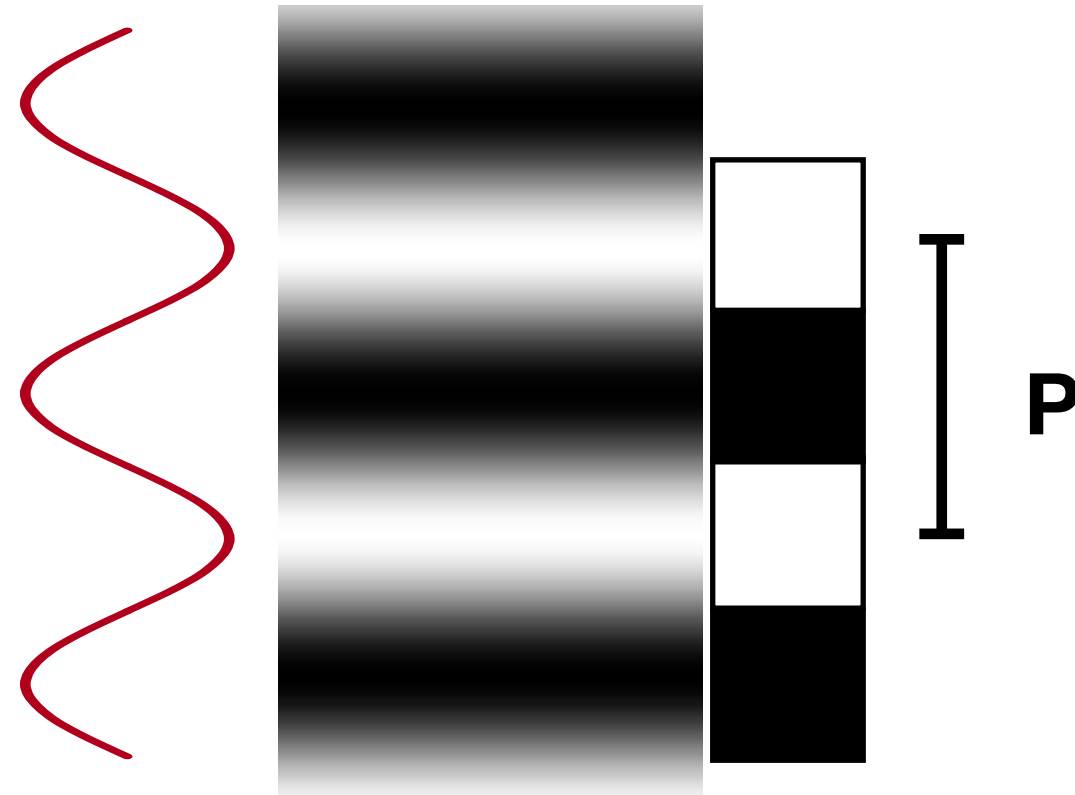
$$\frac{P}{D_1} > 0.0003$$



iPhone 12 Pro Max	iPhone 11 Pro Max
6.7"	6.5"
iPhone 12 Pro	iPhone 11 Pro
6.1"	5.8"
Super Retina XDR display ¹	Super Retina XDR display ¹

iPhone 12 Pro Max Example

$$\frac{P}{D_1} > 0.0003$$



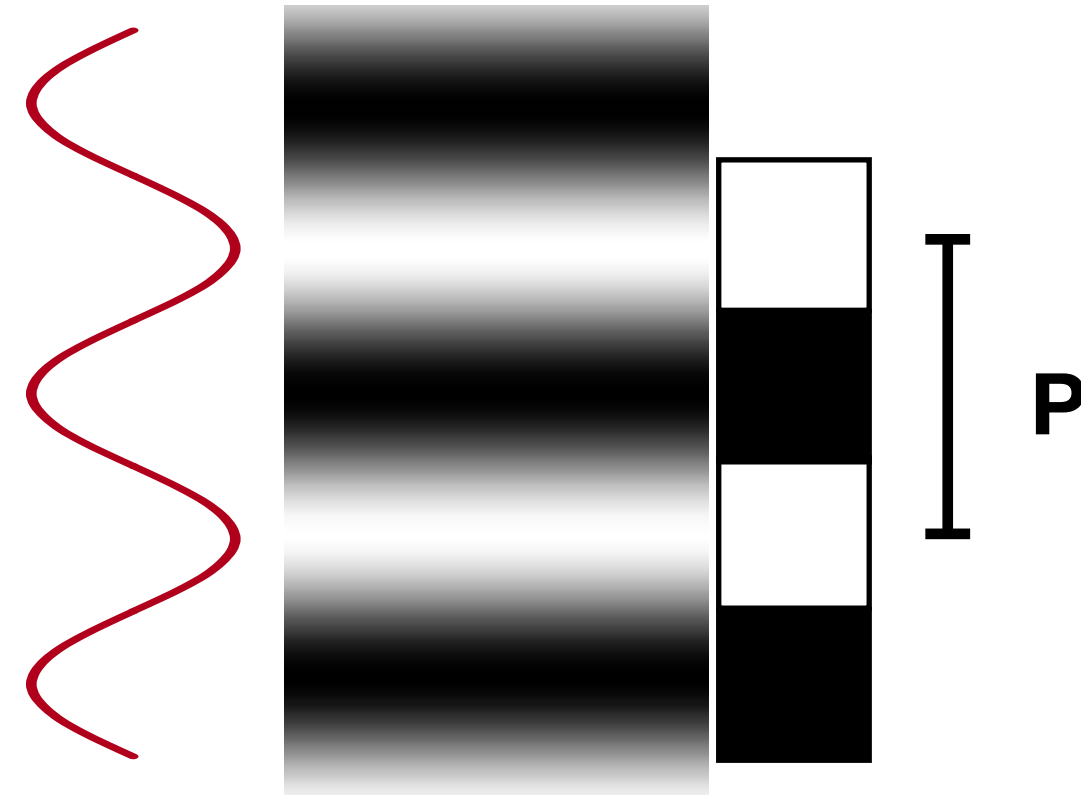
Display alternative black and white pixels, basically use square wave to approximate sine wave.

The highest frequency that can be displayed on this device.

iPhone 12 Pro Max	iPhone 11 Pro Max
6.7"	6.5"
iPhone 12 Pro	iPhone 11 Pro
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Super Retina XDR display ¹	Super Retina XDR display ¹

iPhone 12 Pro Max Example

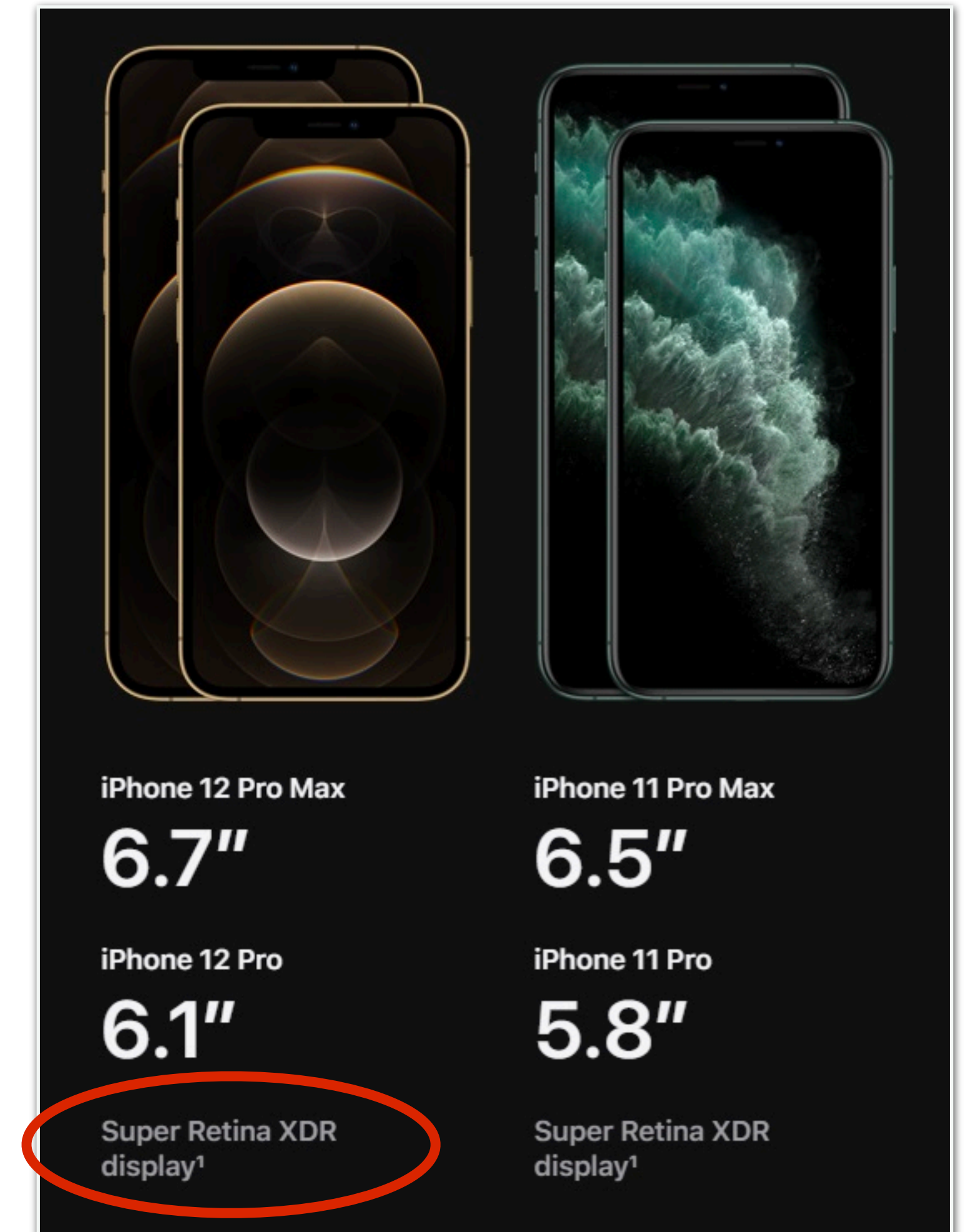
$$\frac{P}{D_1} > 0.0003$$



Display alternative black and white pixels, basically use square wave to approximate sine wave.

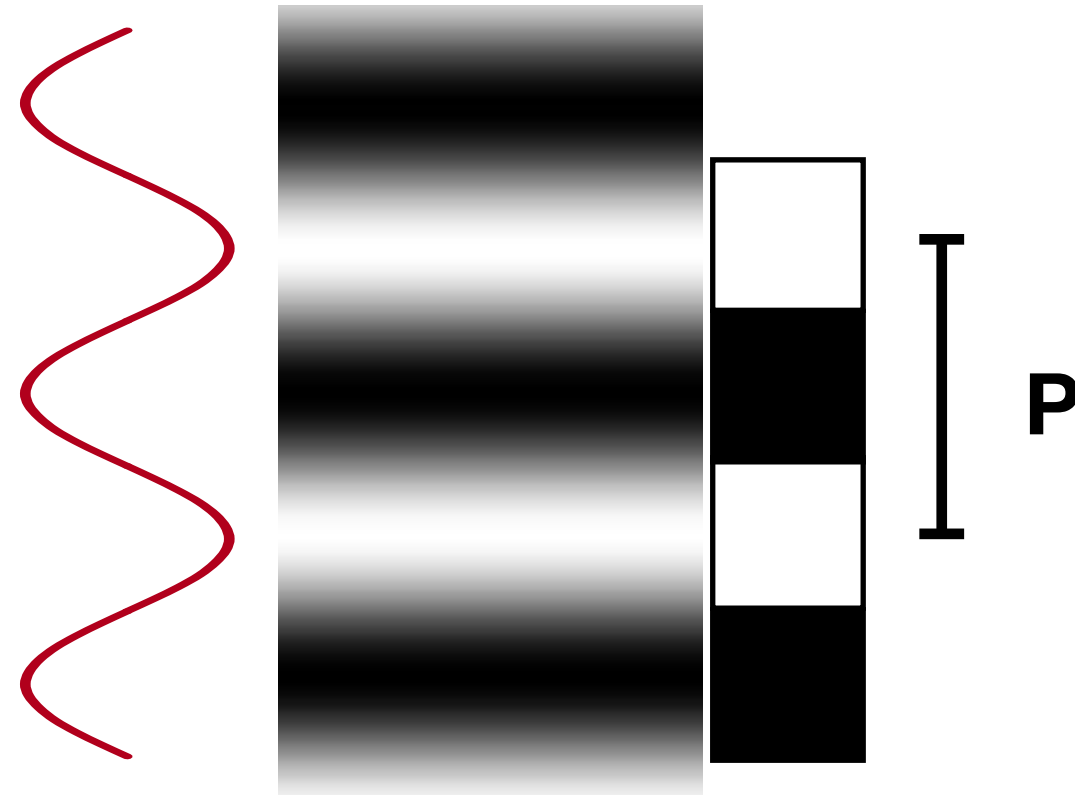
The highest frequency that can be displayed on this device.

- 1284 x 2778 pixels, 6.33 inch height



iPhone 12 Pro Max Example

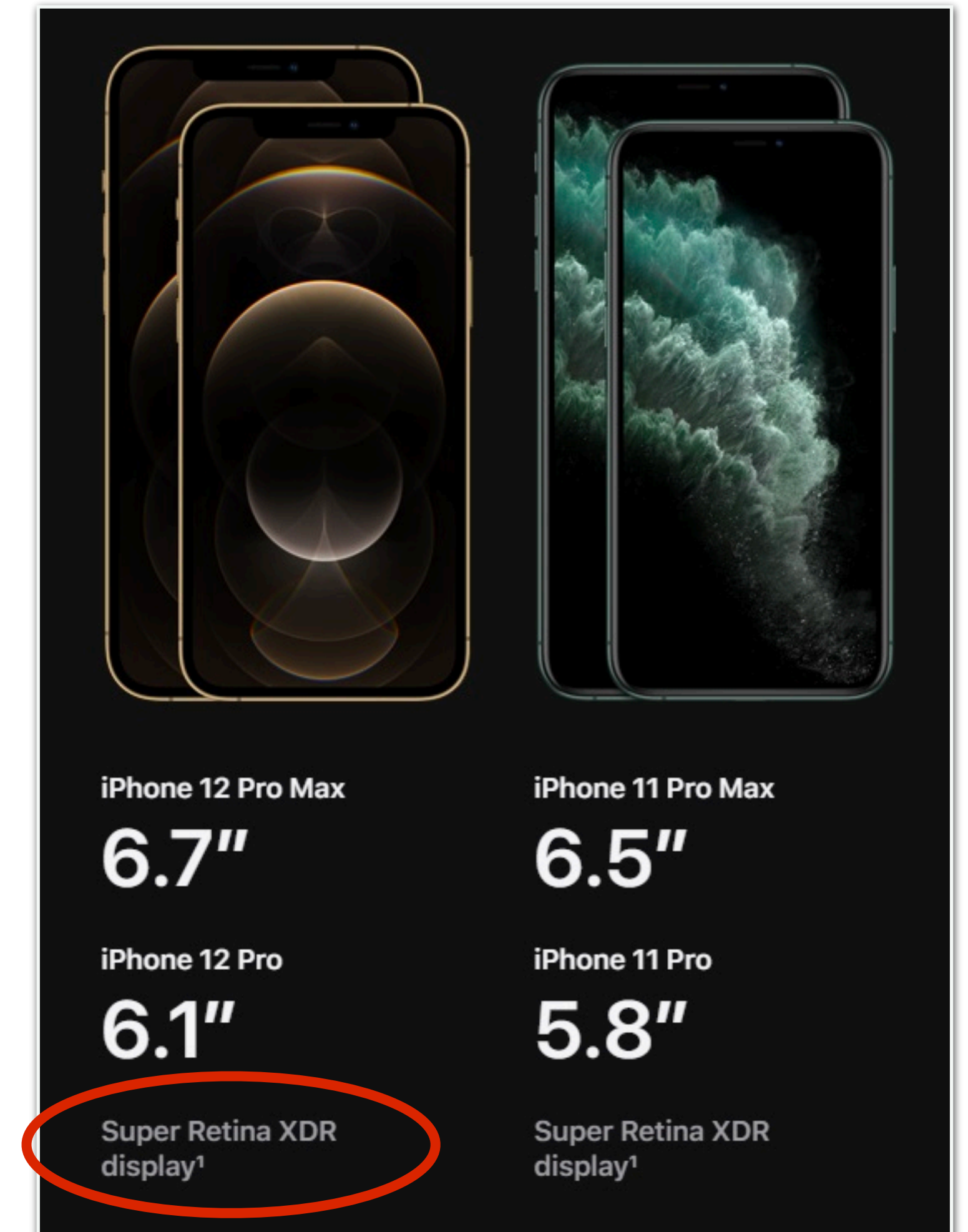
$$\frac{P}{D_1} > 0.0003$$



Display alternative black and white pixels, basically use square wave to approximate sine wave.

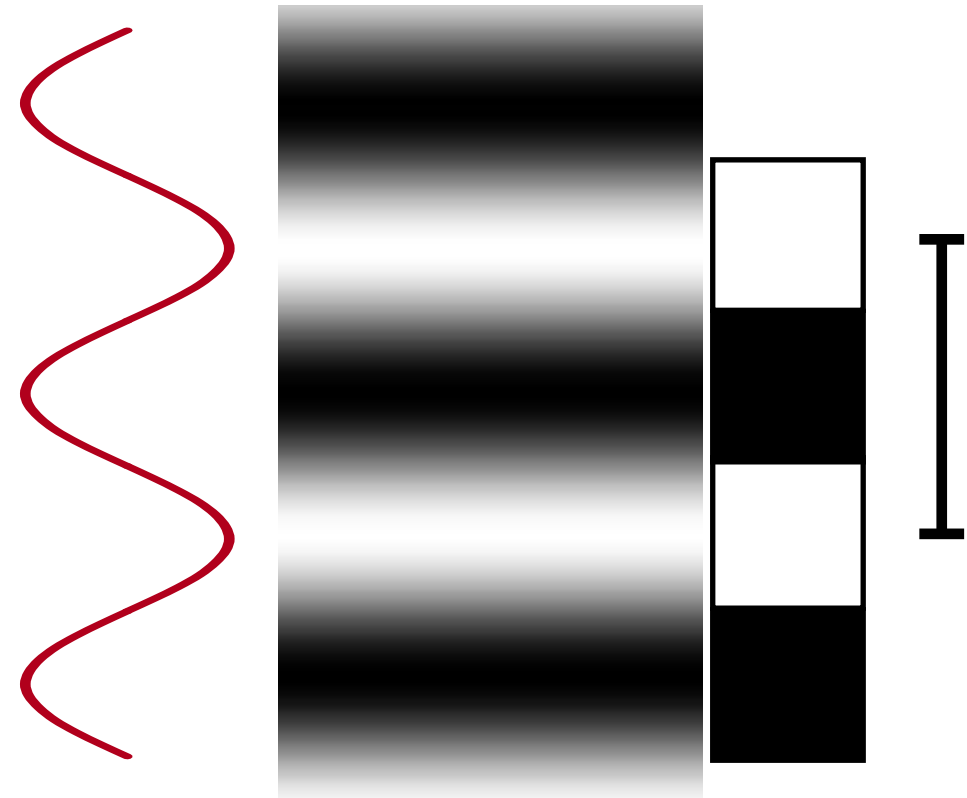
The highest frequency that can be displayed on this device.

- 1284 x 2778 pixels, 6.33 inch height
- $P = 2 (6.33'' / 2778)$



iPhone 12 Pro Max Example

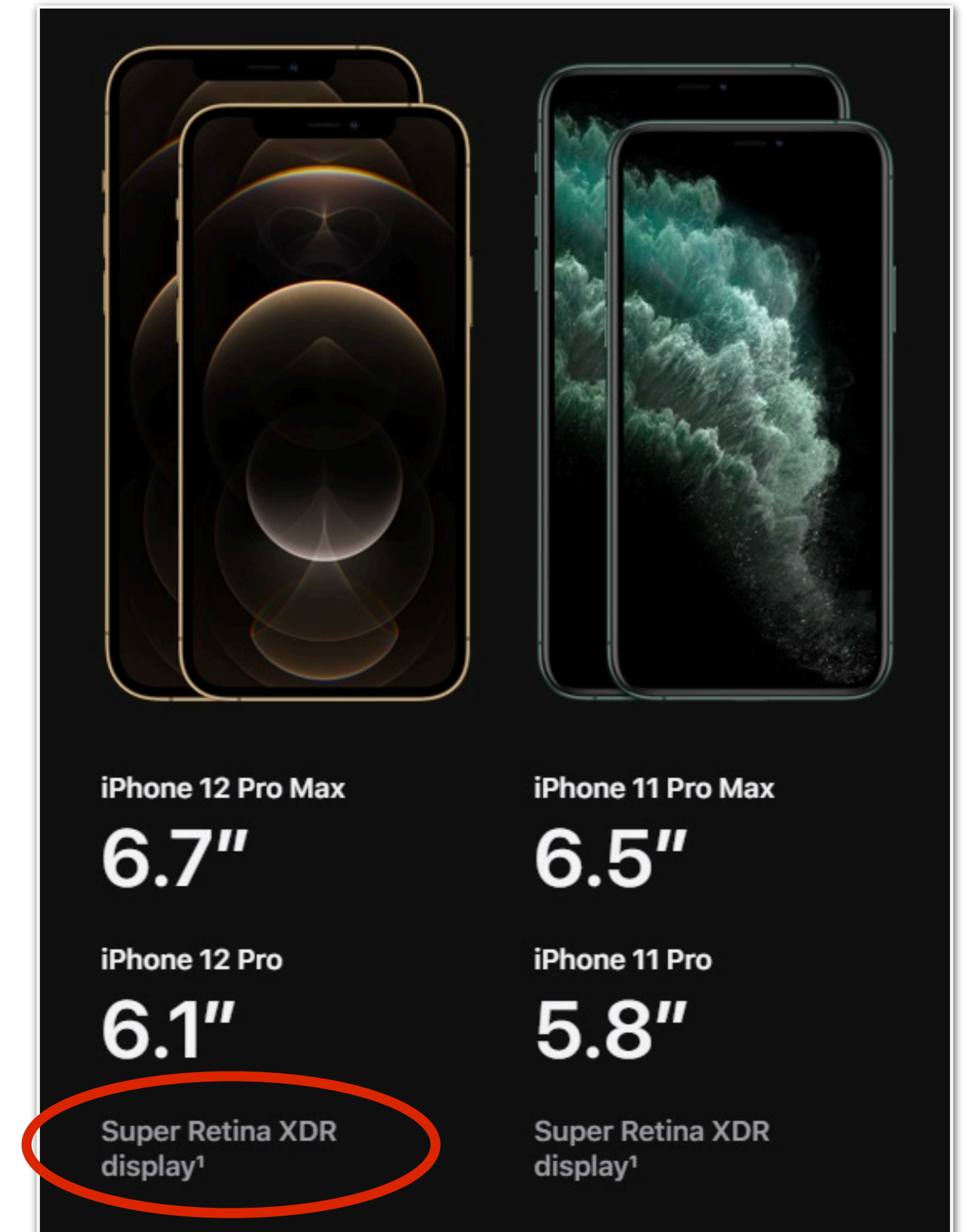
$$\frac{P}{D_1} > 0.0003$$



Display alternative black and white pixels, basically use square wave to approximate sine wave.

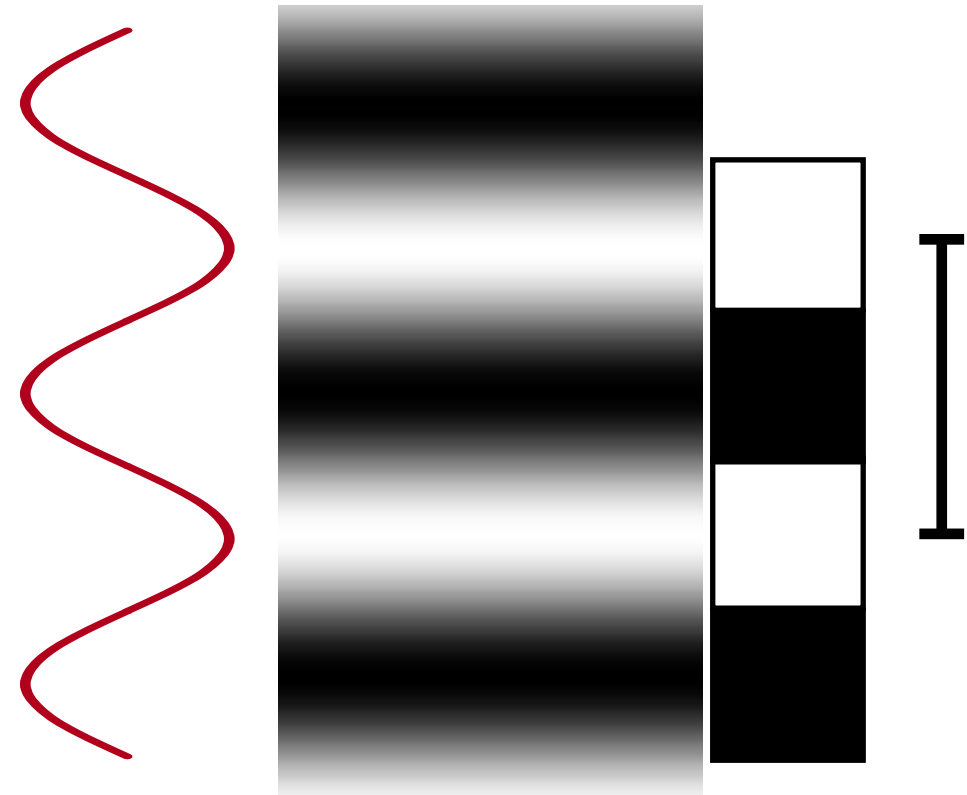
The highest frequency that can be displayed on this device.

- 1284 x 2778 pixels, 6.33 inch height
- $P = 2 (6.33'' / 2778)$
- Viewing distance (D_1) 12 inches



iPhone 12 Pro Max Example

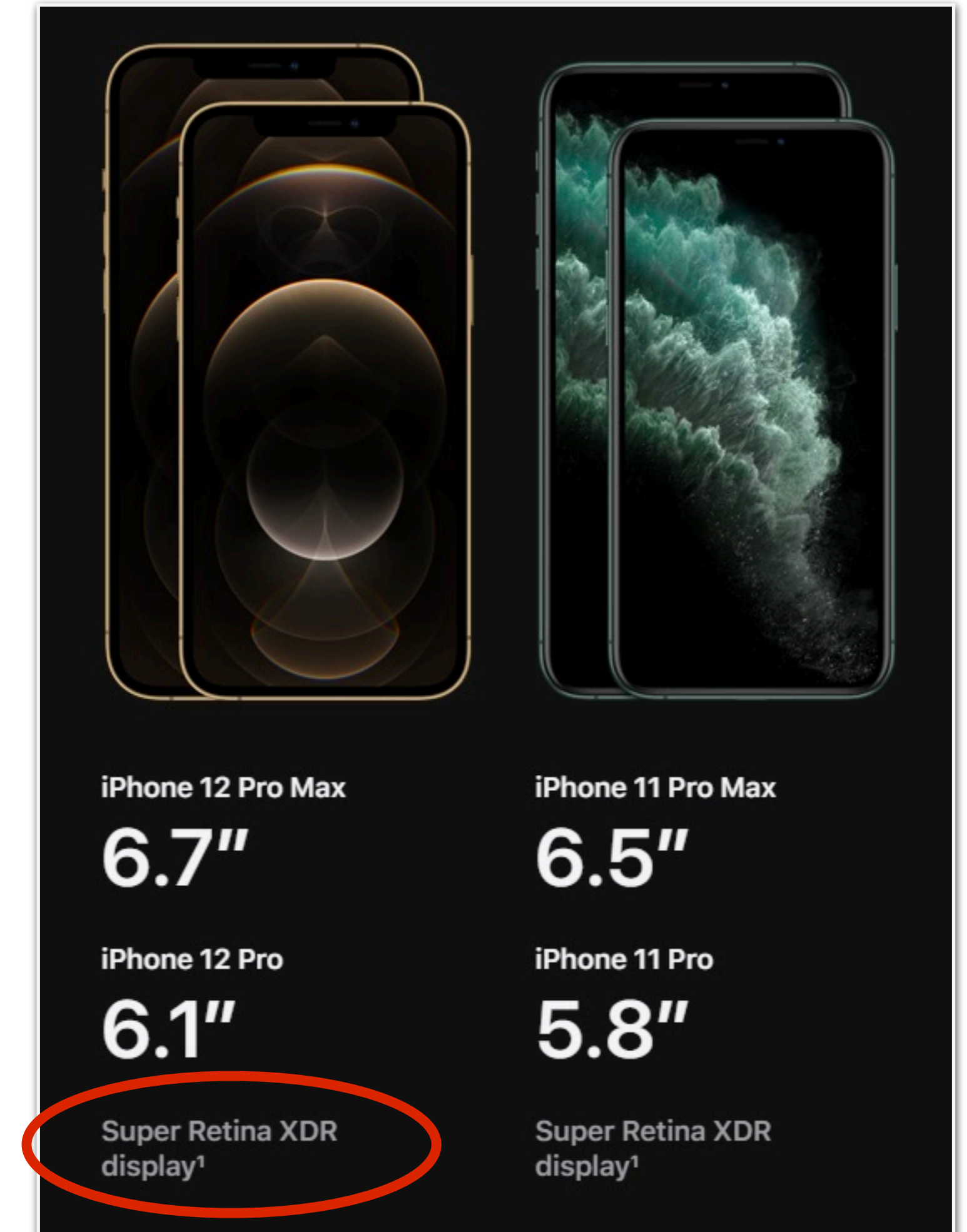
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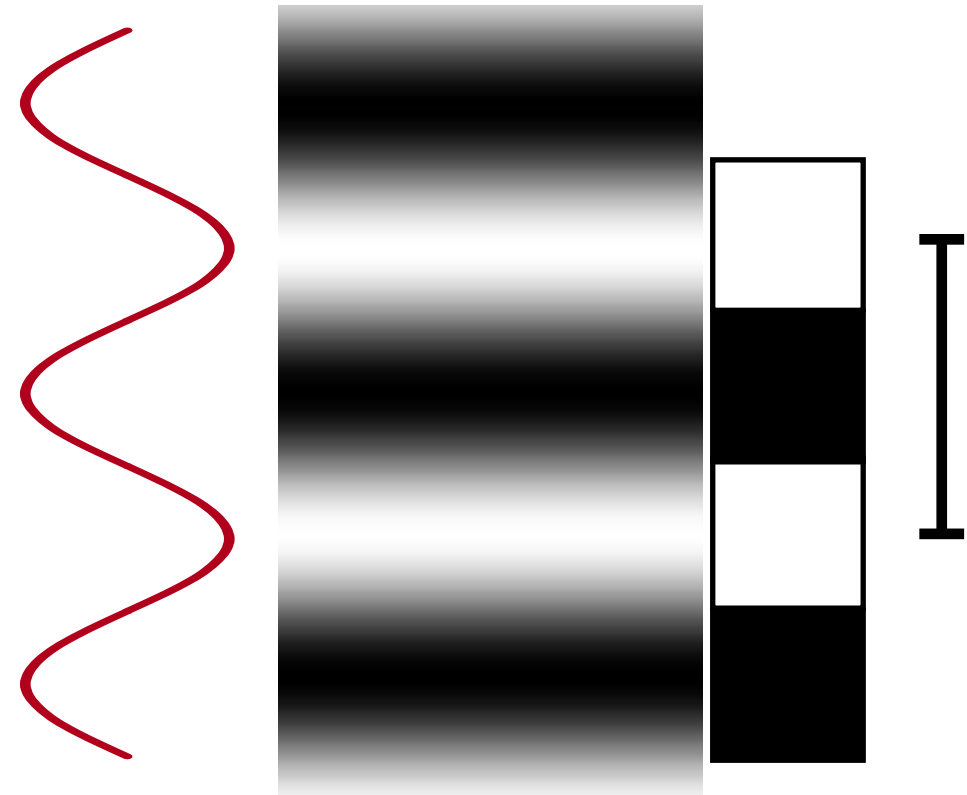
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- 1284 x 2778 pixels, 6.33 inch height
- $P = 2 (6.33'' / 2778)$
- Viewing distance (D_1) 12 inches
- $P/D_1 \approx 0.00038$



iPhone 12 Pro Max Example

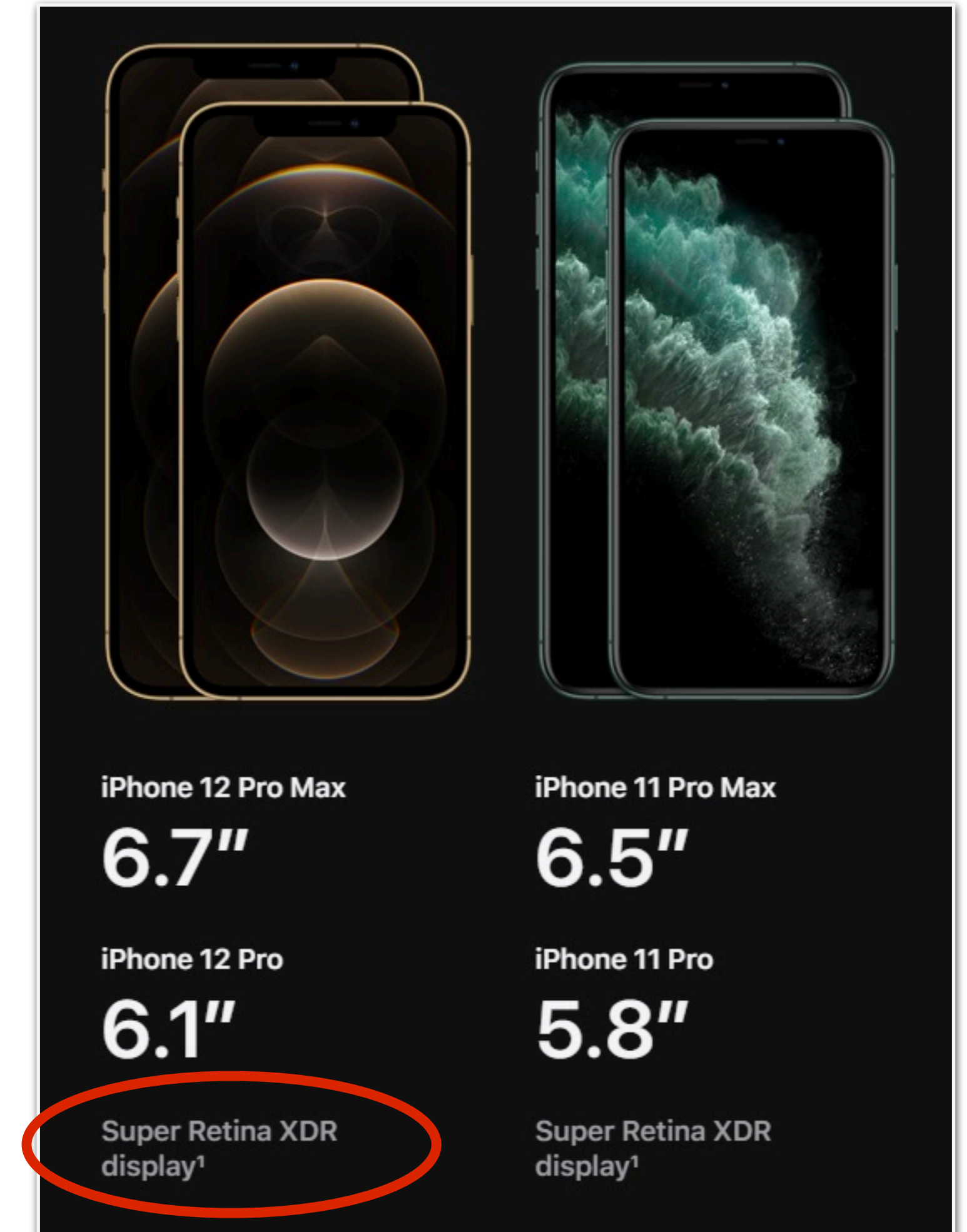
$$\frac{P}{D_1} > 0.0003$$



Display alternative black and white pixels, basically use square wave to approximate sine wave.

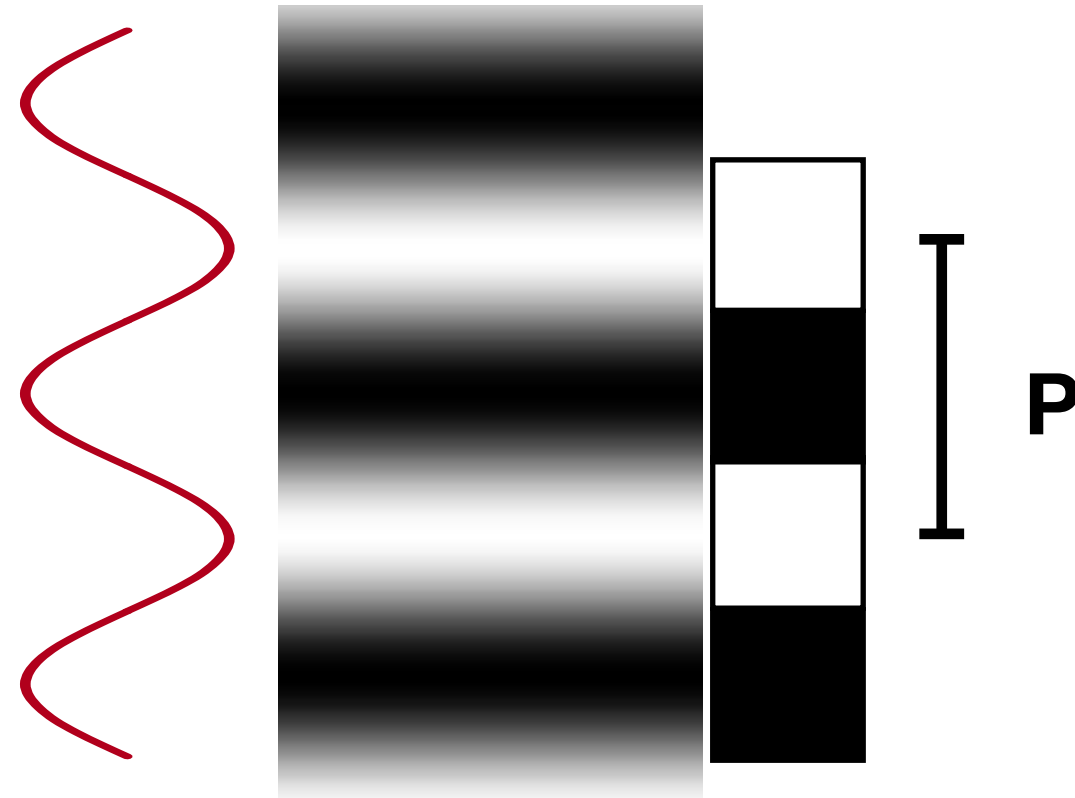
The highest frequency that can be displayed on this device.

- 1284 x 2778 pixels, 6.33 inch height
- $P = 2 (6.33'' / 2778)$
- Viewing distance (D_1) 12 inches
- $P/D_1 \approx 0.00038$
- Just short of being "super retina"



MacBook Air (Mid 2012) Example

$$\frac{P}{D_1} > 0.0003$$



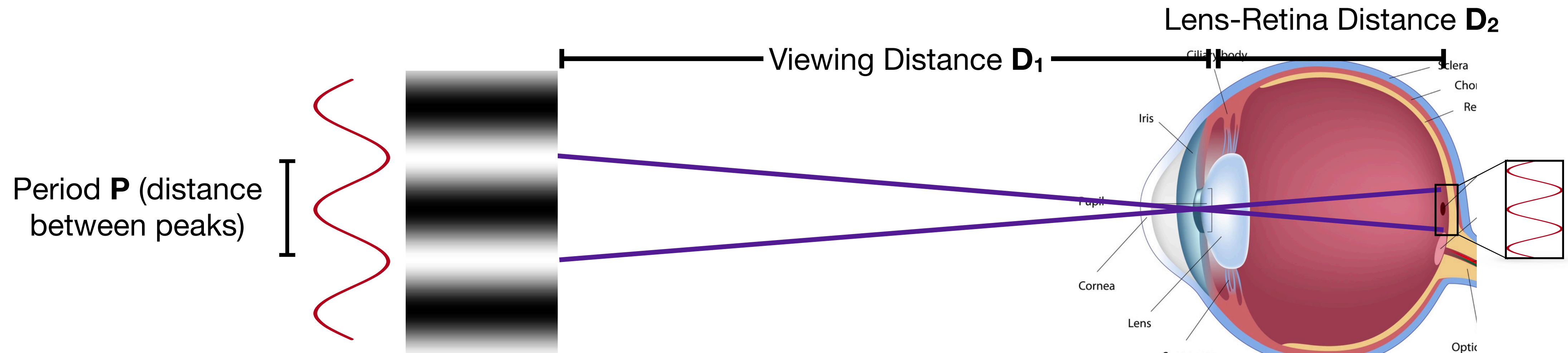
Display alternative black and white pixels, basically use square wave to approximate sine wave.

The highest frequency that can be displayed on this device.



- 1440 x 900 pixels, 8.94 inch height
- $P = 2 (8.94'' / 900)$
- Viewing distance (D_1) 18 inches
- $P/D_1 \approx 0.0011$
- Much lower spatial frequency. No where close to "retinal resolution".

Spatial Resolution Changes with Distance

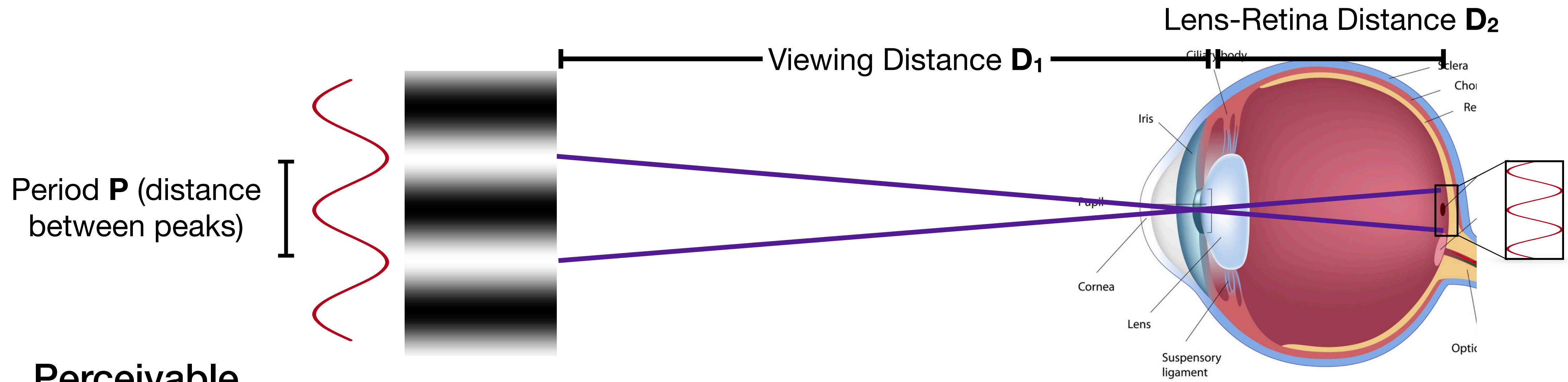


Period P (distance between peaks)

Perceivable threshold

$$\frac{P}{D_1} > \frac{2L}{D_2}$$

Spatial Resolution Changes with Distance

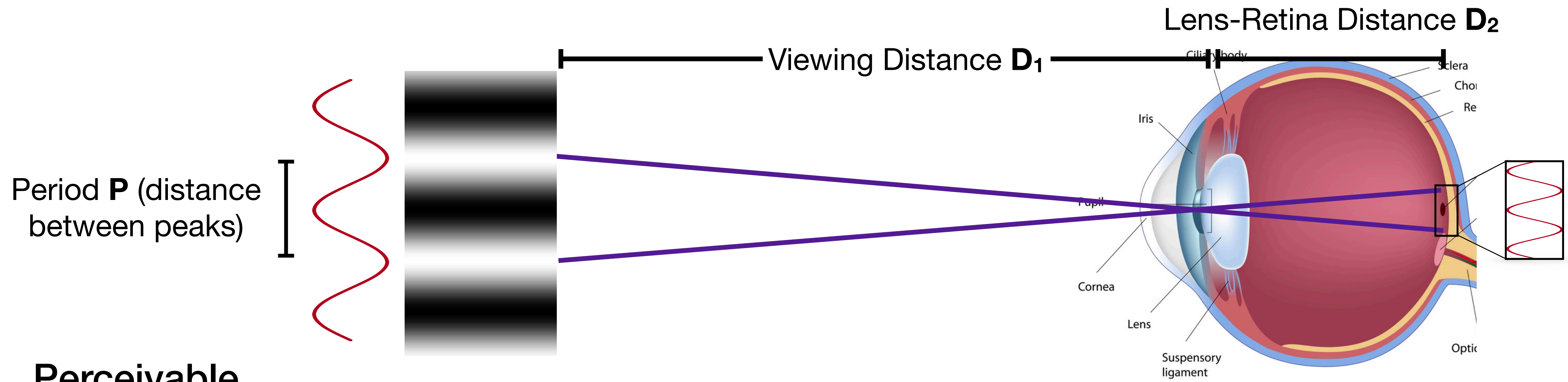


Perceivable threshold

Increasing viewing distance reduces perceptibility, because the spatial frequency on the retina increases

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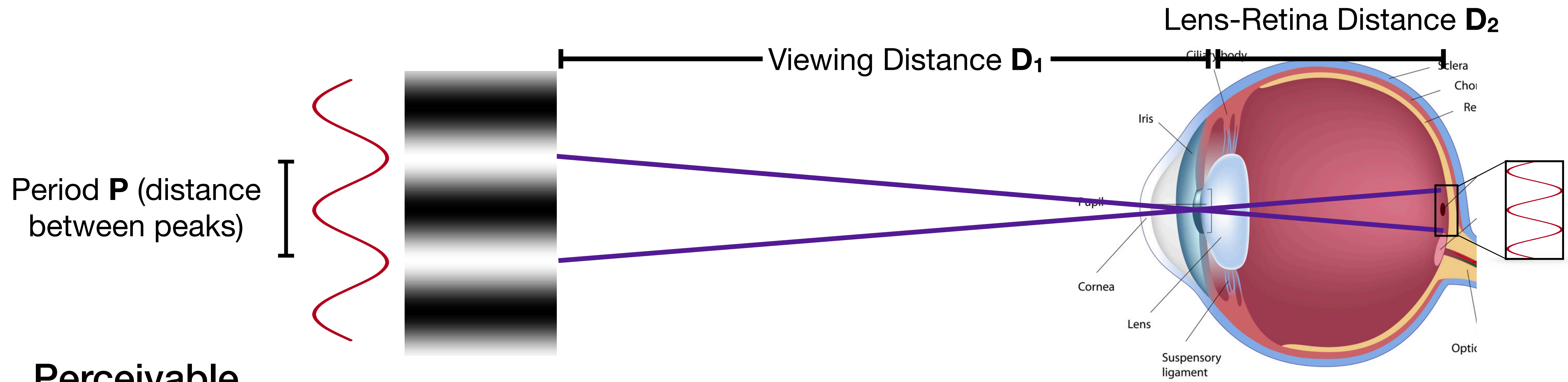
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Decreasing view distance increases perceptibility until...

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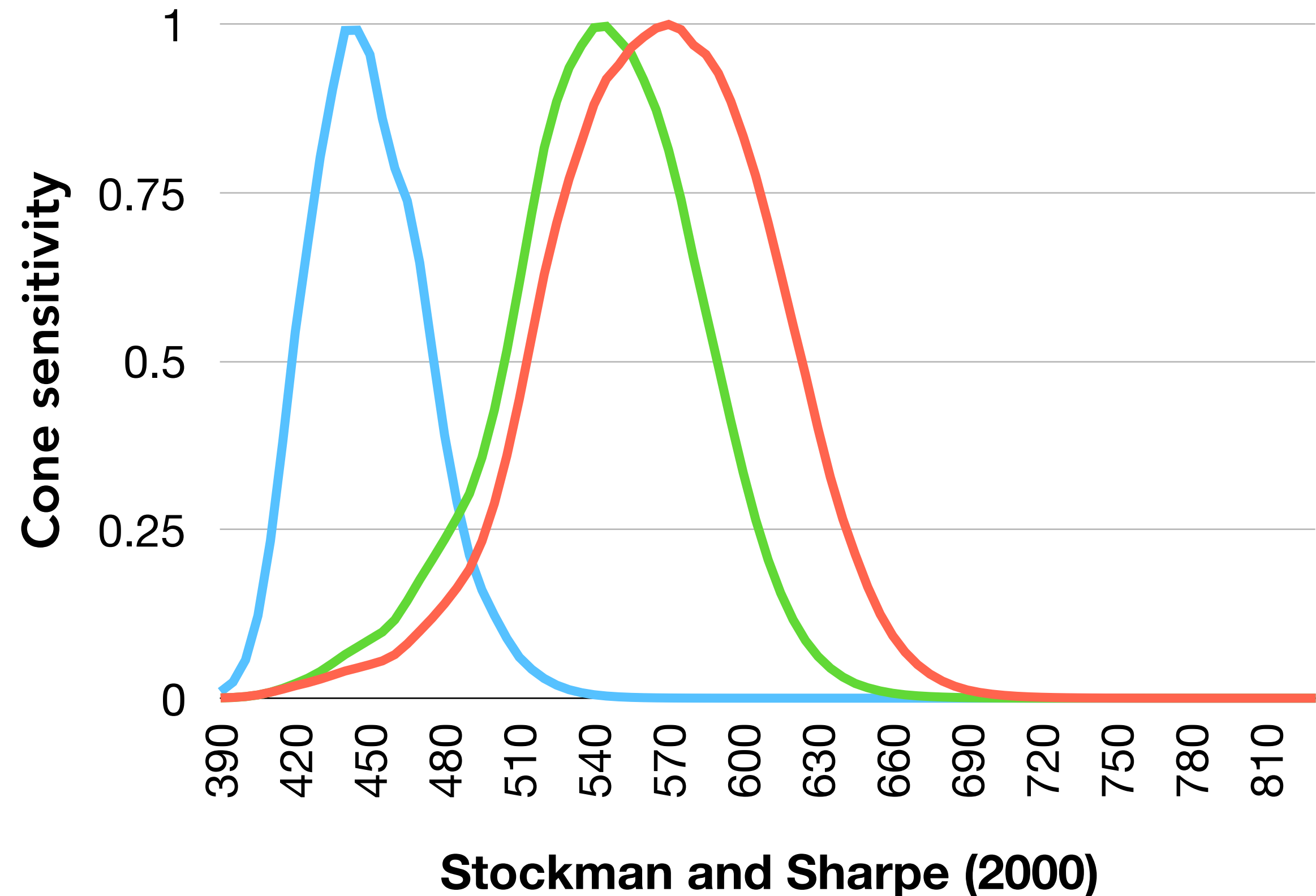
Decreasing view distance increases perceptibility until...

- the object is too close and the lens lacks the optical power to focus

Color Vision

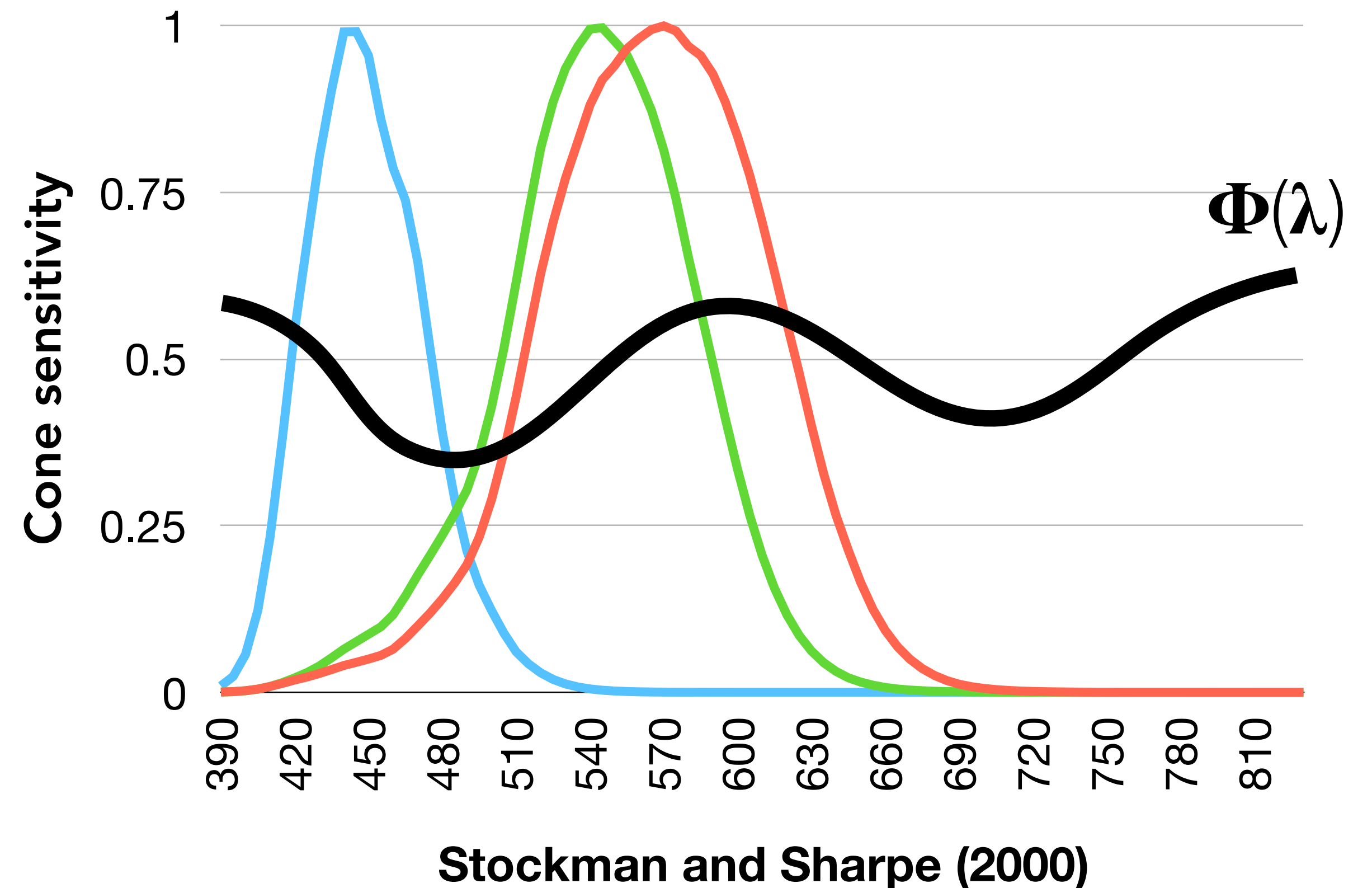
Total Cone Response

Given a light $\Phi(\lambda)$, we can calculate the **total responses** of the L, M, and S cone cells. The three signals are called the **tristimulus values** of the light.



Total Cone Response

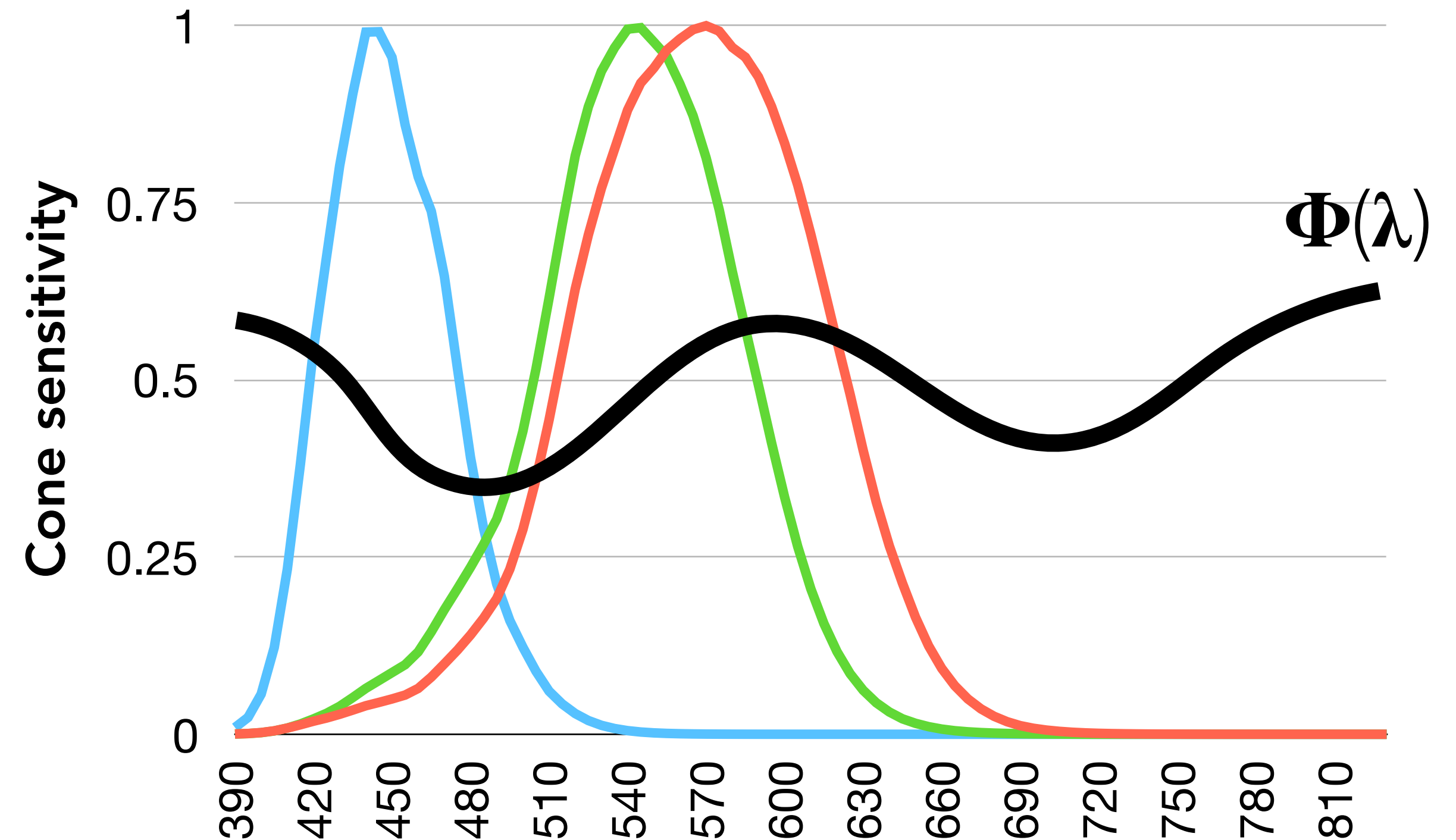
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Total Cone Response

Given a light $\Phi(\lambda)$, we can calculate the **total responses** of the L, M, and S cone cells. The three signals are called the **tristimulus values** of the light.

$$L = \int_{\lambda} \Phi(\lambda) L(\lambda) d\lambda$$



Stockman and Sharpe (2000)

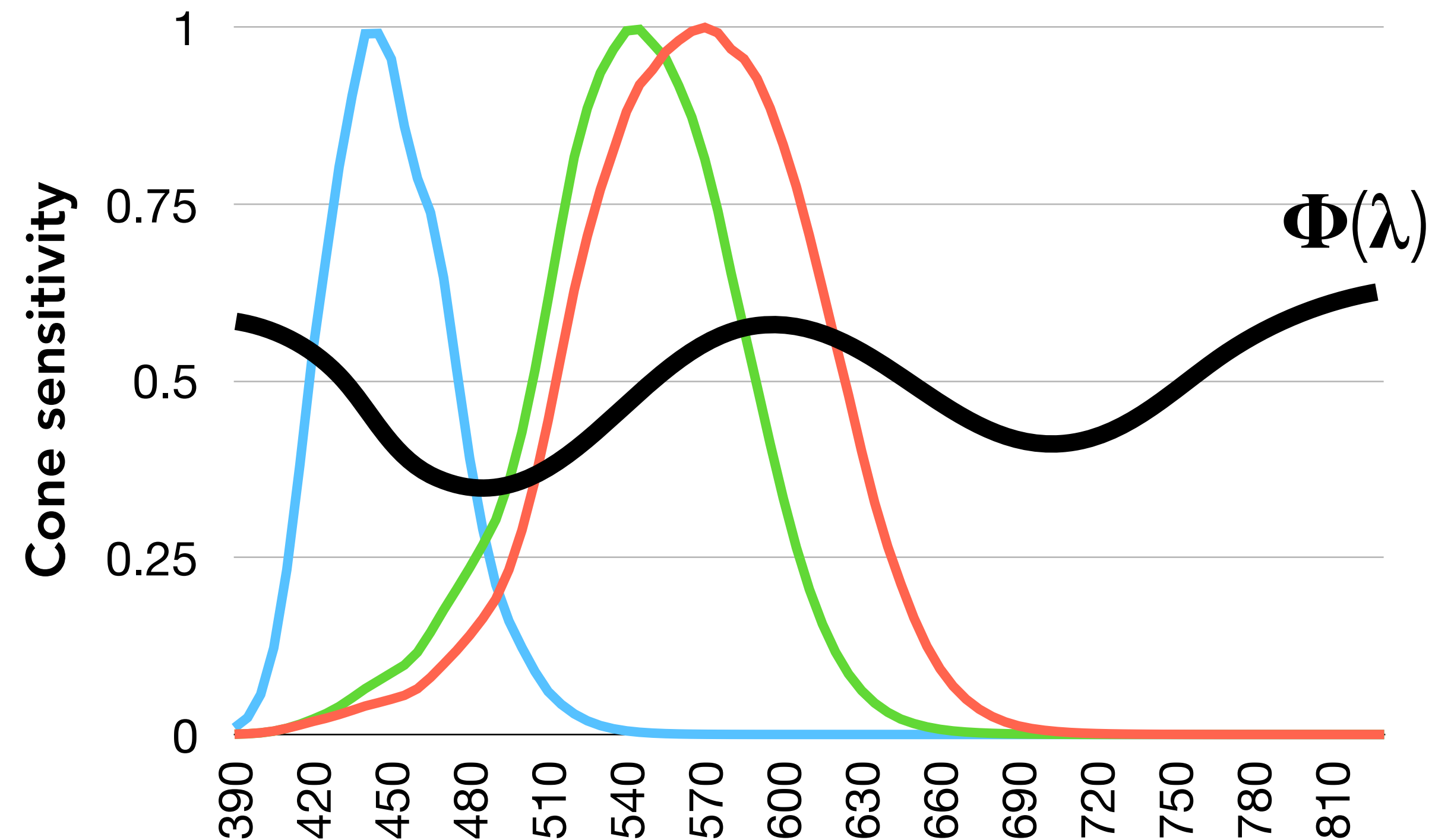
Total Cone Response

Given a light $\Phi(\lambda)$, we can calculate the **total responses** of the L, M, and S cone cells. The three signals are called the **tristimulus values** of the light.

$$L = \sum_{\lambda=380}^{780} \Phi(\lambda)L(\lambda)$$

In practice both the SPD and the cone responses are given at discrete intervals.

$$L = \int_{\lambda} \Phi(\lambda)L(\lambda)d\lambda$$



Stockman and Sharpe (2000)

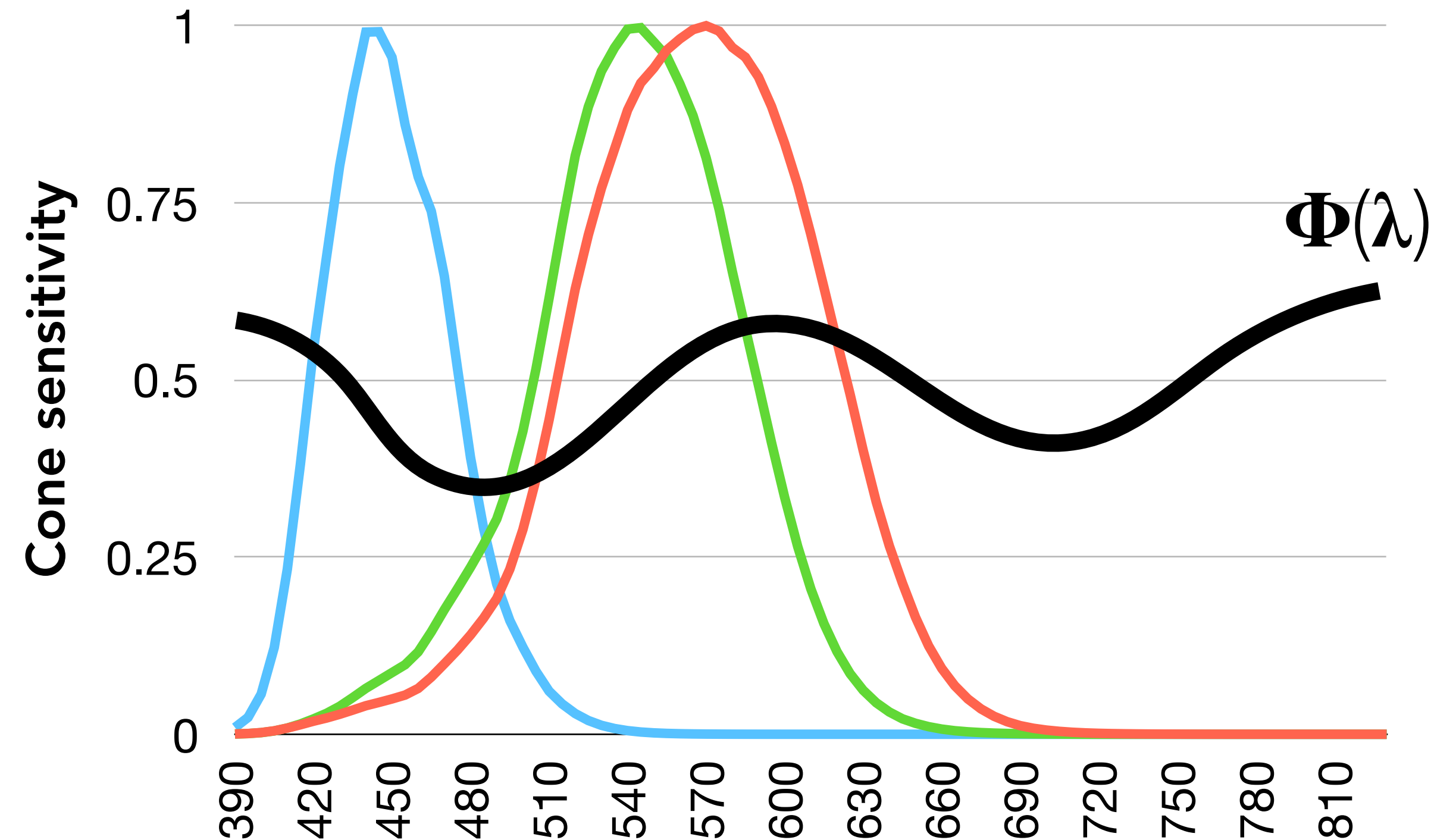
Total Cone Response

Any light could be expressed as a $\langle L, M, S \rangle$ triplet.

$$L = \sum_{\lambda=380}^{780} \Phi(\lambda)L(\lambda)$$

$$M = \sum_{\lambda=380}^{780} \Phi(\lambda)M(\lambda)$$

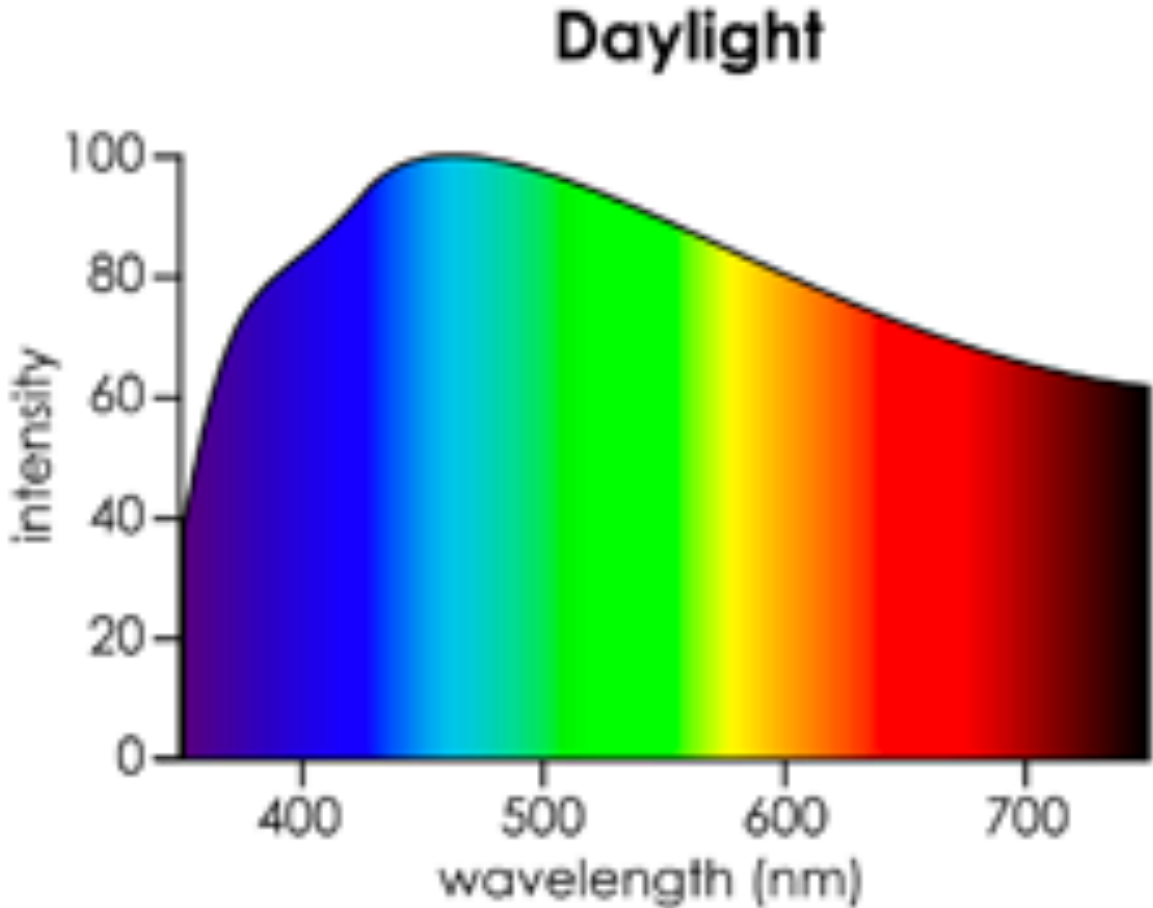
$$S = \sum_{\lambda=380}^{780} \Phi(\lambda)S(\lambda)$$



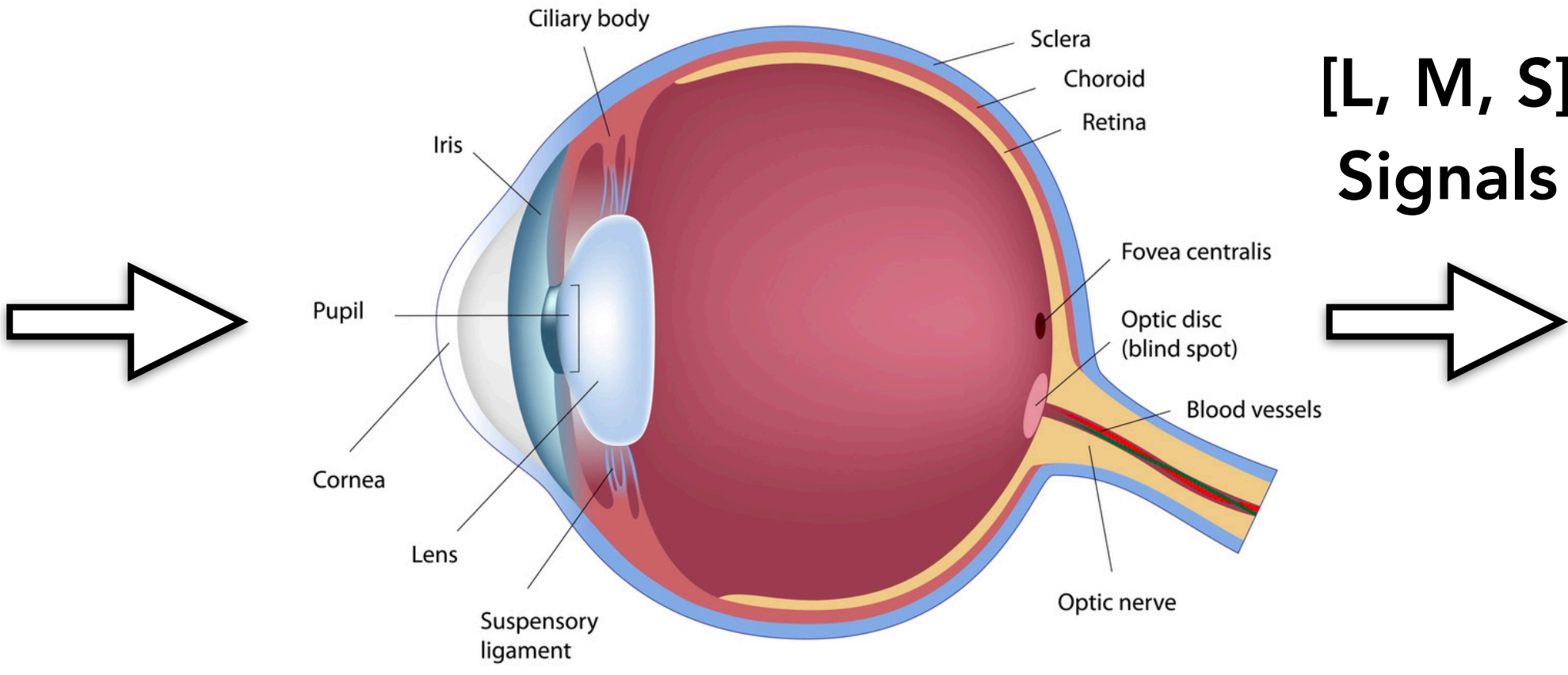
Stockman and Sharpe (2000)

Brain Receives Only the LMS Signals, not Light!

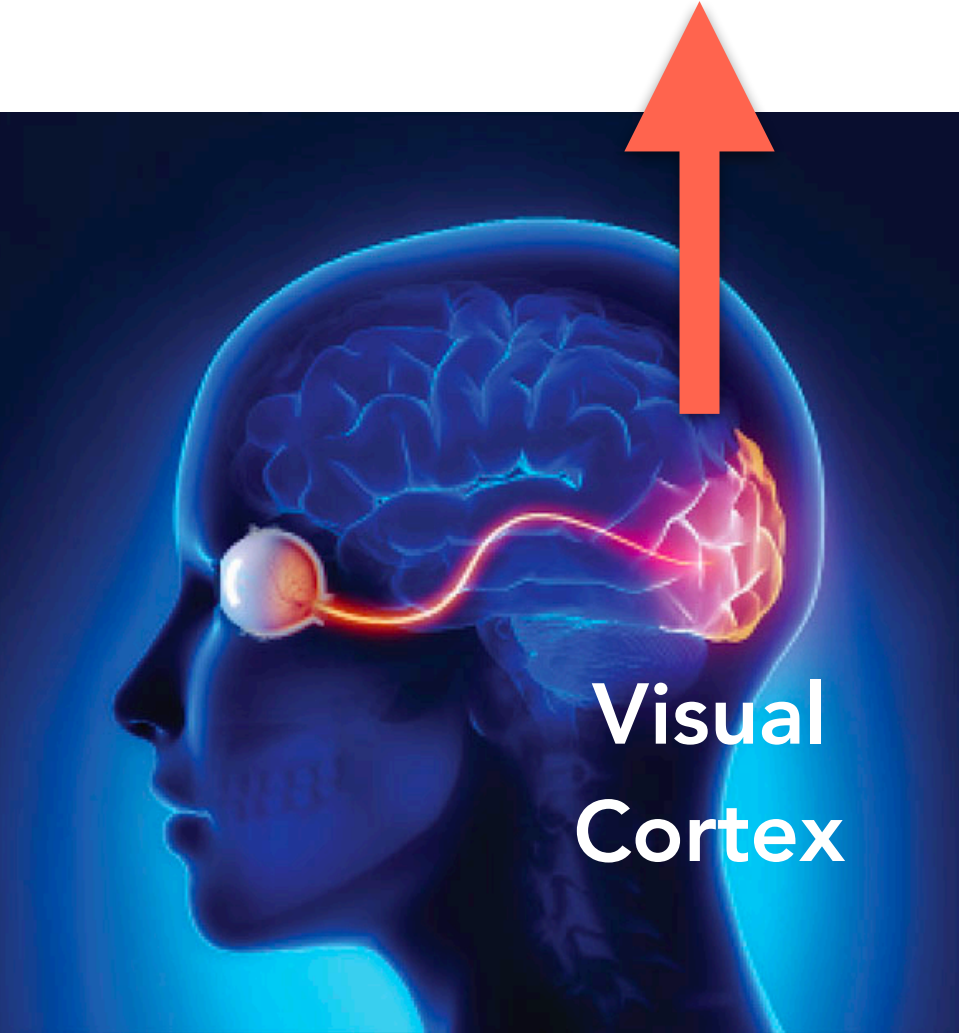
Light Spectrum



Eye

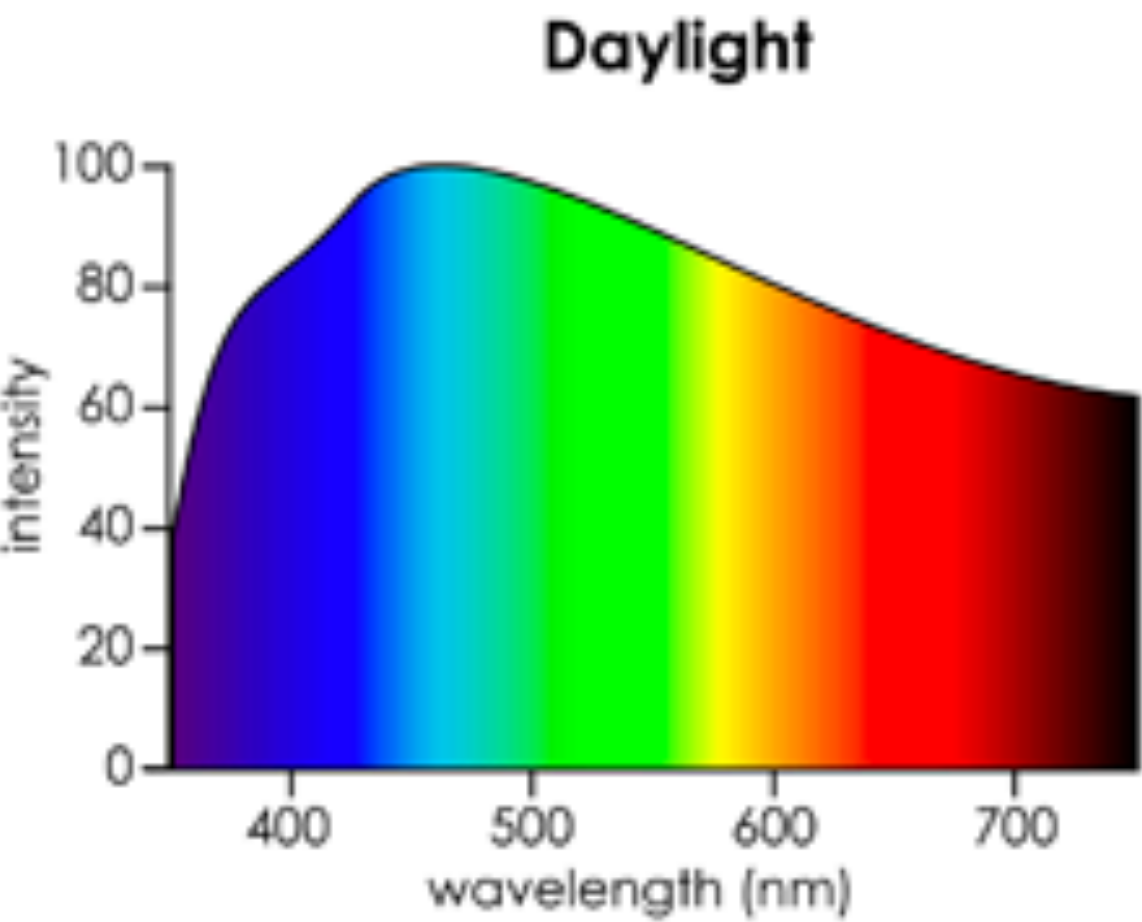


A color sensation!

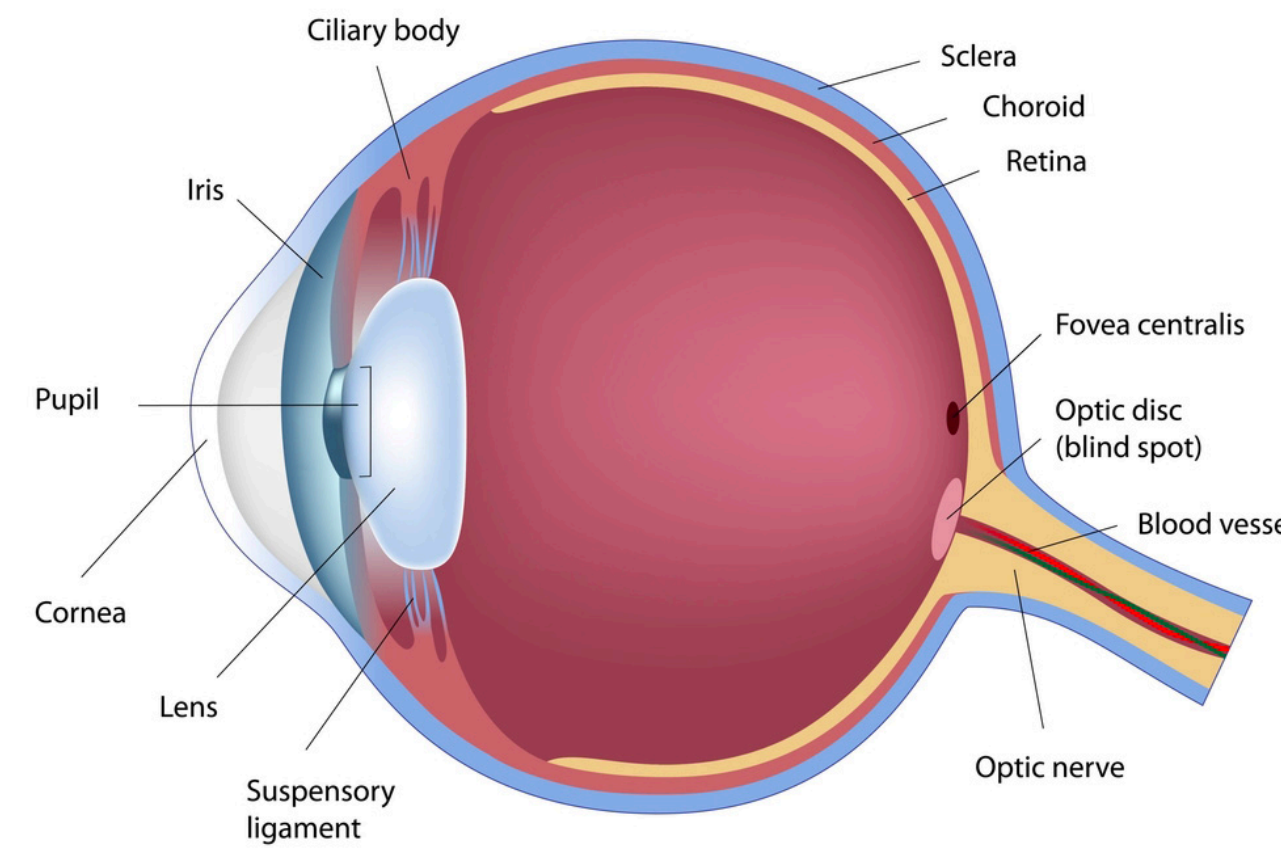


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Light Spectrum

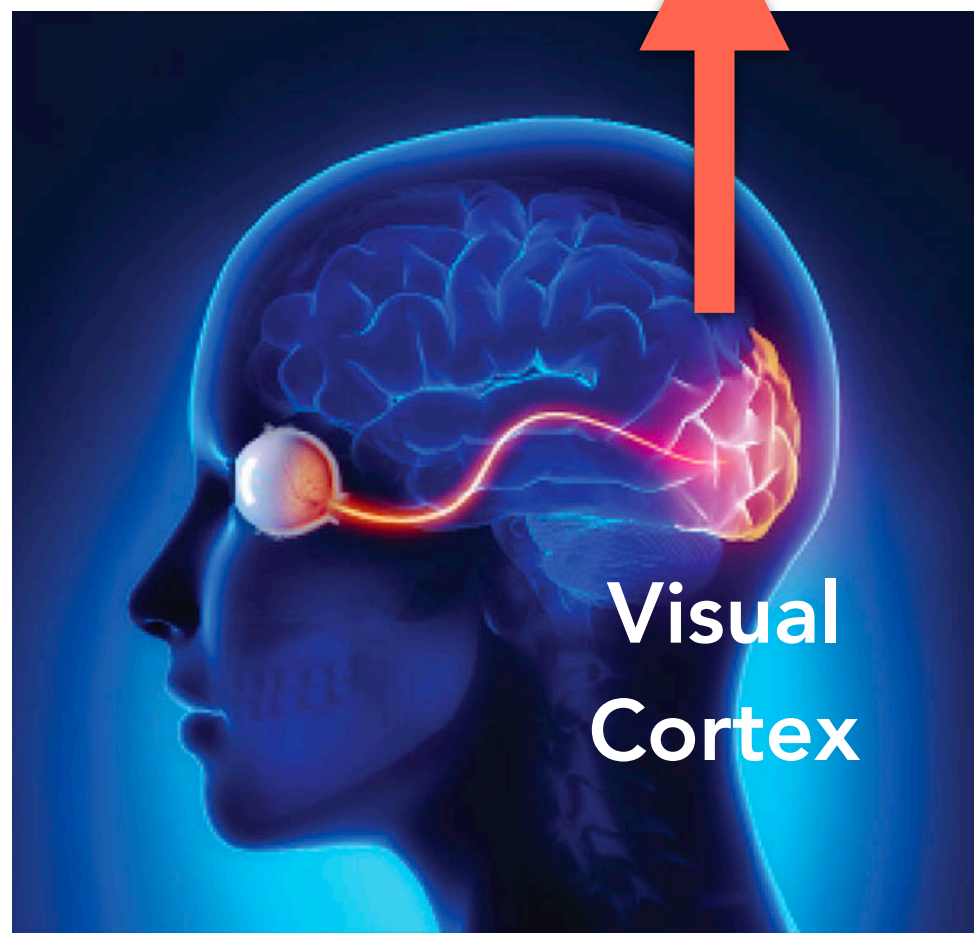


Eye



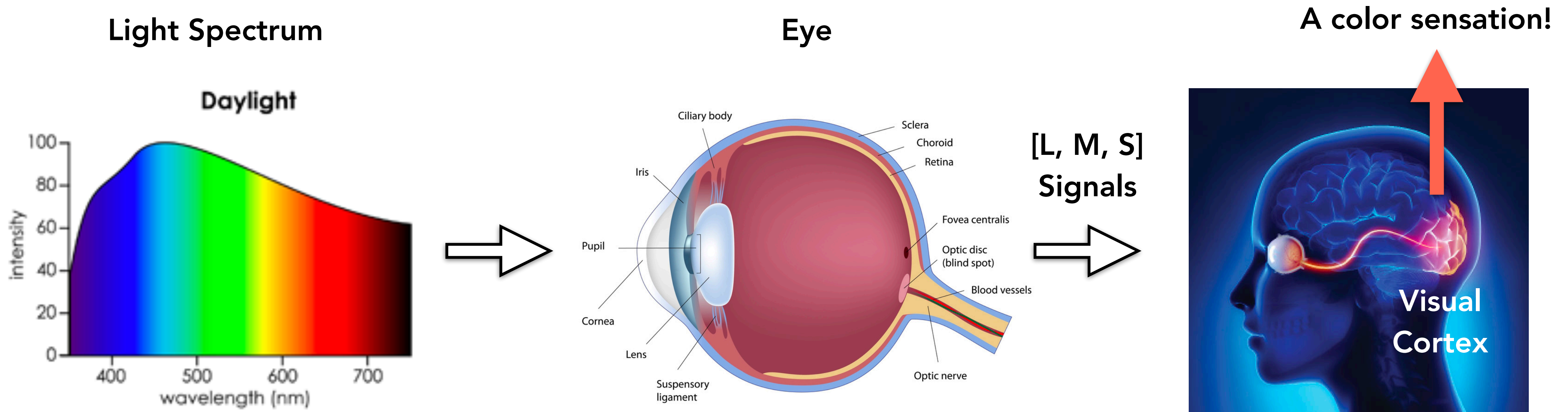
[L, M, S]
Signals

A color sensation!



A particular **LMS combination** leads to a particular **color sensation**.

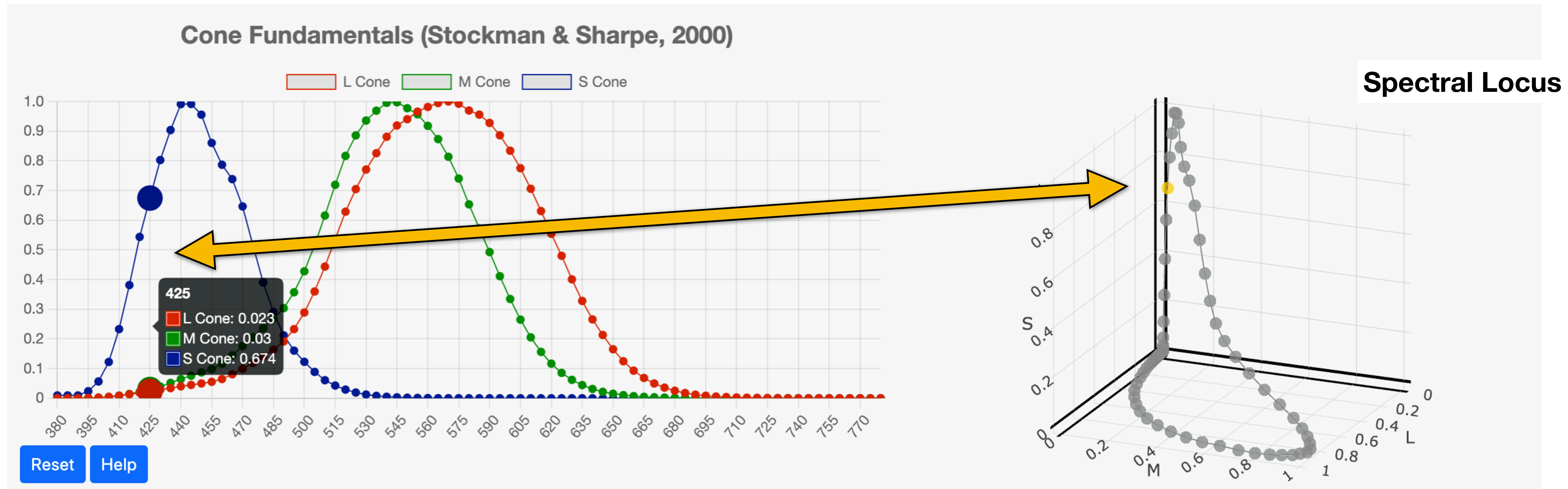
Brain Receives Only the LMS Signals, not Light!



A particular **LMS combination** leads to a particular **color sensation**.

This is a huge dimensionality reduction: the light spectrum is a very high dimensional vector (numerous wavelengths), but is transformed by the retina to only a three-dimensional vector!

Visualizing Spectral Lights in the LMS Color Space



Play with an interactive app here: <https://www.cs.rochester.edu/courses/572/colorvis/cone2cmf.html>

The LMS Color Space

By expressing any light's color as a $\langle L, M, S \rangle$ point, we have inherently defined a 3D color space, in which every color is a point.

See visualization here: <https://www.cs.rochester.edu/courses/572/colorvis/chromaticity.html>.

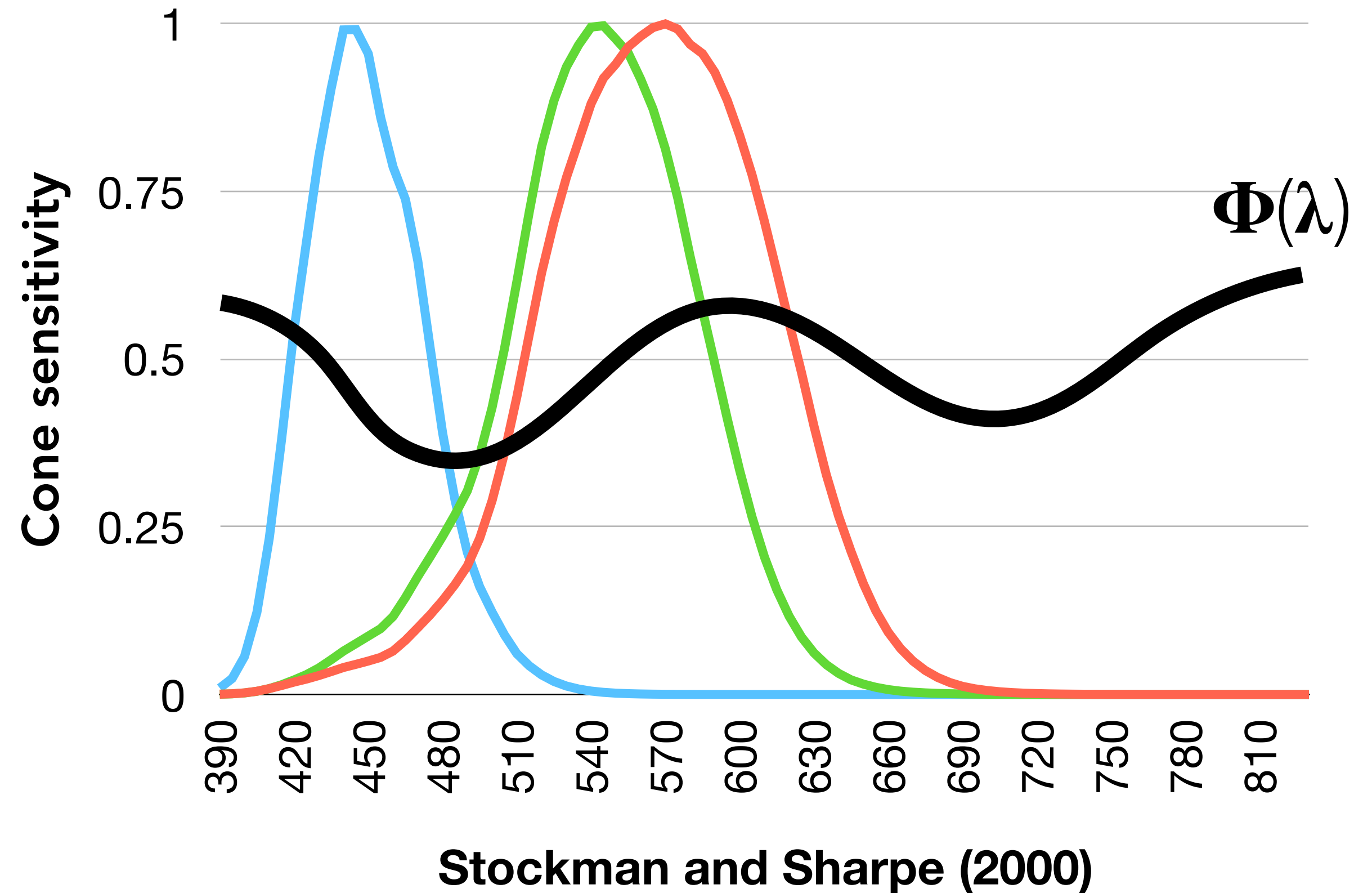
What if we uniformly double the power at each wavelength? How would the corresponding $\langle L, M, S \rangle$ change?

Two Fascinating Questions

1. Does any arbitrary point in the LMS space correspond to a color?
2. Will there be multiple lights that correspond to the same point, i.e., color in the LMS space?

Recall: Calculating Total Cone Responses

$$L = \sum_{\lambda=380}^{780} \Phi(\lambda)L(\lambda)$$
$$M = \sum_{\lambda=380}^{780} \Phi(\lambda)M(\lambda)$$
$$S = \sum_{\lambda=380}^{780} \Phi(\lambda)S(\lambda)$$



Matrix Perspective

$$L = \sum_{\lambda=380}^{780} \Phi(\lambda)L(\lambda)$$

$$M = \sum_{\lambda=380}^{780} \Phi(\lambda)M(\lambda)$$

$$S = \sum_{\lambda=380}^{780} \Phi(\lambda)S(\lambda)$$

$$\begin{bmatrix} L_{380}, L_{381}, \dots, L_{780} \\ M_{380}, M_{381}, \dots, M_{780} \\ S_{380}, S_{381}, \dots, S_{780} \end{bmatrix} \times \begin{bmatrix} \Phi_{380} \\ \Phi_{381} \\ \vdots \\ \Phi_{780} \end{bmatrix} = \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

Matrix Perspective

It's an under-determined system of linear equations. **In general**, it has infinitely many solutions.

In general, there are infinitely many lights that correspond to the same color. This is called **metamerism**. More on this soon.

$$\begin{bmatrix} L_{380}, L_{381}, \dots, L_{780} \\ M_{380}, M_{381}, \dots, M_{780} \\ S_{380}, S_{381}, \dots, S_{780} \end{bmatrix} \times \begin{bmatrix} \Phi_{380} \\ \Phi_{381} \\ \vdots \\ \Phi_{780} \end{bmatrix} = \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

The first matrix is known. Given the [L, M, S] vector, can you solve for the Φ vector?

Matrix Perspective

It's an under-determined system of linear equations. **In general**, it has infinitely many solutions.

In general, there are infinitely many lights that correspond to the same color. This is called **metamerism**. More on this soon.

An additional constraint: Φ must all be non-negative, since there is no negative power!

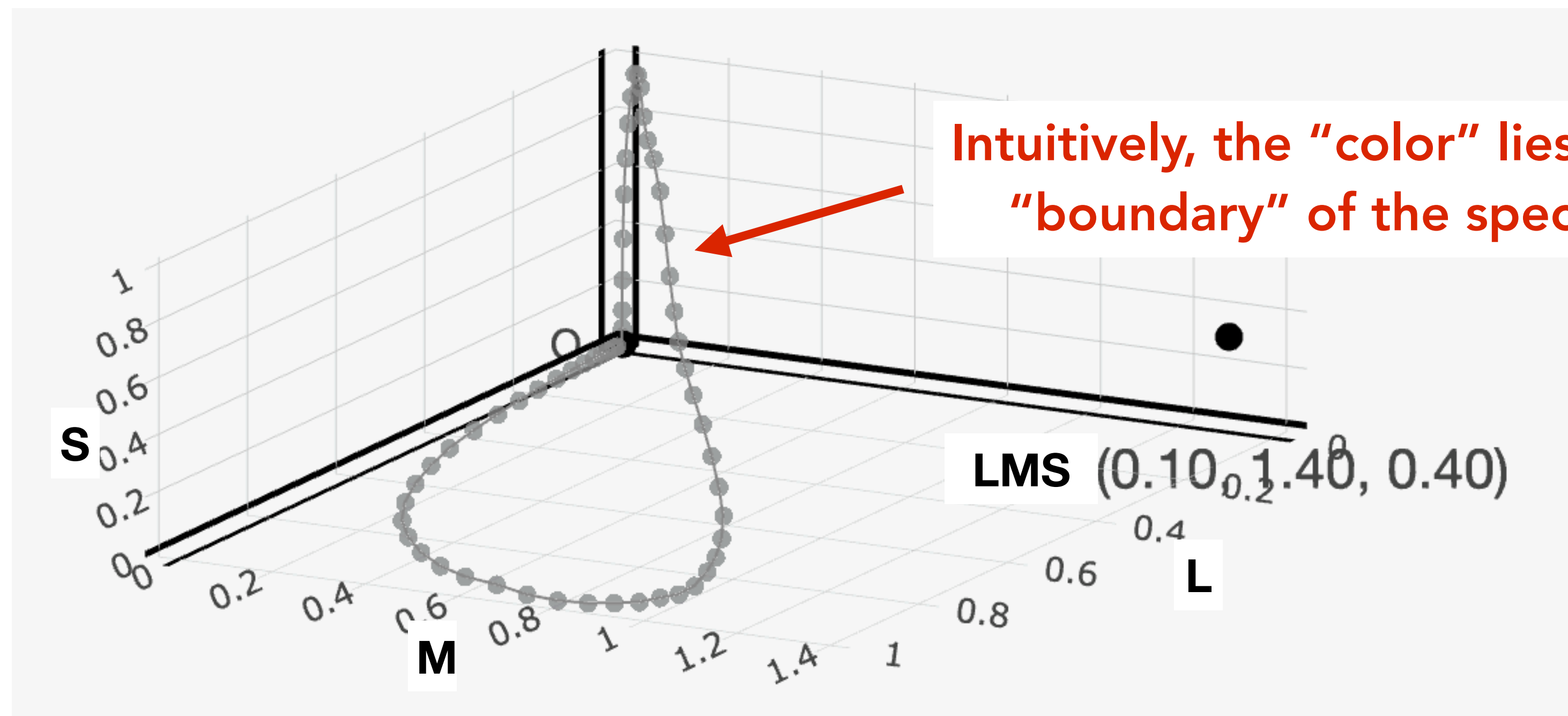
$$\begin{bmatrix} L_{380}, L_{381}, \dots, L_{780} \\ M_{380}, M_{381}, \dots, M_{780} \\ S_{380}, S_{381}, \dots, S_{780} \end{bmatrix} \times \begin{bmatrix} \Phi_{380} \\ \Phi_{381} \\ \vdots \\ \Phi_{780} \end{bmatrix} = \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

The first matrix is known. Given the [L, M, S] vector, can you solve for the Φ vector?

But Can There Be No Solution?

One scenario: negative L, M, or S responses. All cone responses are physical quantities, which must be positive.

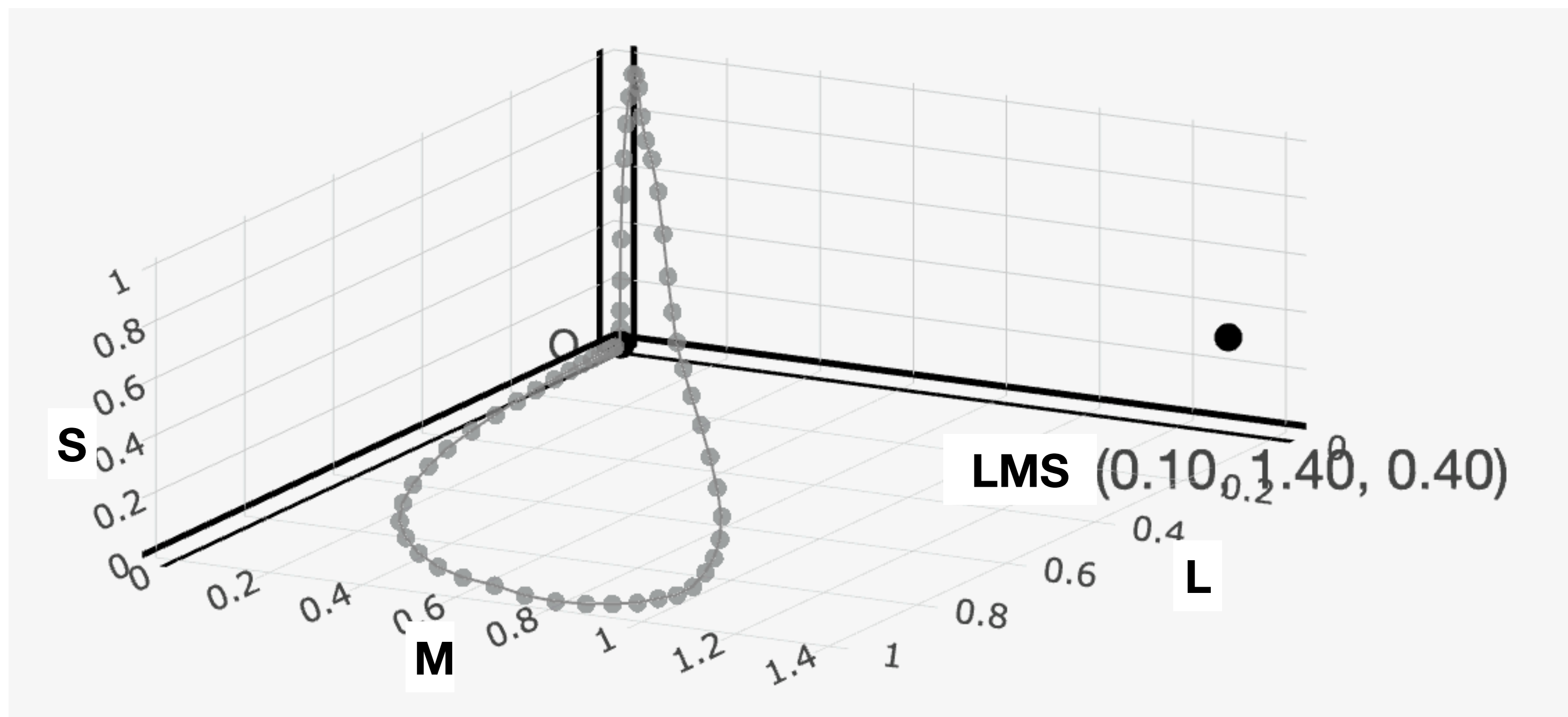
Another scenario: impossible cone response combinations.



Play with an interactive app here: <https://www.cs.rochester.edu/courses/572/colorvis/chromaticity.html>

Imaginary Color

An impossible cone combination still corresponds to a point ("color") in the LMS space. We call it an imaginary color. It's not a color that can be produced by physical lights; it requires negative power in the light spectrum.



Play with an interactive app here: <https://www.cs.rochester.edu/courses/572/colorvis/chromaticity.html>

Color Gamut of Human Visual System (HVS)

HVS Color Gamut: the set of all the colors that human can see.

The HVS color gamut encapsulates all the LMS points for which the system of linear equations below has a non-negative solution.

$$\begin{bmatrix} L_{380}, L_{381}, \dots, L_{780} \\ M_{380}, M_{381}, \dots, M_{780} \\ S_{380}, S_{381}, \dots, S_{780} \end{bmatrix} \times \begin{bmatrix} \Phi_{380} \\ \Phi_{381} \\ \vdots \\ \Phi_{780} \end{bmatrix} = \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

Two Fascinating Questions

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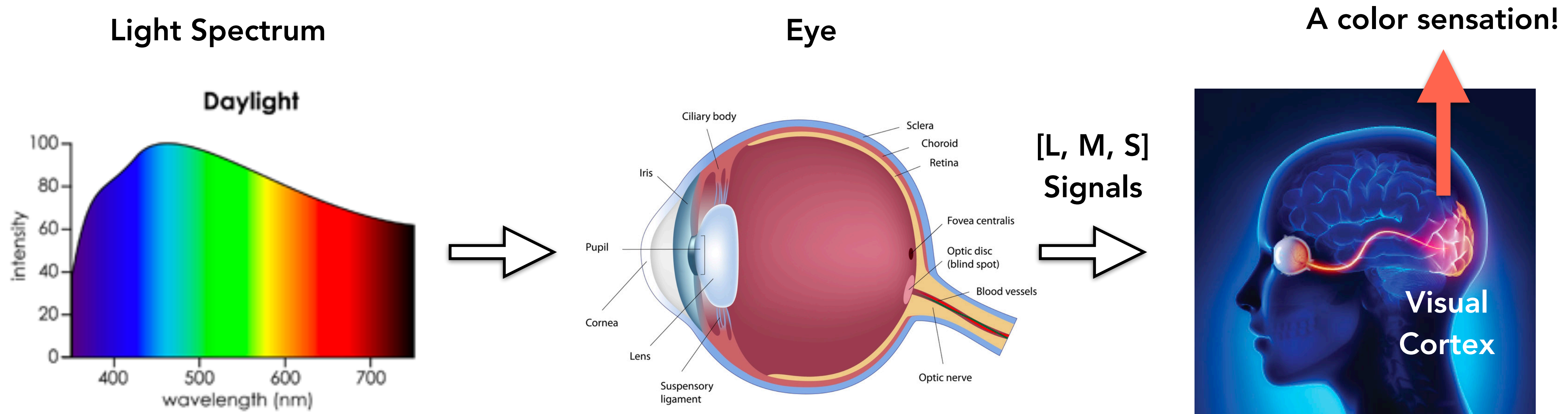
- No

2. Will there be multiple lights that correspond to the same point, i.e., color in the LMS space?

- Yes. In fact, there are infinitely many.

Metamerism

Intuition

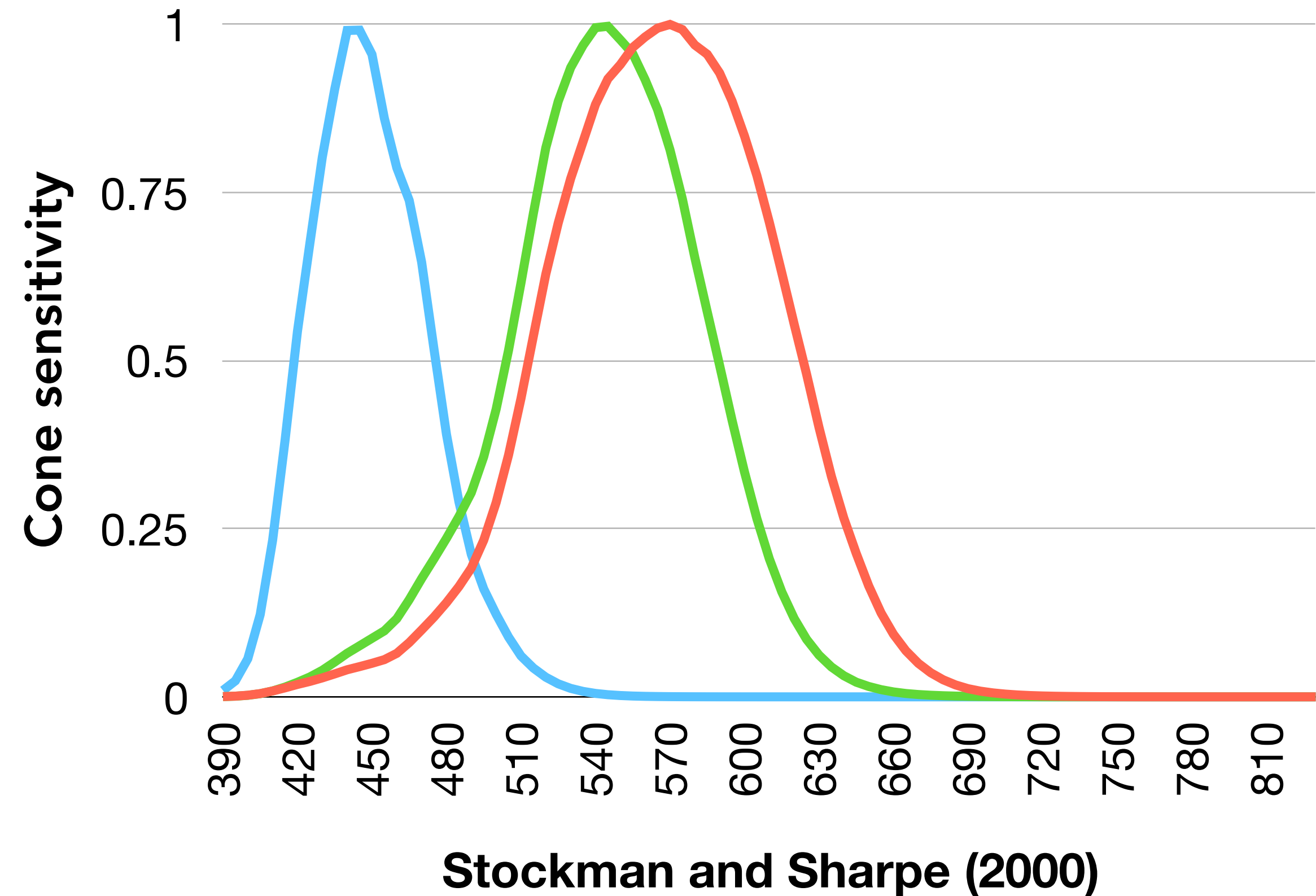


Lights that produce the same LMS signals are ***perceived*** by the brain as the same "color", even if their spectra might be totally different!

...because retinal processing is a dramatic dimensionality reduction.

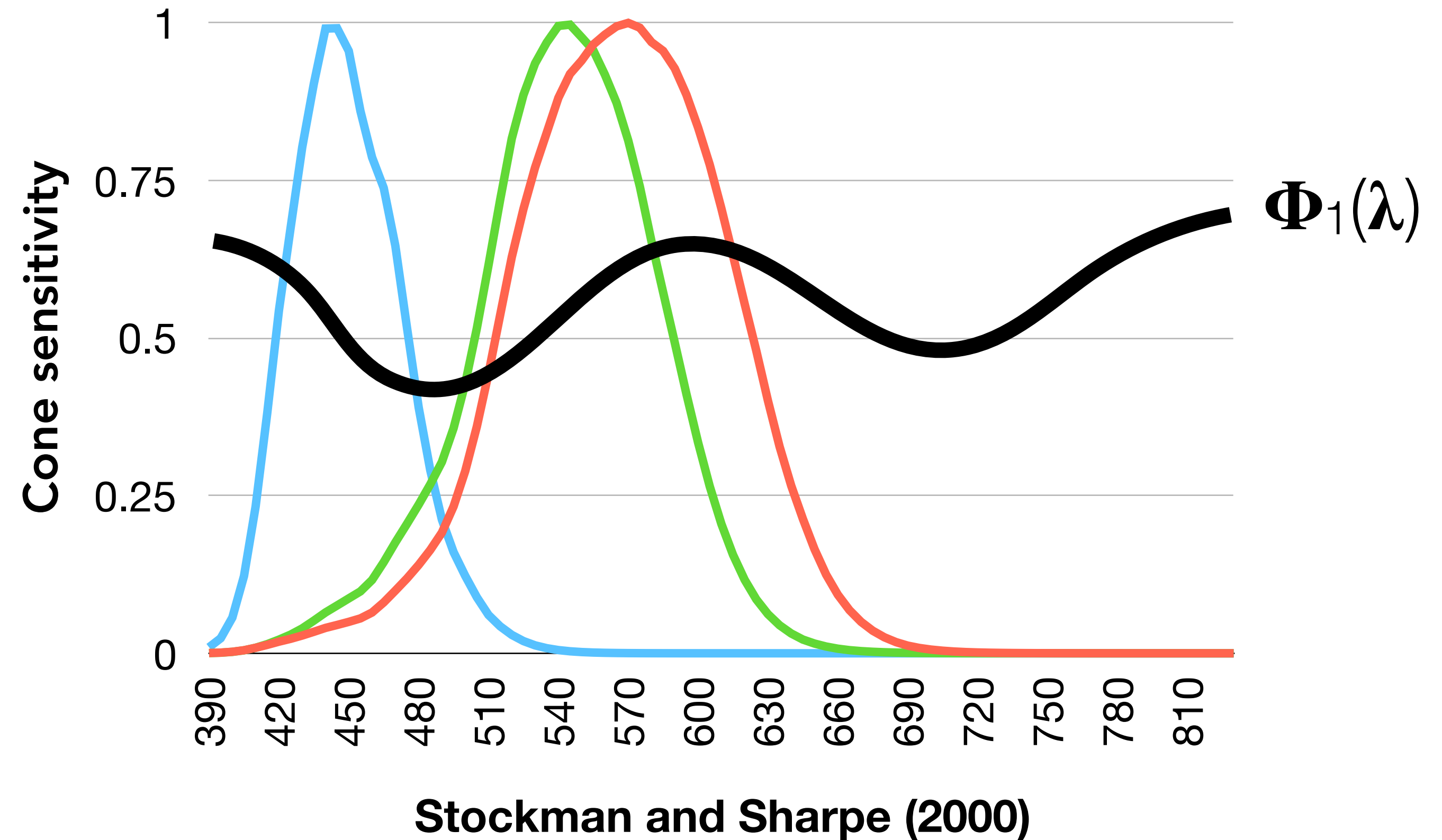
Metamerism

The two lights are metamers of each other if their LMS responses match.



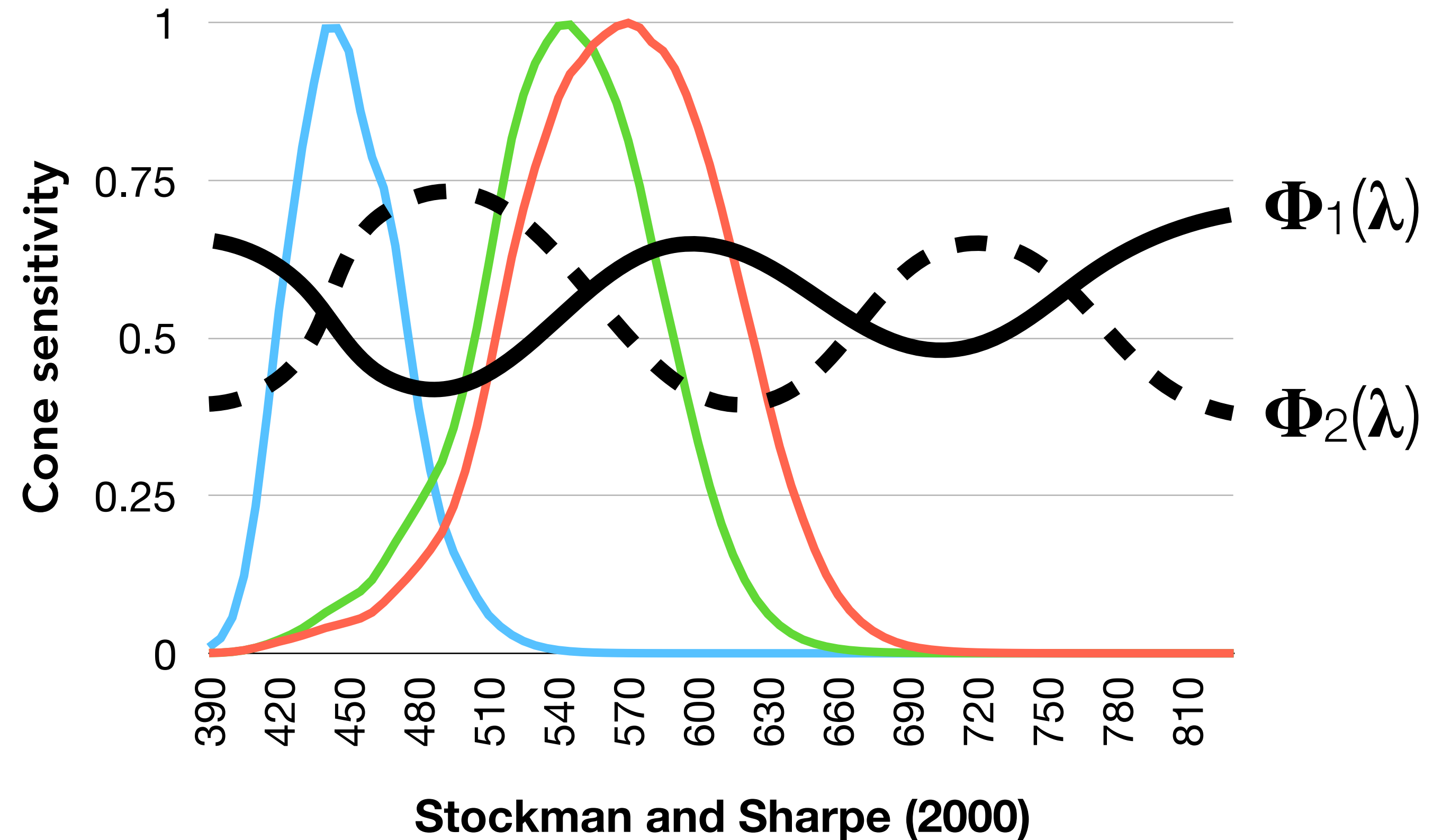
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Metamerism

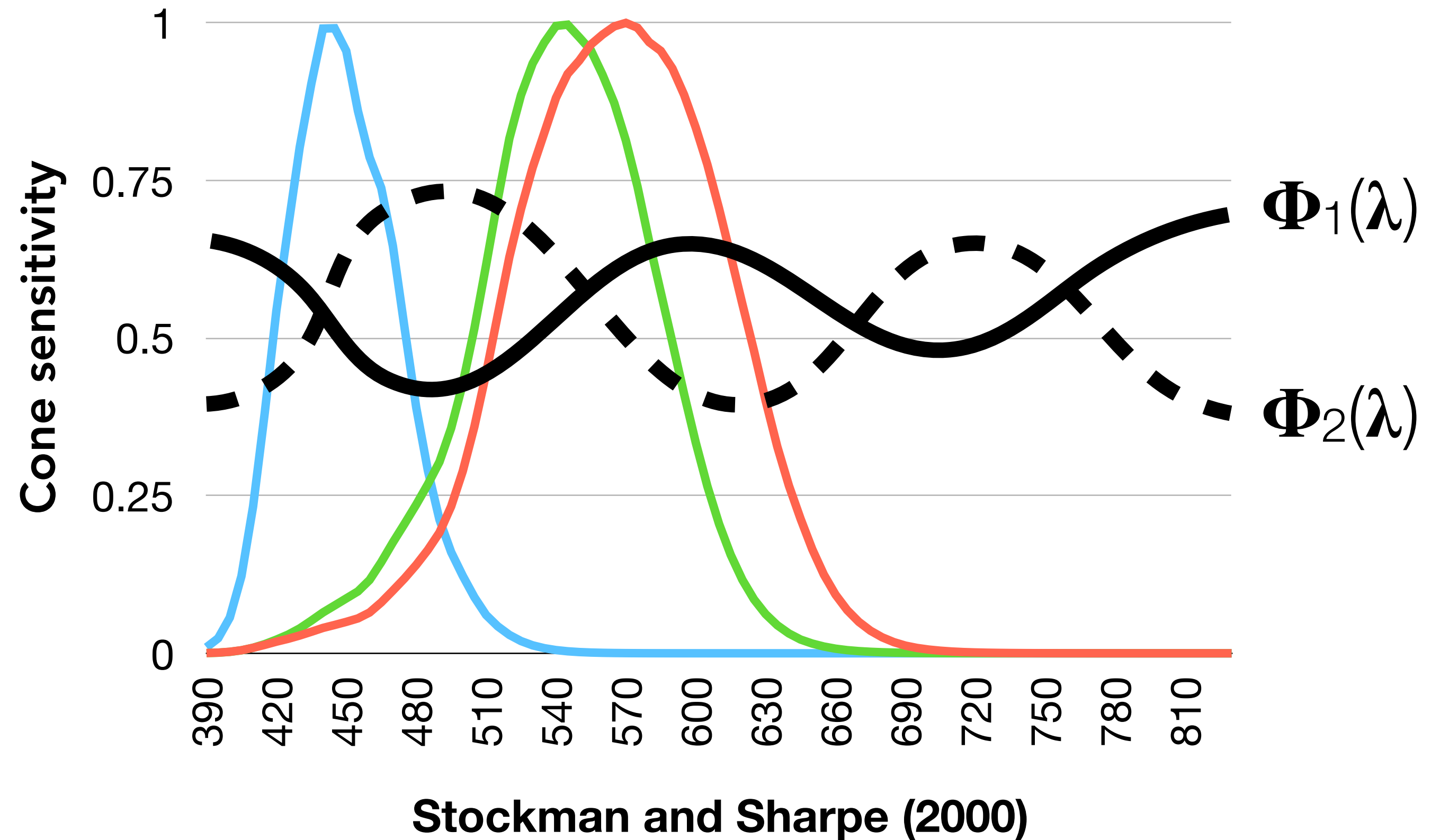
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Metamerism

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$$\sum_{\lambda} \Phi_1(\lambda)L(\lambda) = \sum_{\lambda} \Phi_2(\lambda)L(\lambda)$$
$$\sum_{\lambda} \Phi_1(\lambda)M(\lambda) = \sum_{\lambda} \Phi_2(\lambda)M(\lambda)$$
$$\sum_{\lambda} \Phi_1(\lambda)S(\lambda) = \sum_{\lambda} \Phi_2(\lambda)S(\lambda)$$



Metamerism

Does metamerism exist? That is, given a spectrum Φ , can we always find at least one Φ' that is the metamer of Φ ? Can we find multiple?

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But people were able to find metamers as early as late 19th century, through a clever ***color matching*** experiment.

Studying color matching experiment will provide us with immense insights.

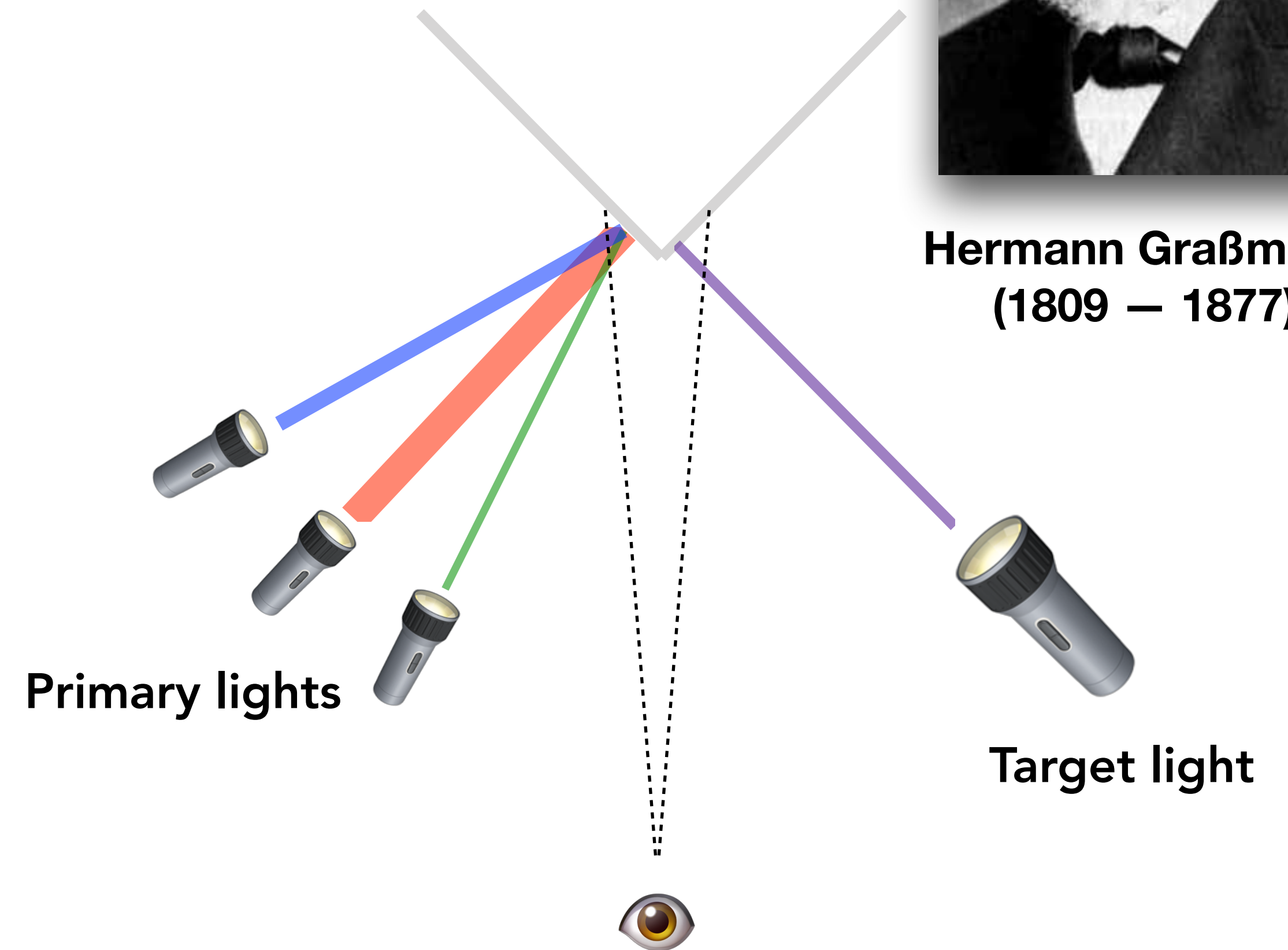
Color Matching Experiments

You are given three monochromatic (spectral) lights with known spectra pointing to the same point.

- They are called the *primary lights*.



Hermann Graßmann
(1809 – 1877)



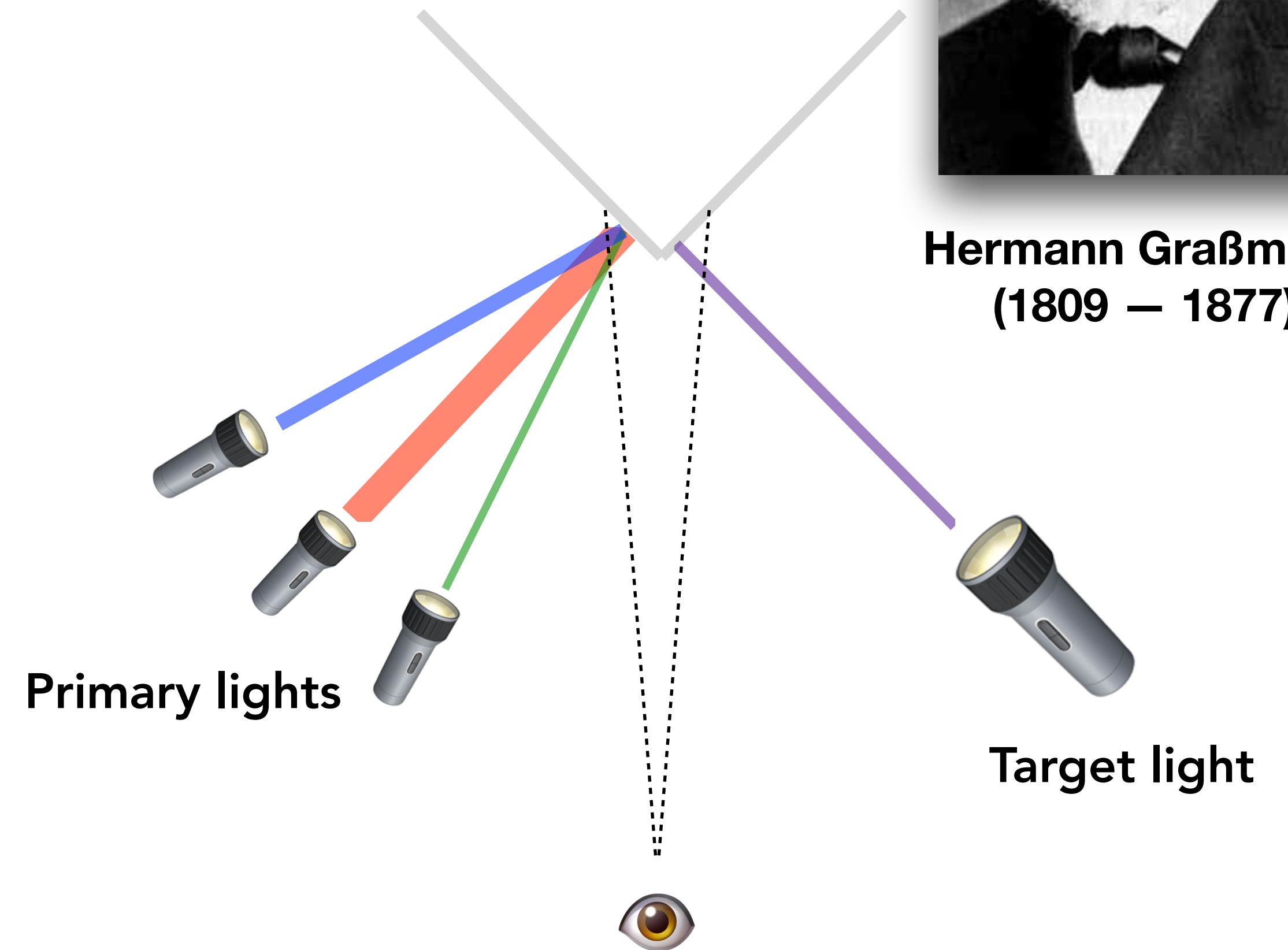
Color matching experiment; circa 1835

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You are also given a target light with unknown spectrum.



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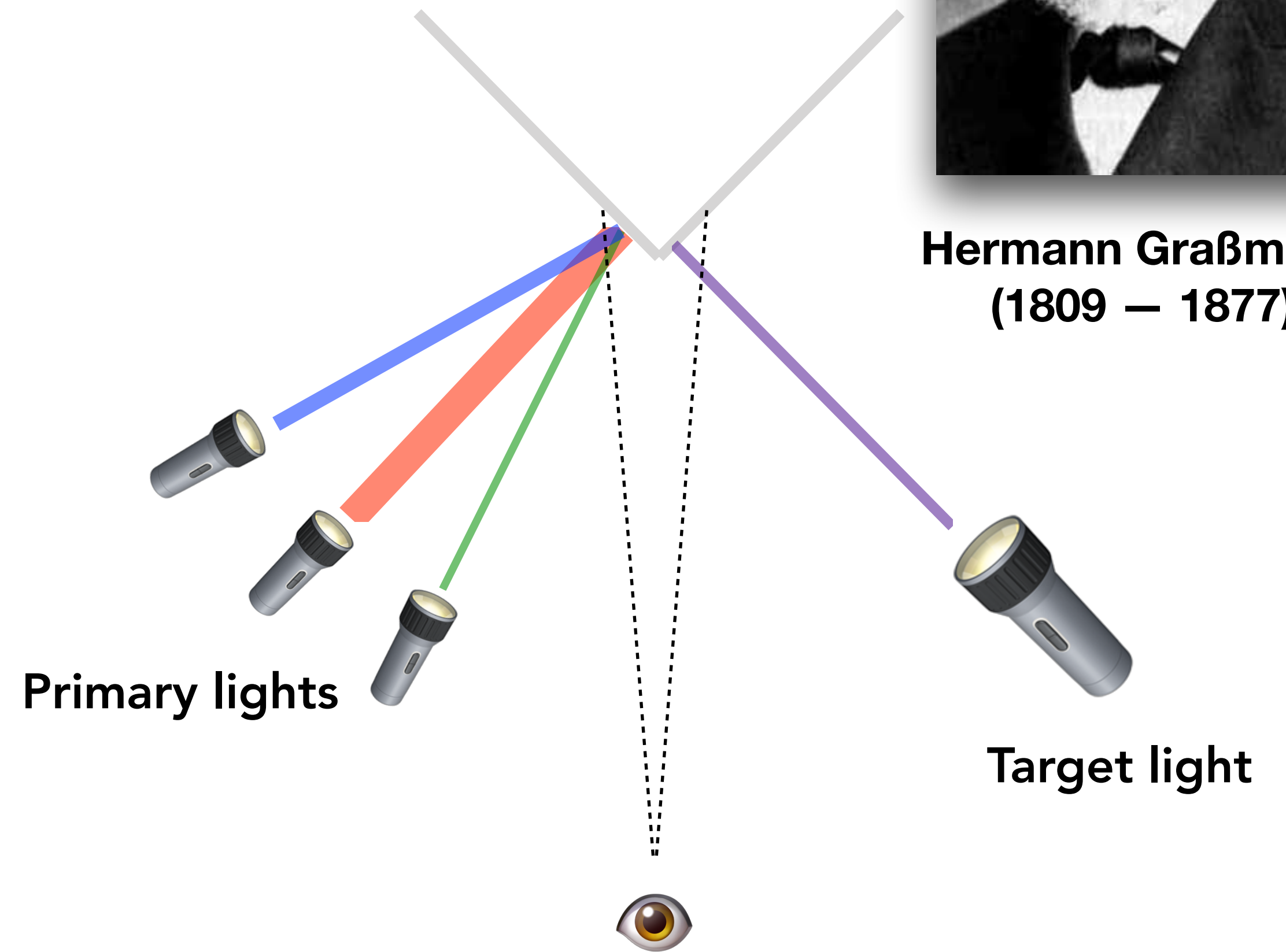
- They are called the *primary lights*.

You are also given a target light with unknown spectrum.

Tune each primary light's power so that the color of the primary light mix matches the color of the target light.



Hermann Graßmann
(1809 – 1877)



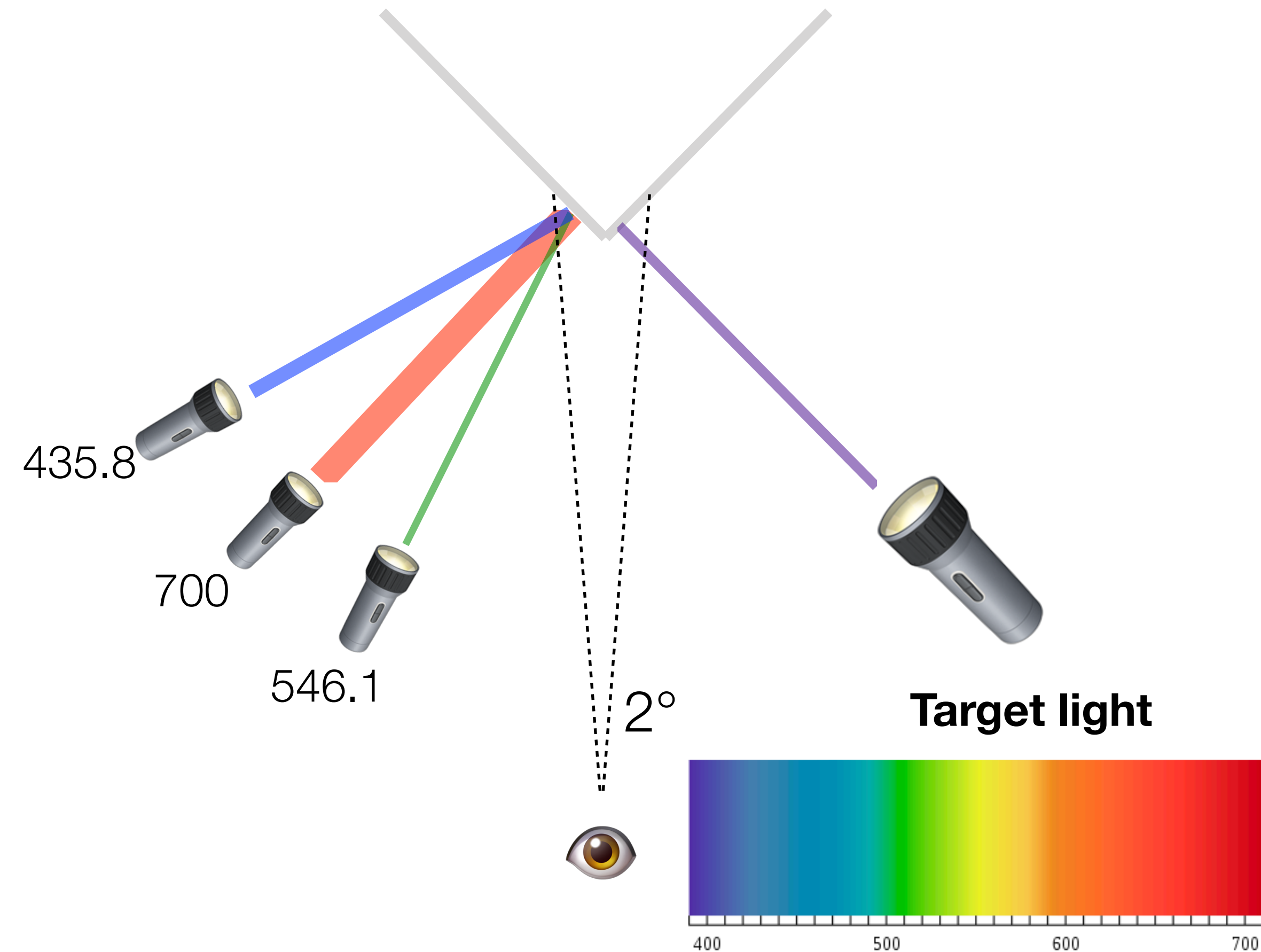
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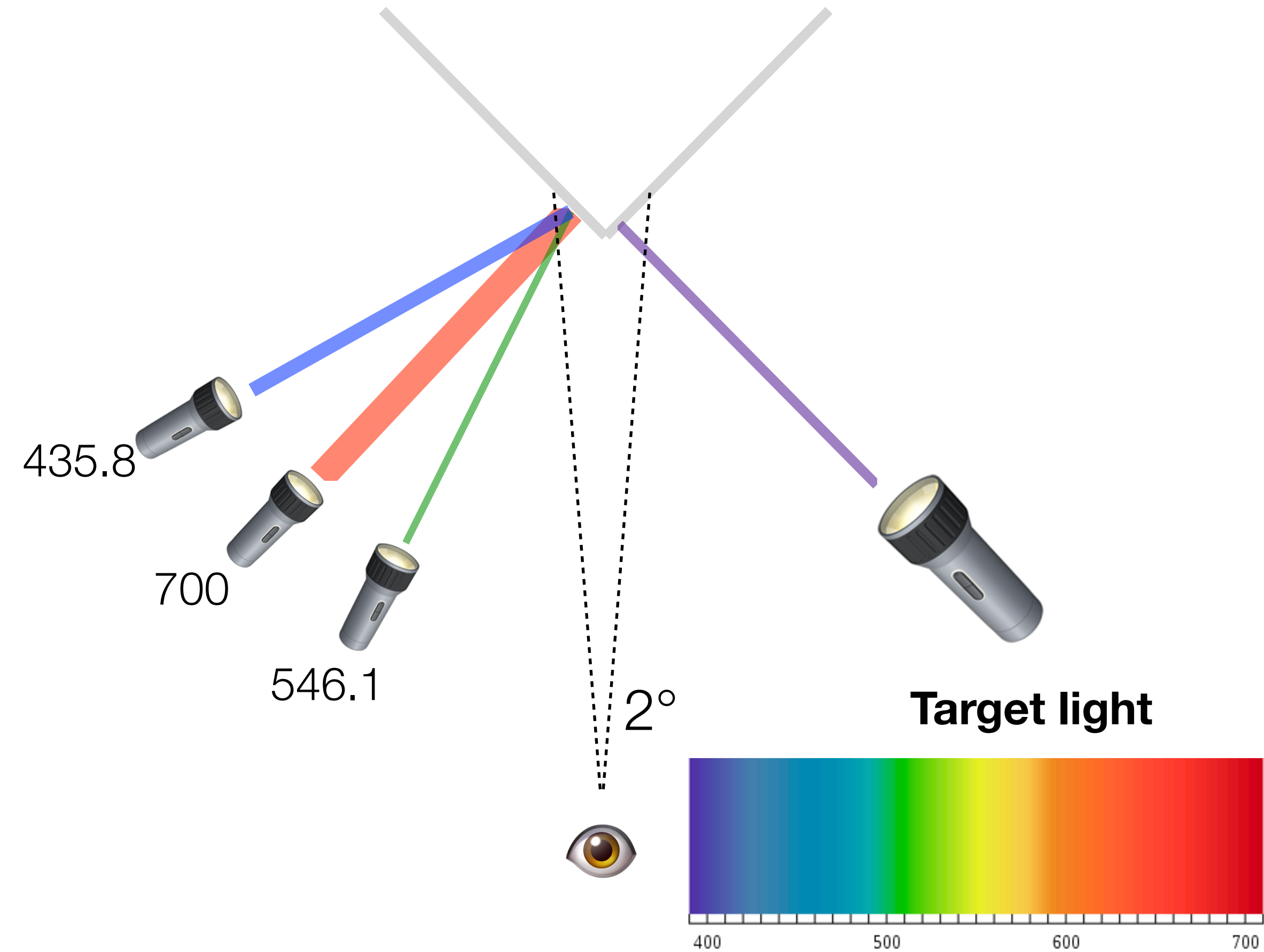
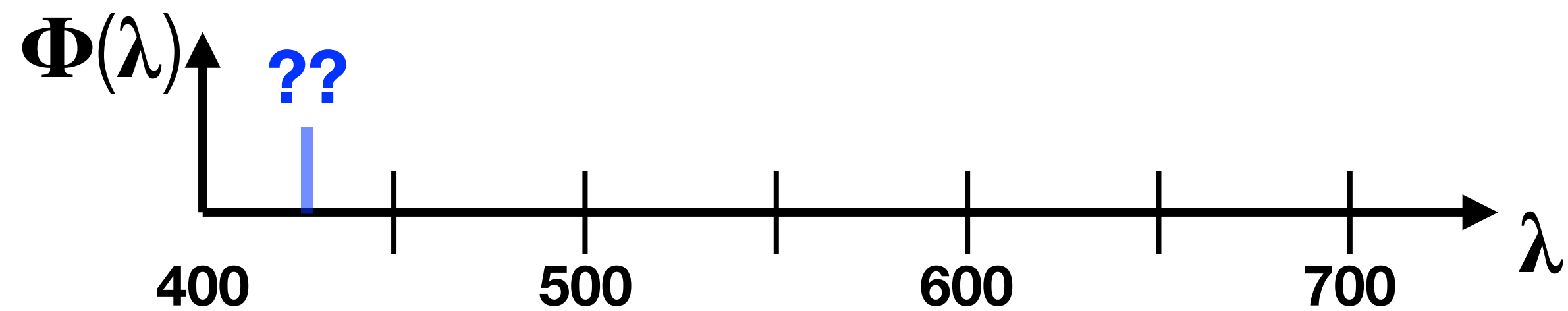
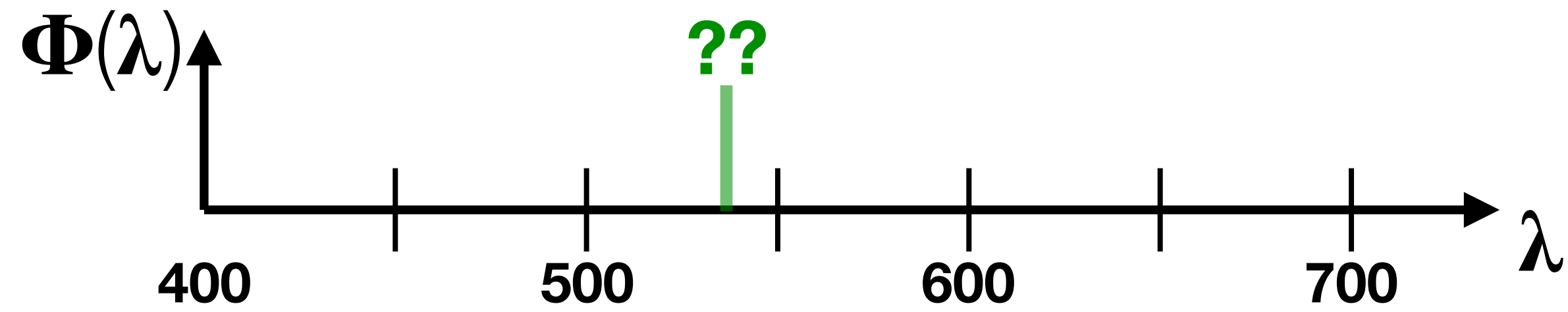
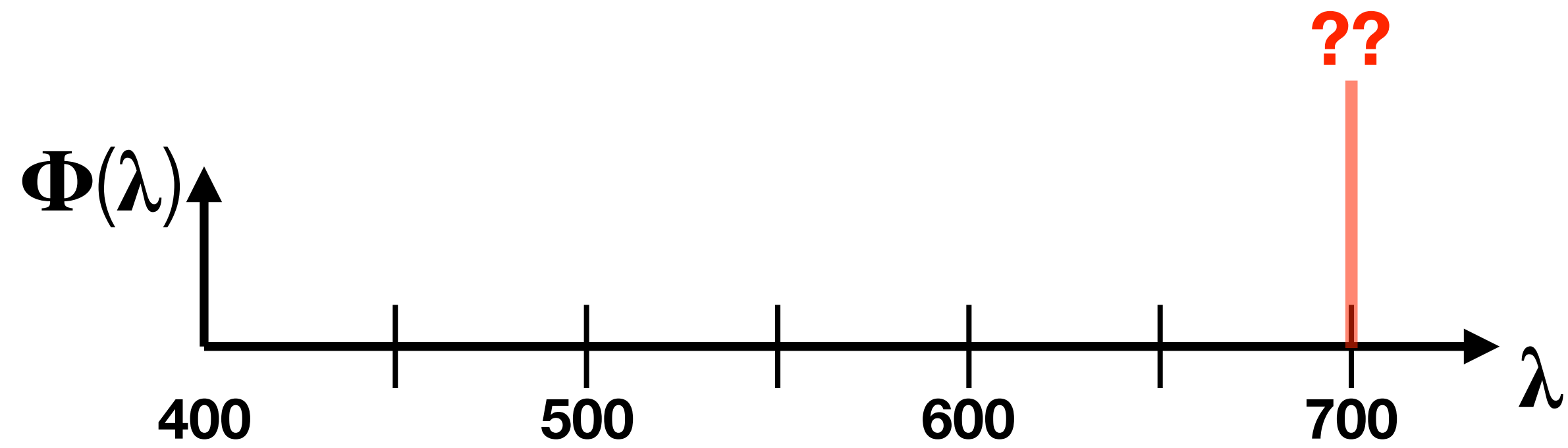
The three primary lights used in CIE 1931 standard: $\lambda=435.8$ nm, 546.1 nm, 700 nm; roughly blue, green, and red.

CIE 1931 limited viewing angle to be 2° so that only the fovea gets light.

It swept all the spectral lights in the visible spectrum as the target light (at a 5-nm interval).

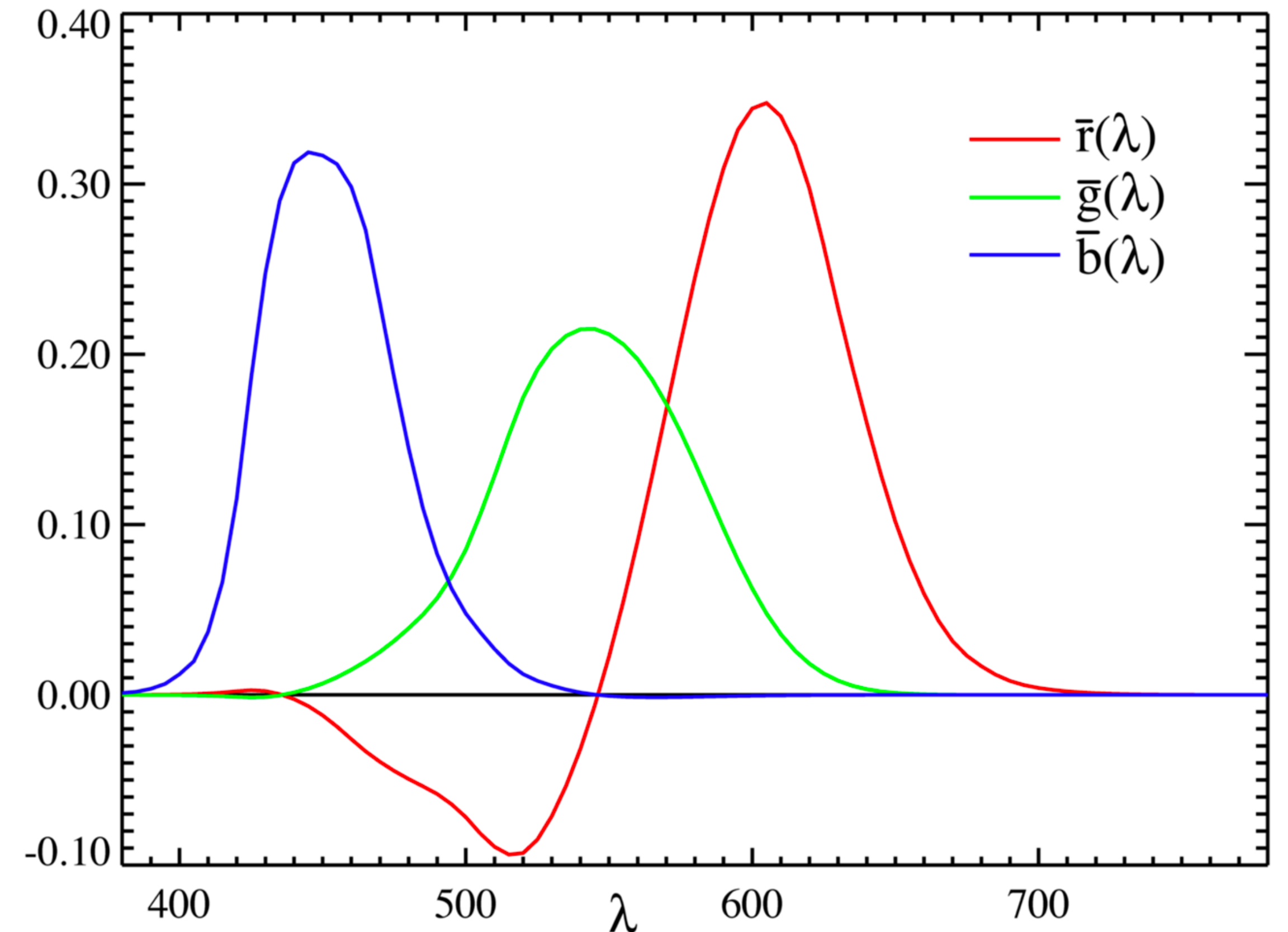


Color Matching Experiments



The Results are the Color Matching Functions

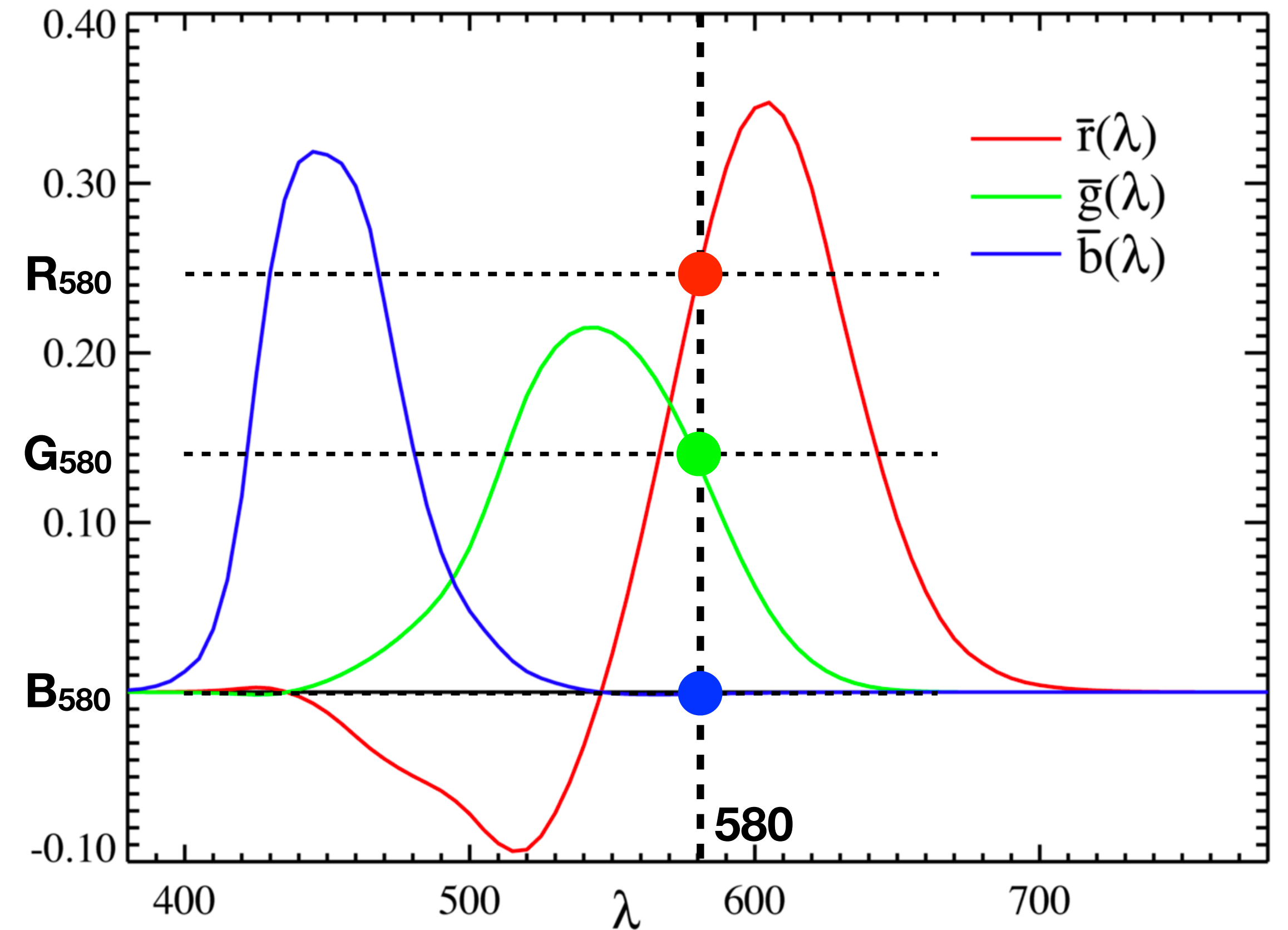
For **one unit power** of each target light, how many units of each primary light are needed to match the color.



CIE 1931 2° RGB Color Matching Functions

The Results are the Color Matching Functions

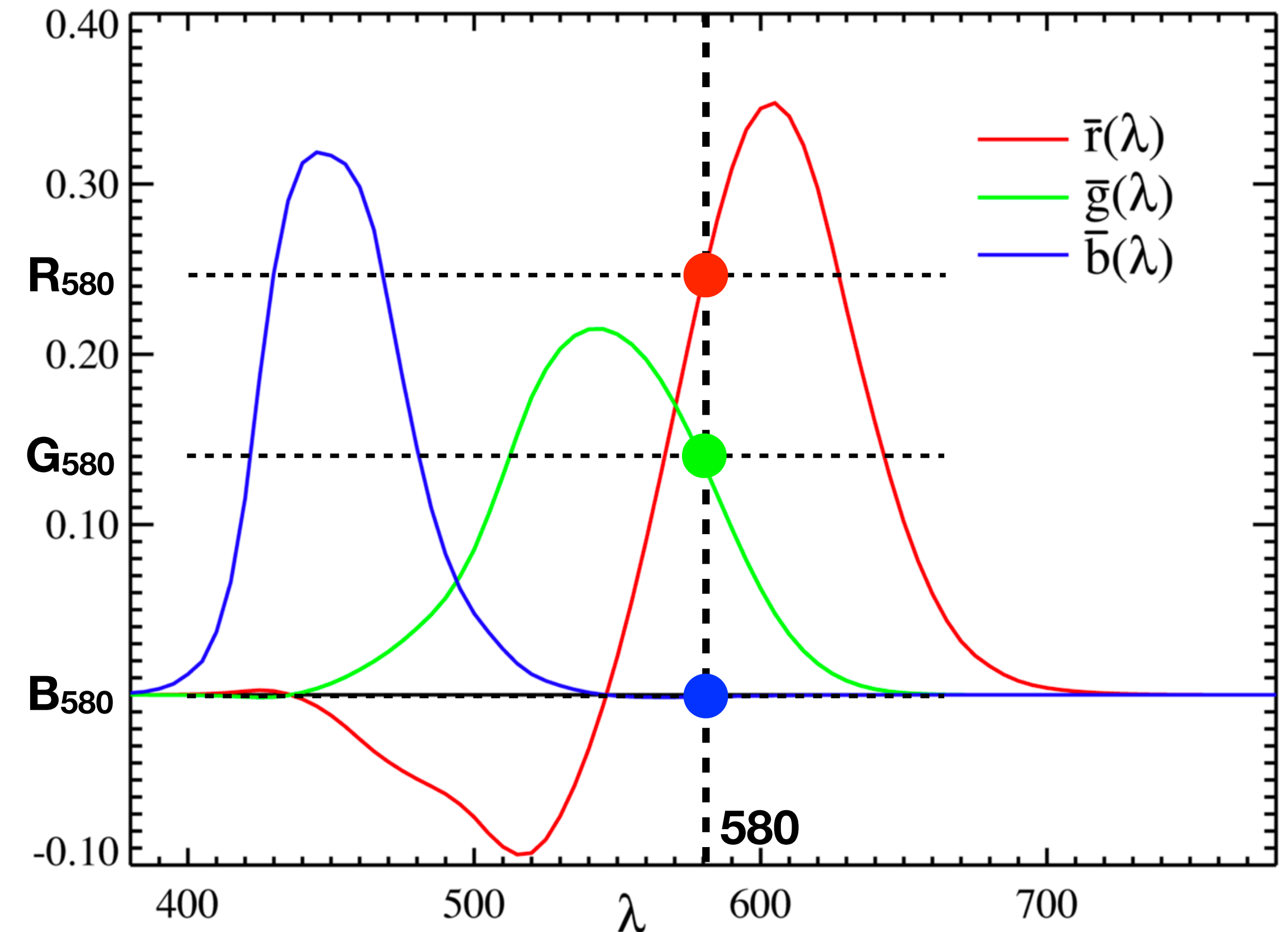
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CIE 1931 2° RGB Color Matching Functions

The Results are the Color Matching Functions

For **one unit power** of each target light, how many units of each primary light are needed to match the color.



If the target light λ has a power of Φ_{580} , the amounts of the primary lights we need are:

$$\Phi_{580} \times R_{580}, \Phi_{580} \times G_{580}, \Phi_{580} \times B_{580}$$

CIE 1931 2° RGB Color Matching Functions

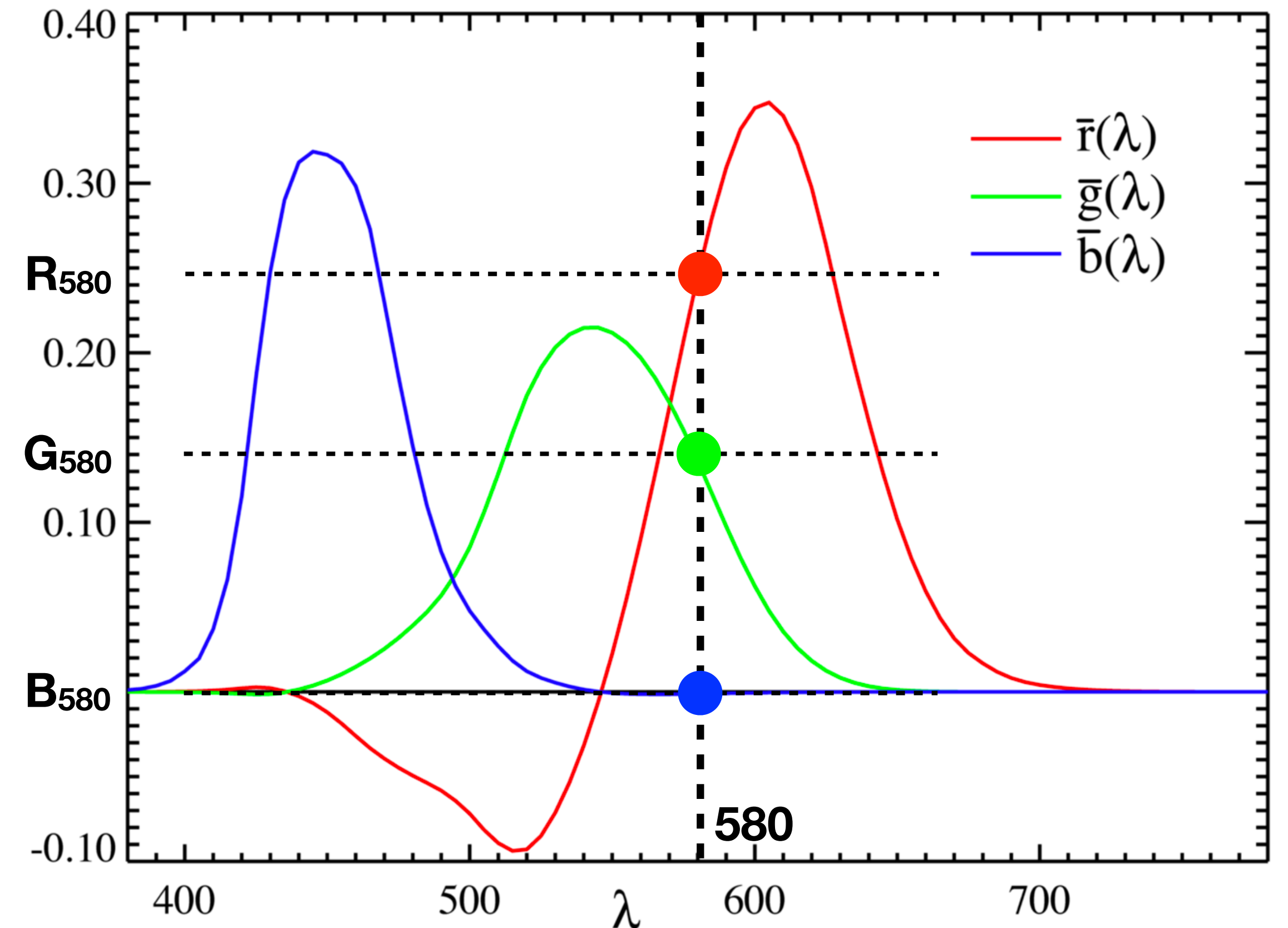
The Results are the Color Matching Functions

For **one unit power** of each target light, how many units of each primary light are needed to match the color.

The RGB values of a target color are called the tristimulus values of the color in the CIE 1931 RGB color space.

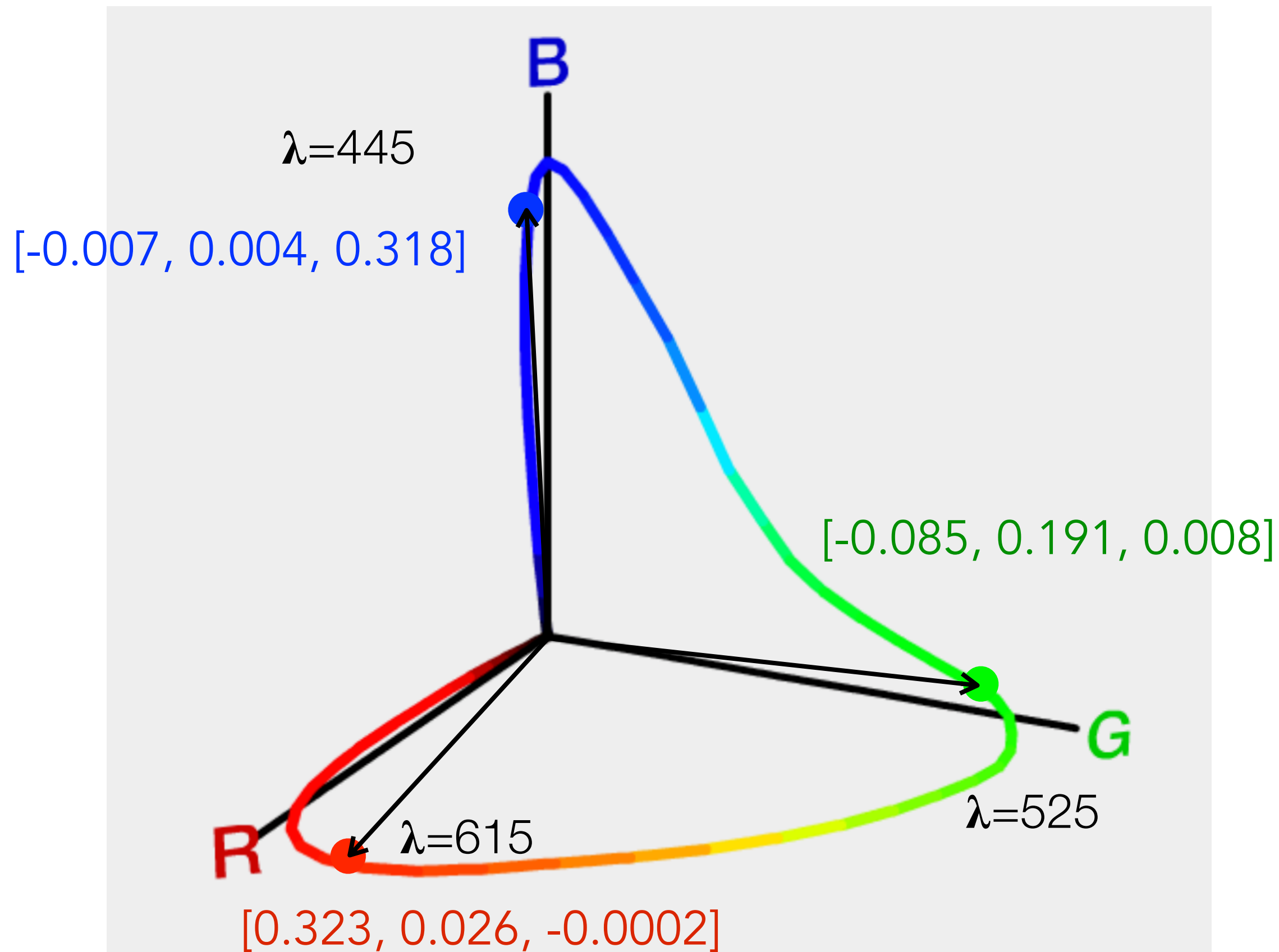
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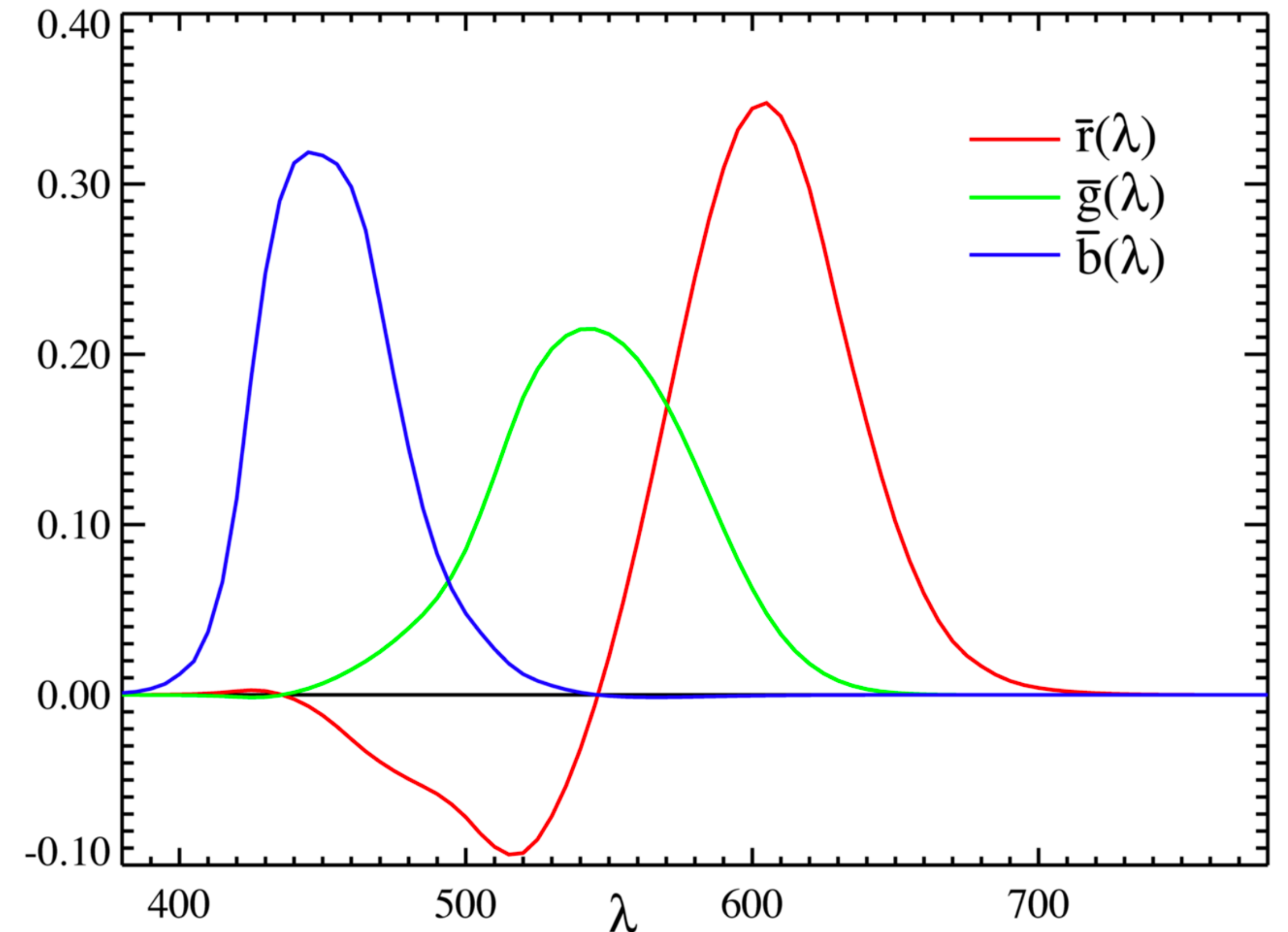


CIE 1931 2° RGB Color Matching Functions

Spectral Locus in the CIE 1931 RGB Space



Spectral locus in CIE 1931 RGB



CIE 1931 2° RGB Color Matching Functions

Expressing Arbitrary Light Using CMFs

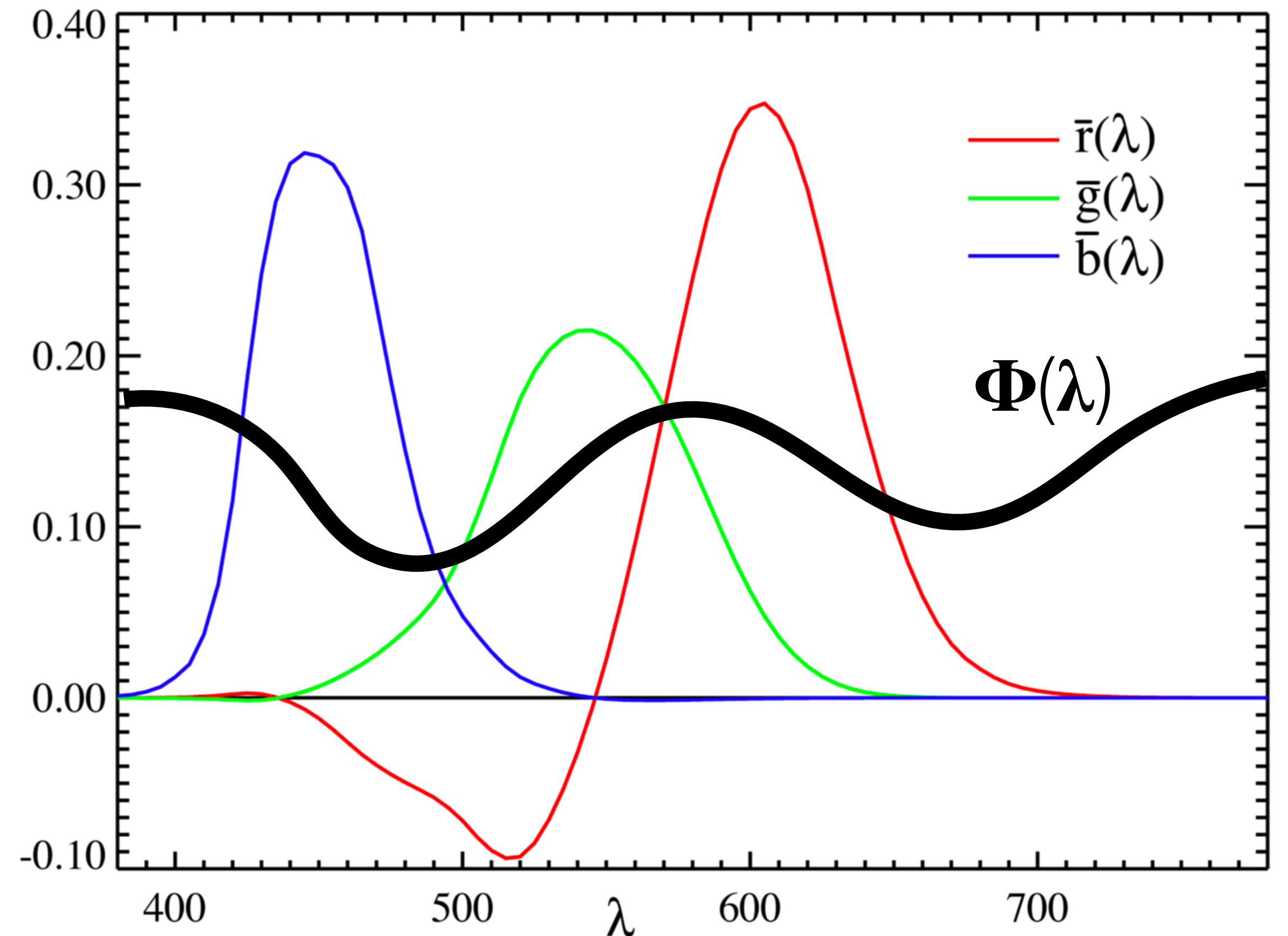
How much total R, G, and B needed to match the color of an arbitrary light Φ ?

Calculate it wavelength-by-wavelength and sum it up.

$$R = \sum_{\lambda} \Phi(\lambda) \bar{r}(\lambda)$$

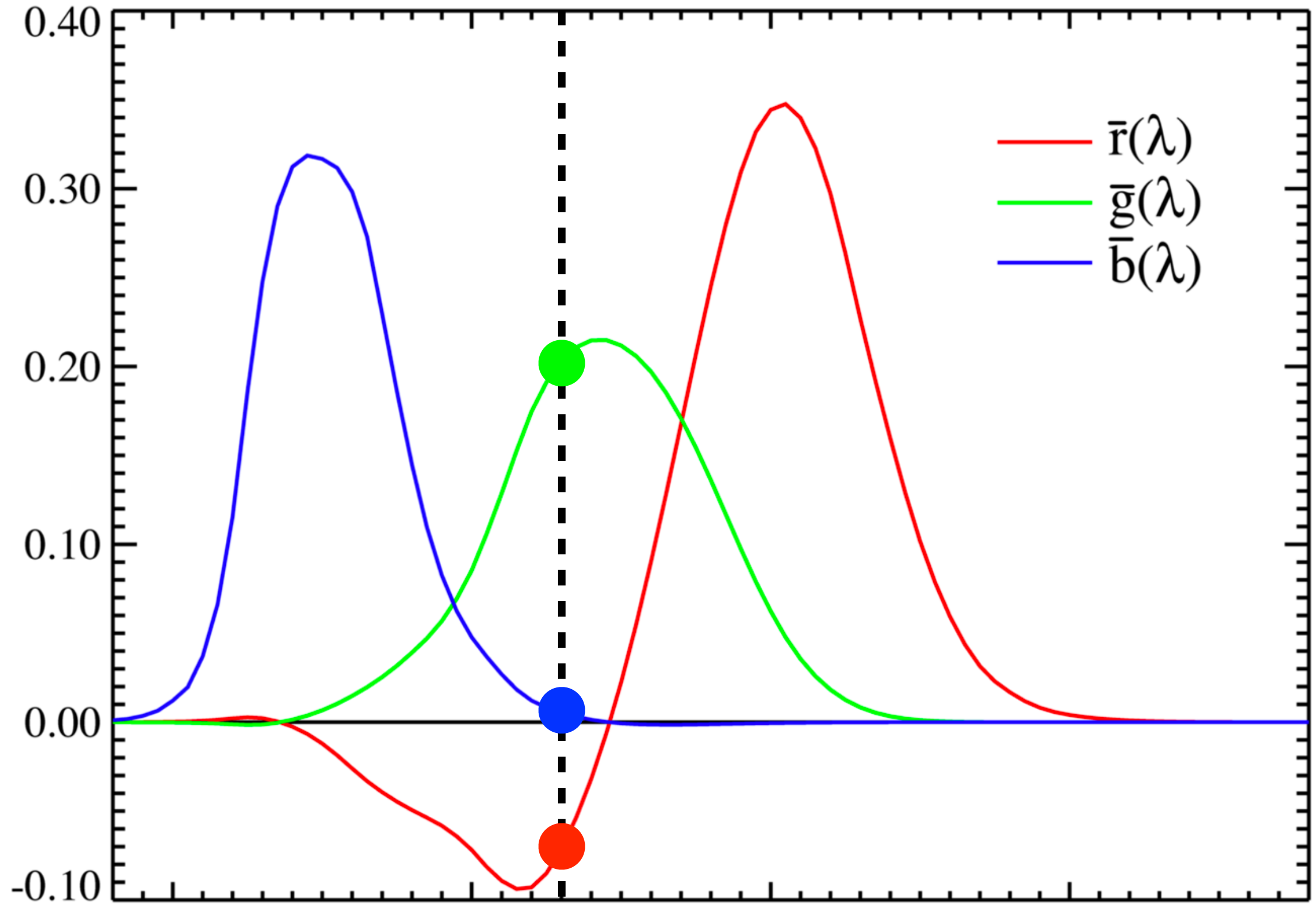
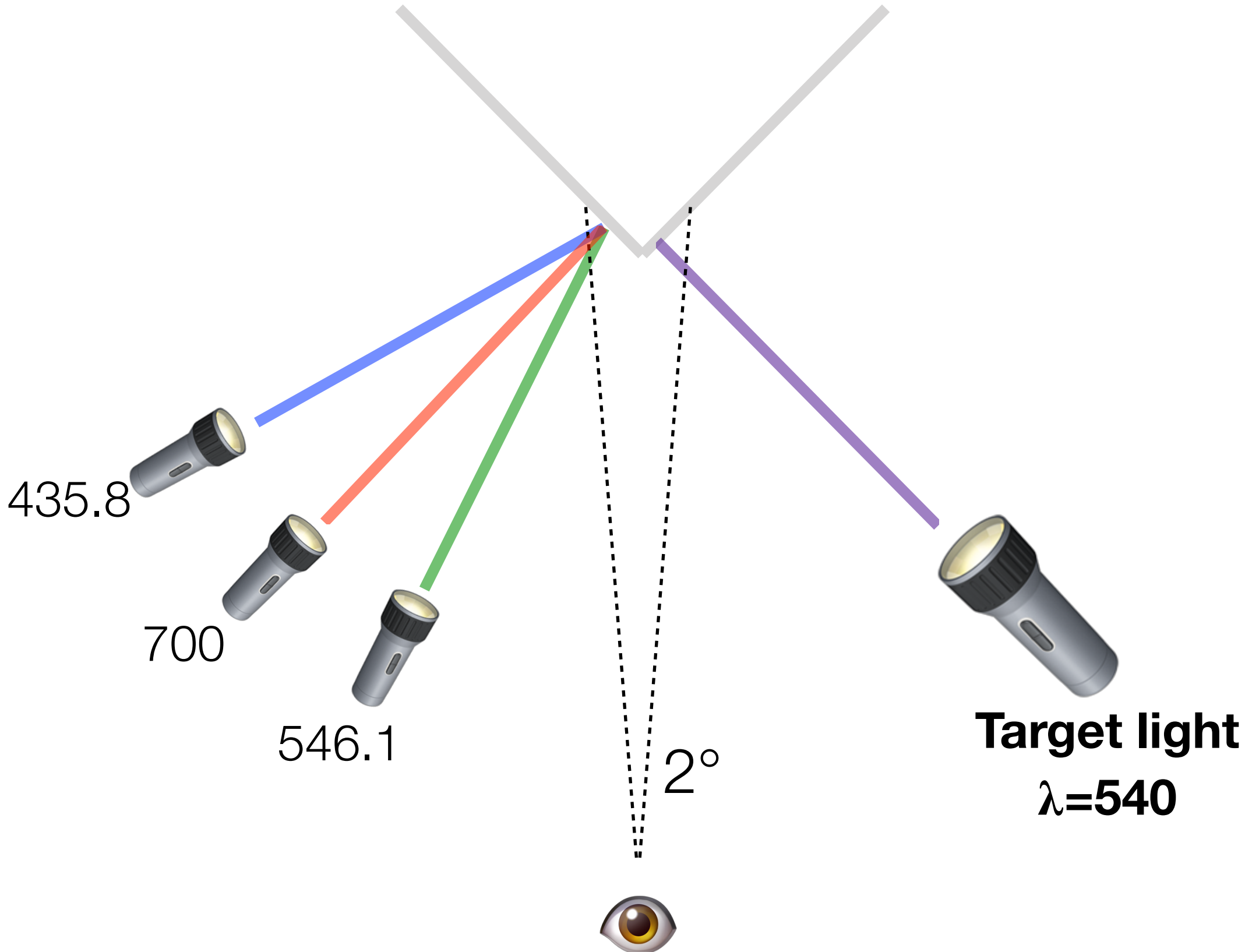
$$G = \sum_{\lambda} \Phi(\lambda) \bar{g}(\lambda)$$

$$B = \sum_{\lambda} \Phi(\lambda) \bar{b}(\lambda)$$



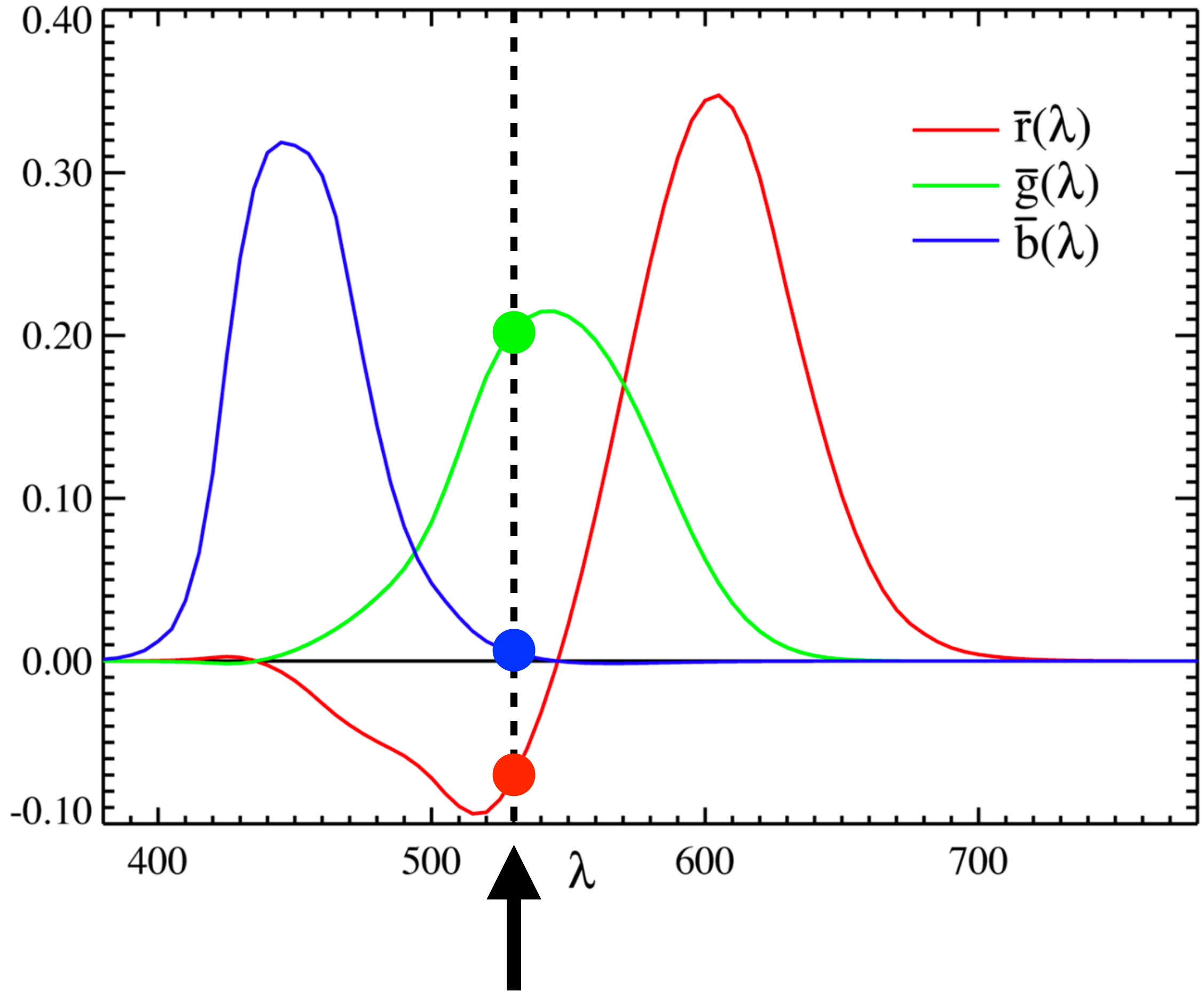
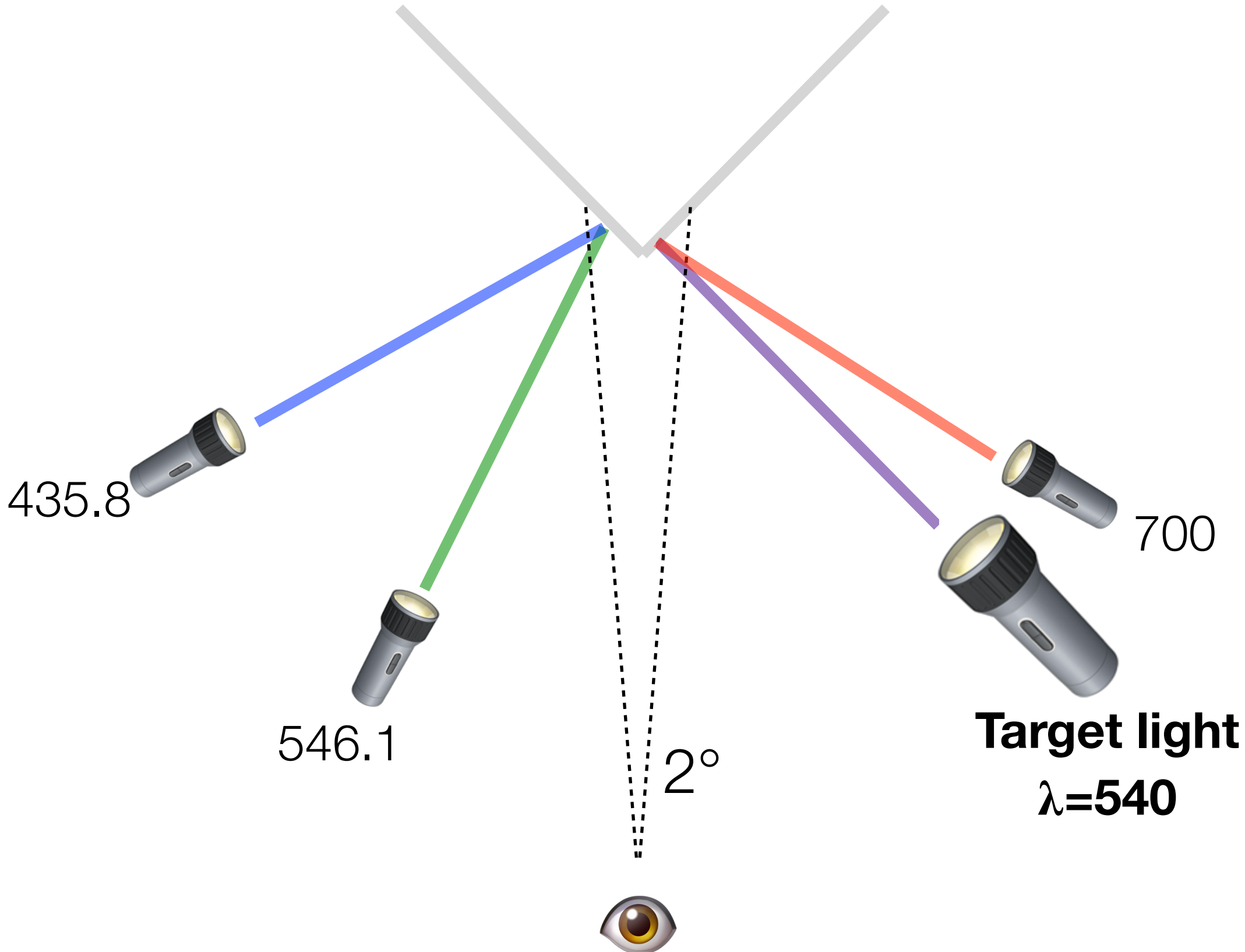
CIE 1931 2° RGB Color Matching Functions

Negative Color?



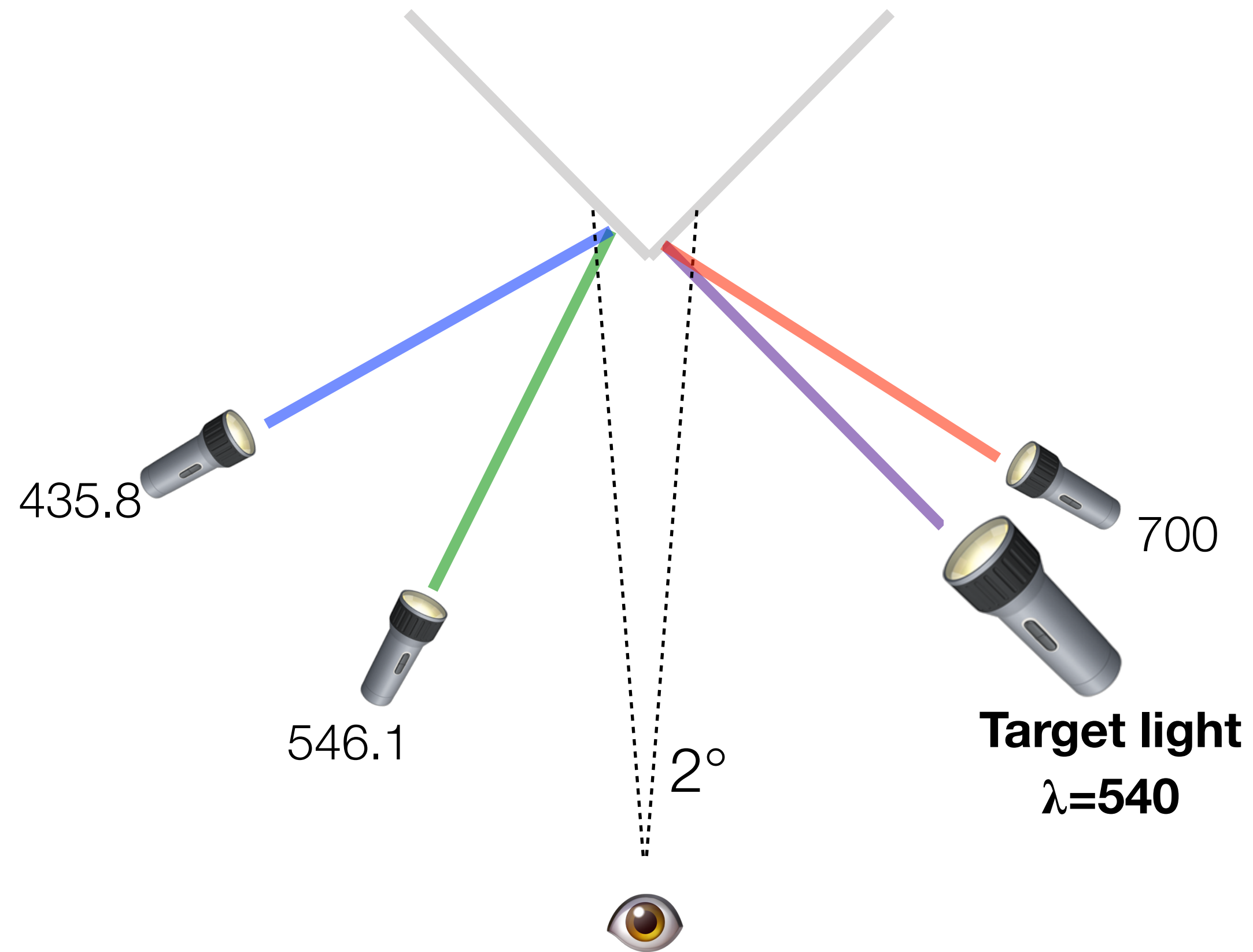
To match color of $\lambda=540$ nm, we need a bit of negative red???

Negative Color?



To match color of $\lambda=540$ nm, we need a bit of negative red???

Negative Values in Color Matching Functions

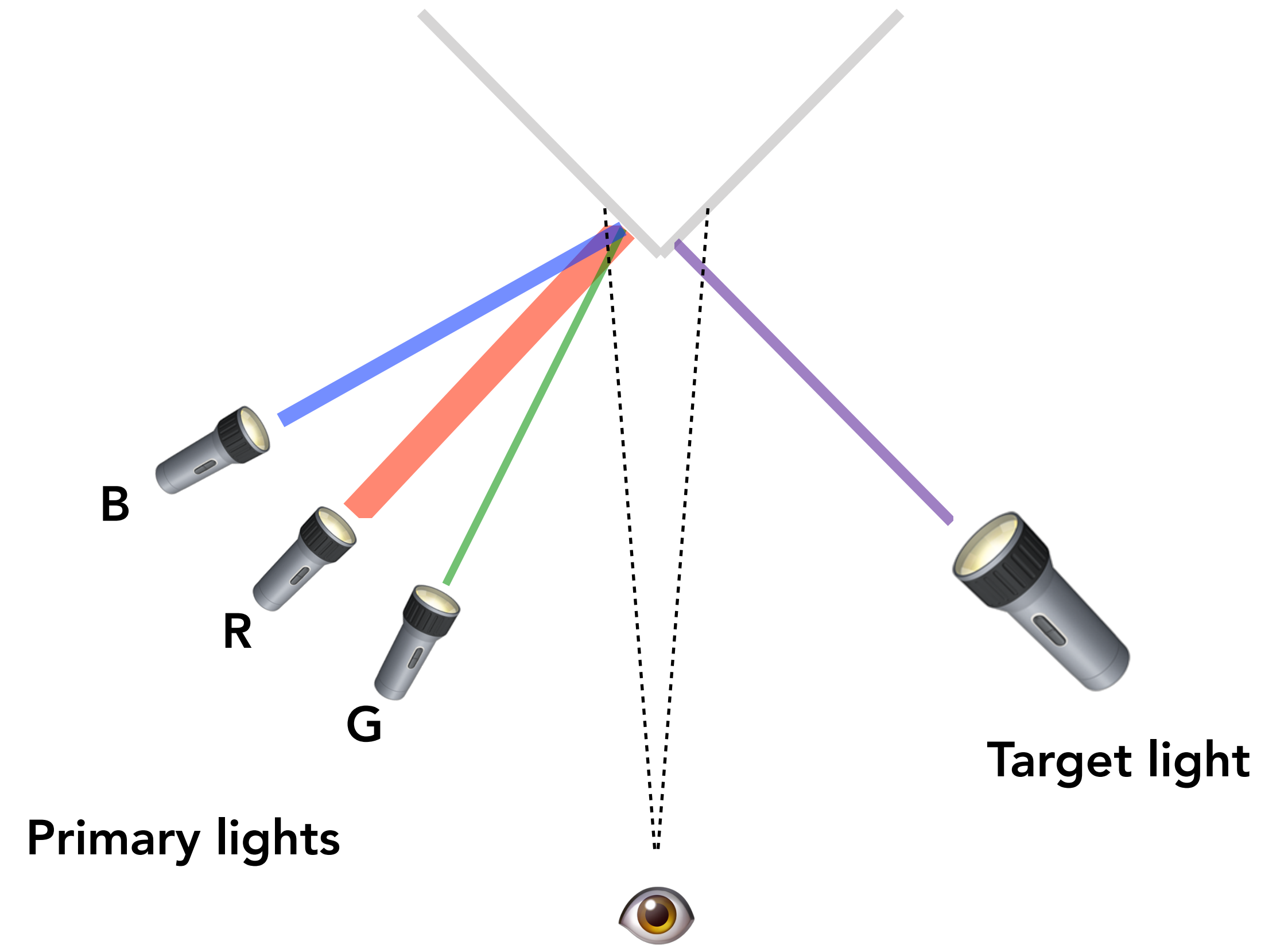


No combination of RGB matches the target light, but adding some R with the target light matches the color of some combination of G and B.

Color of light $\lambda=540$ nm can't be reproduced by R, G, B. Doesn't mean $\lambda=540$ nm doesn't exist; it just can't be generated from the three chosen primary lights. More later (gamut).

How To Explain Color Matching Experiments?

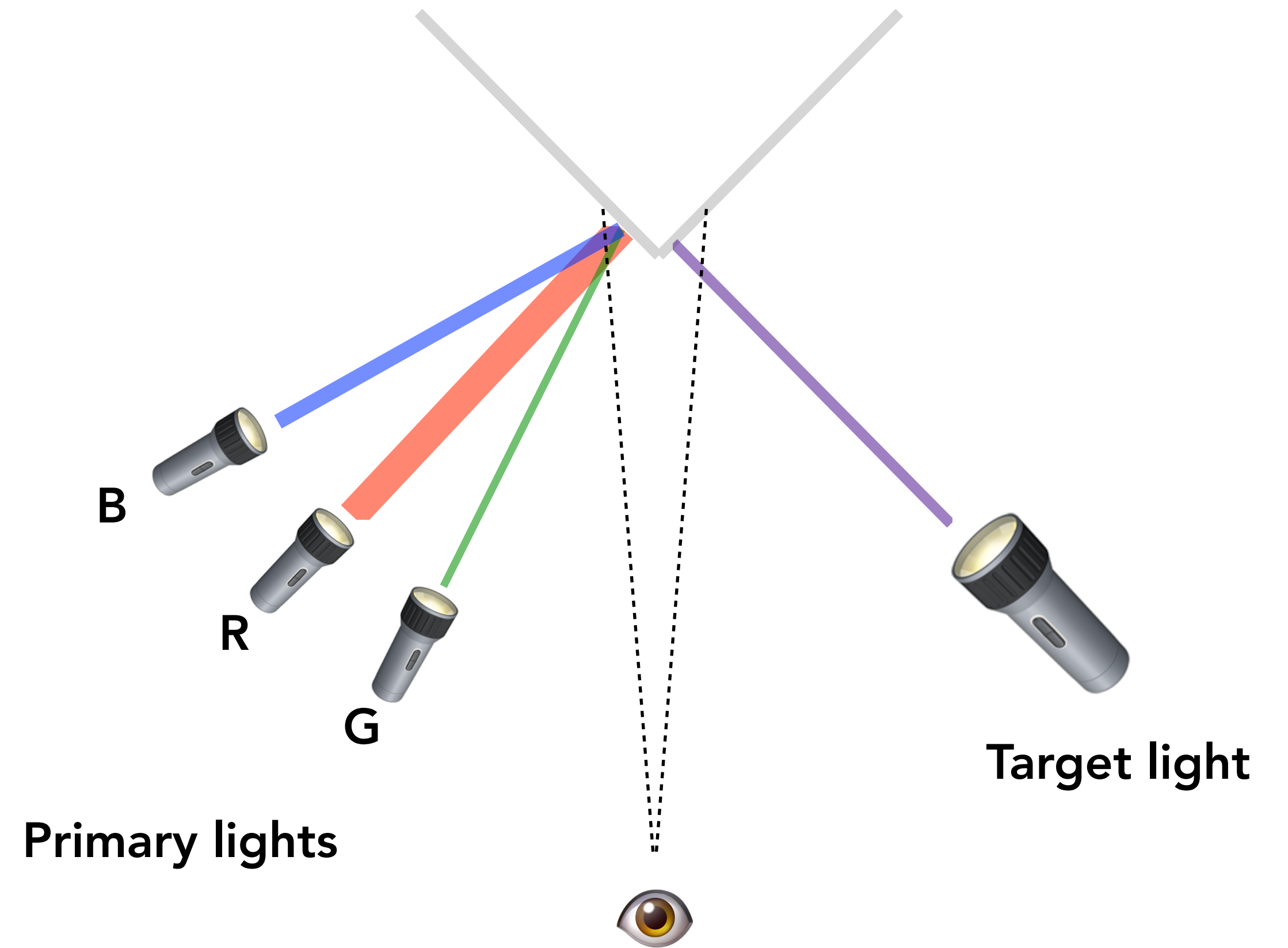
Why do color matching experiments work? Can we explain that?



How To Explain Color Matching Experiments?

Why do color matching experiments work? Can we explain that?

Given a target light $\Phi(\lambda)$ and the primary lights, can we calculate the tristimulus values for $\Phi(\lambda)$ *without* the color matching experiment?

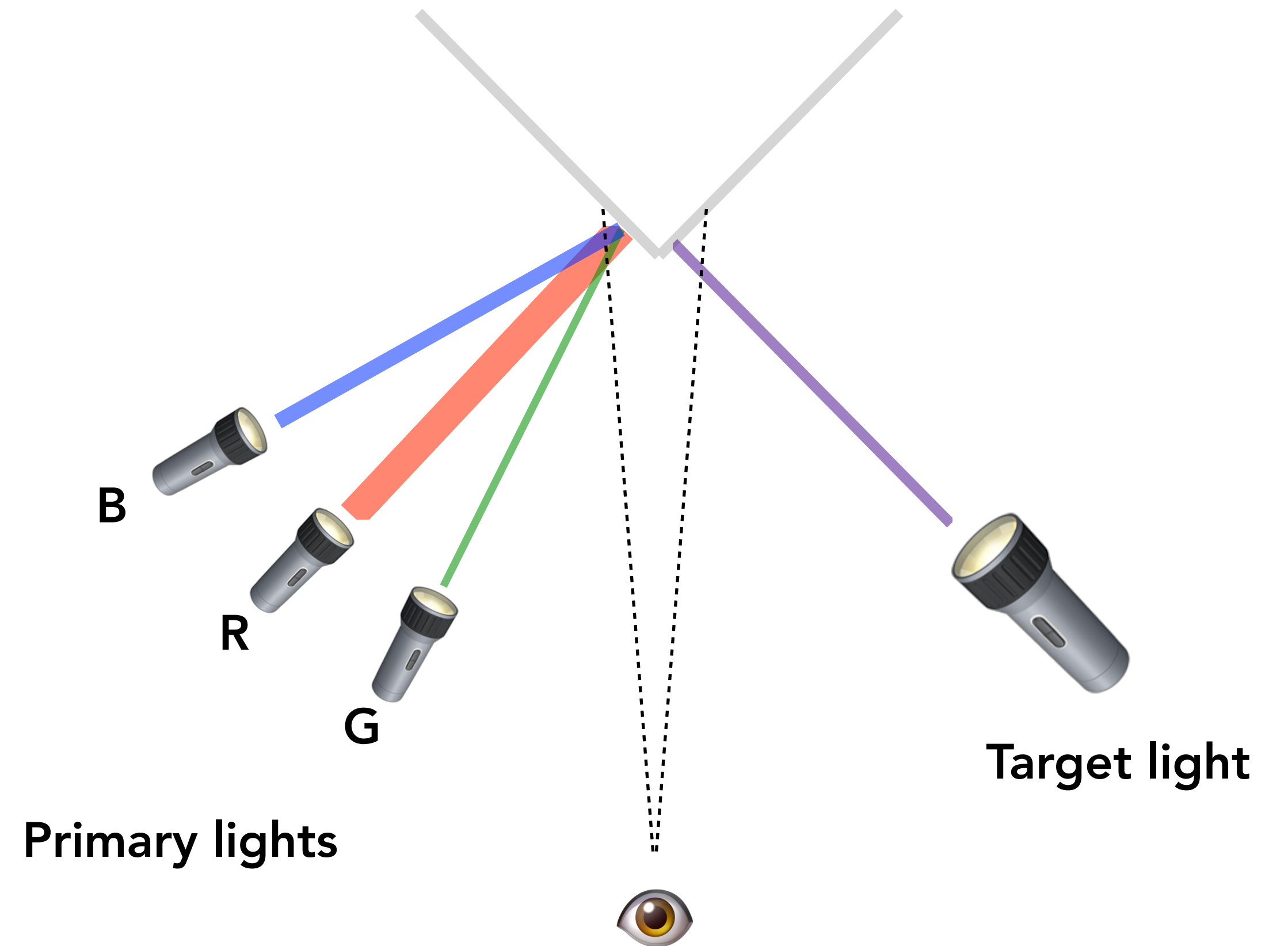


How To Explain Color Matching Experiments?

Why do color matching experiments work? Can we explain that?

Given a target light $\Phi(\lambda)$ and the primary lights, can we calculate the tristimulus values for $\Phi(\lambda)$ *without* the color matching experiment?

Key: the total L, M and S cone responses of the three primary lights must match the total cone responses of the target light.



Intuition

**L cone response of target light = L cone response of primary R
+ L cone response of primary G
+ L cone response of primary B**

**M cone response of target light = M cone response of primary R
+ M cone response of primary G
+ M cone response of primary B**

**S cone response of target light = S cone response of primary R
+ S cone response of primary G
+ S cone response of primary B**

Matching L Cone Responses

$$= \sum \Phi(\lambda)L(\lambda)$$

The L cone response
of the target light

Matching L Cone Responses

$$\sum R(\lambda)L(\lambda) \quad \sum G(\lambda)L(\lambda) \quad \sum B(\lambda)L(\lambda) \quad = \quad \sum \Phi(\lambda)L(\lambda)$$

Amount of L cone responses of one
unit of each primary light



The L cone response
of the target light

Matching L Cone Responses

$$\sum R(\lambda)L(\lambda) \times r + \sum G(\lambda)L(\lambda) \times g + \sum B(\lambda)L(\lambda) \times b = \sum \Phi(\lambda)L(\lambda)$$

Amount of L cone responses of one
unit of each primary light

The L cone response
of the target light

Matching L Cone Responses

Amount of primary lights needed (unknown)

$$\sum R(\lambda)L(\lambda) \times r + \sum G(\lambda)L(\lambda) \times g + \sum B(\lambda)L(\lambda) \times b = \sum \Phi(\lambda)L(\lambda)$$

Amount of L cone responses of one unit of each primary light

The L cone response of the target light

Matching All Cone Responses

$$\sum R(\lambda)L(\lambda) \times r + \sum G(\lambda)L(\lambda) \times g + \sum B(\lambda)L(\lambda) \times b = \sum \Phi(\lambda)L(\lambda)$$

$$\sum R(\lambda)M(\lambda) \times r + \sum G(\lambda)M(\lambda) \times g + \sum B(\lambda)M(\lambda) \times b = \sum \Phi(\lambda)M(\lambda)$$

$$\sum R(\lambda)S(\lambda) \times r + \sum G(\lambda)S(\lambda) \times g + \sum B(\lambda)S(\lambda) \times b = \sum \Phi(\lambda)S(\lambda)$$

Equivalent to Solving a System of Linear Equations

Equivalent to solving the system of linear equations with 3 equations and 3 variables (\mathbf{r} , \mathbf{g} , \mathbf{b}). The system has a unique solution (generally).

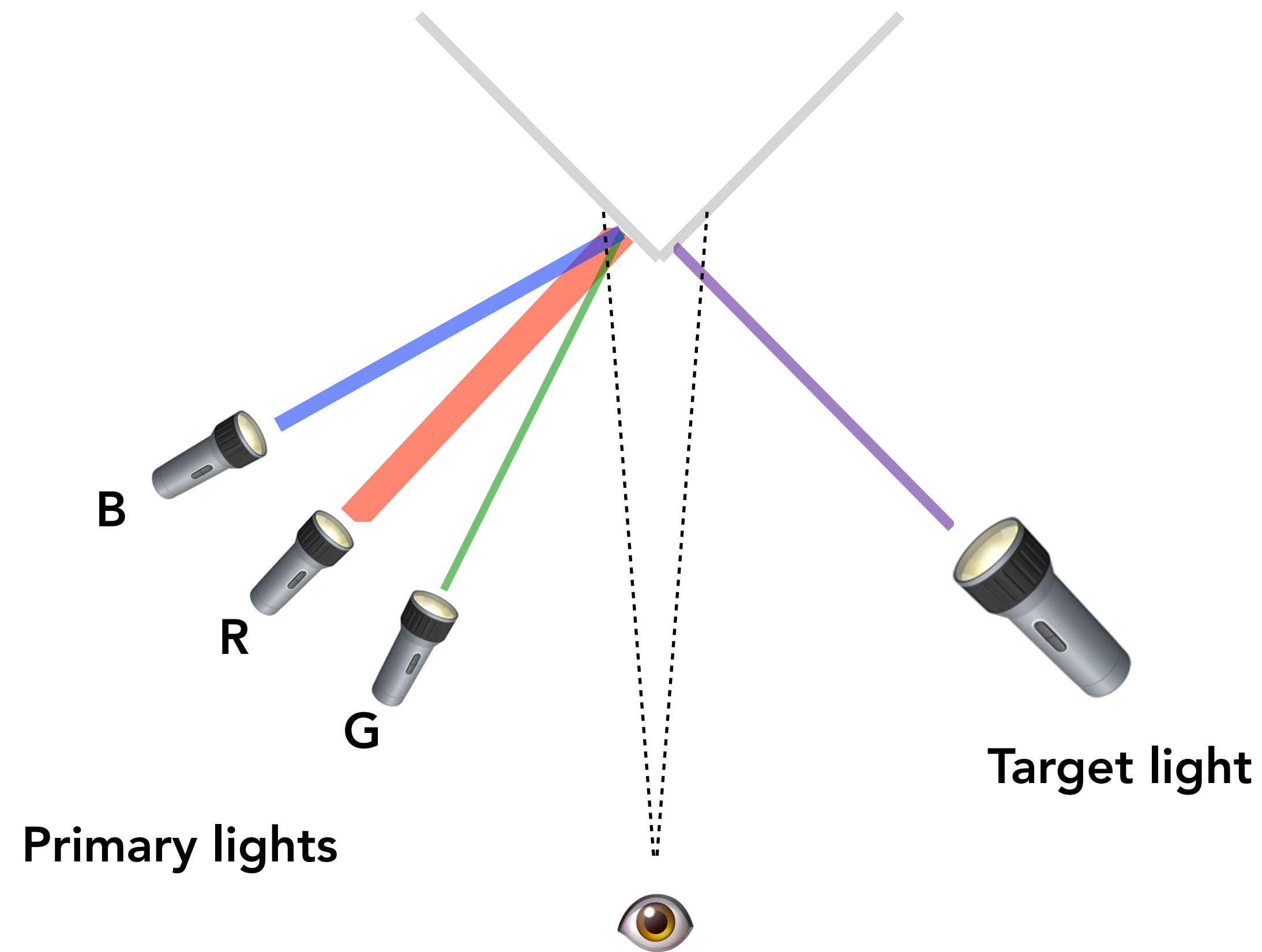
$$\begin{bmatrix} \sum R(\lambda)L(\lambda) & \sum G(\lambda)L(\lambda) & \sum B(\lambda)L(\lambda) \\ \sum R(\lambda)M(\lambda) & \sum G(\lambda)M(\lambda) & \sum B(\lambda)M(\lambda) \\ \sum R(\lambda)S(\lambda) & \sum G(\lambda)S(\lambda) & \sum B(\lambda)S(\lambda) \end{bmatrix} \times \begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} \sum \Phi(\lambda)L(\lambda) \\ \sum \Phi(\lambda)M(\lambda) \\ \sum \Phi(\lambda)S(\lambda) \end{bmatrix}$$

Why Three Primary Colors?

If only two primary colors, the system of equations has **two** variables with **three** equations, which generally doesn't have a solution, i.e., no way of finding a match, generally. Over-determined system.

More than three primaries: more variables than equations, so there are infinitely many ways of combining the primary lights to match the target color. Under-determined system.

Trichromatic Theory of Color



Conjectured by Thomas Young as early as 1801, and expanded by Helmholtz and Maxwell in 1860.

- Color matching experiments empirically verified it.
- Experimental measurement of cone sensitivities much later.

Pervasive applications:

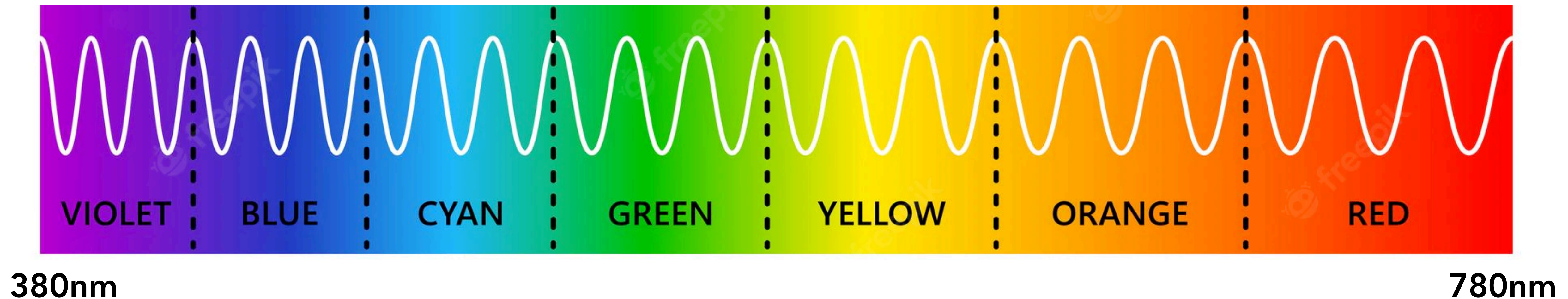
- Color reproduction chain: display, print
- Capture color: camera

A Note on "RGB"

There is no one single definition of "red", "green", and "blue".

Usually "RGB" is just synonyms for "primary lights", which usually are so chosen that they *look* red, green, and blue.

Primary lights **don't** have to be monochromatic spectral lights.



A RGB Color Space

Given the exact spectra of the “RGB” primary lights $R(\lambda)$, $G(\lambda)$, and $B(\lambda)$, we have a color space.

- What this means is that the color of any arbitrary light $\Phi(\lambda)$ can be expressed as a point in that 3D coordinate system.

That is, we can mathematically study colors now — colorimetry.

$$\begin{bmatrix} \sum R(\lambda)L(\lambda) & \sum G(\lambda)L(\lambda) & \sum B(\lambda)L(\lambda) \\ \sum R(\lambda)M(\lambda) & \sum G(\lambda)M(\lambda) & \sum B(\lambda)M(\lambda) \\ \sum R(\lambda)S(\lambda) & \sum G(\lambda)S(\lambda) & \sum B(\lambda)S(\lambda) \end{bmatrix} \times \begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} \sum \Phi(\lambda)L(\lambda) \\ \sum \Phi(\lambda)M(\lambda) \\ \sum \Phi(\lambda)S(\lambda) \end{bmatrix}$$

Geometric Transformation Perspective

So far we've seen two ways to express a color in a 3D space:

$$\begin{bmatrix} \sum \Phi(\lambda)L(\lambda) \\ \sum \Phi(\lambda)M(\lambda) \\ \sum \Phi(\lambda)S(\lambda) \end{bmatrix} = \begin{bmatrix} l \\ m \\ s \end{bmatrix}$$

Geometric Transformation Perspective

So far we've seen two ways to express a color in a 3D space:

- A color can be expressed as its three cone responses (l, m, s)

$$\begin{bmatrix} \sum \Phi(\lambda)L(\lambda) \\ \sum \Phi(\lambda)M(\lambda) \\ \sum \Phi(\lambda)S(\lambda) \end{bmatrix} = \begin{bmatrix} l \\ m \\ s \end{bmatrix}$$

Geometric Transformation Perspective

So far we've seen two ways to express a color in a 3D space:

- A color can be expressed as its three cone responses (l, m, s)
- A color can also be expressed as the amount of primary lights (r, g, b) needed

$$\begin{bmatrix} \sum \Phi(\lambda)L(\lambda) \\ \sum \Phi(\lambda)M(\lambda) \\ \sum \Phi(\lambda)S(\lambda) \end{bmatrix} = \begin{bmatrix} l \\ m \\ s \end{bmatrix}$$

Geometric Transformation Perspective

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$$\begin{bmatrix} \sum R(\lambda)L(\lambda) & \sum G(\lambda)L(\lambda) & \sum B(\lambda)L(\lambda) \\ \sum R(\lambda)M(\lambda) & \sum G(\lambda)M(\lambda) & \sum B(\lambda)M(\lambda) \\ \sum R(\lambda)S(\lambda) & \sum G(\lambda)S(\lambda) & \sum B(\lambda)S(\lambda) \end{bmatrix} \times \begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} \sum \Phi(\lambda)L(\lambda) \\ \sum \Phi(\lambda)M(\lambda) \\ \sum \Phi(\lambda)S(\lambda) \end{bmatrix} = \begin{bmatrix} l \\ m \\ s \end{bmatrix}$$

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- A color can be expressed as its three cone responses (l, m, s)
- A color can also be expressed as the amount of primary lights (r, g, b) needed

How are the two expressions related?

$$\begin{bmatrix} \sum R(\lambda)L(\lambda) & \sum G(\lambda)L(\lambda) & \sum B(\lambda)L(\lambda) \\ \sum R(\lambda)M(\lambda) & \sum G(\lambda)M(\lambda) & \sum B(\lambda)M(\lambda) \\ \sum R(\lambda)S(\lambda) & \sum G(\lambda)S(\lambda) & \sum B(\lambda)S(\lambda) \end{bmatrix} \times \begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} \sum \Phi(\lambda)L(\lambda) \\ \sum \Phi(\lambda)M(\lambda) \\ \sum \Phi(\lambda)S(\lambda) \end{bmatrix} = \begin{bmatrix} l \\ m \\ s \end{bmatrix}$$

Geometric Transformation Perspective

Expressing a color from the $[l, m, s]$ coordinates to the $[r, g, b]$ coordinates is a geometric transformation.

Or equivalently, it's a transformation of the coordinate systems (color spaces)!

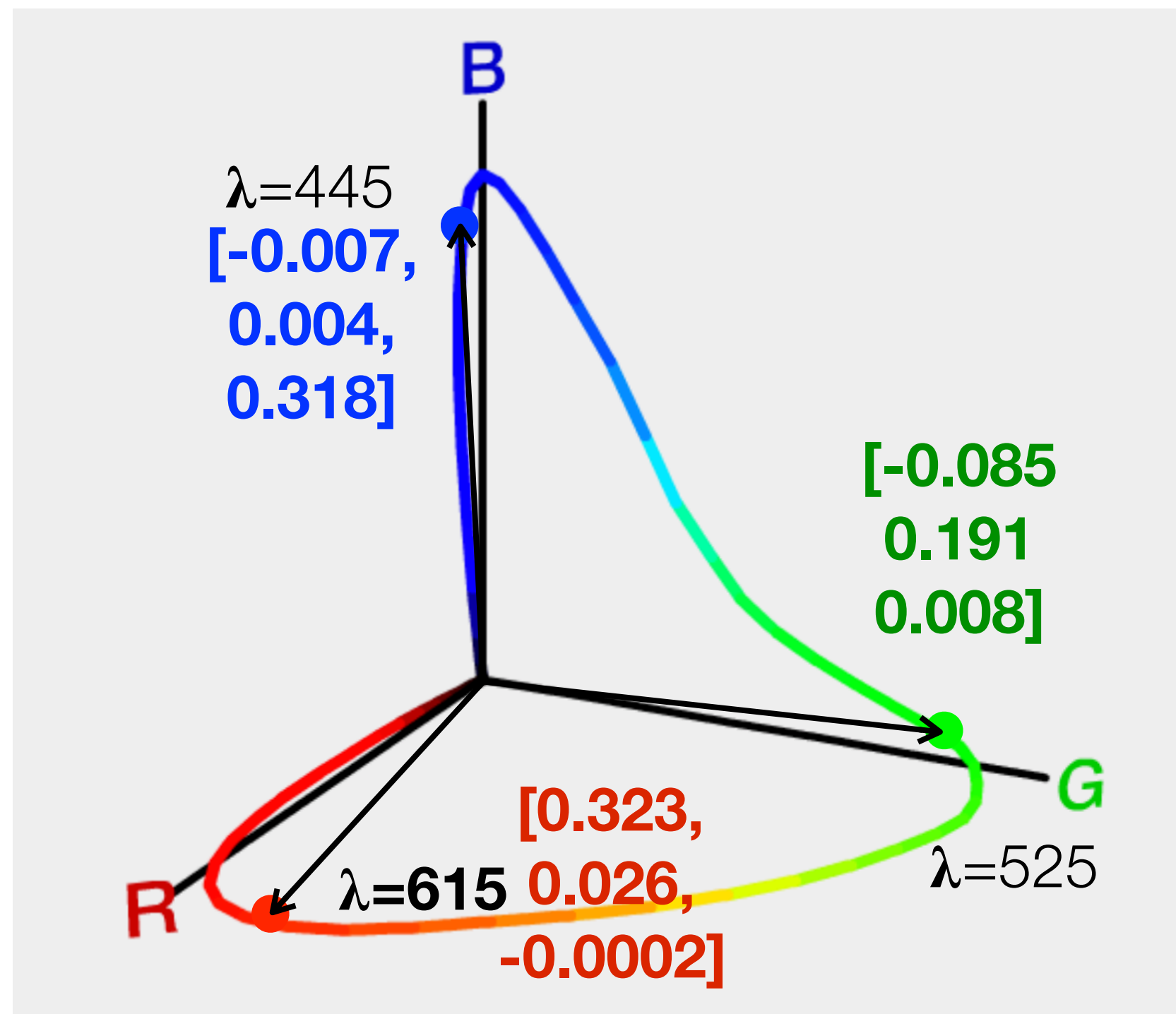
- Because the transformation matrix is constant, metamers in RGB space are metamers in LMS space too.

Transformation matrix from the RGB space to the LMS space

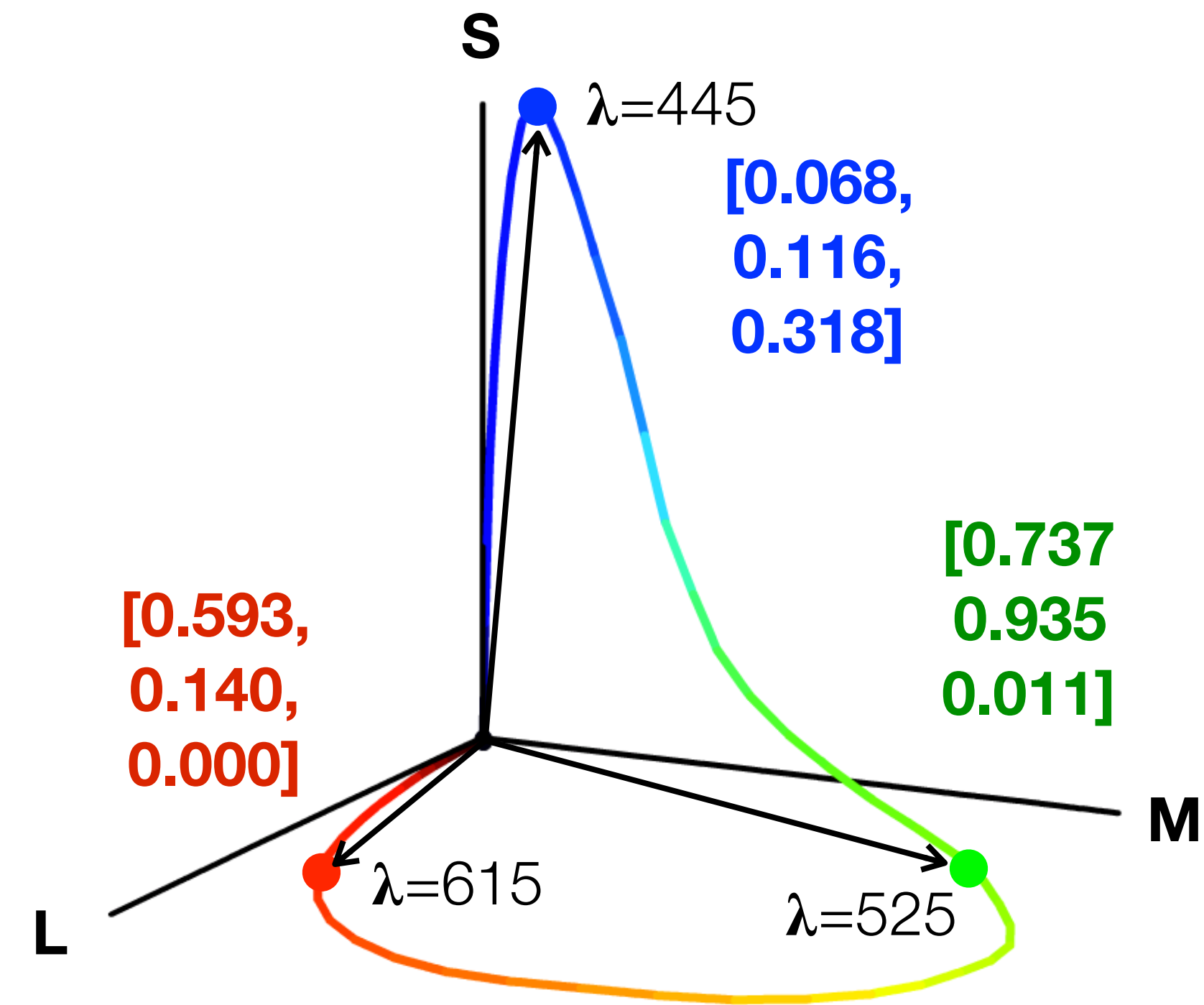
$$\begin{bmatrix} \sum R(\lambda)L(\lambda) & \sum G(\lambda)L(\lambda) & \sum B(\lambda)L(\lambda) \\ \sum R(\lambda)M(\lambda) & \sum G(\lambda)M(\lambda) & \sum B(\lambda)M(\lambda) \\ \sum R(\lambda)S(\lambda) & \sum G(\lambda)S(\lambda) & \sum B(\lambda)S(\lambda) \end{bmatrix} \times \begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} \sum \Phi(\lambda)L(\lambda) \\ \sum \Phi(\lambda)M(\lambda) \\ \sum \Phi(\lambda)S(\lambda) \end{bmatrix} = \begin{bmatrix} l \\ m \\ s \end{bmatrix}$$

CIE 1931 RGB \leftrightarrow LMS Spaces

Spectral lights in the CIE 1931 RGB space



Spectral lights in the LMS space



Color Deficiency & Color Perception in Animals

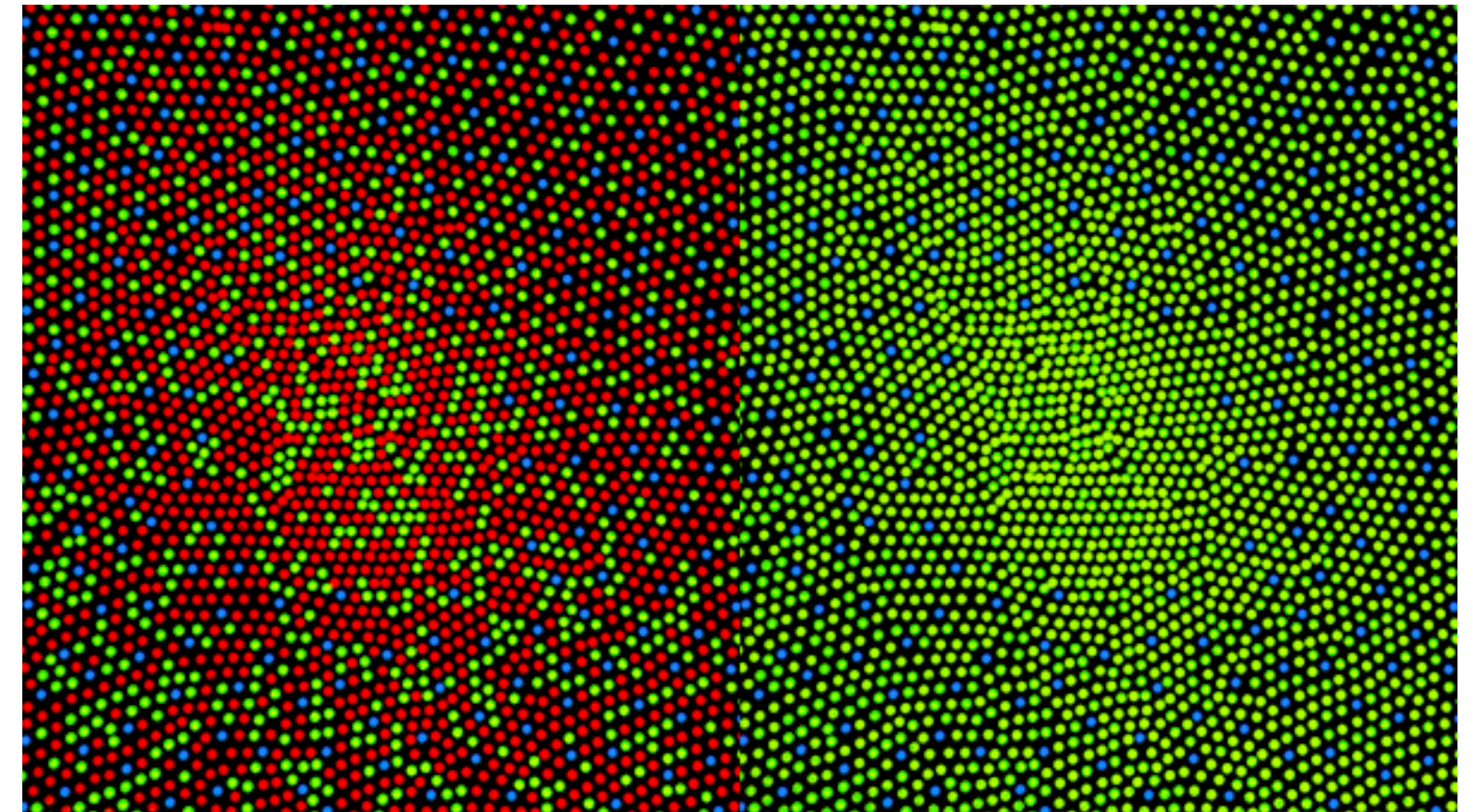
Color Deficiency

Normal vision is trichromatic; that is, there are three kinds of cone cells.

Color blindness: one or two kinds of cone cells are missing.

Dichromat: missing one cone type

- Protanopia: missing L cones
- Deuteranopia: missing M cones
- Tritanopia: missing S cones.



Fovea cone cell distribution in a **trichromat** vision.

Fovea cone cell distribution of a **Protanopia** vision, where **L cones** are missing.

And with three, we enjoy trichromacy:



With two, we
are dichromatic:

Protanopia (missing red sensor)



Tritanopia (missing blue sensor)



Deuteranopia (missing green sensor)



Thus, if we had only one photoreceptor, we would be colour-blind...



Examples: night vision (rod achromatopsia)

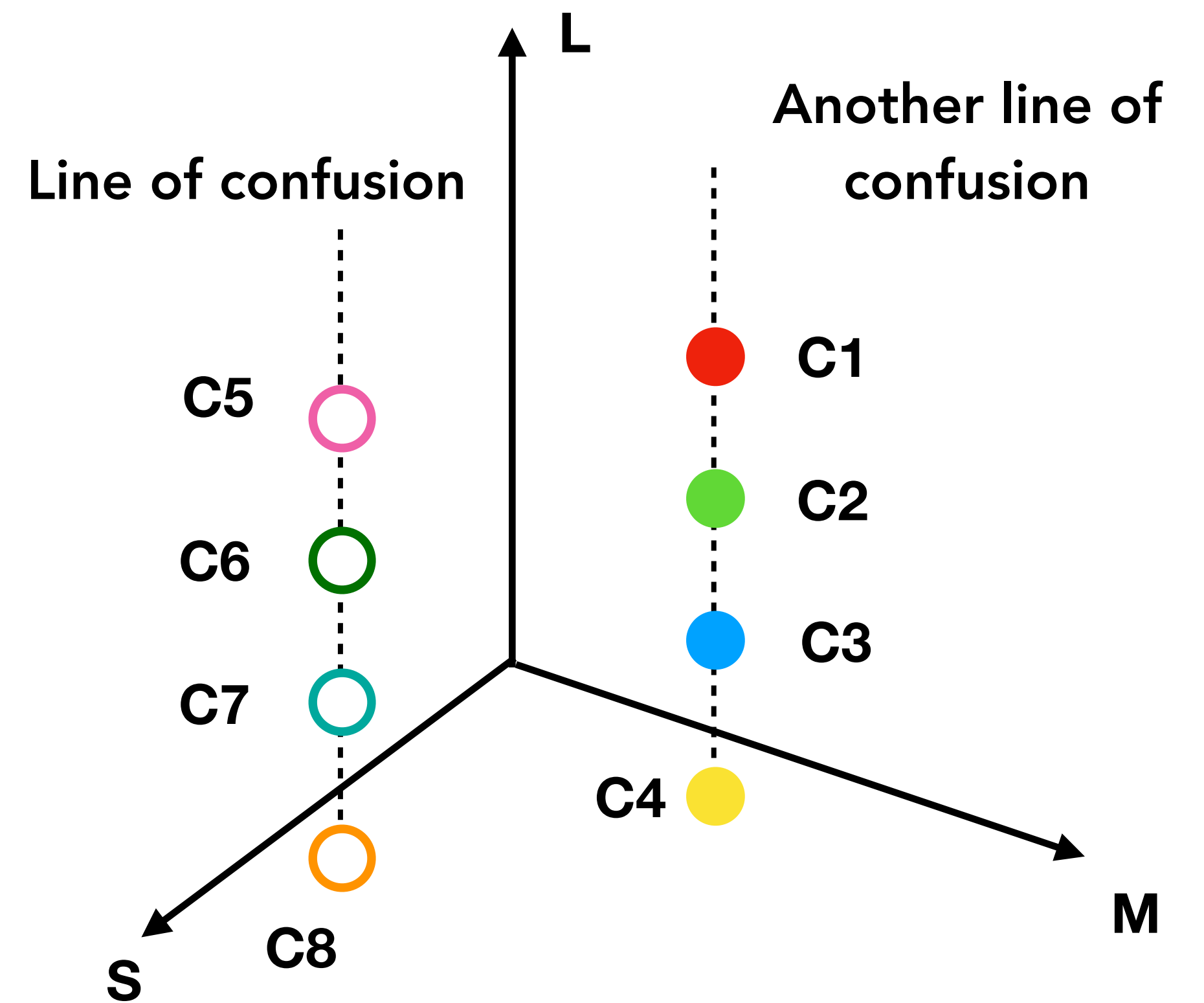
Line of Confusion

For a protanope (which lacks L cones), all the colors that have the same M and S cone responses but **differ in L** would be perceived as the same color.

All the colors on a line parallel to the L-axis are seen as the same color by a protanope.

- C1~C4 are seen as the same color
- C5~C8 are seen as the same, but another color

But which color on a line of confusion does a protanope actually see?



* This is an orthographic projection

Isochromes From Unilateral Dichromats

Isochromes: colors that even a dichromat can correctly perceive.

Typically it's impossible to know which colors are isochromes, since one can never describe/tell one's color sensation.

Fortunately, there are extremely rare people who are unilateral **dichromats:** their one eye has trichromatic vision and the other eye is dichromatic. Color matching between the two eyes by a unilateral dichromat would allow us to identify isochromes.

It's believed/hypothesized that all isochromes live on the same plane.

Isochromes From Unilateral Dichromats

RED-GREEN BLINDNESS CONFINED TO ONE EYE

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(Received 10 September 1975; accepted 15 September 1975)

Abstract—We report experimental studies of an individual with different types of color vision in his two eyes. The left eye is typically deuteranomalous, with good wavelength discrimination. The right eye is typically deuteranopic, both in its wavelength discrimination and in its color matching functions. Luminosity curves of left and right eyes are indistinguishable, both being within the normal range. Binocular matches show that the colors seen with the dichromatic eye range from “orange” (610 nm) at long wavelengths to “greenish blue” (473 or 474 nm) at short wavelengths. Possible physiological and genetic mechanisms are discussed.

INTRODUCTION

The nature of the link between deuteranopia and normal color vision has been a central problem in color theory for more than a hundred years, and many experiments have attempted to clarify it. Thanks to the investigations of Pitt (1935) and others, reviewed by

Holmgren (1881) and discussed more recently by Walls and Mathews (1952). Both von Hippel and Holmgren found evidence of substantially reduced sensitivity to long wavelengths in the red-green blind eye, so this observer was probably a unilateral protanope. A second case of unilateral red-green blindness, possibly congenital, was reported by von



Peter Lennie, UR

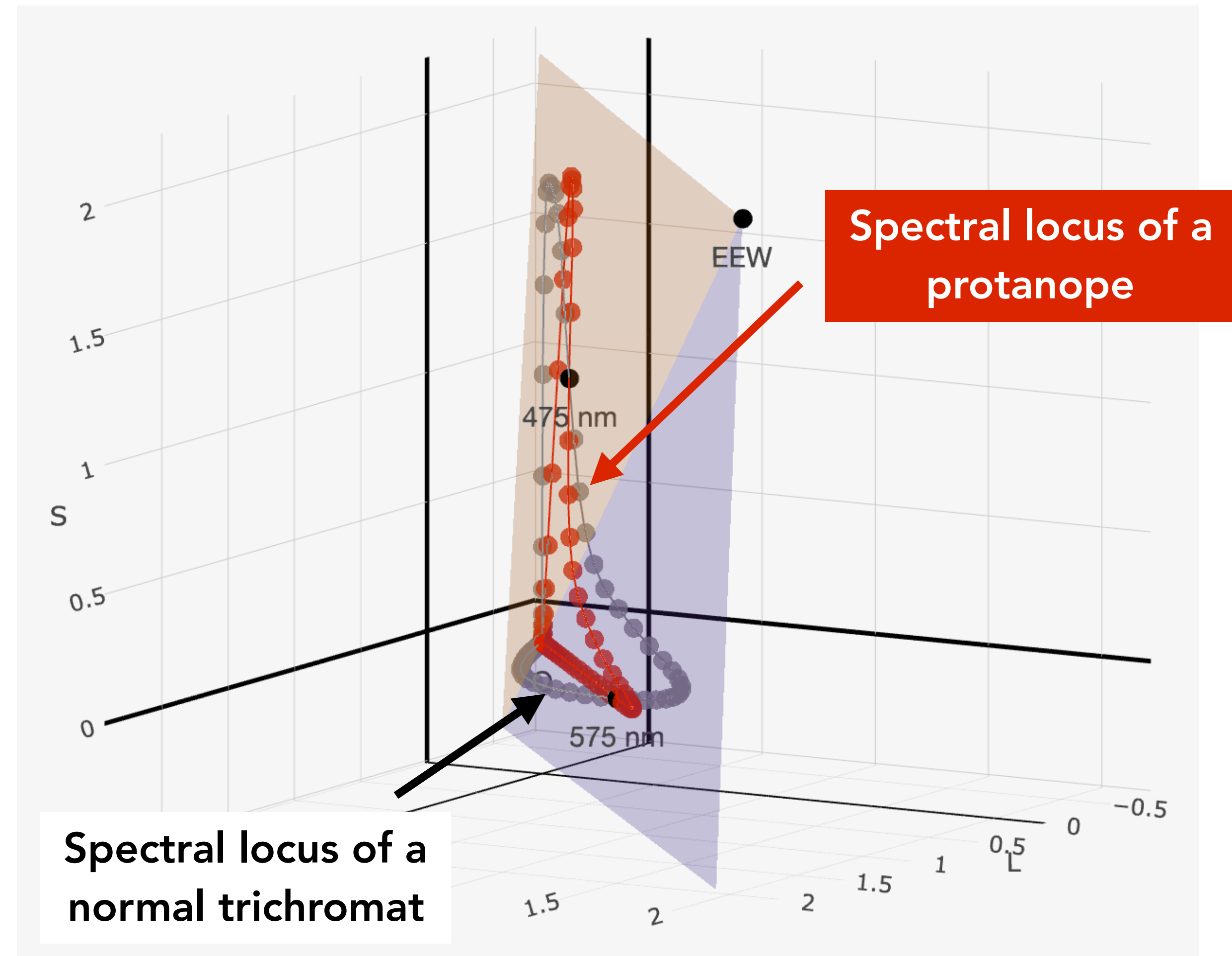
The Geometry of a Dichromatic Vision

See the visualization here: <https://www.cs.rochester.edu/courses/572/colorvis/colorblind.html>.

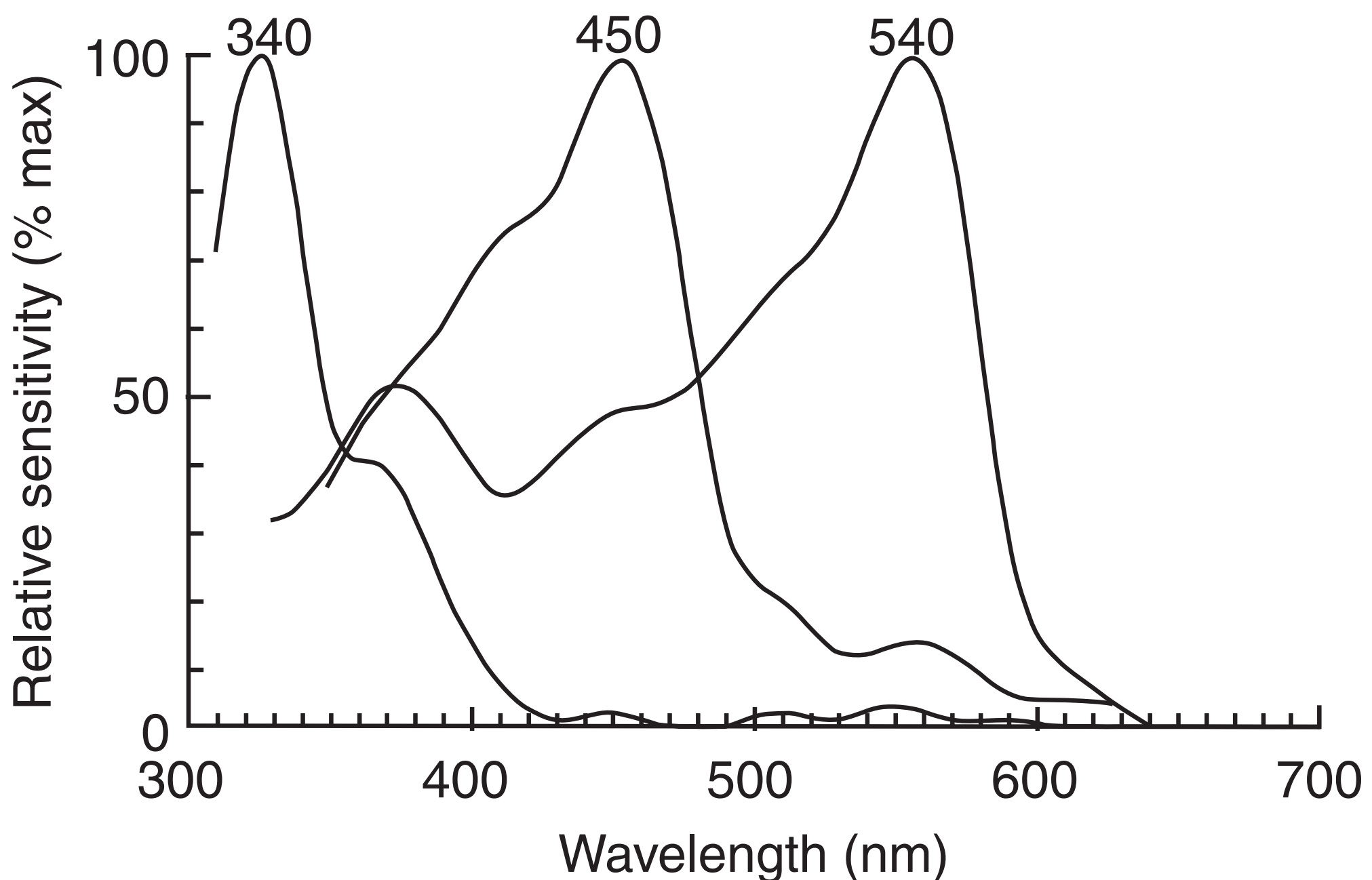
Intercept the lines of confusion with the plane of isochromes.

Based on [Brettel et al. \(1997\)](#).

- It actually used two planes.
- Widely cited but also challenged.

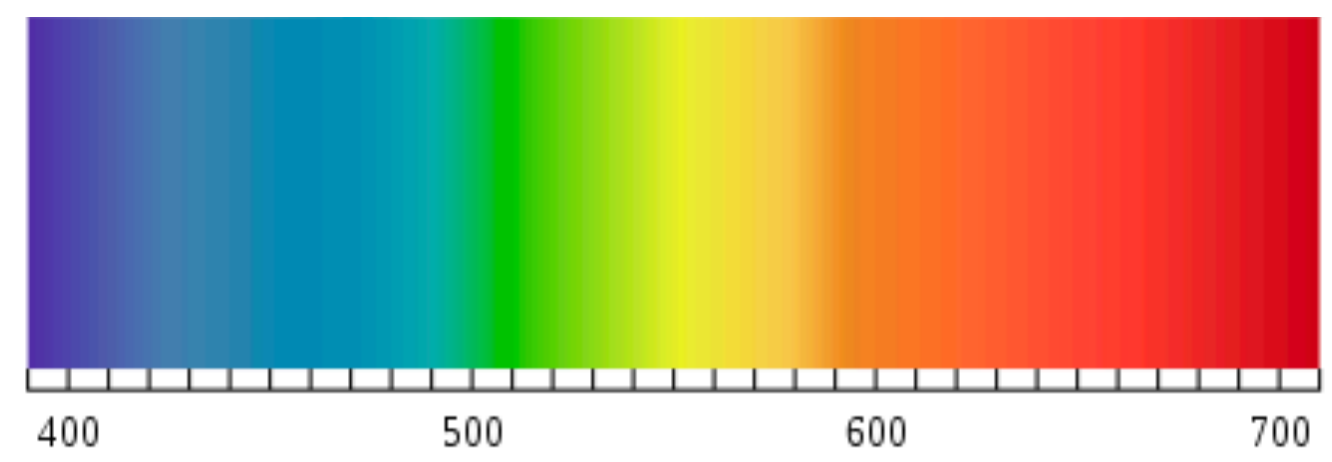


Bee Photoreceptor Sensitivities



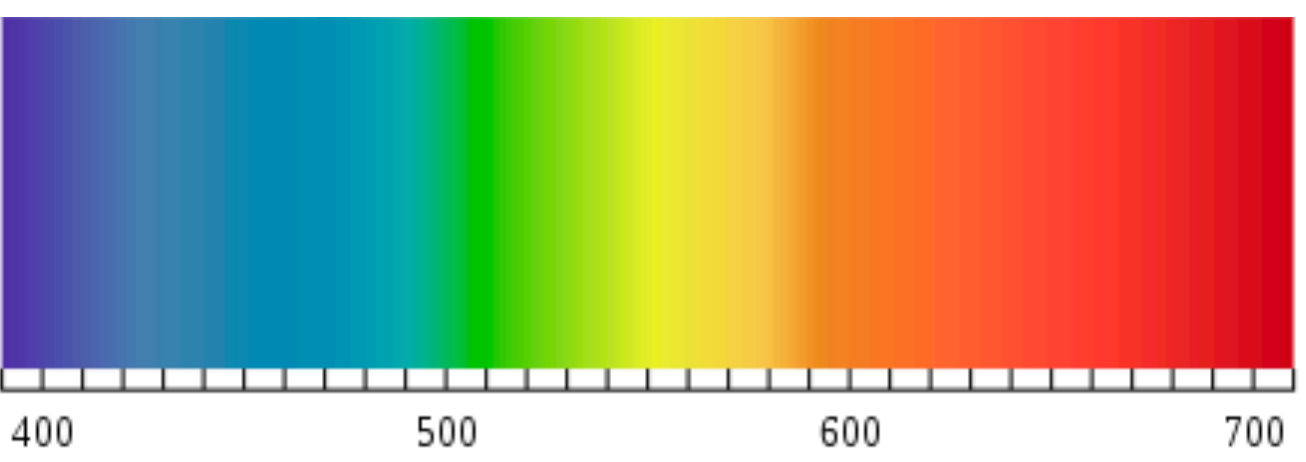
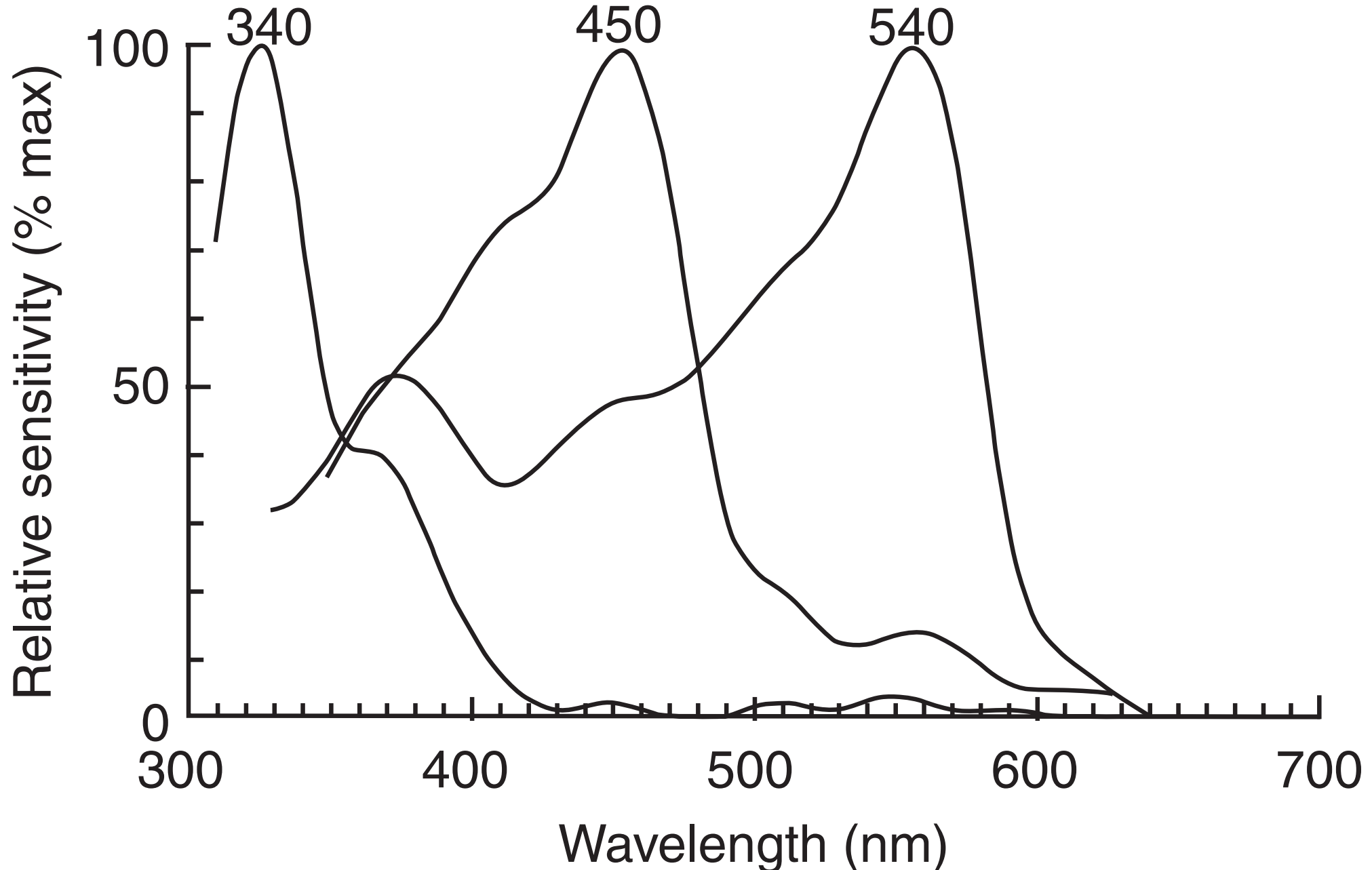
Bees have three photoreceptors, but their sensitivities are shifted toward lower wavelengths.

So they can see UV lights but can't distinguish between red and black.



Bee Photoreceptor Sensitivities

Karl von Frisch (Nobel laureate, 1973)

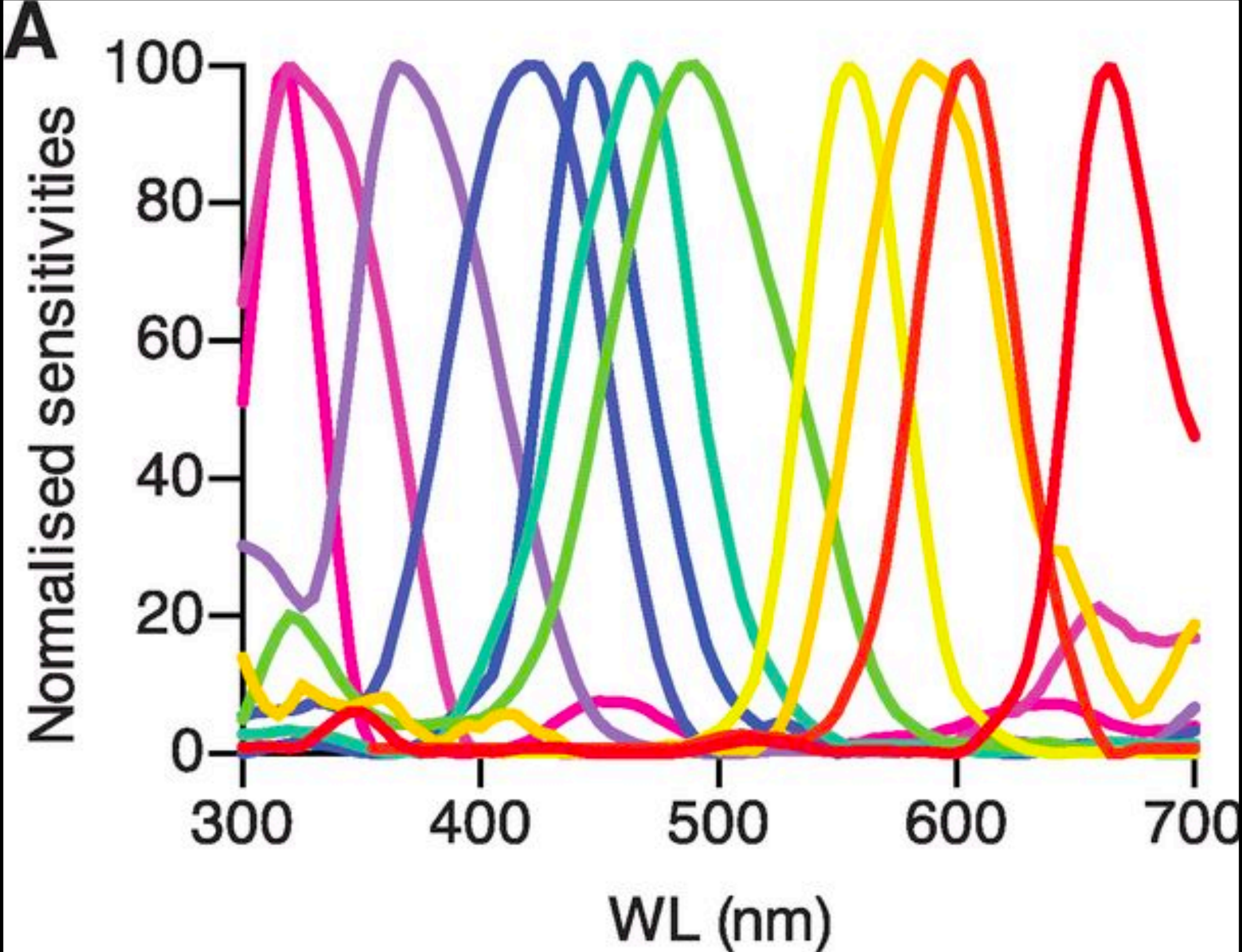


Bees have three photoreceptors, their sensitivities are shifted to lower wavelengths.

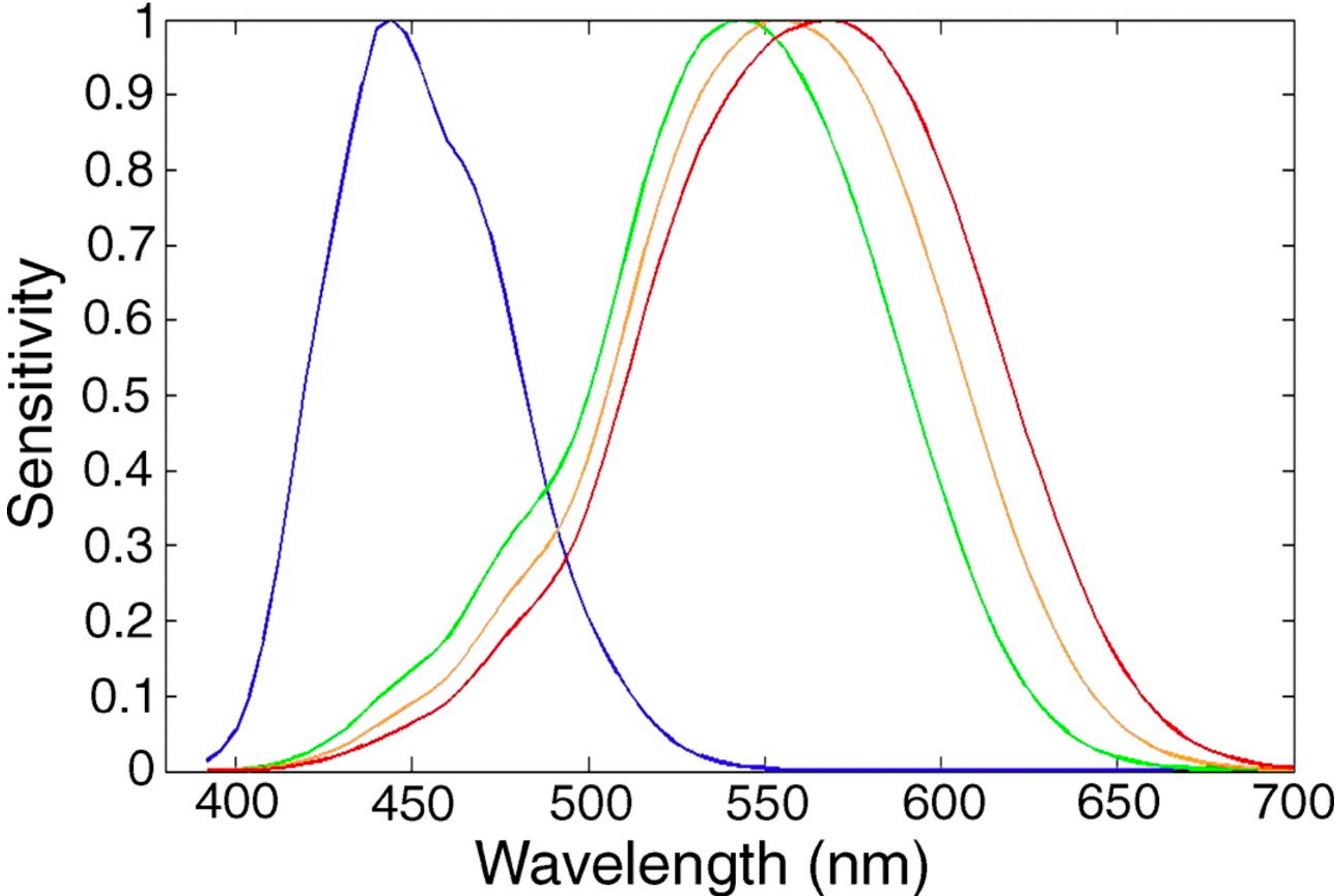
So they can see UV lights but can't distinguish between red and black.

Karl von Frisch, one of the first to experimentally show bees see color.

Mantis Shrimp



Human Tetrachromacy



The dimensionality of color vision in carriers of anomalous trichromacy

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Samir S. Deeb

Departments of Medicine and Genome Sciences,
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Department of Experimental Psychology, University of
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J. D. Mollon

Department of Experimental Psychology, University of
Cambridge, Cambridge, UK



Some 12% of women are carriers of the mild, X-linked forms of color vision deficiencies called “anomalous trichromacy.” Owing to random X chromosome inactivation, their retinæ must contain four classes of cone rather than the normal three; and it has previously been speculated that these female carriers might be tetrachromatic, capable of discriminating spectral stimuli that are indistinguishable to the normal trichromat. However, the existing evidence is sparse and inconclusive. Here, we address the question using (a) a forced-choice version of the Rayleigh test, (b) a test using multidimensional scaling to reveal directly the dimensionality of the participants’ color space, and (c) molecular genetic analyses to estimate the X-linked cone peak sensitivities of a selected sample of strong candidates for tetrachromacy. Our results suggest that most carriers of color anomaly do not exhibit four-dimensional color vision, and so we believe that anomalous trichromacy is unlikely to be maintained by an advantage to the carriers in discriminating colors. However, 1 of 24 obligate carriers of deuteranomaly exhibited tetrachromatic behavior on all our tests; this participant has three well-separated cone photopigments in the long-wave spectral region in addition to her short-wave cone. We assess the likelihood that behavioral tetrachromacy exists in the human population.

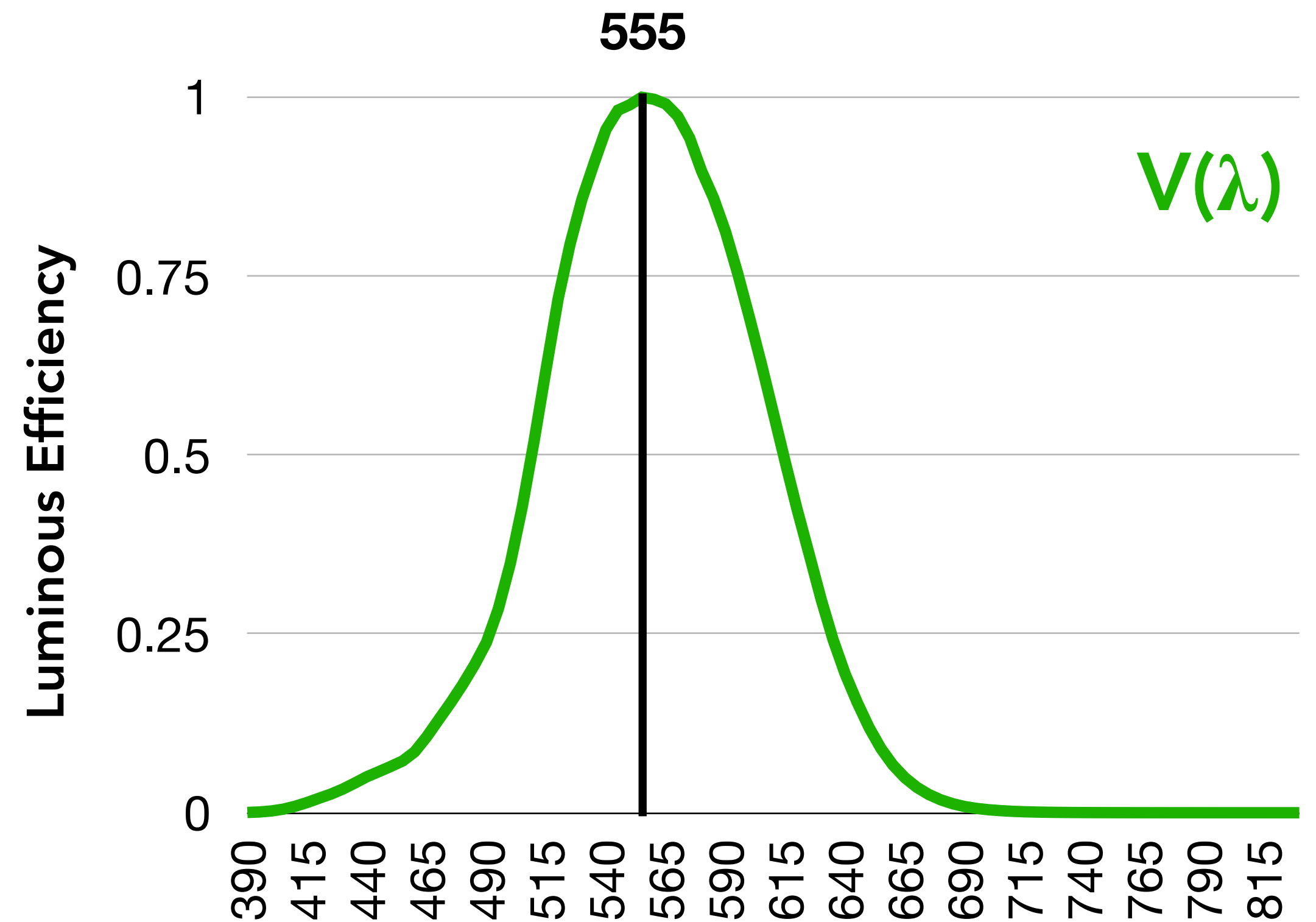
Luminous Efficiency

Luminous Efficiency Function

Informally, quantifies the overall cone responses under different wavelengths.

- Given 1 Watt of light at a wavelength, how much cone response is generated?
- Experimentally measured and then fit as a weighted sum of the three individual cone sensitivity functions.

Human eyes under daylight (photopic vision) are most sensitive to green-ish light (LEF peaks at 555 nm).



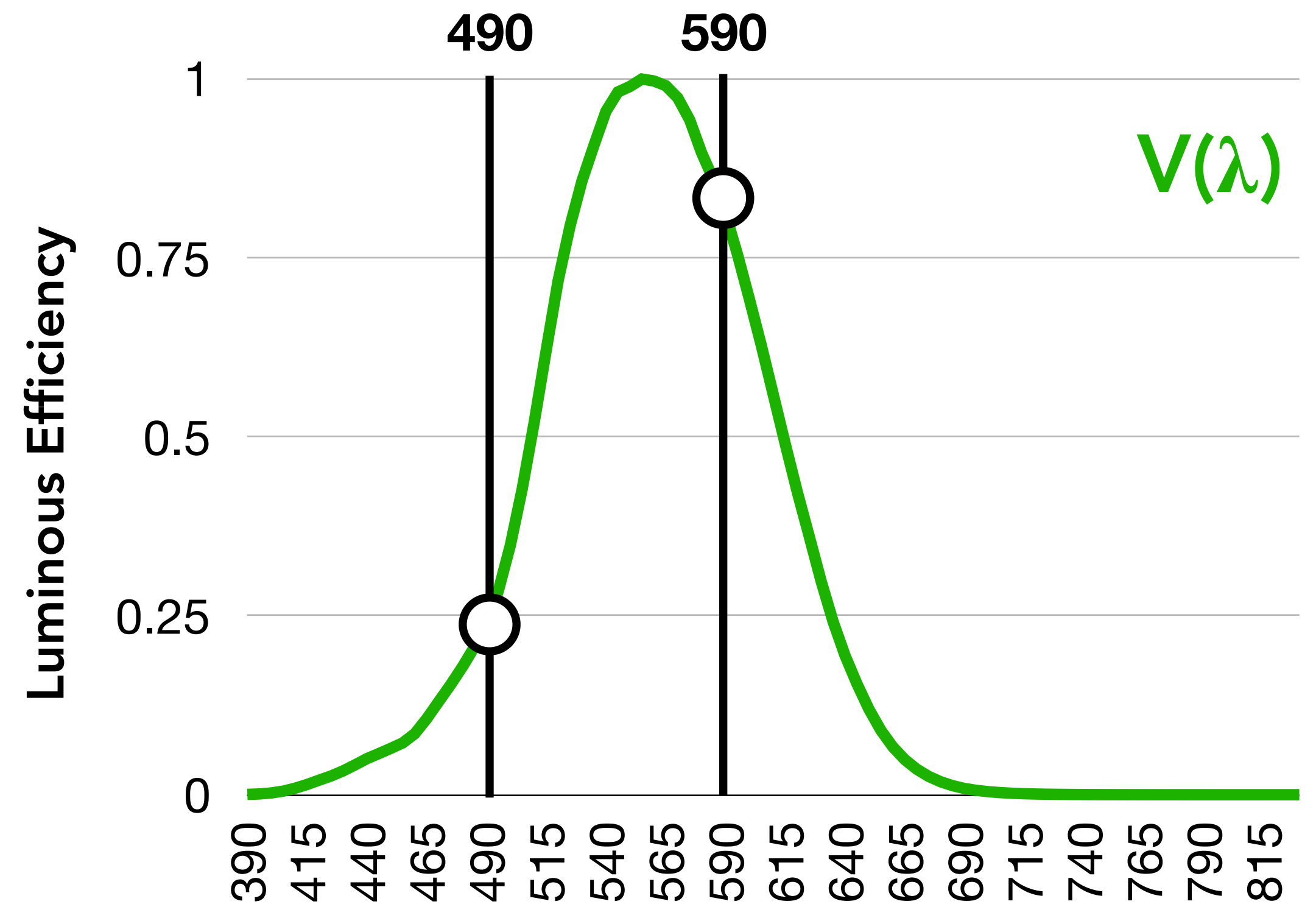
CIE 2008 Photopic LEF

Luminous Efficiency Function

If two spectral light $\lambda=590$ (green-ish) and $\lambda=490$ (blue-ish) have the **same power**, the former would look brighter

- because cones are more sensitive to 590 nm than to 490 nm.

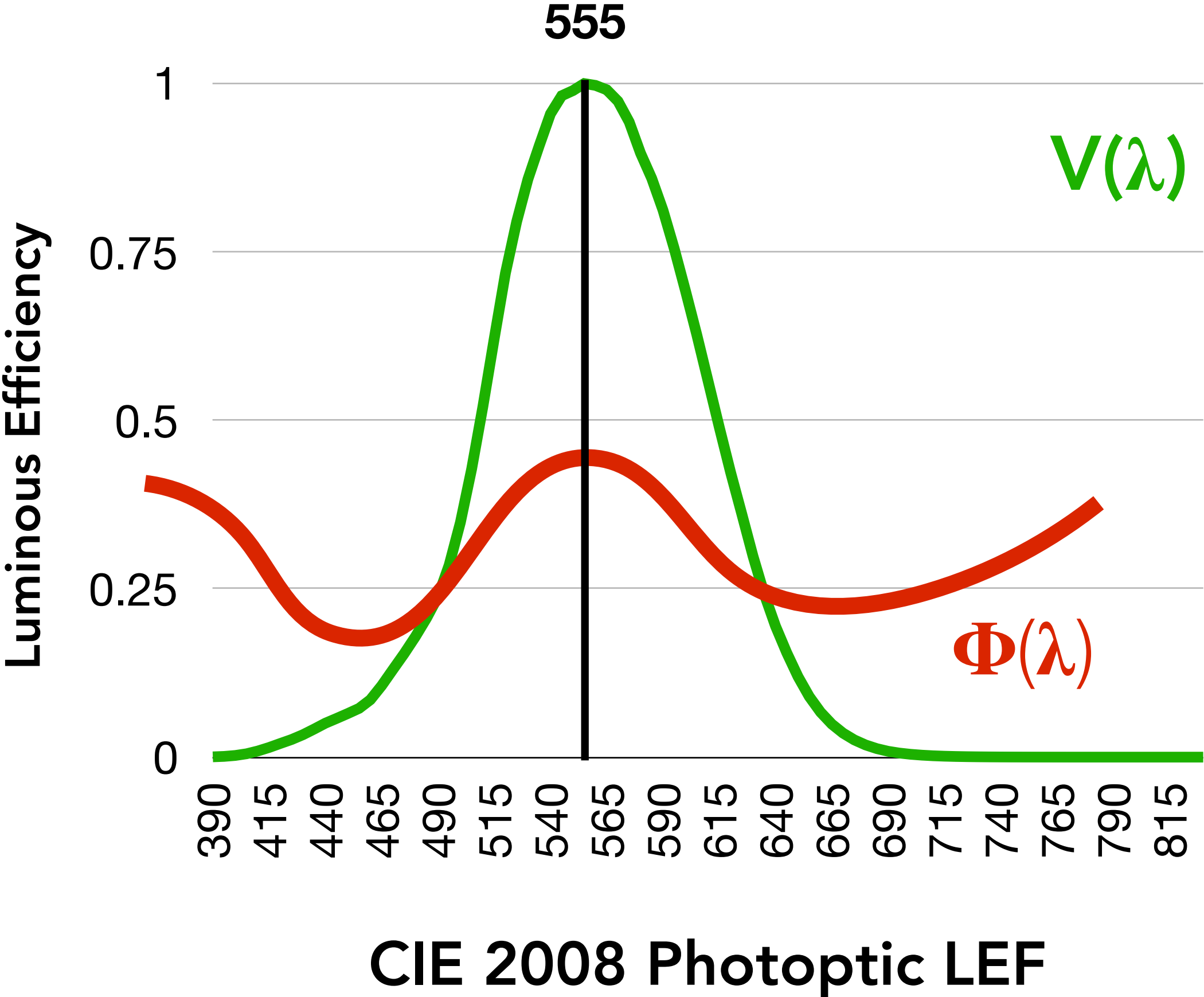
If luminous efficiency is, say, 0.8, it means 1 Watt of spectral light is translated to 0.8 Watt of "**perceived**" power.



CIE 2008 Photopic LEF

Luminous Flux

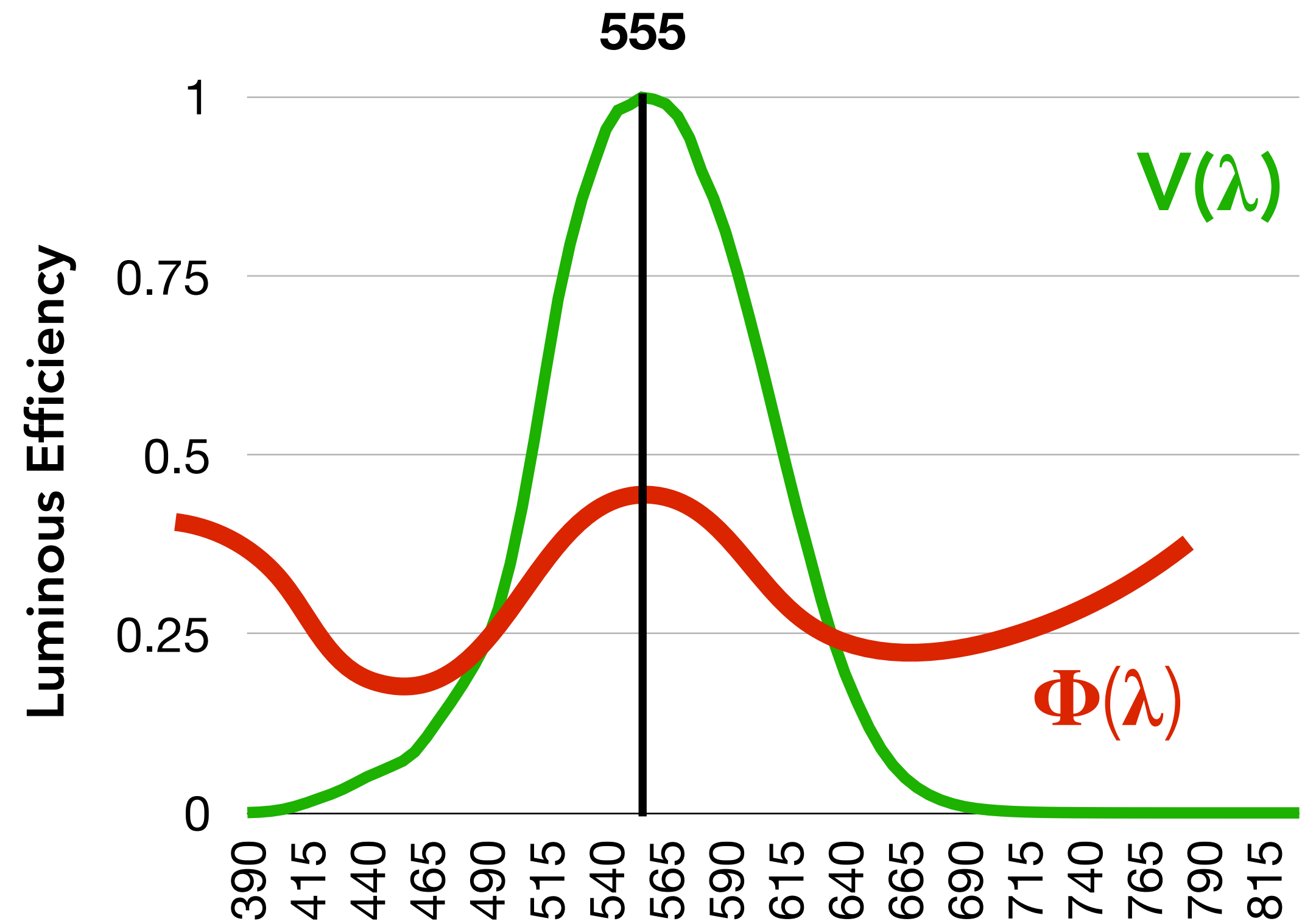
How would we calculate the "perceived" power of an arbitrary light?



Luminous Flux

$$Y = 683 \frac{\text{lm}}{\text{W}} \int \Phi(\lambda) V(\lambda) d\lambda$$

How would we calculate the "perceived" power of an arbitrary light?



CIE 2008 Photopic LEF

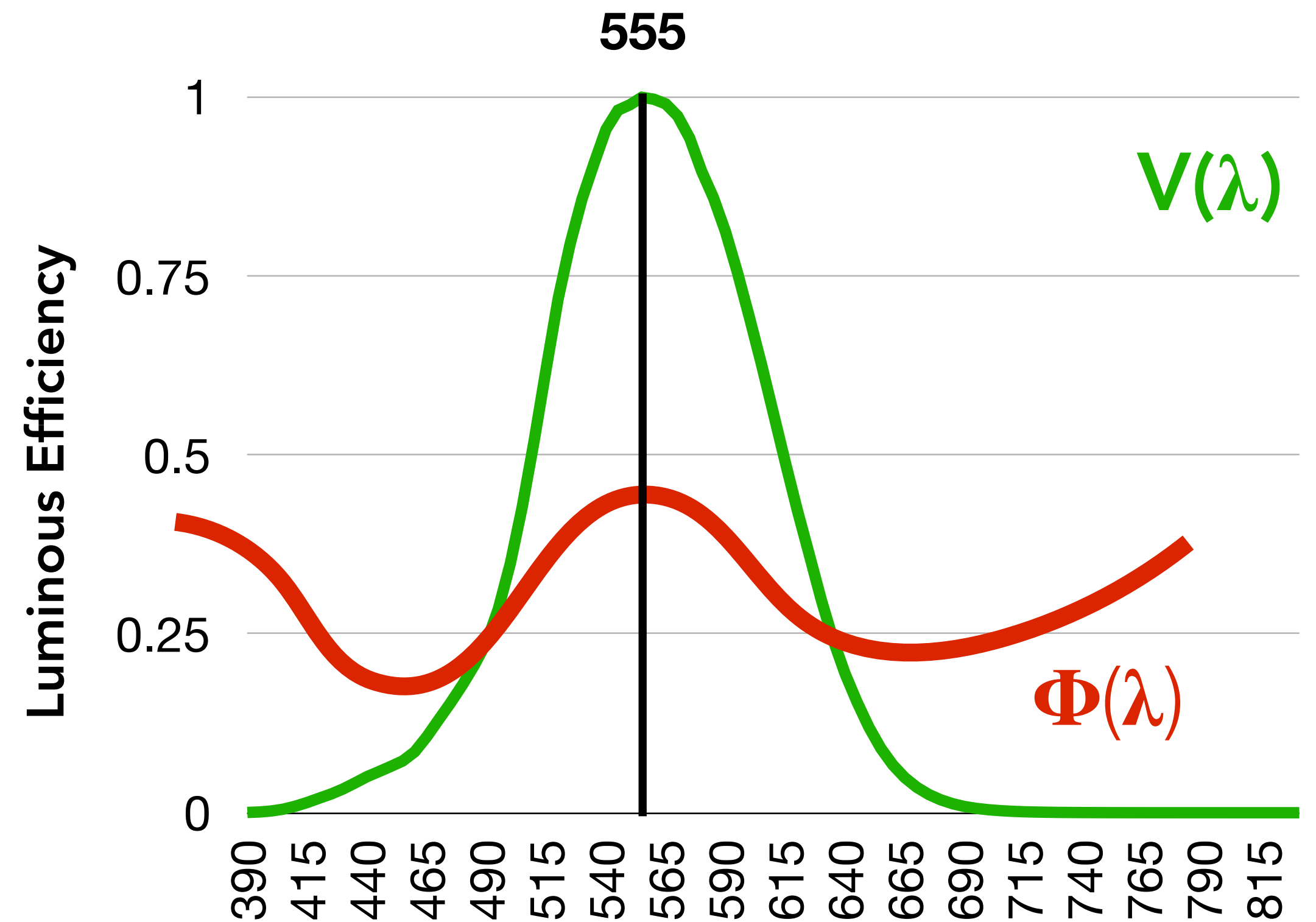
Luminous Flux

$$Y = 683 \frac{\text{lm}}{\text{W}} \int \Phi(\lambda) V(\lambda) d\lambda$$

How would we calculate the “perceived” power of an arbitrary light?

Luminous flux’s unit is **lumen** (lm).

- Spectral light $\lambda=555$ nm emitting $1/683$ W power generates 1 lumen.

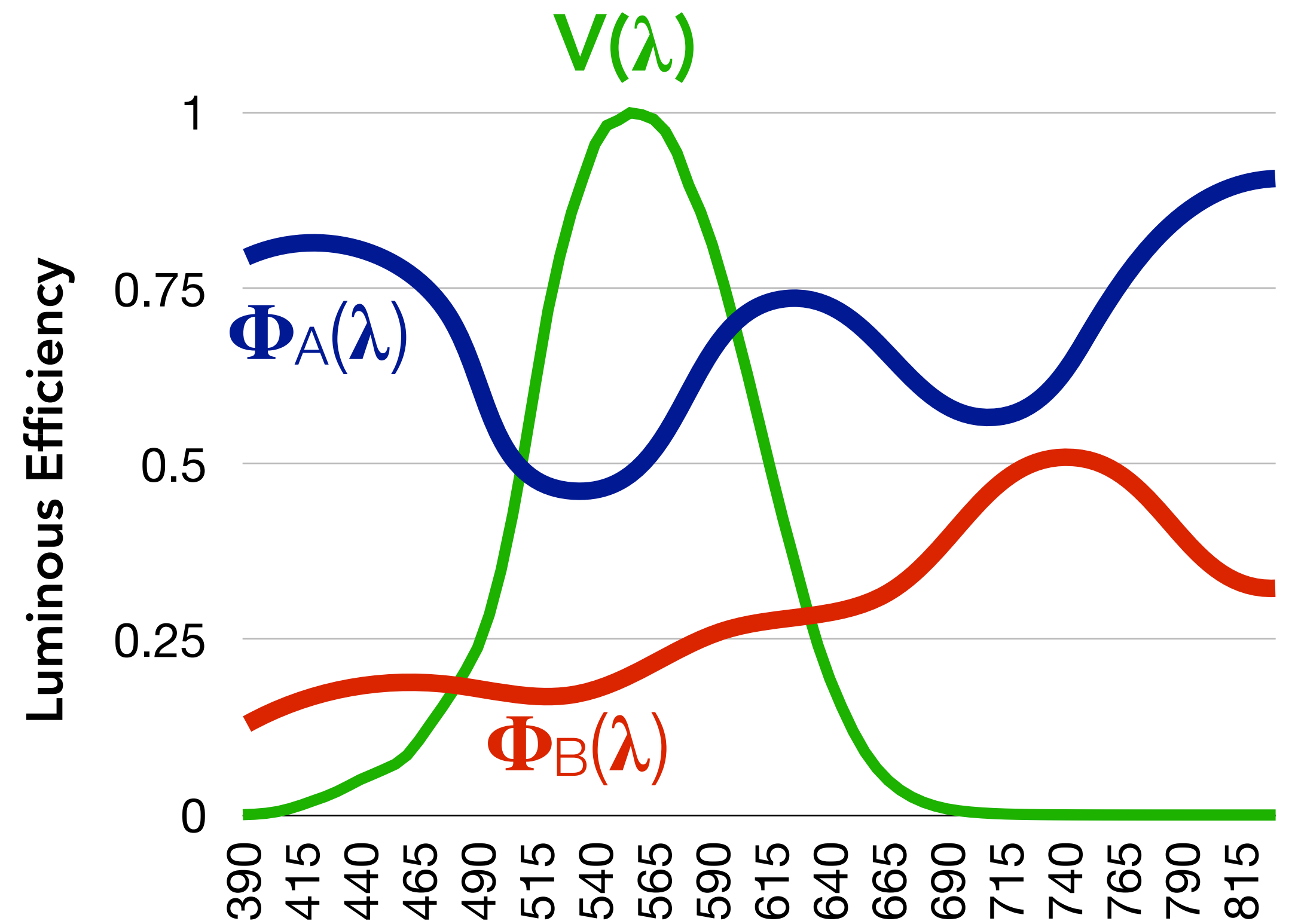


CIE 2008 Photopic LEF

Matching Brightness


If light A's luminous flux is, say, 5X higher than that of light B, we can match light A's brightness by increasing the power of light B by 5X.

Critically, this does NOT mean light A is five times brighter than light B!




CIE 2008 Photopic LEF

Power vs. Luminous Flux

		Lumen				
		250+	450+	800+	1100+	1600+
	Standard	25W	40W	60W	75W	100W
	Halogen	18W	29W	43W	53W	72W
	CFL	6W	10W	13W	18W	23W
	LED	4W	5W	10W	15W	20W

Power vs. Luminous Flux

What's the difference?

Lumen		250+	450+	800+	1100+	1600+
 Standard		25W	40W	60W	75W	100W
 Halogen		18W	29W	43W	53W	72W
 CFL		6W	10W	13W	18W	23W
 LED		4W	5W	10W	15W	20W