

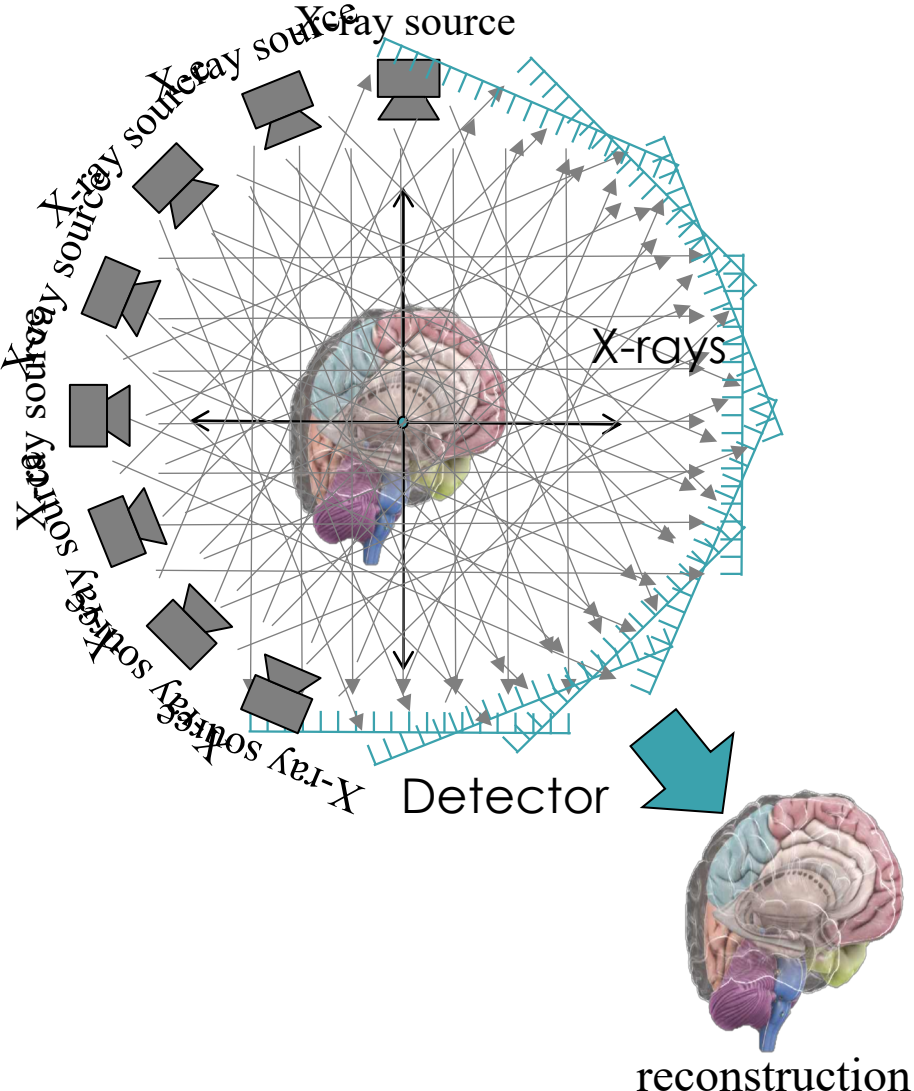
Visual Computing and Tomographic Imaging Principles

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Tomographic Imaging

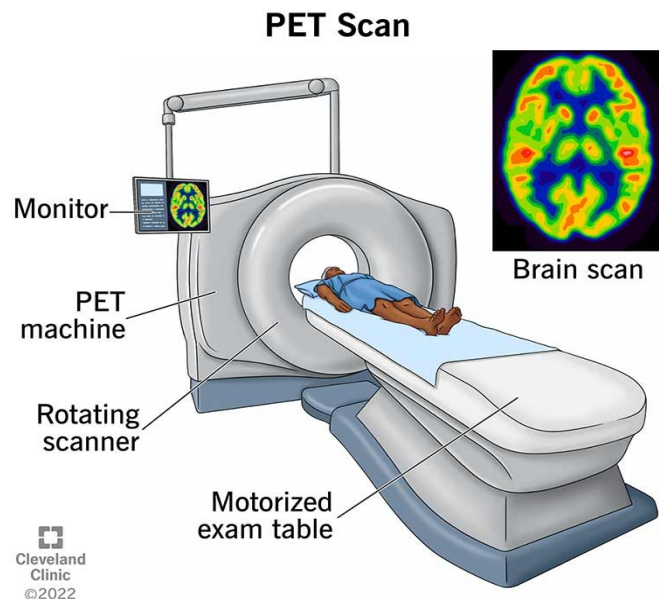


Tomography in Medical Imaging

Positron Emission Tomography (PET)

Computed Tomography (CT) Aka, Cat Scan

X-ray Radiograph



PET

CT

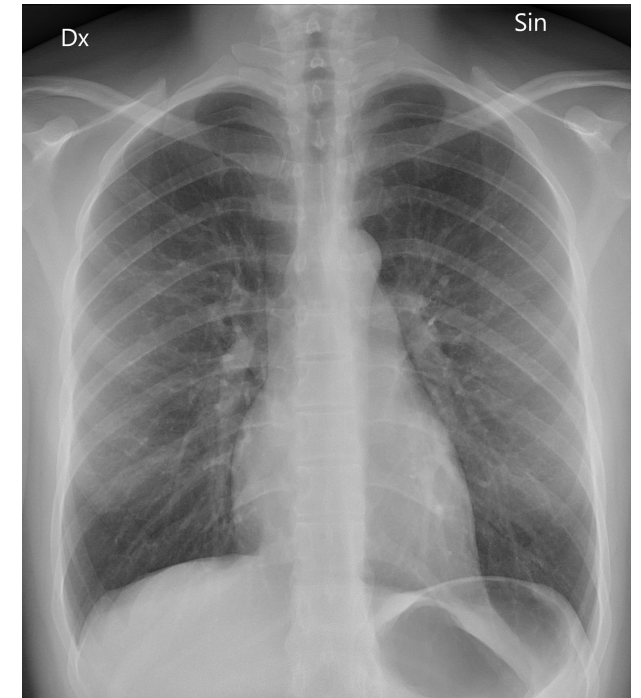
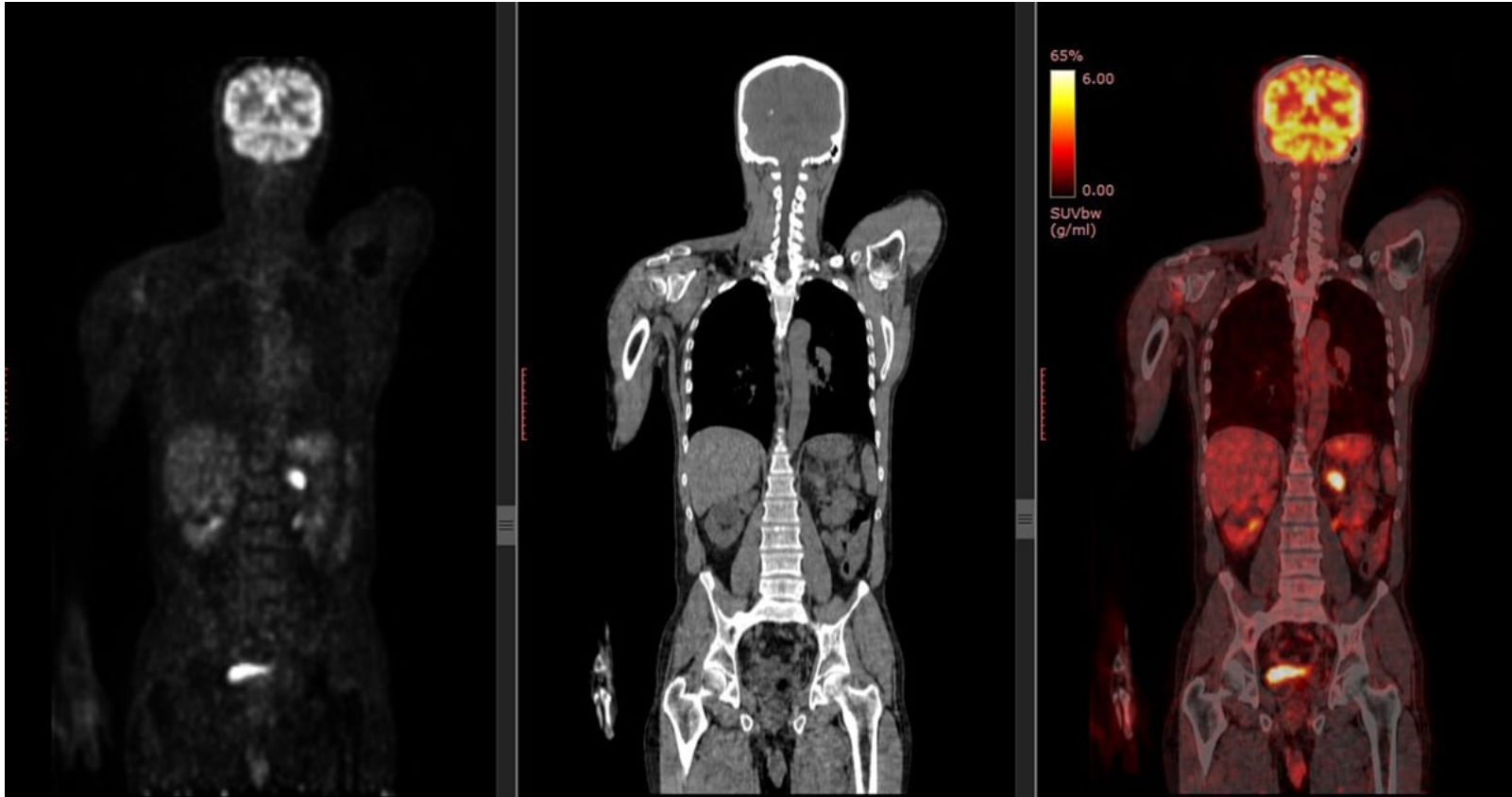
X-ray Radiograph

Tomography in Medical Imaging

Positron Emission Tomography (PET)

Medical Computed Tomography (CT)
Aka, Cat Scan

X-ray Radiograph



PET (functional)

CT
(anatomical)

PET + CT
(functional + anatomical)

Chest X-ray
Radiograph
(anatomical)

Tomography in Medical Imaging

Medical Computed Tomography (CT)

- X-ray
- 3D image volume
- External source
- Anatomical information


Positron Emission Tomography (PET)

- Gamma ray instead of x-ray
- 3D image volume (low resolution)
- The ray source is within the patient body
- Functional information

X-ray Radiograph

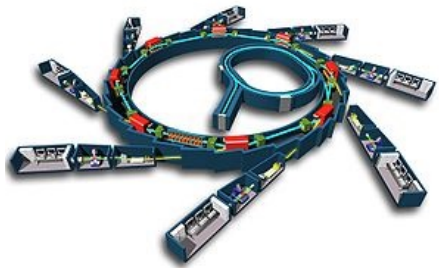
- x-ray
- A single 2D image
- External source
- A single view angle version of CT
- Anatomical information

Medical Imaging Modality

- X-ray CT
 - X-ray Radiograph
 - PET, and SPECT
- 
- tomography applications
- Magnetic Resonance Imaging (MRI)
 - Functional MRI
 - Ultrasound (not general)

More than half of all medical imaging scans are tomography applications

Other Non-Medical Tomography Applications



Synchrotron Imaging



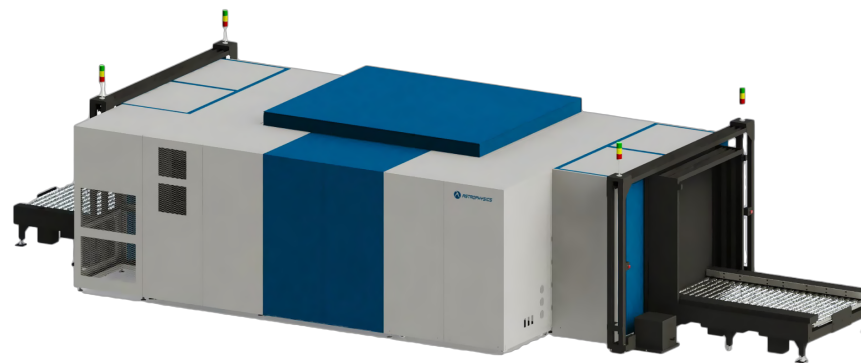
Electron Microscopy



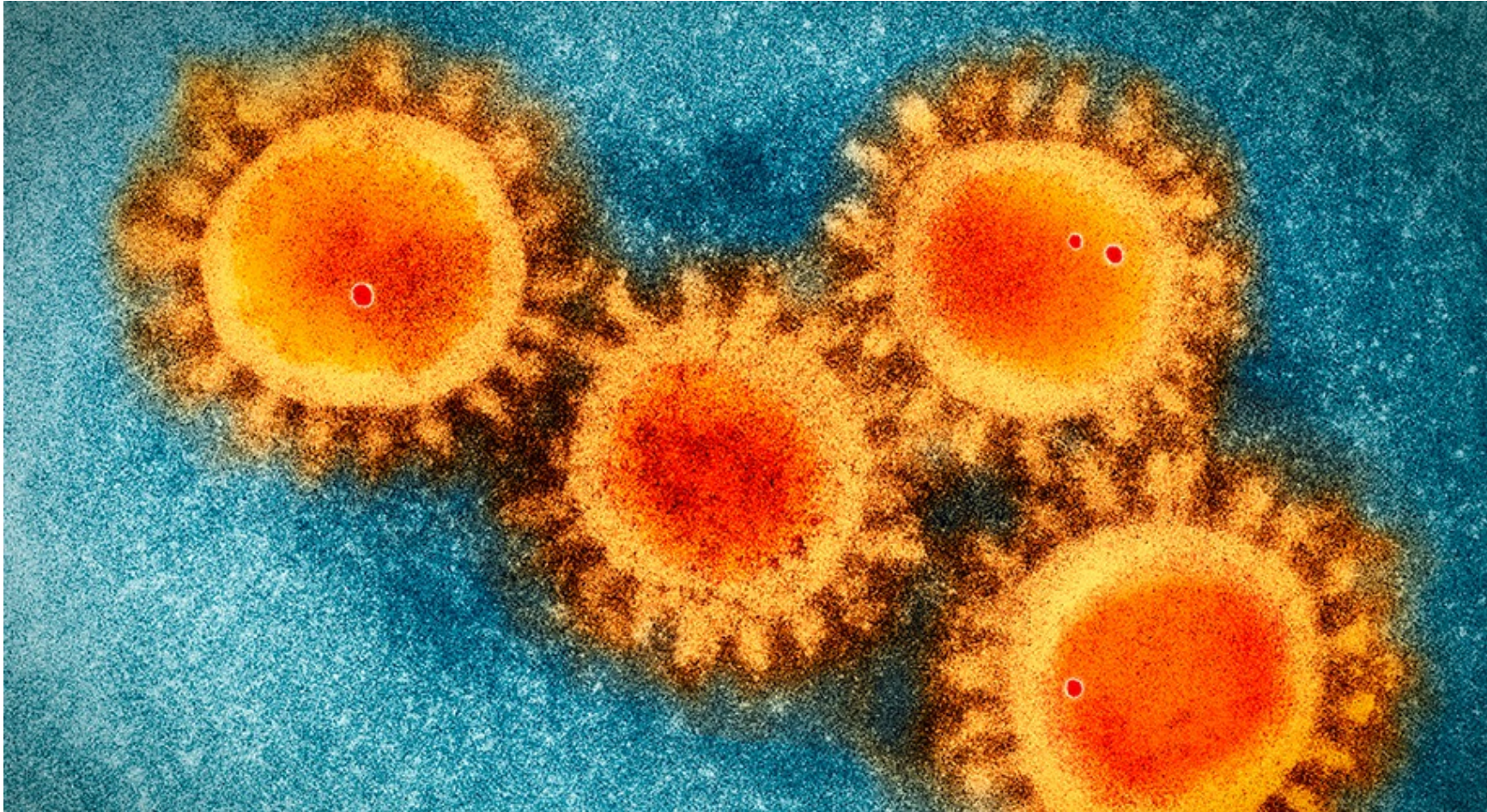
Airport Security Baggage Scanner



X-ray Microscopy



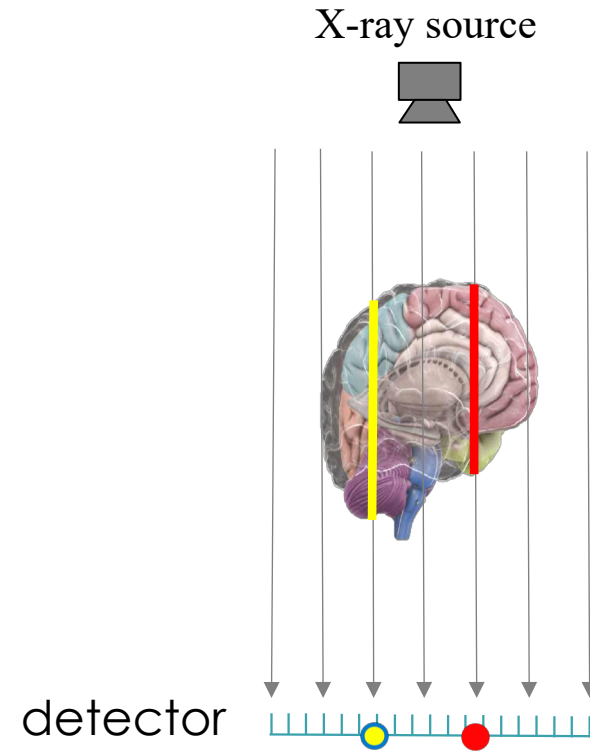
Seaport Cargo Screening



Covid virus imaged under
an electron microscope

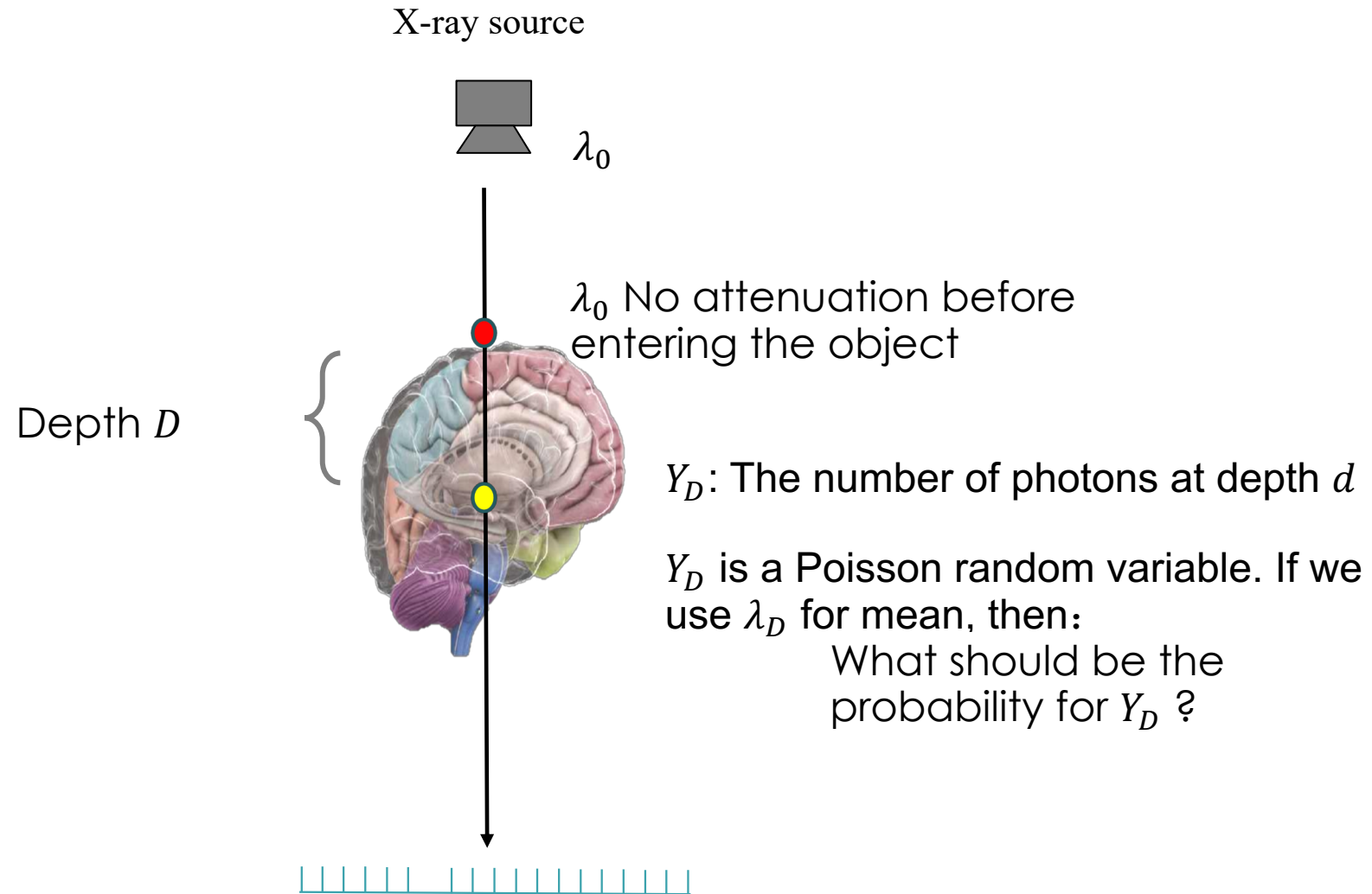
covid virus is corona virus,
“crown”

X-ray Projections and Photon Attenuation



Each projection represents the integral of all densities along the path of the X-ray

X-ray Projections and Photon Attenuation



X-ray Projections and Photon Attenuation

How can we compute projections from the photon statistics?

As X-ray photons pass through material, they are absorbed.

The rate of absorption is proportional to the number of photons and the density of the material (Beer-Lambert Law)

If the material density at depth D is μ_D in units of cm^{-1} , and λ_D is the mean for the number of photons at depth D , then the rate of photon absorption should be:

$$\frac{\partial \lambda_D}{\partial D} = ?$$
$$\frac{\partial \lambda_D}{\partial D} = -\mu_D \lambda_D$$

Then $\lambda_D = \lambda_0 e^{-\int_0^D \mu(t) dt}$

Mean for the number of photons at depth D

Mean Incident photons (photons from the source)

material density at depth t

X-ray Projections and Photon Attenuation

$$\lambda_D = \lambda_0 e^{-\underbrace{\int_0^D \mu(t) dt}_{\text{projection}}}$$

$$y = \int_0^D \mu(t) dt = -\log \frac{\lambda_D}{\lambda_0},$$

y is the projection, and it represents the integral of all densities along the path of the X-ray
 λ_D is still unknown. Cannot scan the same patient many times to compute the mean

$$y = \int_0^D \mu(t) dt = -\log \frac{\lambda_D}{\lambda_0} \approx -\log \frac{Y_D}{\lambda_0}, \quad \leftarrow \text{The number of measured photons}$$

X-ray Projections and Dark Current Correction

Dark current: small electric current can flow through photosensitive devices, even when the photon source is turned off.

The photons carried in the dark current is called dark scan photons, or dark current photons.

$y \approx -\log \frac{Y_D - \lambda_{dark}}{\lambda_0 - \lambda_{dark}}$, λ_{dark} is the mean for the number of dark current photons.

- λ_0 is the mean for the number of incident photons. To measure it, remove the patient but with the X-ray source on, and compute the mean photon counts from repeated empty scans.
- λ_{dark} is the mean for the number of dark current photons. To measure it, remove the patient and completely turned off the X-ray source, and then compute the mean for the number of photons.
- Y_D is a single-time photon measurement, with the patient in the scanner, and the X-ray turned on.

Beer-Lambert Law Limitation

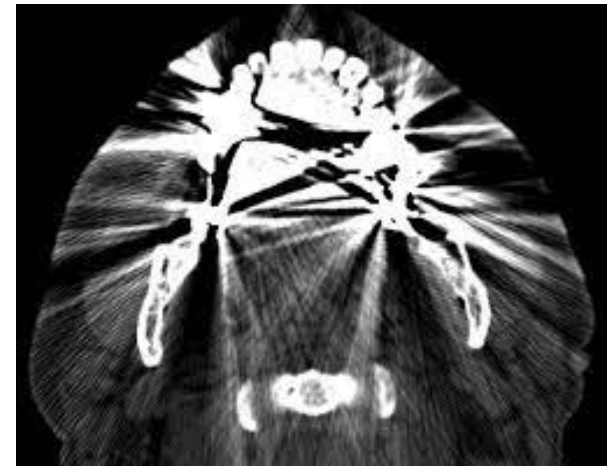
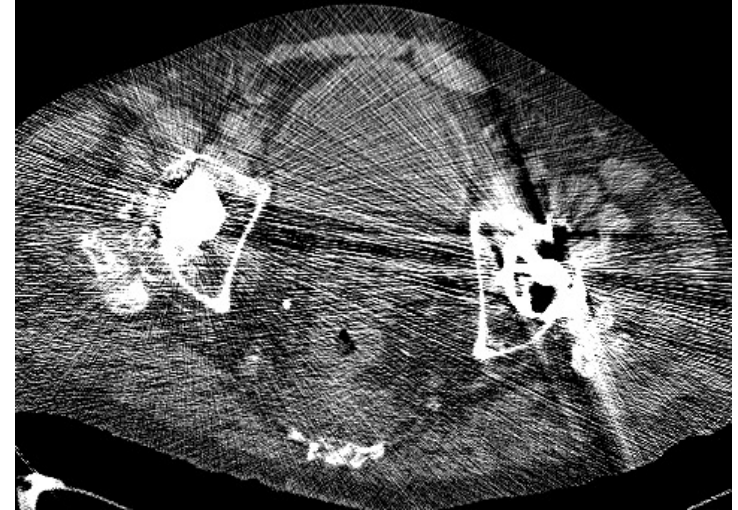
$y \approx -\log \frac{Y_D - \lambda_{dark}}{\lambda_0 - \lambda_{dark}}$, assumes that Y_D and λ_0 are taken at the same X-ray energy.

When the object is very dense, such as metal:

- Scattering
- the X-ray energy for the photons changes

We call it the “Metal Artifact”

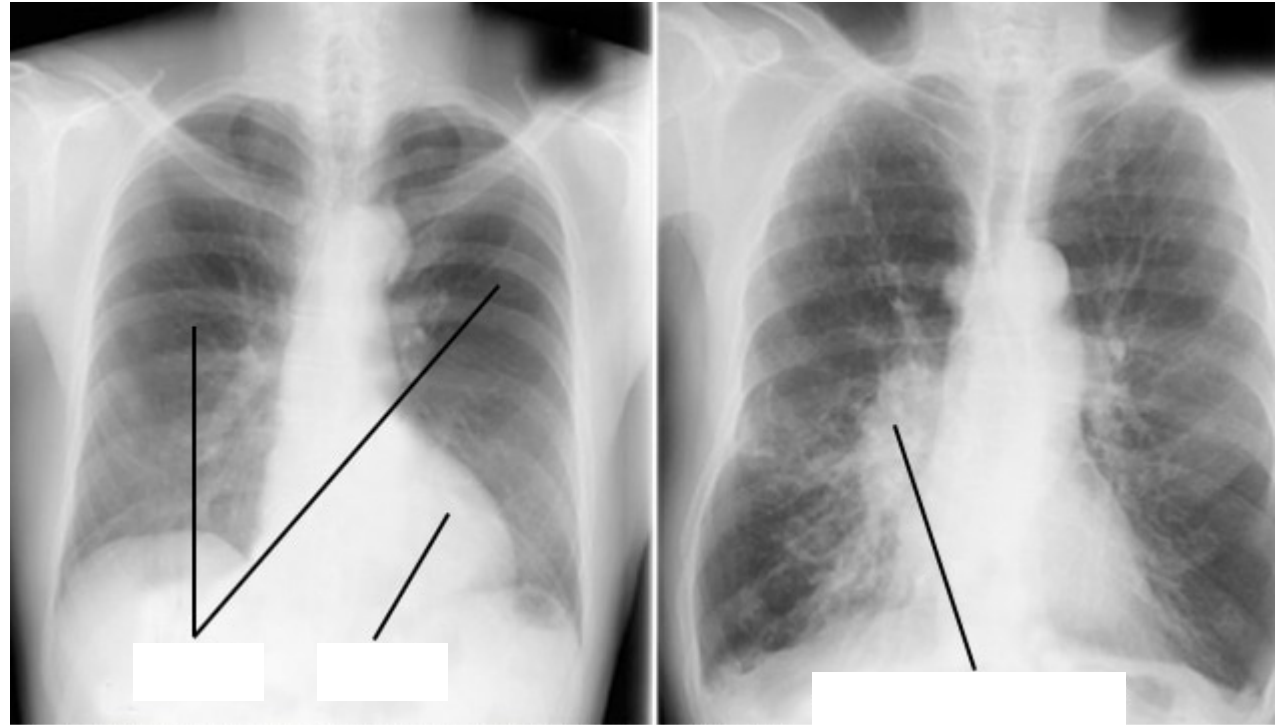
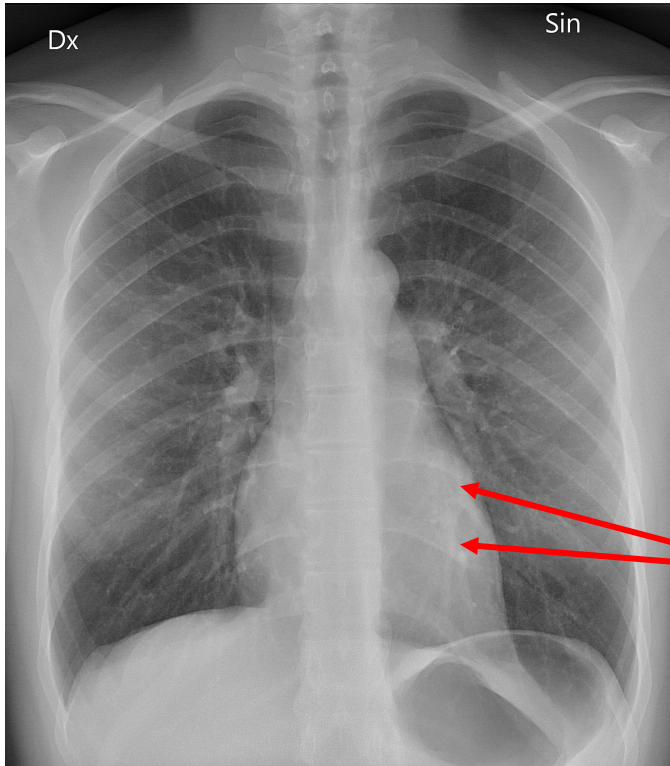
Still a big challenge nowadays



Interesting Facts



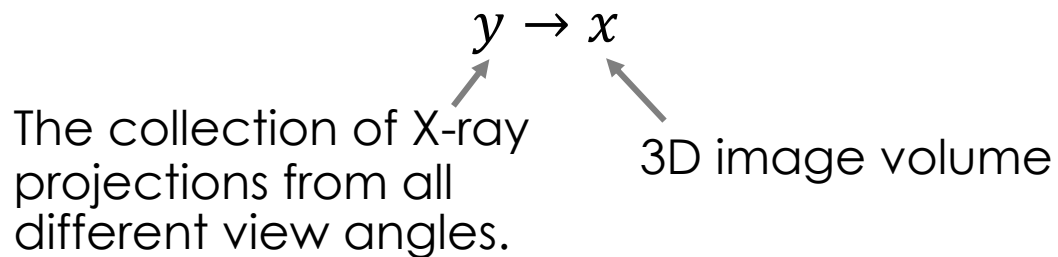
Medical physicists calibrate the tomography scanners daily. One of the main task is to measure λ_0 and λ_{dark} .



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X-ray Radiograph and CT Comparison

- X-ray radiograph image is the single-view X-ray projection
 - 2D
 - Each image pixel is the integral of densities
- CT final image volume
 - 3D image volume
 - Each image voxel is the discrete density at the location of the voxel
 - Require image reconstruction from the integral of data to discrete data



Two Approaches for Reconstruction

Filtered-back Projection (FBP)

- Fourier transform and inverse Fourier transform
- Feldkamp (FDK) Algorithm, Siemens ADMIRE, GE ASiR and etc. (clinical-standard IR)

advantage: light-weight, easy to implement

disadvantage: loss of resolution, and more image artifacts

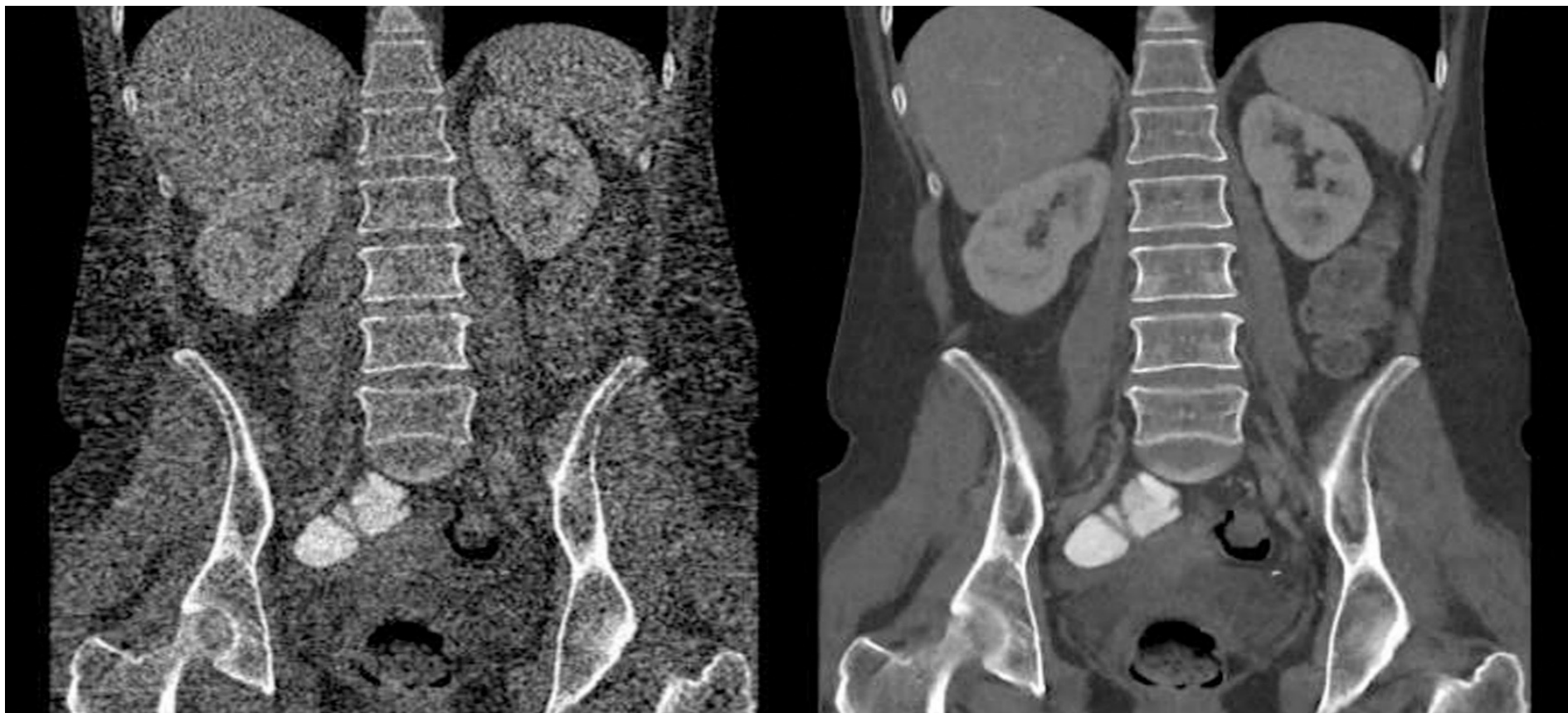
Model Based Iterative Reconstruction (MBIR)

- Physics-geometry forward model
- Image spatial property prior model (assumption)
- Linear algebra and Bayesian estimation

advantage: higher spatial resolution, fewer artifacts

disadvantage: complexity, computation time, memory requirement

Model Based Iterative Reconstruction vs FBP

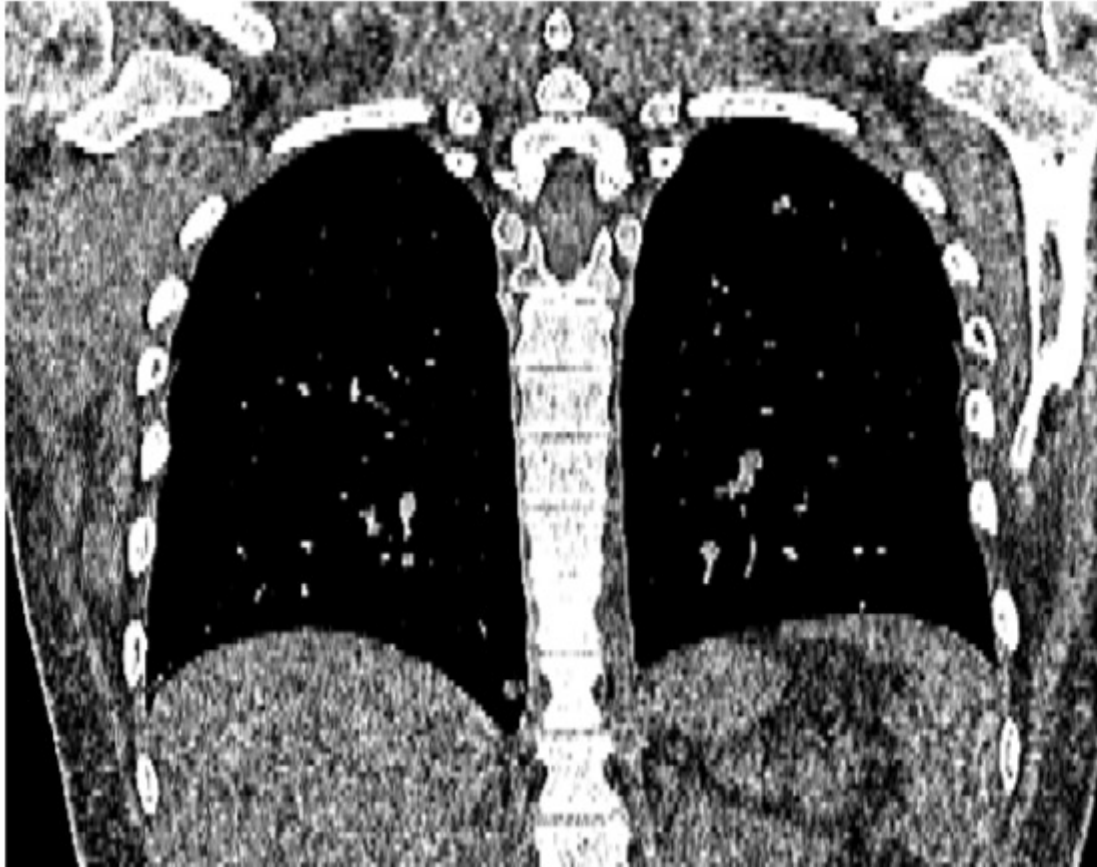


traditional reconstruction (FBP)

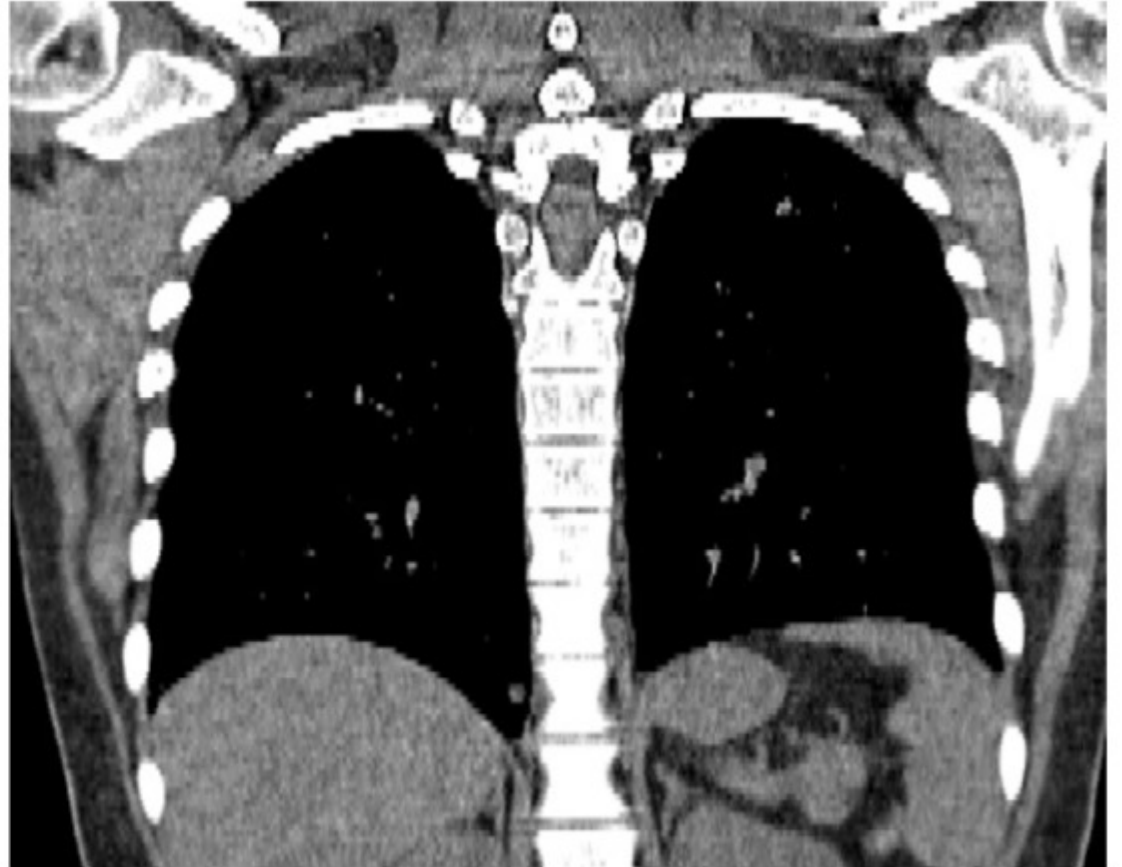
MBIR reconstruction

1/4 the X-ray dosage!

Model Based Iterative Reconstruction vs Siemens Commercial Product

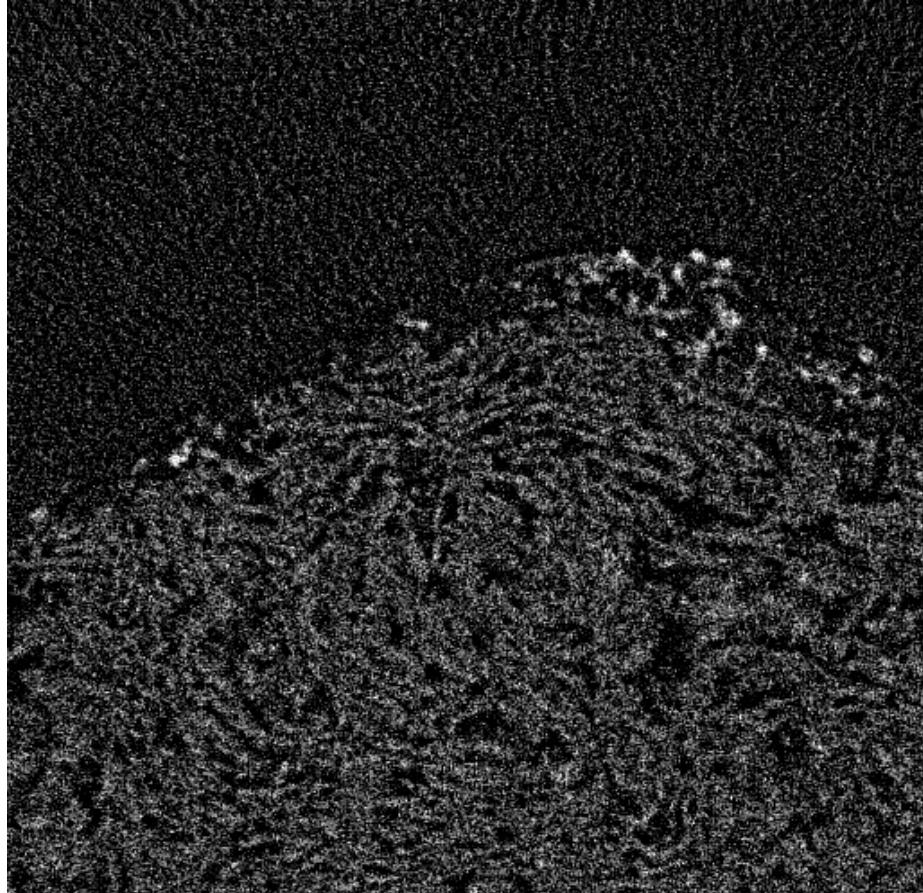


Siemens State-of-the-art
Siemens ADMIRE

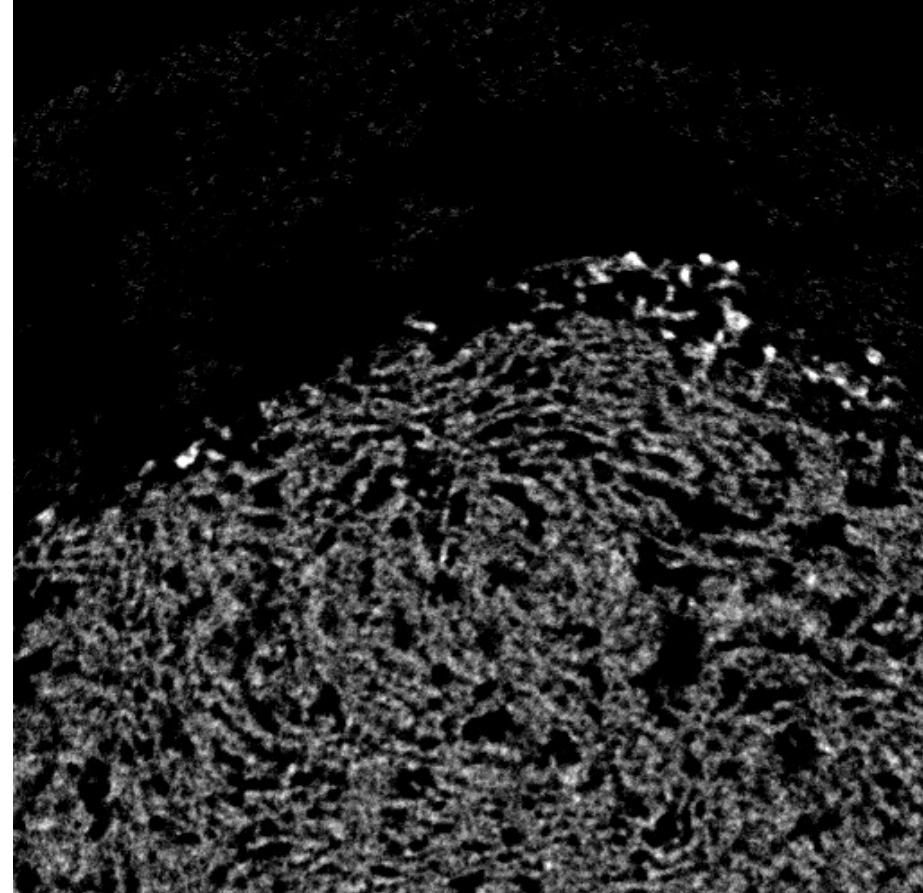


MBIR

MBIR vs FBP for Low-Dose Material Imaging

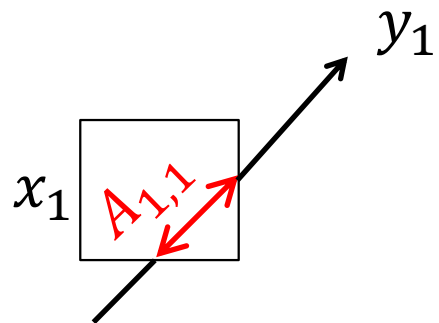


FBP

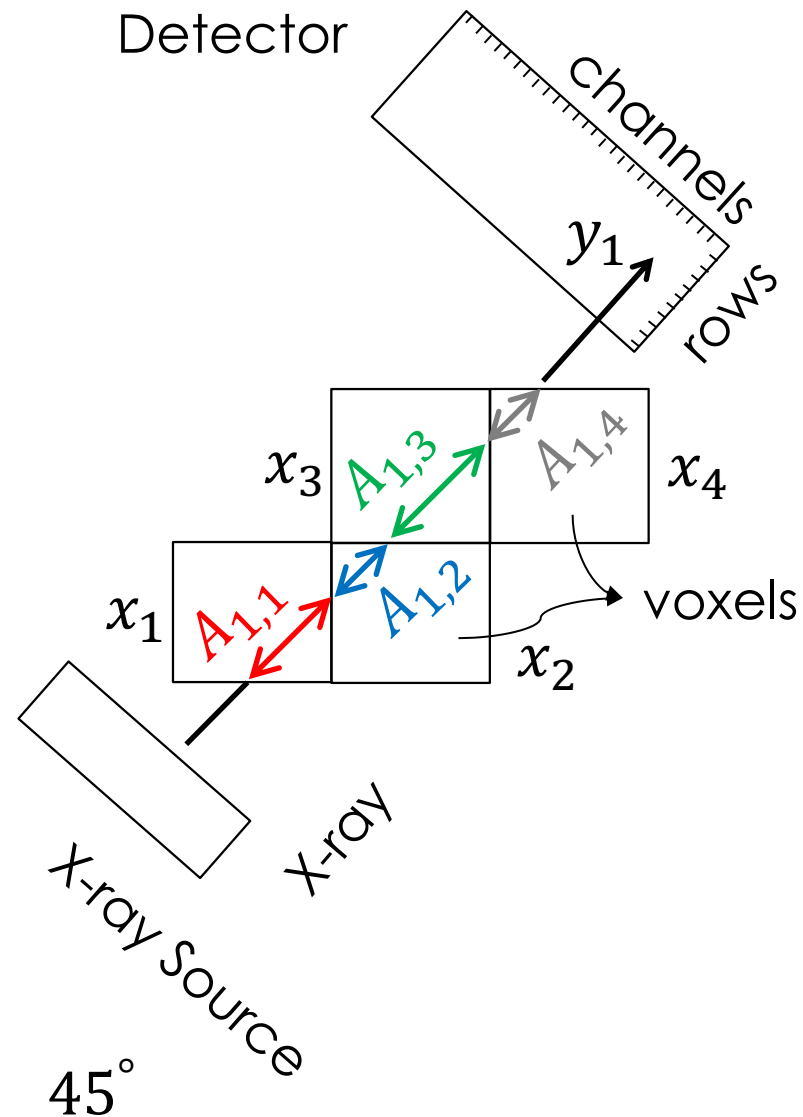
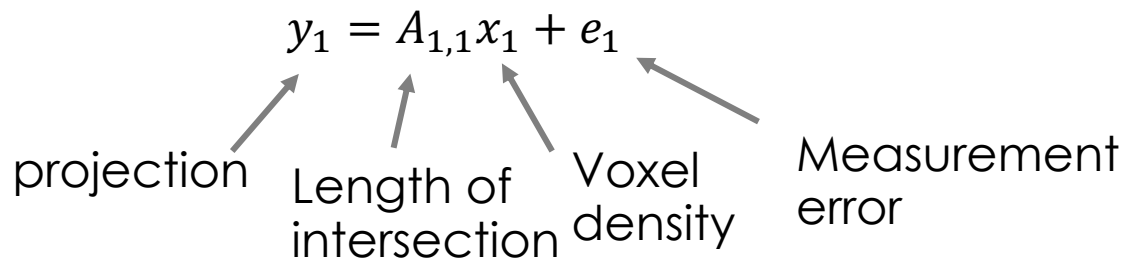


MBIR

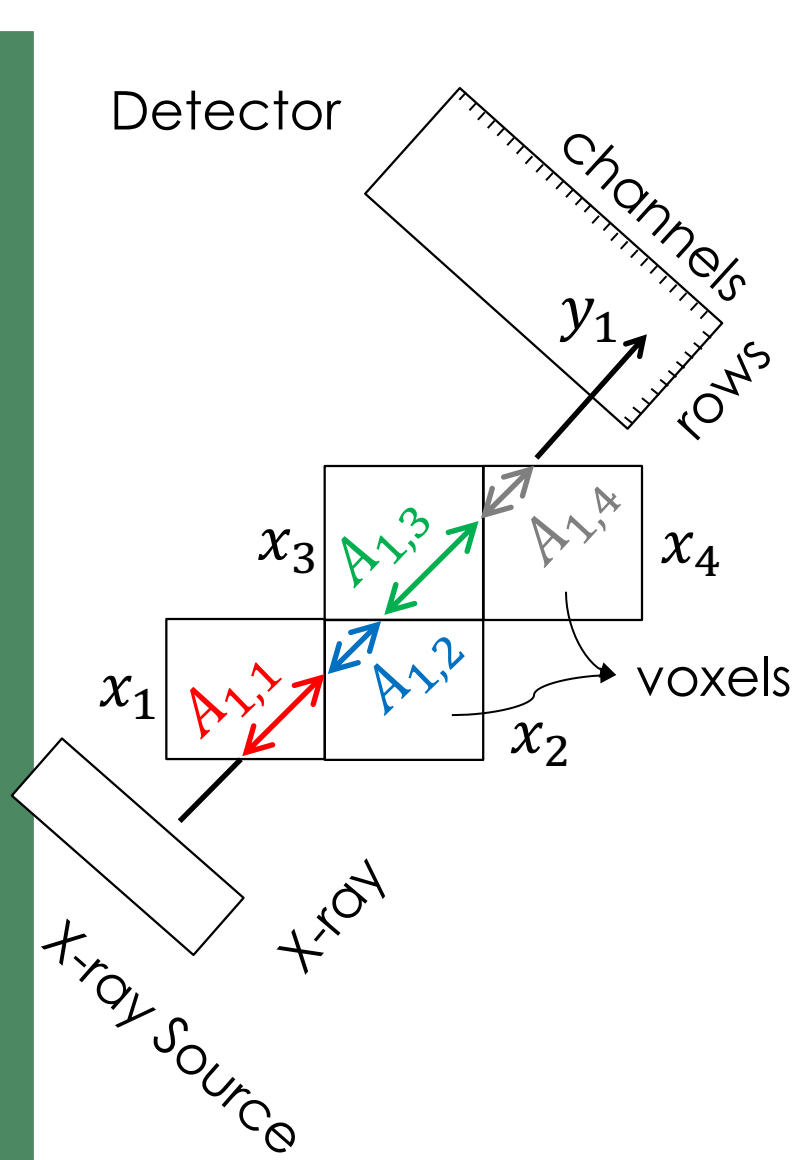
Iron Hydroxide dataset (FeOOH), imaged under Xradia 810 X-ray microscope at Air Force Research Lab.



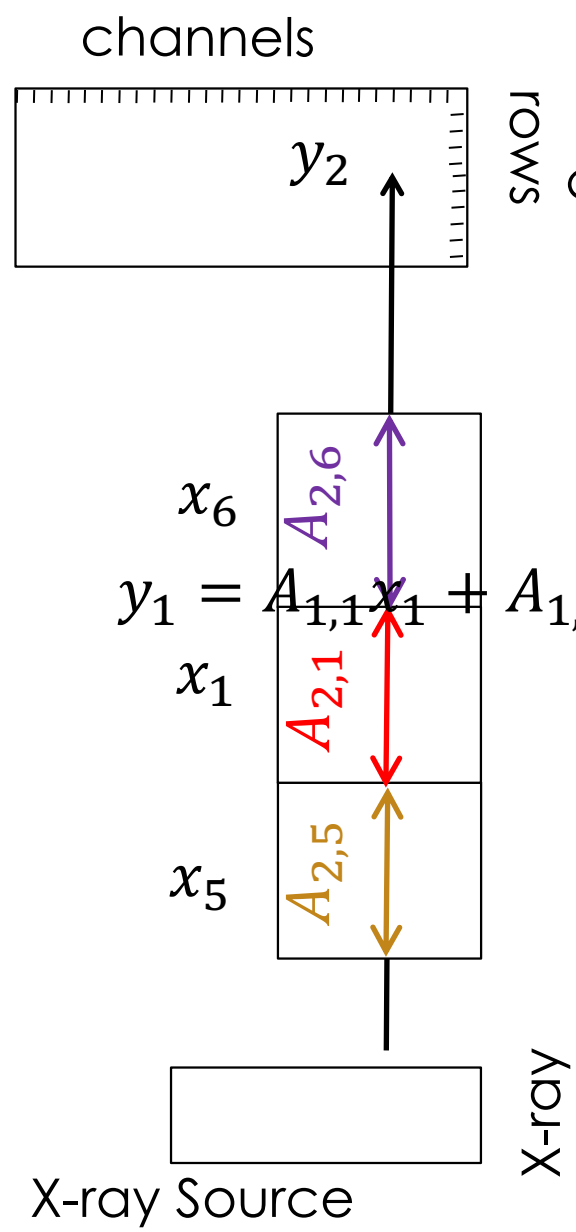
If we assume that the density is uniform within a voxel



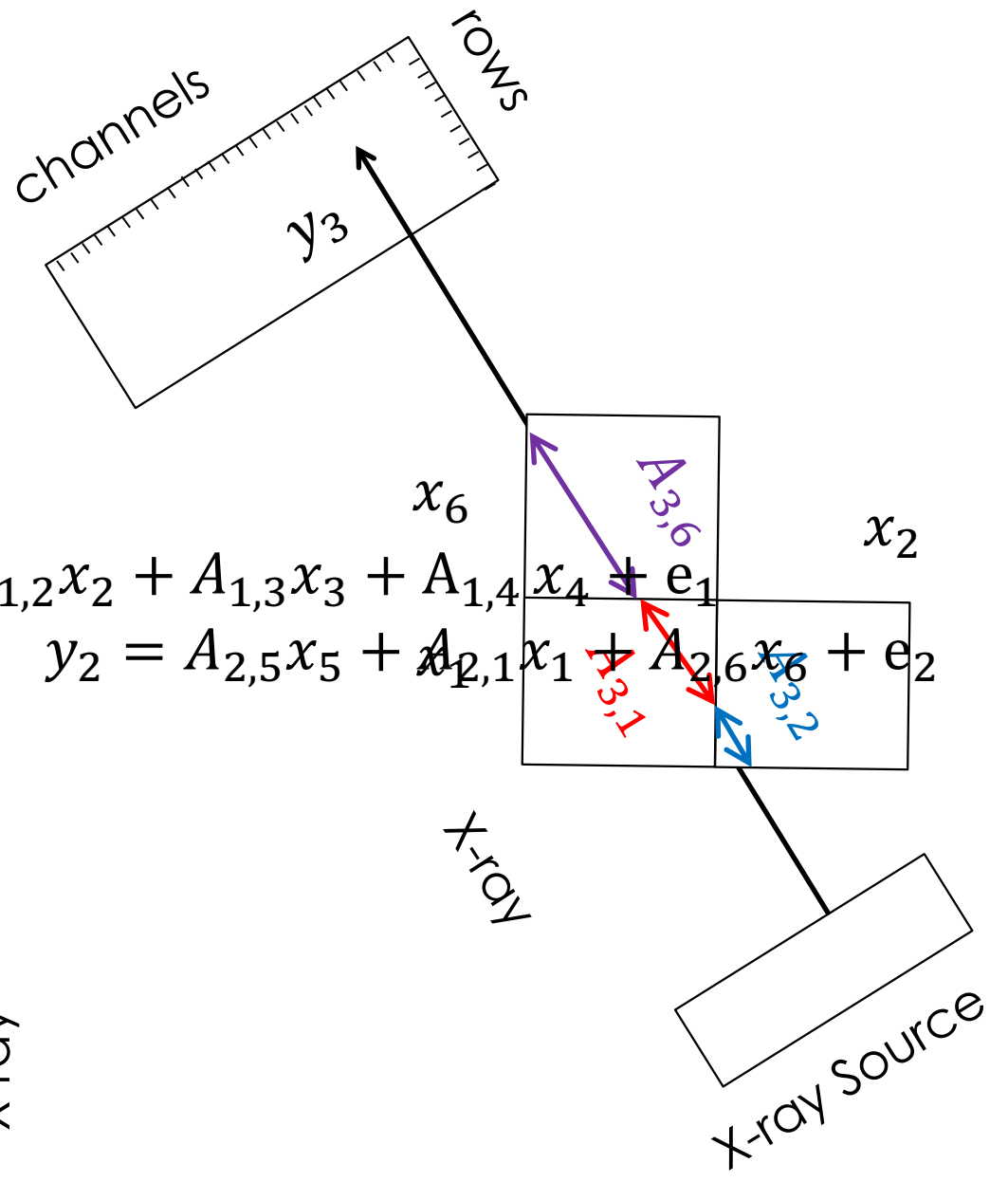
$$y_1 = A_{1,1}x_1 + A_{1,2}x_2 + A_{1,3}x_3 + A_{1,4}x_4 + e_1$$



45°



(b) 90°



(c) 135°

$$y_1 = A_{1,1}x_1 + A_{1,2}x_2 + A_{1,3}x_3 + A_{1,4}x_4 + e_1$$

$$y_2 = A_{2,5}x_5 + A_{2,1}x_1 + A_{2,6}x_6 + e_2$$

$$\begin{aligned}
 y_1 &= A_{1,1}x_1 + A_{1,2}x_2 + A_{1,3}x_3 + A_{1,4}x_4 + e_1 \\
 y_2 &= A_{2,5}x_5 + A_{2,1}x_1 + A_{2,6}x_6 + e_2 \\
 y_3 &= A_{3,2}x_2 + A_{3,1}x_1 + A_{3,6}x_6 + e_3
 \end{aligned}$$



Reconstruction

← $y = Ax + e$

↑
↑
↓

Projections from all view angles
system matrix
measurement error

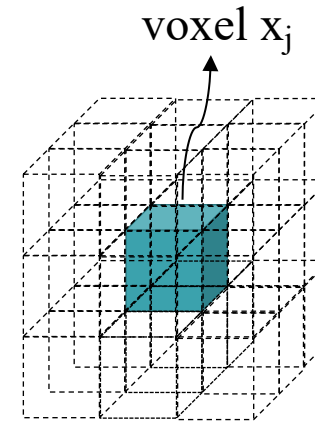
Issues:

- (1) large system matrix.
- (2) error, e , is unknown and cannot be measured.
- (3) For super high-resolution imaging, reconstruction x is undetermined with many solutions.
- (4) For low radiation dose imaging with sparse measurements, x is also undetermined

MBIR Cost Function

$$\hat{x} \leftarrow \arg \min_x \left\{ \underbrace{\frac{1}{2} \|y - Ax\|_\lambda^2}_{\text{Forward model}} + \underbrace{u(x)}_{\text{Prior model}} \right\}$$

weight matrix



26 voxels local neighborhood

- Can be understood as a machine learning problem.
- The cost function is the same for any compressive sensing inverse problem.
- No closed-form solution for the above cost function. Need numerical methods to compute the solution

Weight Matrix

$$\hat{x} \leftarrow \arg \min_x \{ \|y - Ax\|_{\Lambda}^2 + u(x) \}$$

- Λ_i represents the importance of projection y_i to the reconstruction
- $\Lambda_i = \left(\frac{1}{\sigma_{y_i}^2}\right)^p$, where $\sigma_{y_i}^2$ represents the noise variance of projection y_i
- p is between 0 and 1, default to 0.5 for medical imaging

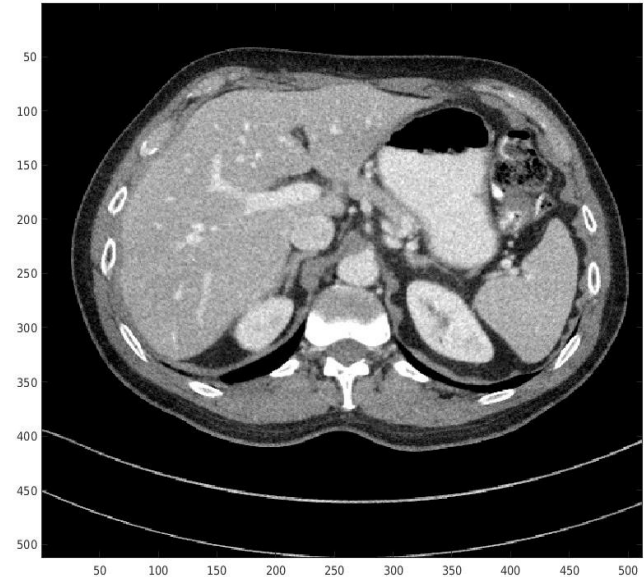
(1) control image texture uniformity

(2) Prevents projection underfitting

- $\sigma_{y_i}^2$ cannot be measured. Need approximation

$$\Lambda_i = \left(\frac{1}{\sigma_{y_i}^2}\right)^p \approx (\lambda_D)^p$$

$p = 0.5$



$p = 1$

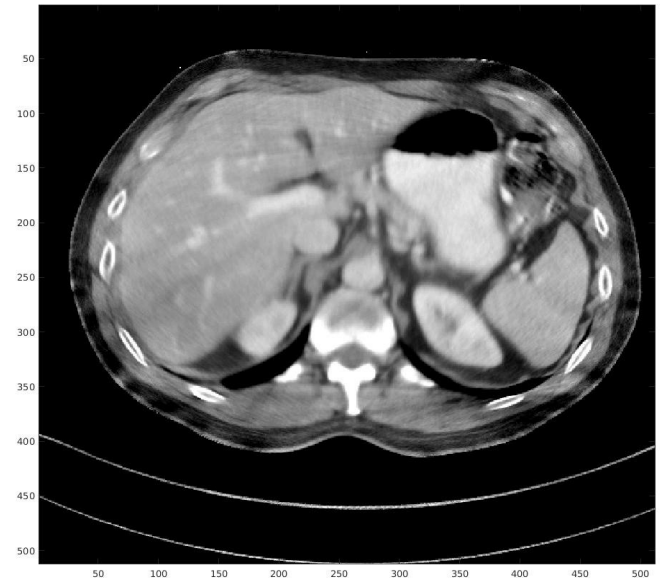
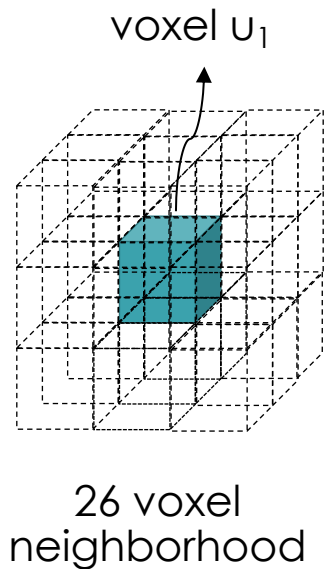


Image Prior Model

- $\hat{x} \leftarrow \arg \min_x \{ \|y - Ax\|_{\Lambda}^2 + u(x) \}$
- Conventional statistical prior model
- Deep learning prior model



phantom

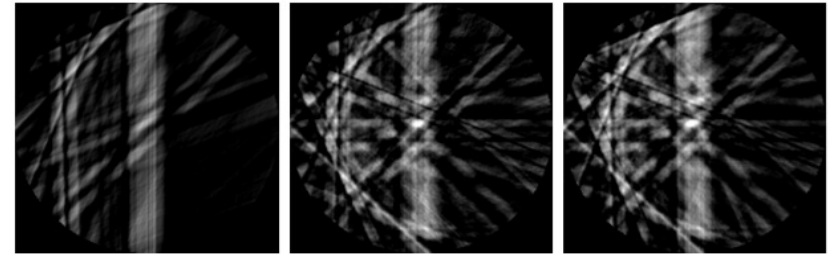
4 views

8 views

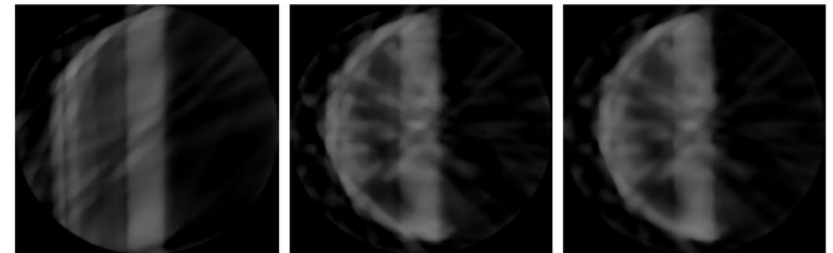
16 views



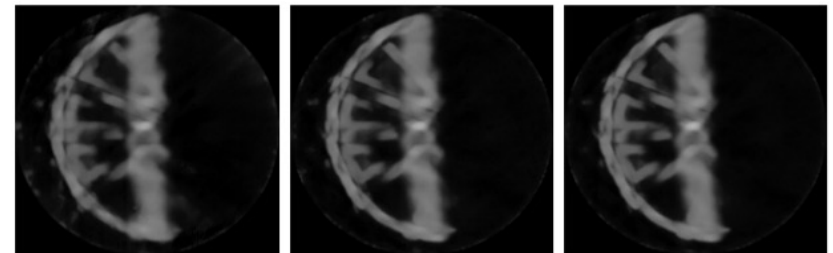
Filtered Back Projection



MBIR + statistical prior



MBIR + deep learning prior model



Summary

- Medical Imaging Modalities
- Tomography applications
- What's X-ray projection
- How the X-ray projections are computed from photon attenuation
- Dark Current Correction
- The Two Approaches for Tomography Reconstruction
- MBIR Formulation for Forward, Prior and Noise Weight Models

For future contact, email me at wangx2@ornl.gov