Lecture 7: Colorimetry

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The Roadmap

Theoretical Preliminaries

Human Visual Systems

Display and Camera

Modeling and Rendering

Overview

Color Vision

Colorimetry

Visual Adaptation

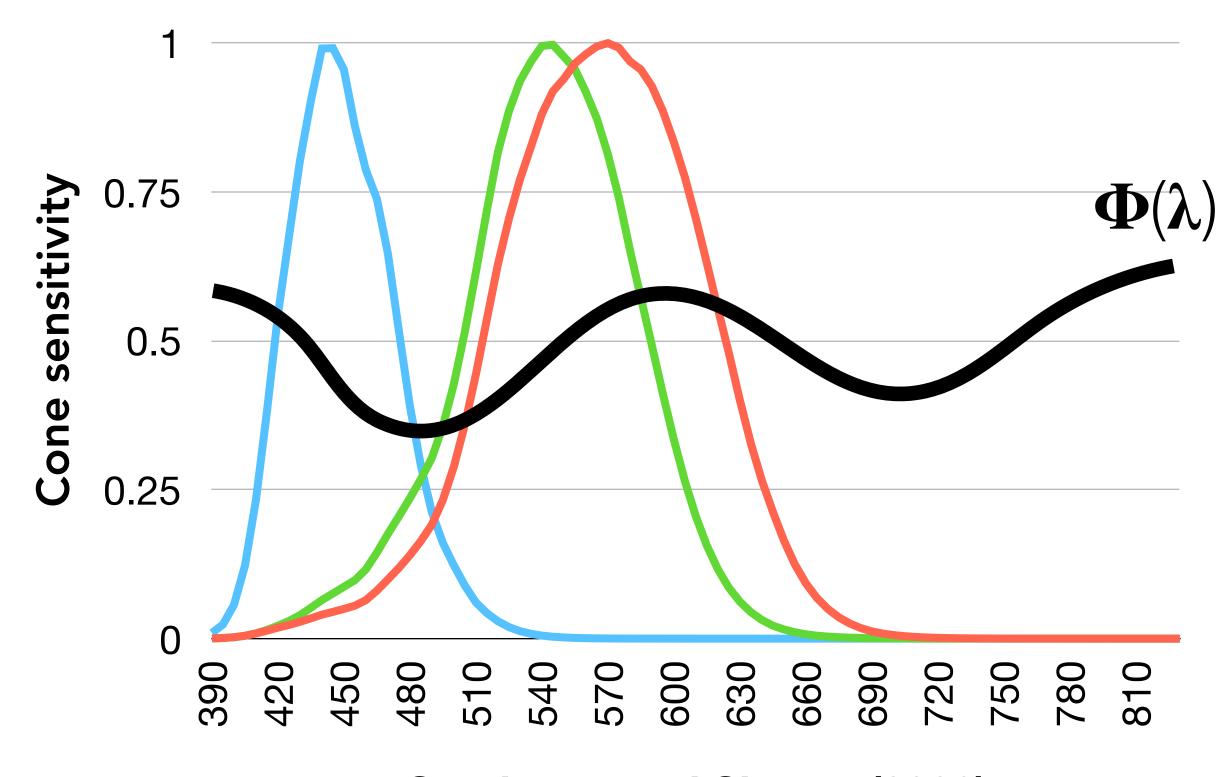
Total Cone Response

Any light could be expressed as a <L, M, S> triplet.

$$L = \sum_{\lambda=380}^{780} \Phi(\lambda)L(\lambda)$$

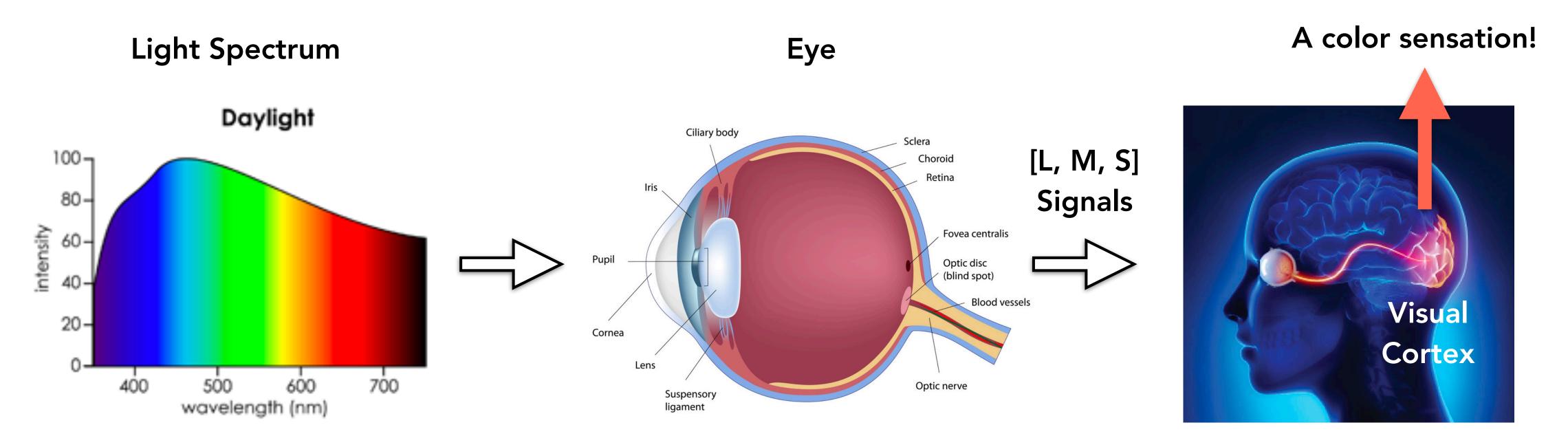
$$M = \sum_{\lambda=380}^{780} \Phi(\lambda) M(\lambda)$$

$$S = \sum_{\lambda=380}^{780} \Phi(\lambda) S(\lambda)$$



Stockman and Sharpe (2000)

Brain Receives Only the LMS Signals, not Light!

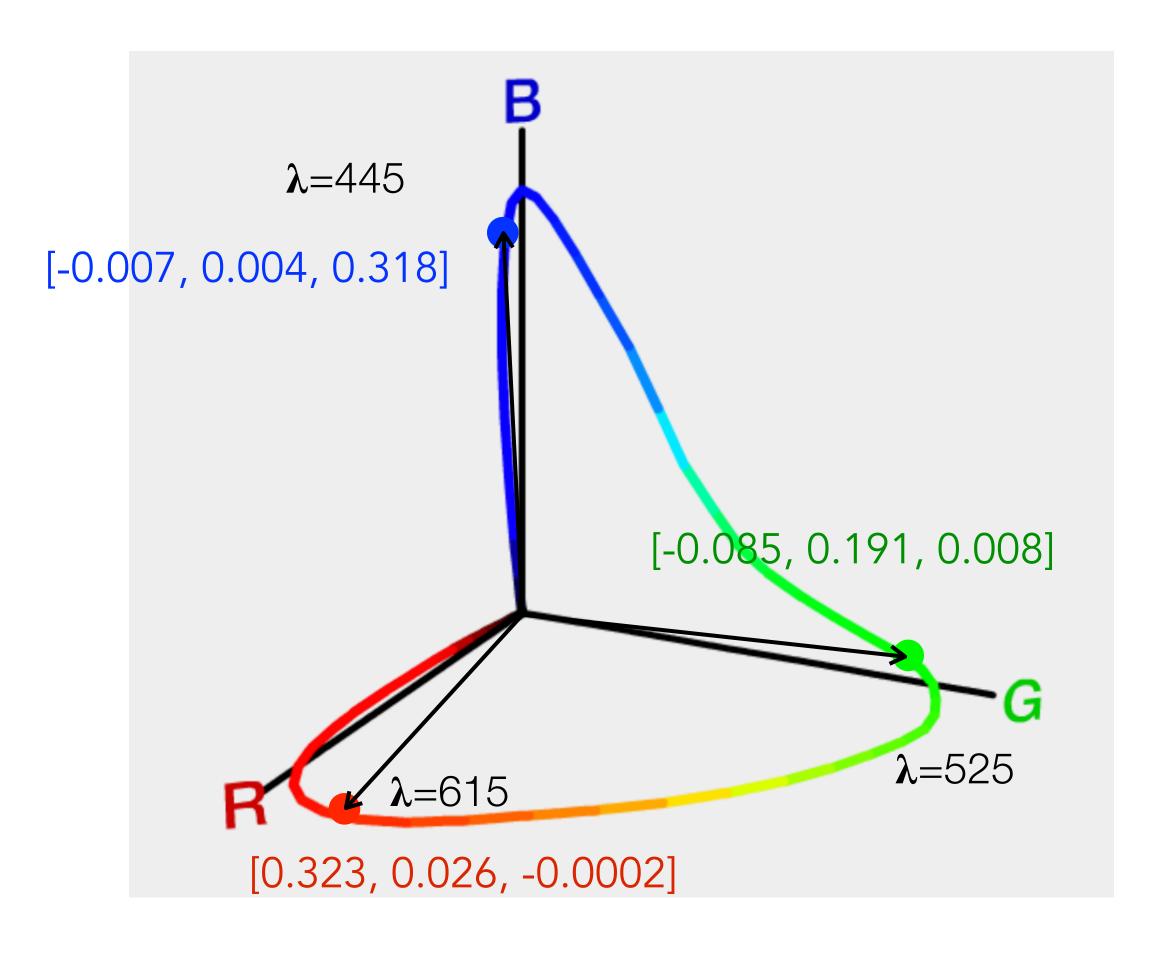


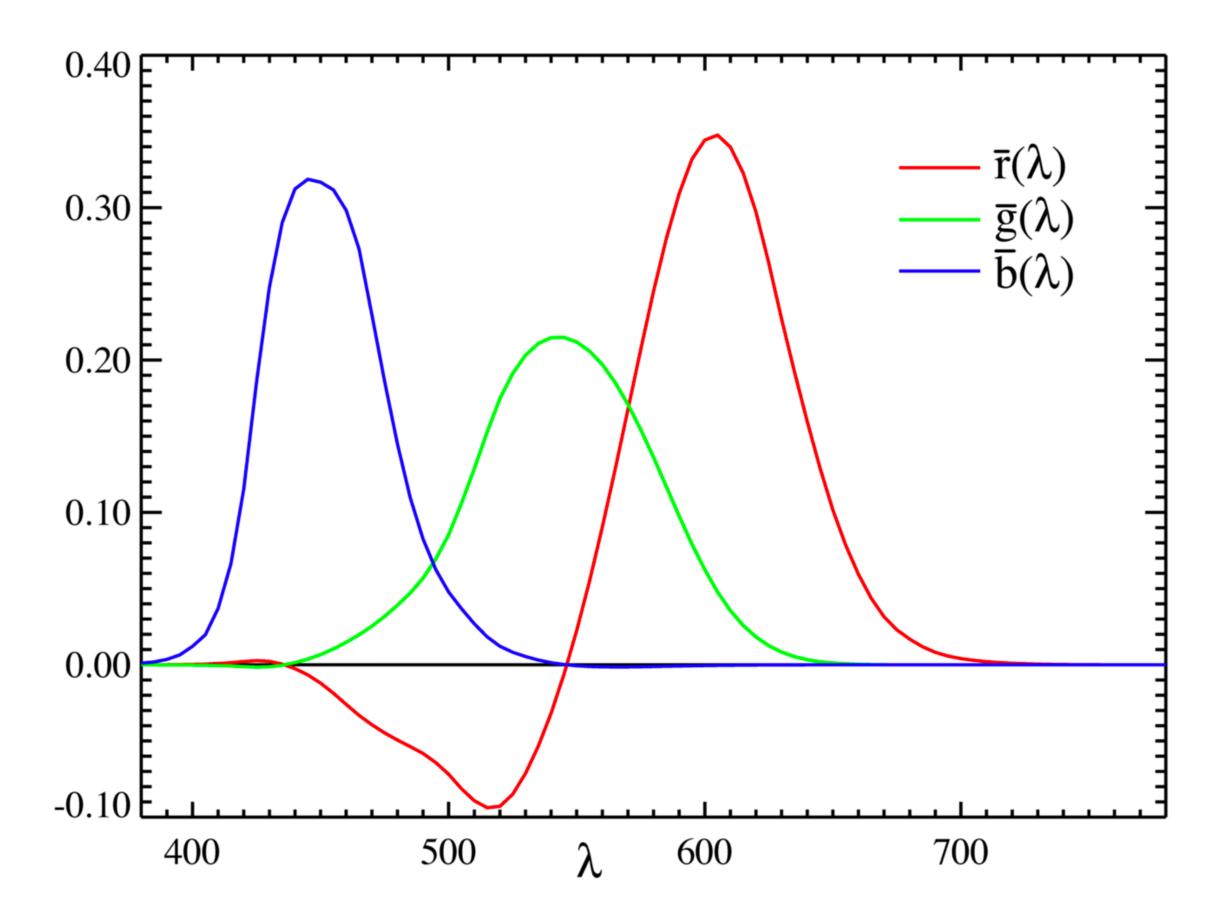
- A particular LMS combination leads to a particular color sensation.
- This is a huge dimensionality reduction: the light spectrum is a very high dimensional vector (numerous wavelengths), but is transformed by the retina to only a three-dimensional vector!

XYZ Color Space

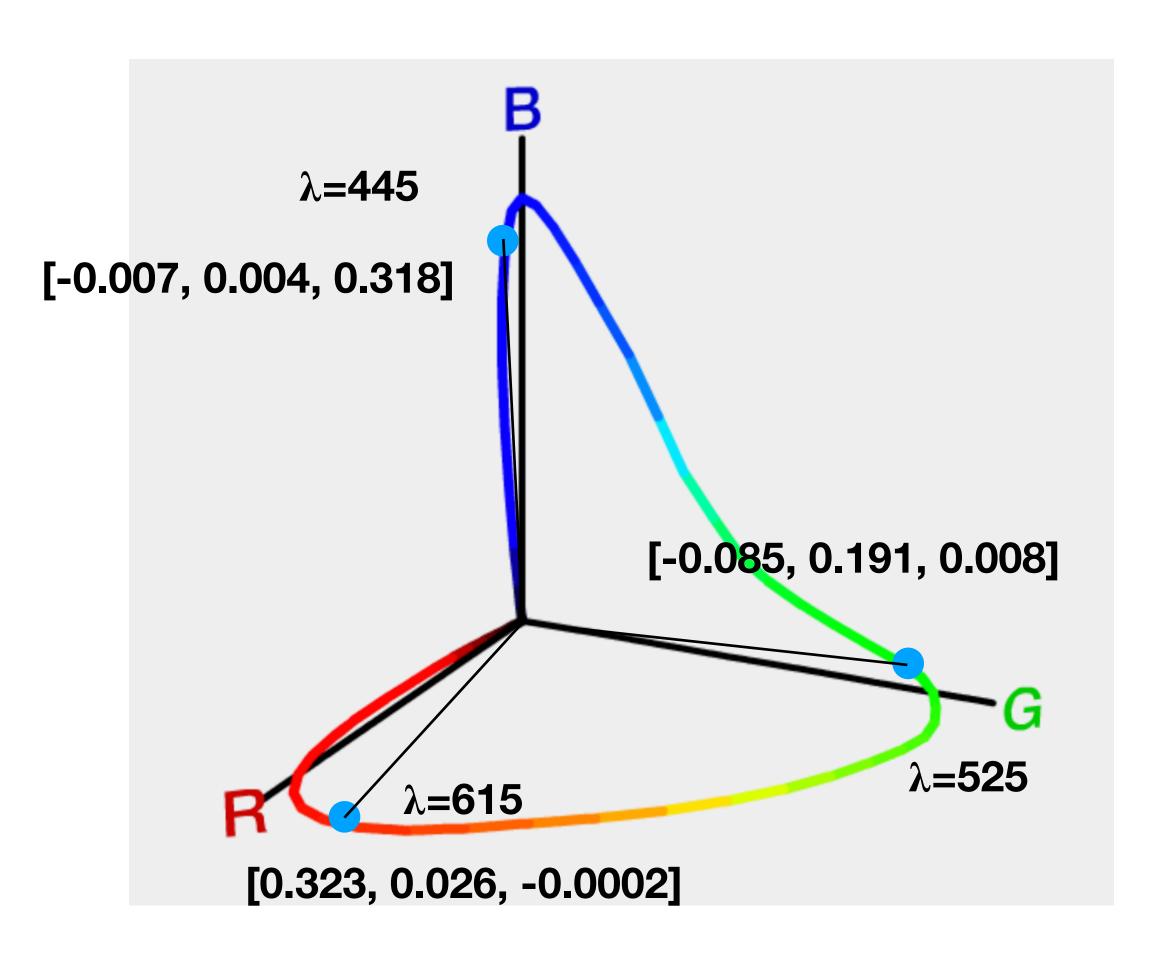
Visualization and math: https://horizon-lab.org/colorvis/xyz.html

Recall: CIE RGB Color Space





A Slight Issue with CIE RGB Color Space



"Issue": Some colors are expressed in negative RGB values, which is mathematically OK but not very intuitive.

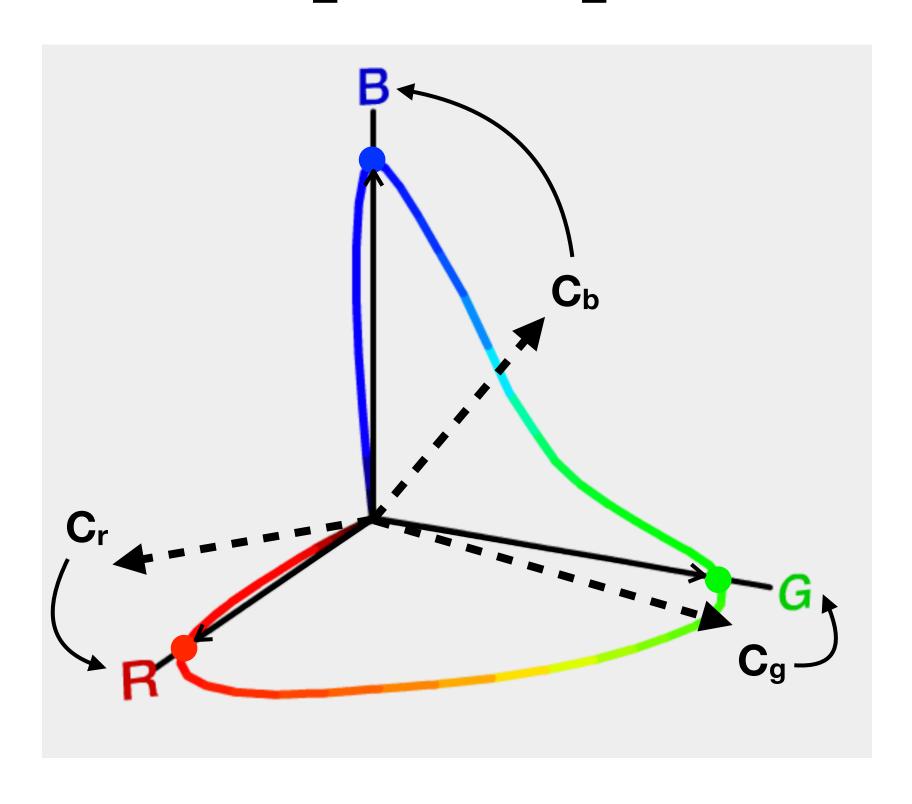
Goal: design a color space so that all colors have positive values.

How? Transforming the coordinate system so that all colors are in the positive quadrant.

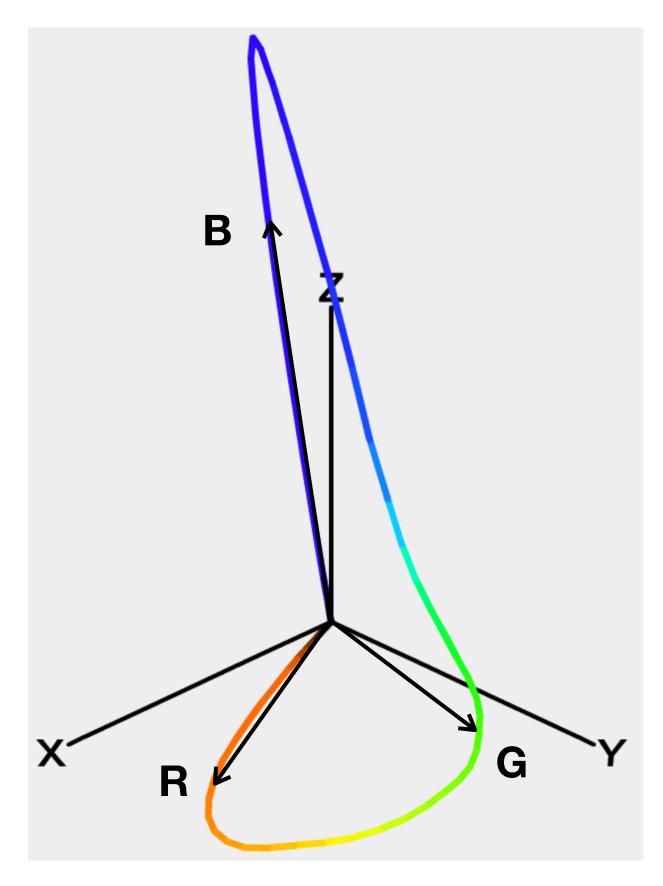
CIE RGB to XYZ Color Space Transformation

- CIE in 1931 constructed the XYZ color space for that purpose.
- Every [R, G, B] value of a color is transformed to a [X, Y, Z] value.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} R_x, G_x, B_x \\ R_y, G_y, B_y \\ R_z, G_z, B_z \end{bmatrix} \times \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$



CIE RGB to XYZ Color Space Transformation

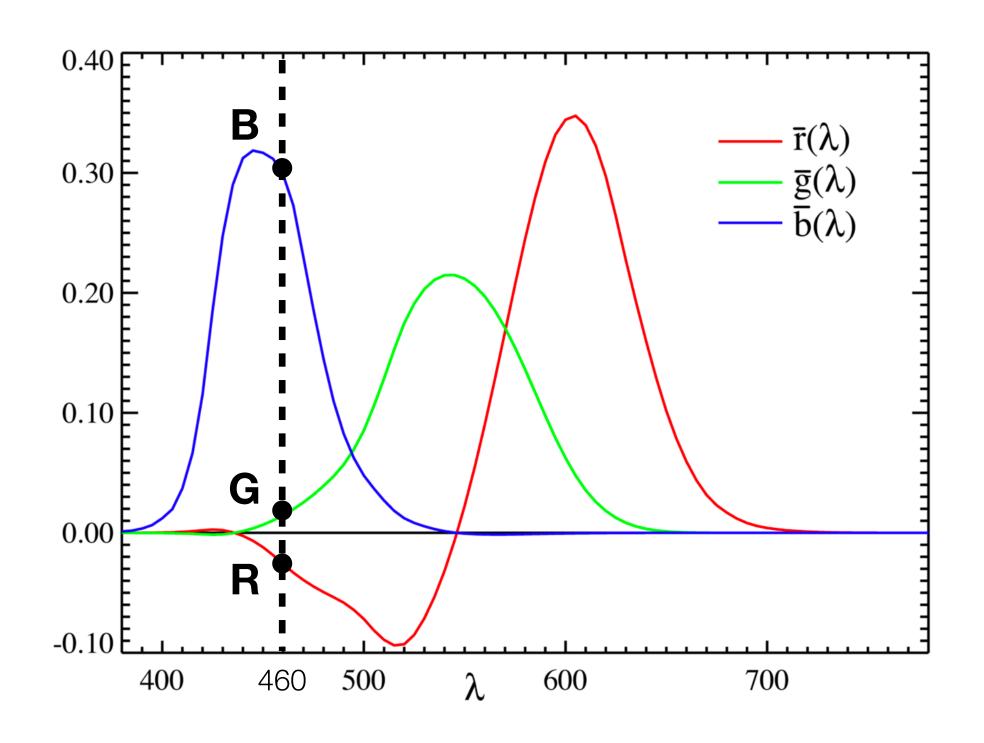


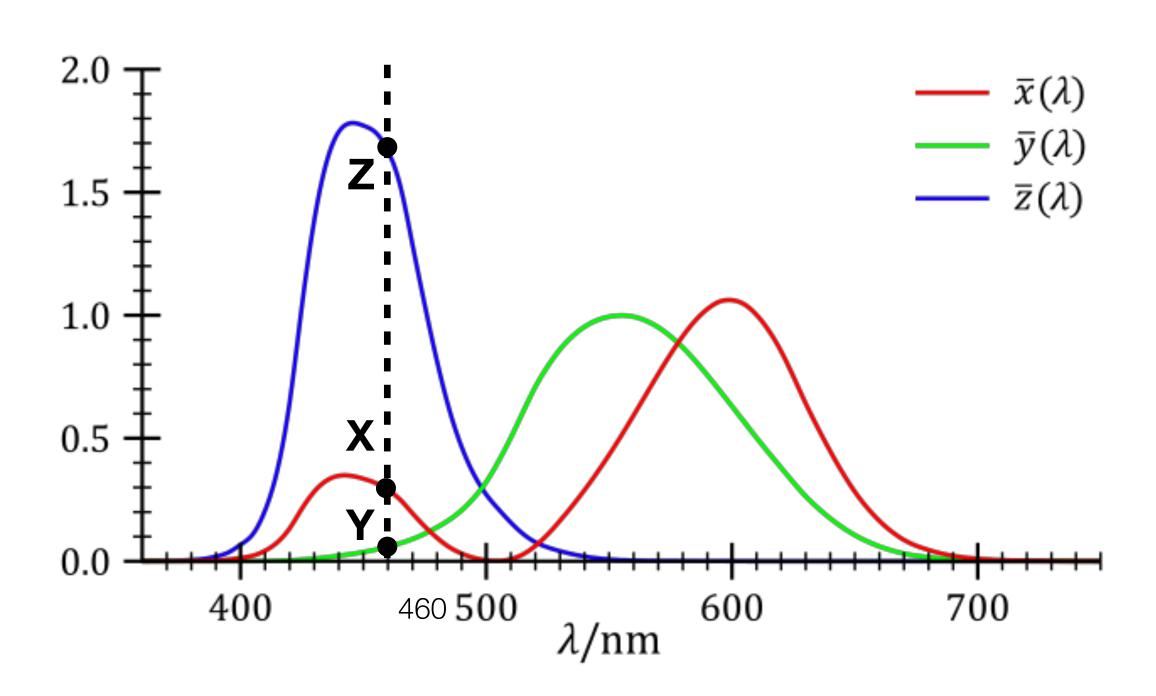
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4887180 & 0.3106803 & 0.2006017 \\ 0.1762044 & 0.8129847 & 0.0108109 \\ 0.0000000 & 0.0102048 & 0.9897952 \end{bmatrix} \times \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 2.3706743 & -0.9000405 & -0.4706338 \\ -0.5138850 & 1.4253036 & 0.0885814 \\ 0.0052982 & -0.0146949 & 1.0093968 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

XYZ to RGB transformation matrix.

XYZ Color Matching Functions





CIE 1931 RGB color matching functions

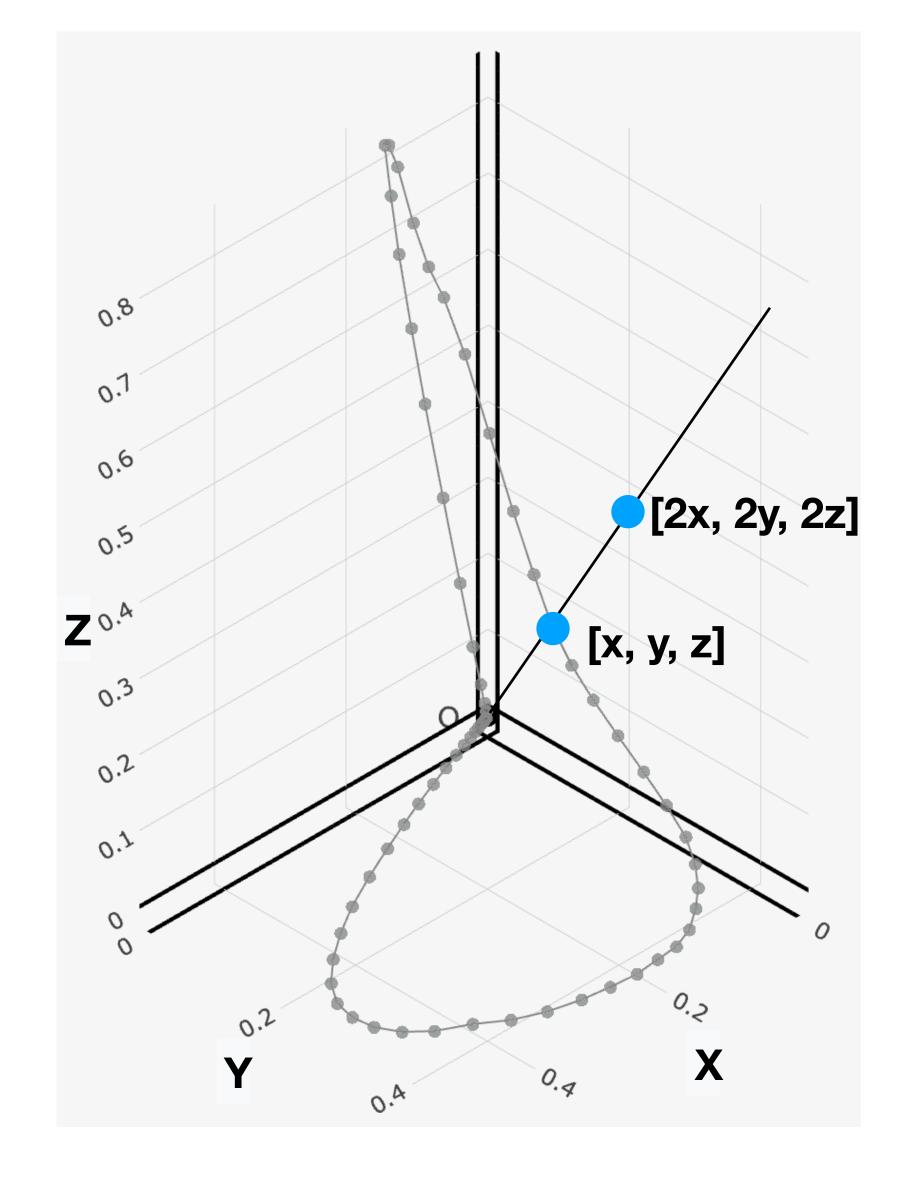
CIE 1931 XYZ color matching functions

Chromaticity and Gamut

Visualization and math: https://horizon-lab.org/colorvis/chromaticity.html

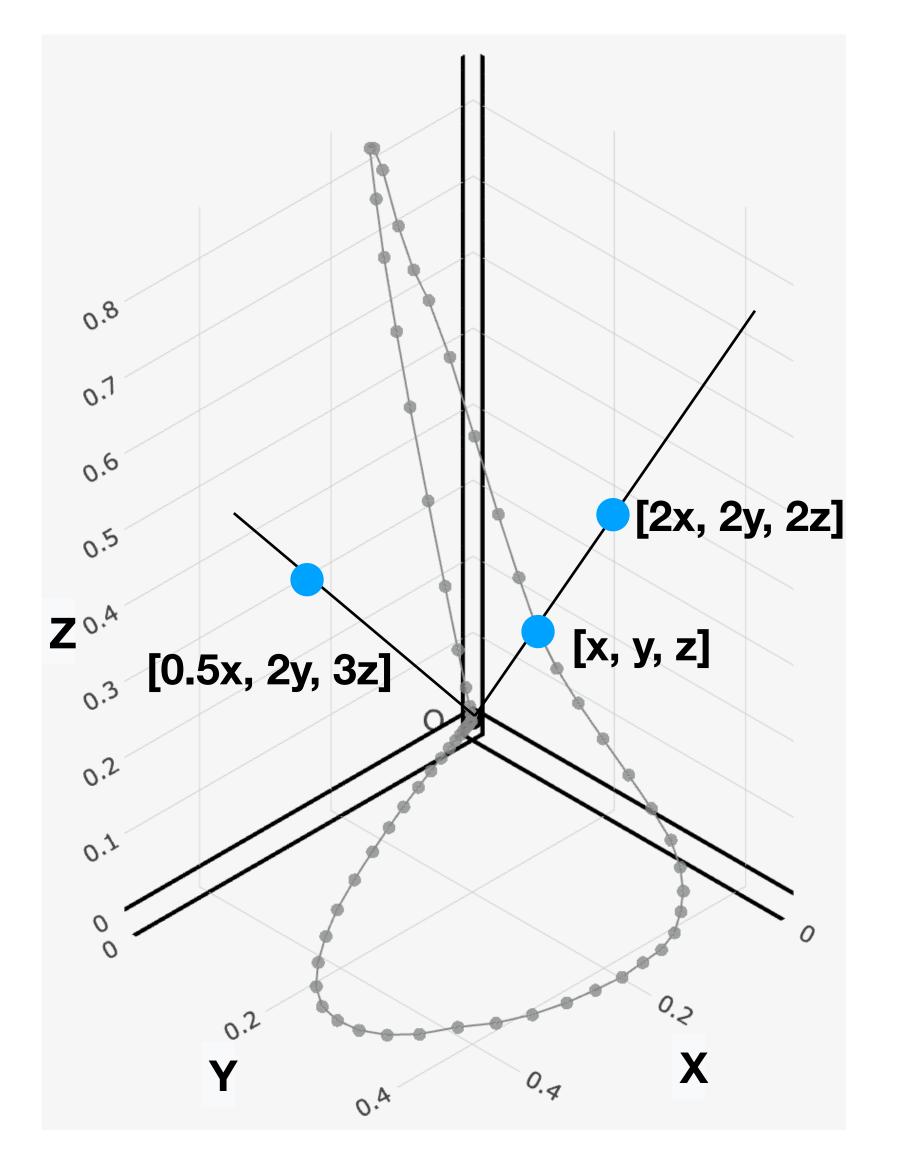
Scaling the Intensity of a Light

- If we double the power of a light, intuitively its "color" doesn't change.
 It just becomes brighter.
- Doubling the power of a color [x, y, z] gets us another color [2x, 2y, 2z]. Both colors have the same ratio of the primary contributions. They lie on the same line that goes through the color space origin.



Separating Intensity From "True" Color

- Another color, say, [0.5x, 2y, 3z], has different primary contributions, and is not simply scaled from the color [x, y, z].
- Goal: separate the objective quantity (intensity) and the quality of a color.



Chromaticity

- Normalize XYZ to xyz.
- xyz are called the chromaticity, the "objective quality" of the color.
 - Can extract chromaticity in any color space by the same calculations.
- It's just a geometric transformation from [X, Y, Z] to [x, y, z]. What kind of transformation is this??

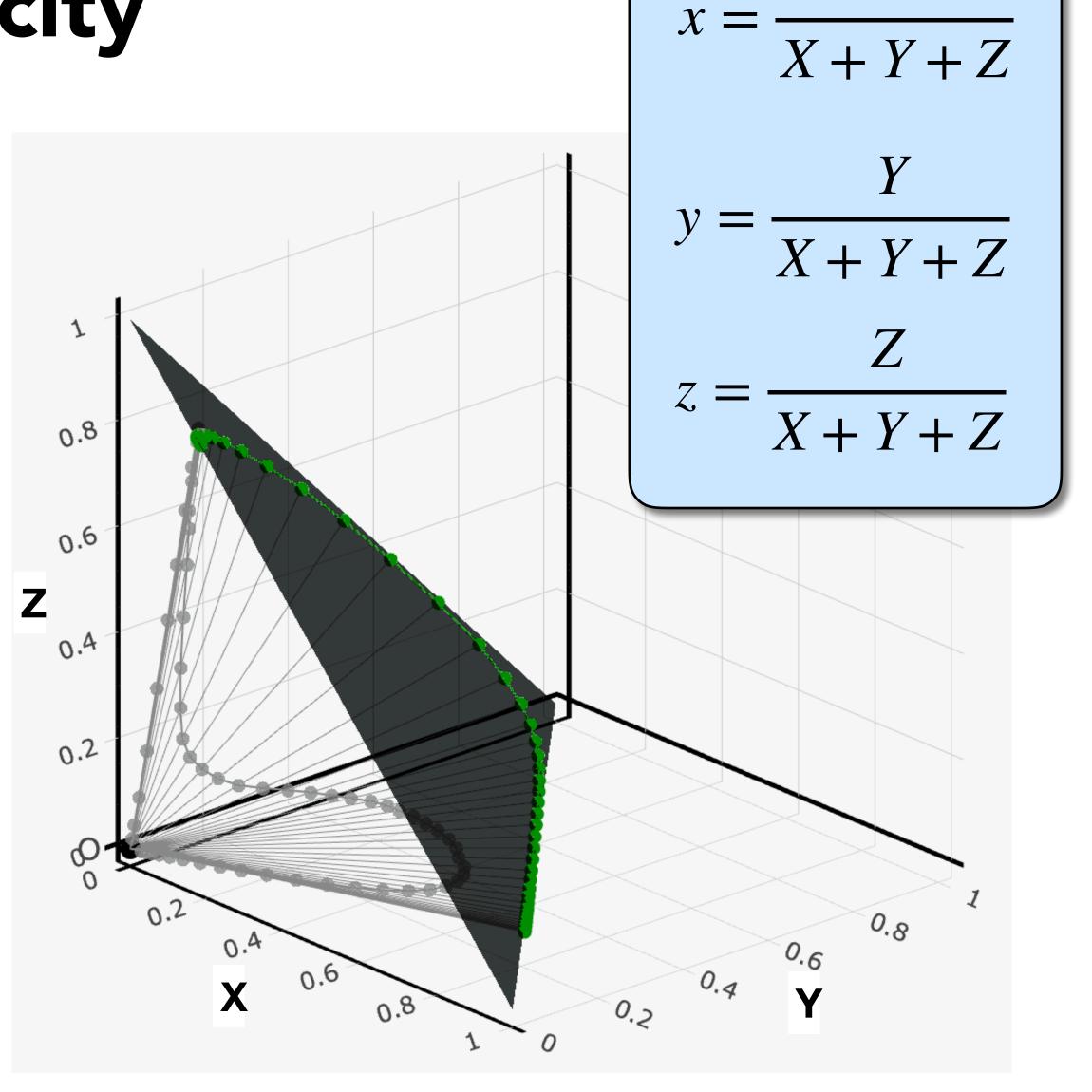
$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

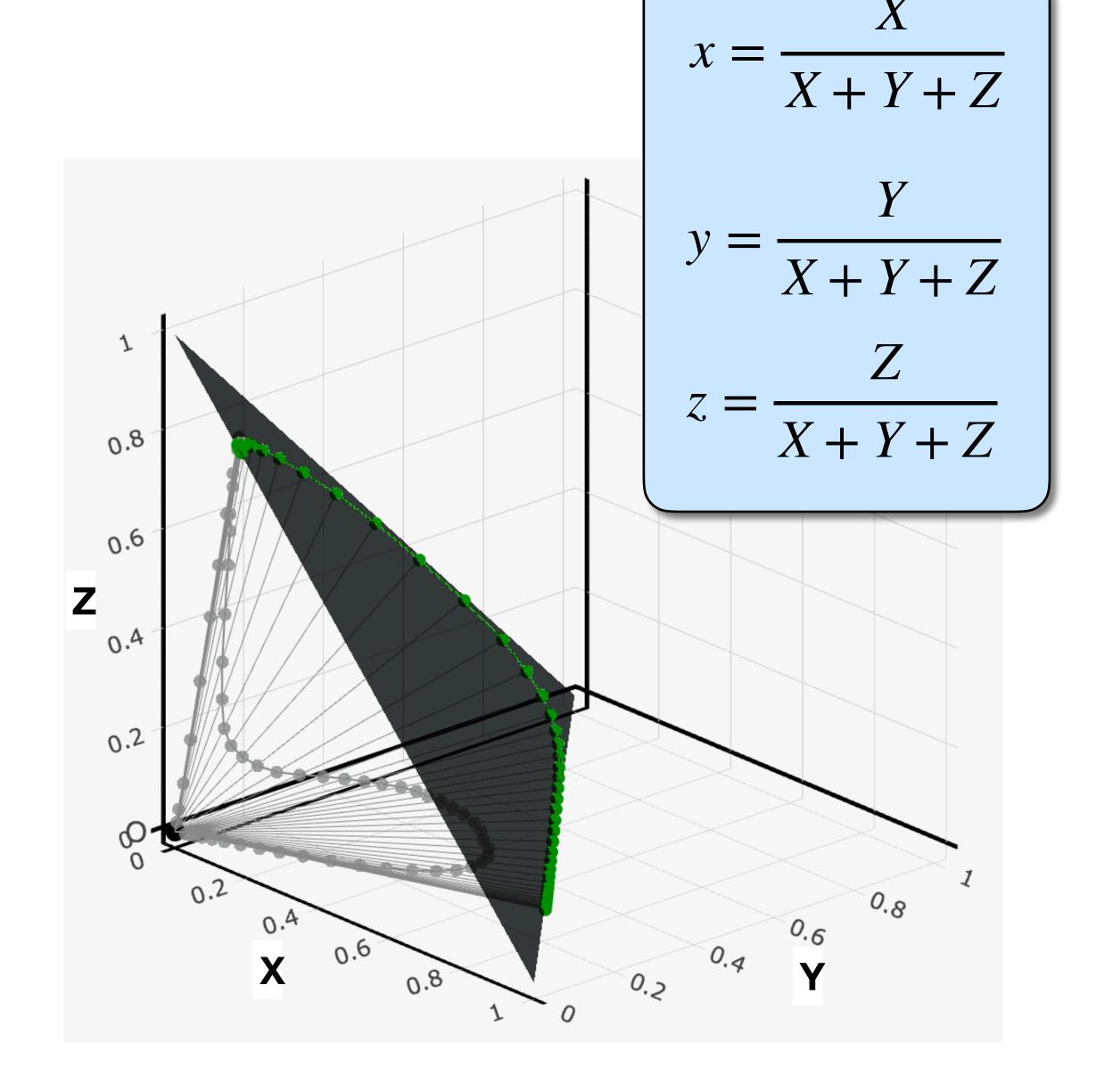
The Geometry of Chromaticity

 We've effectively projected all the colors onto the same plane using what's called the perspective projection, which is exactly what a (pinhole) camera does.

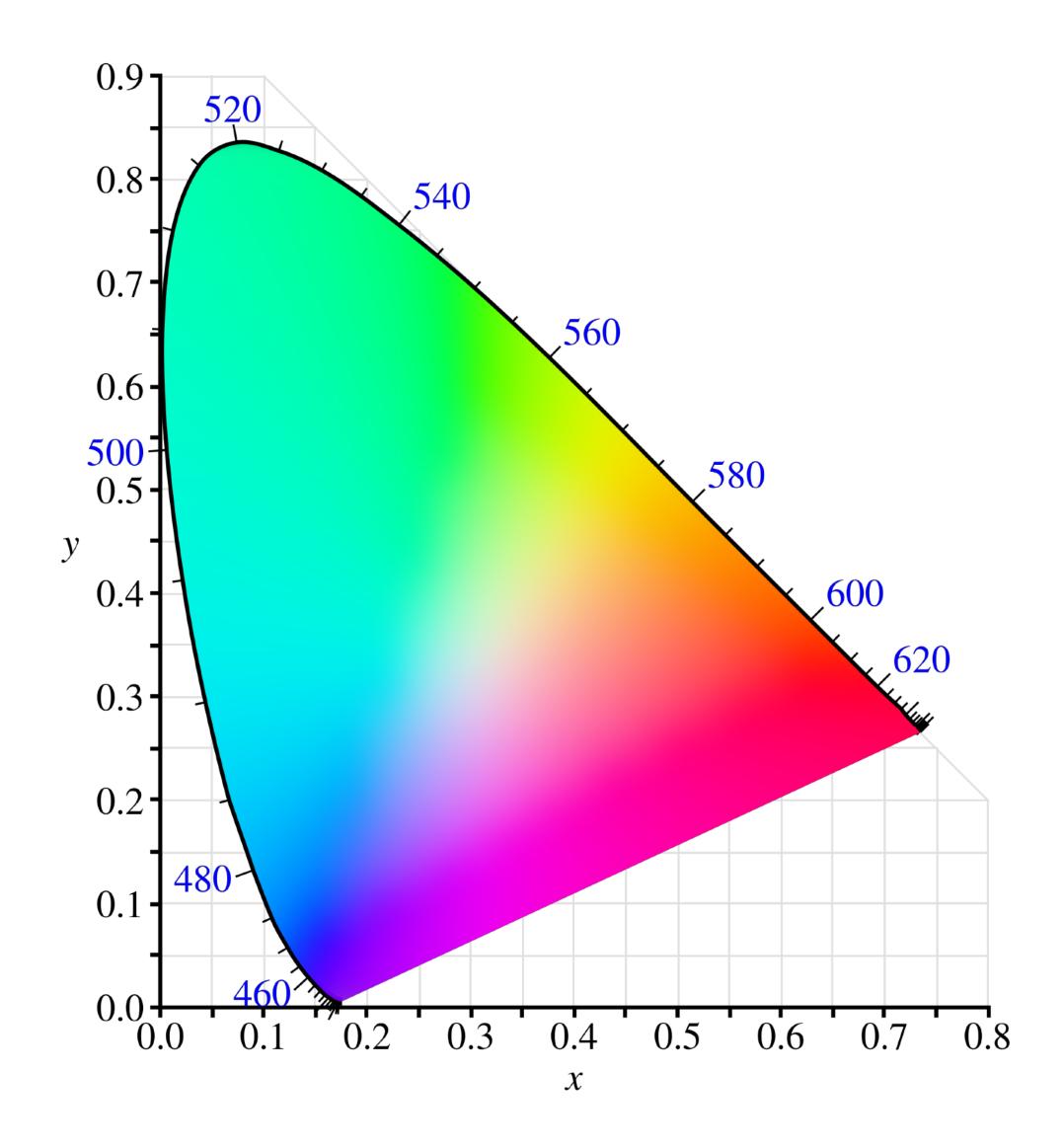


Two Properties

- 1. Any color that has the same X:Y:Z ratio will be mapped to the same point.
 - What points will have the same X:Y:Z ratio?
- 2. Given that x+y+z=1, all the transformed points are now on the same plane.



xy-Chromaticity Plot



Linear transformation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = T_{LMS2XYZ} \times \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

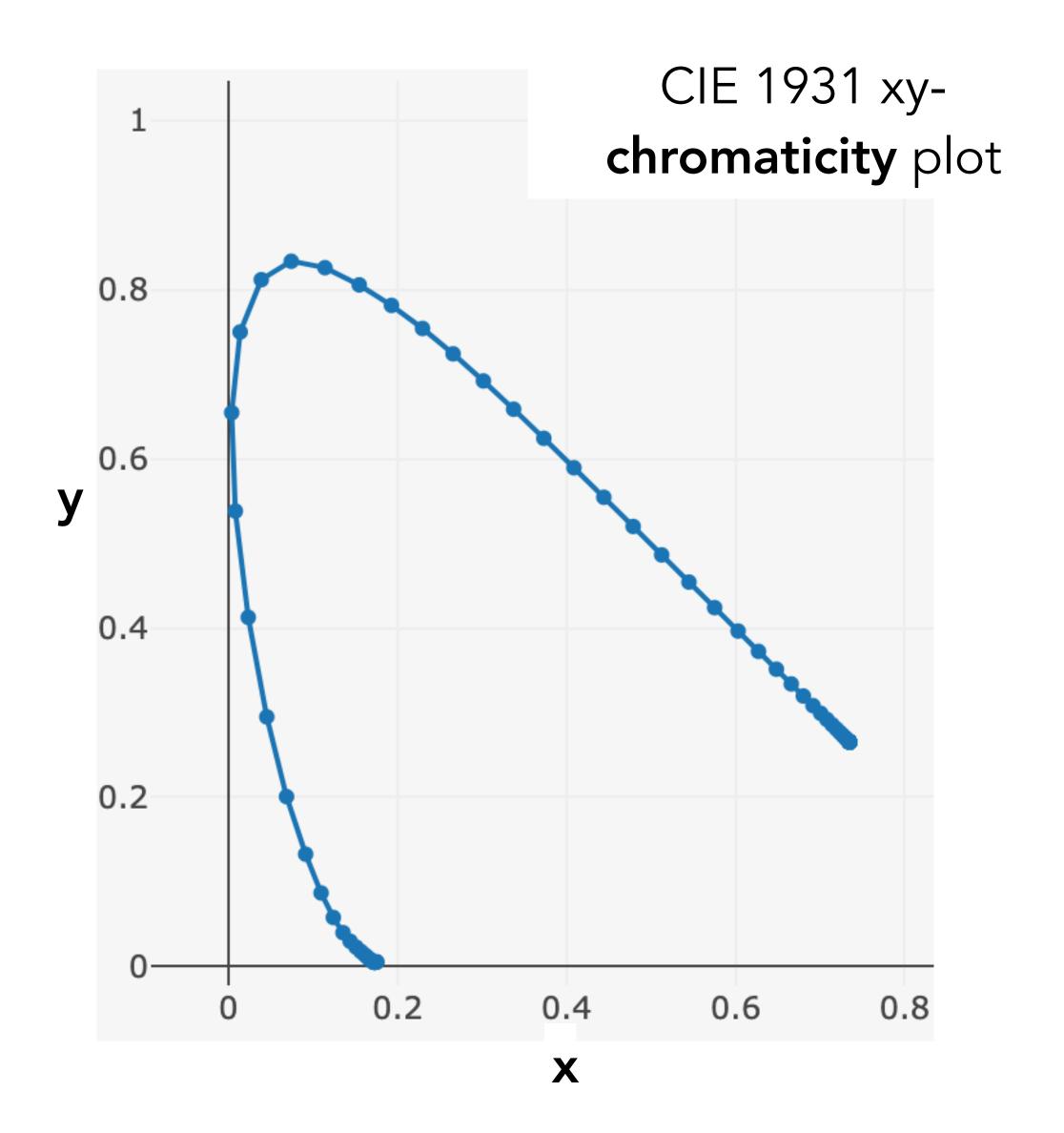
Perspective projection through [0, 0, 0] to the X+Y+Z=1 plane

$$\begin{cases} x = \frac{X}{X + Y + Z} \end{cases}$$

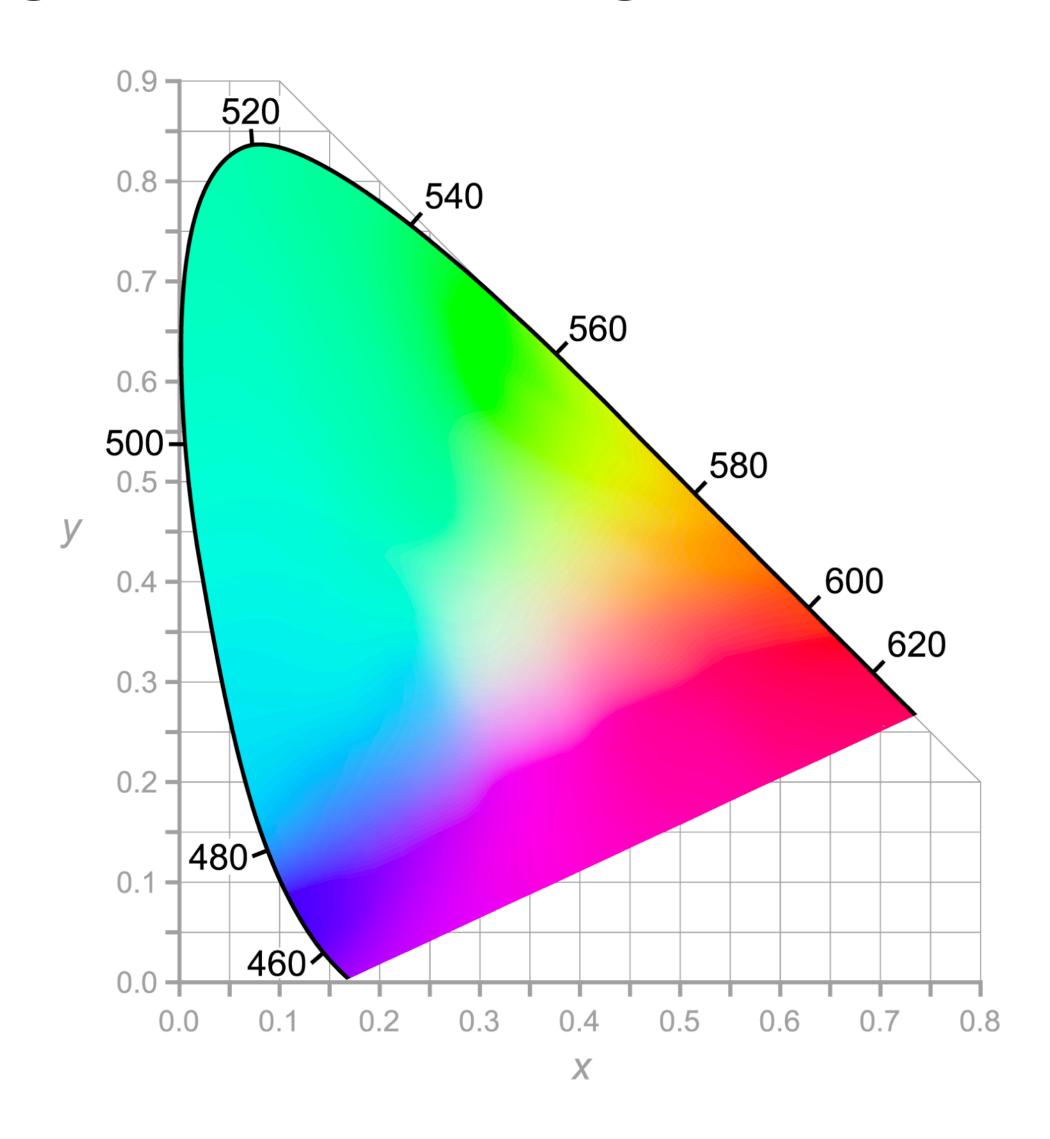
$$y = \frac{Y}{X + Y + Z}$$

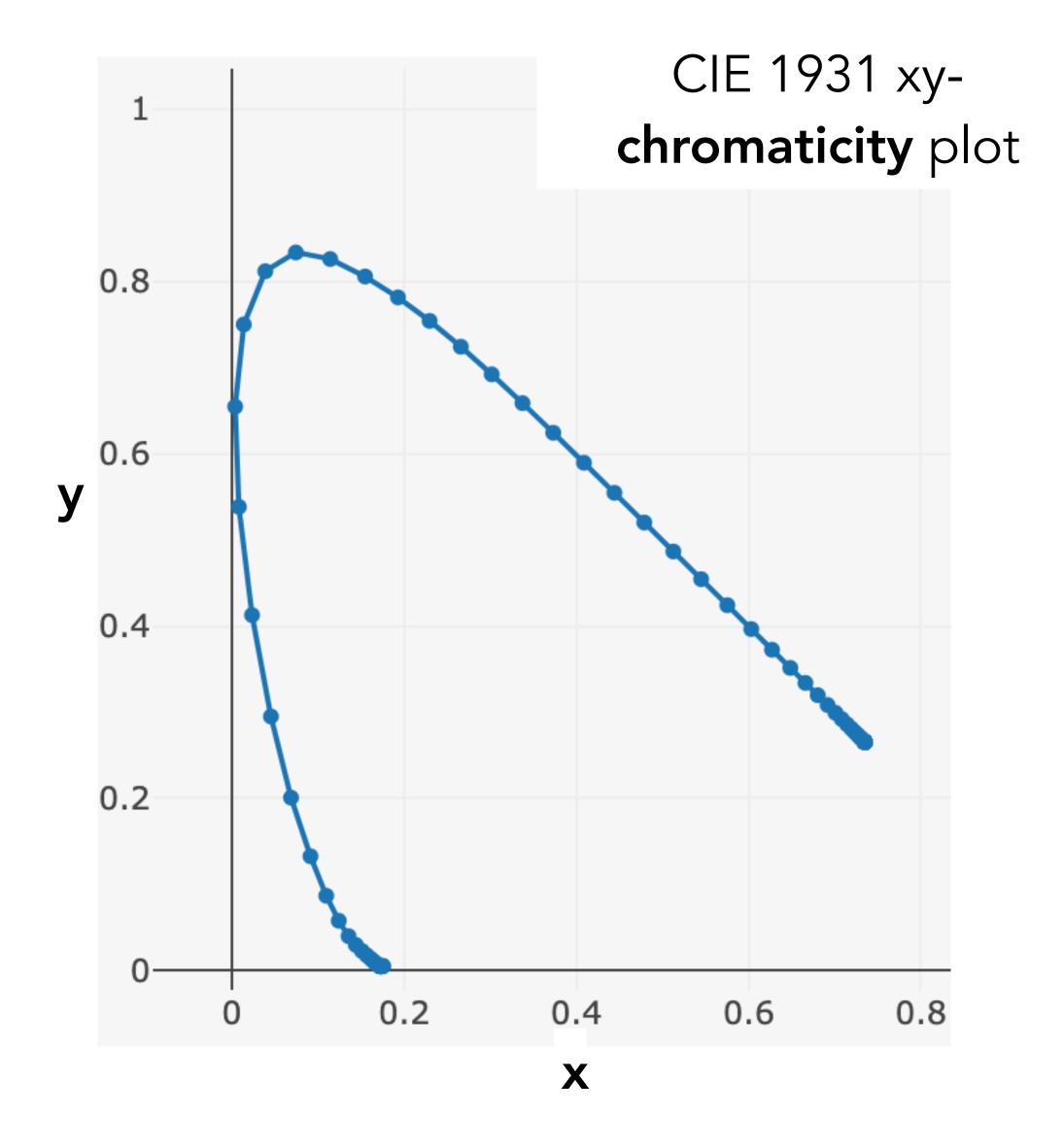
xy-Chromaticity Plot

- Since x+y+z=1, we need only two components to represent a color. That is, we can represent colors in a 2D space (except we lose the intensity information).
- Plot the locus in a 2D xy plot. z is implicit (i.e., z = 1 x y).



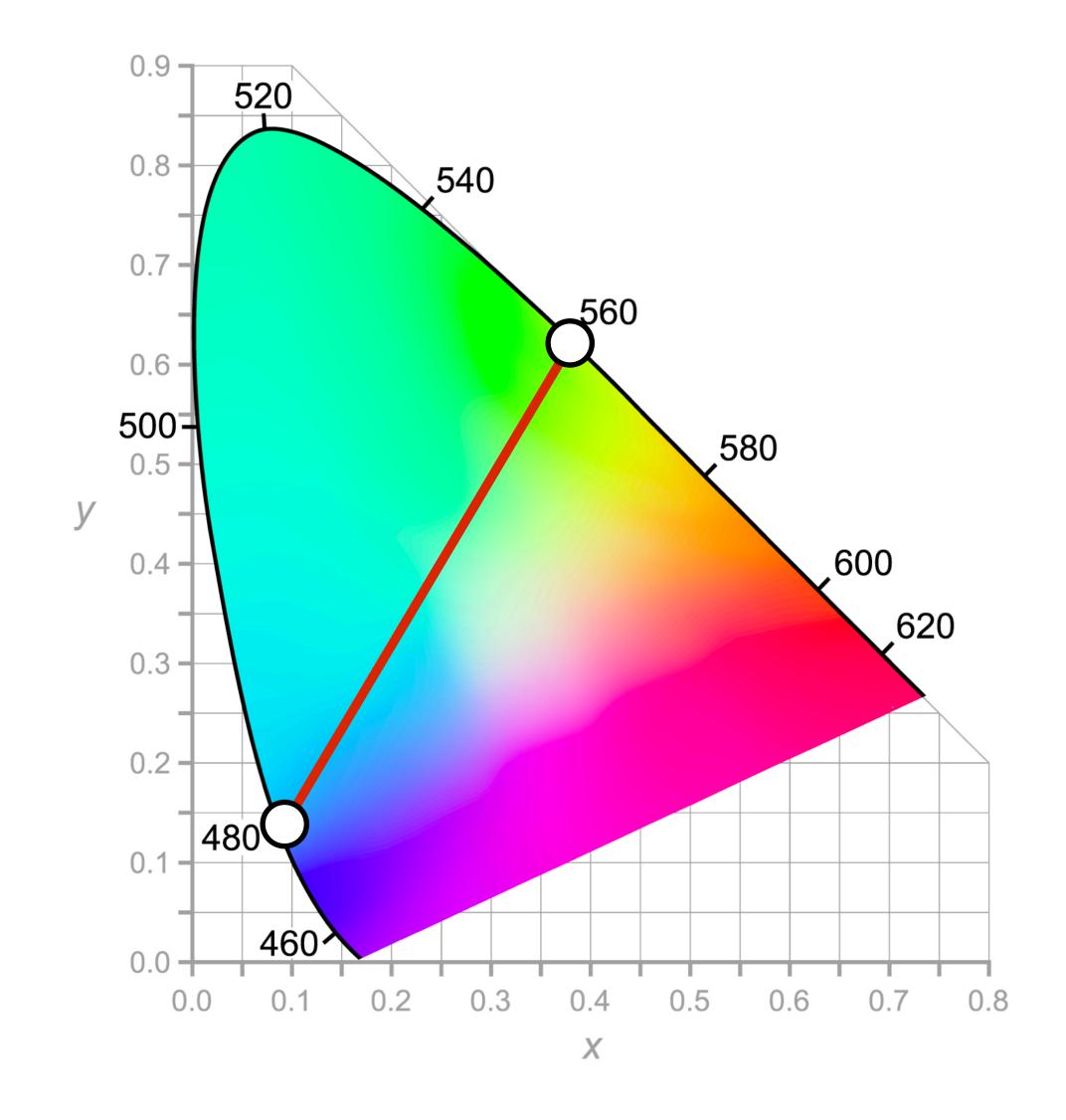
xy-Chromaticity Plot





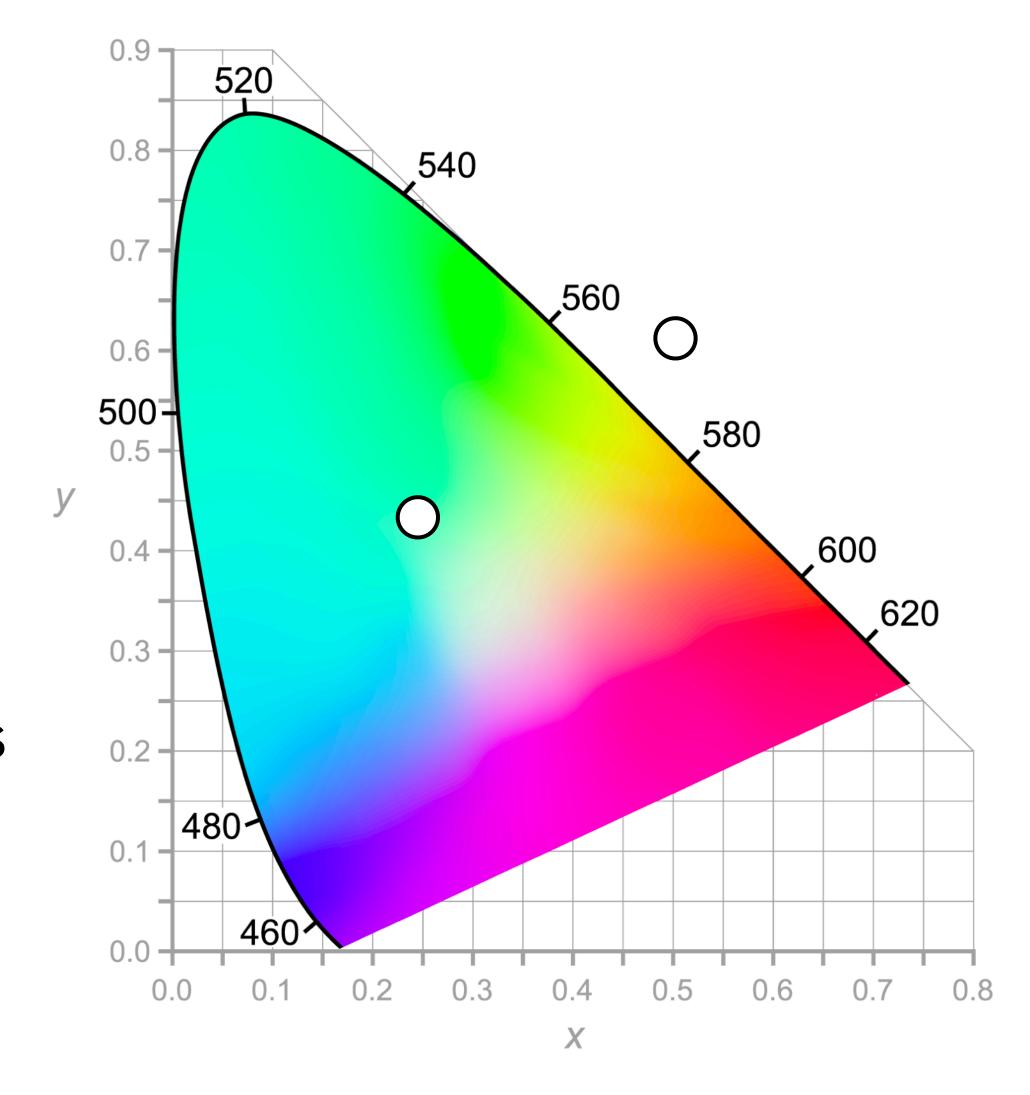
Interpreting the xy-Chromaticity Plot

- The curve is the spectral locus.
- Points on locus: spectral lights.
- Connect two points on the locus: all the colors that are can be produced from mixing those two spectral lights.
 - Because any point on a line segment is a
 positive linear combination of the two end
 points.



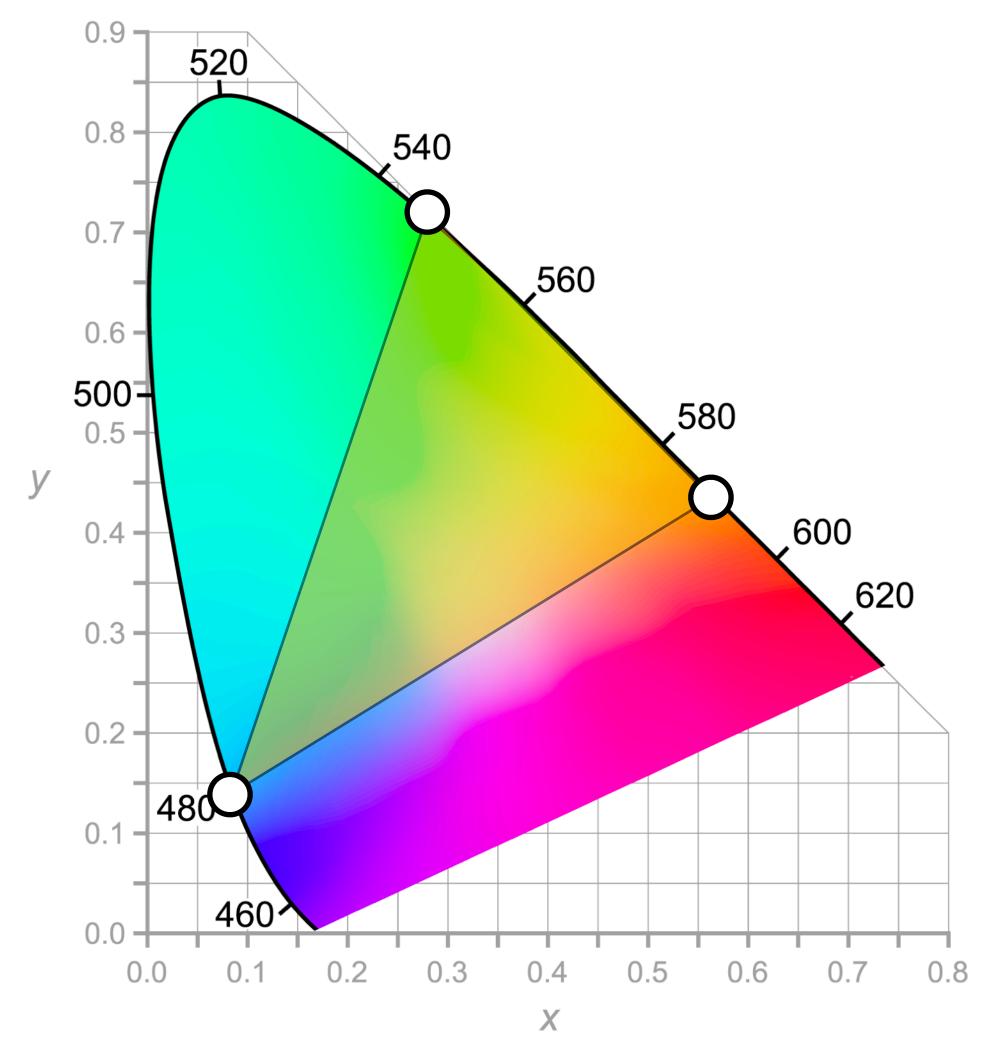
Interpreting the xy-Chromaticity Plot

- Points inside locus: colors of nonspectral lights. They can be produced from some combination of other lights.
- Points outside locus are imaginary colors; can't be produced from any combination of real lights.
- The area enclosed by the spectral locus represents the gamut of human visual system — all the colors we can see.



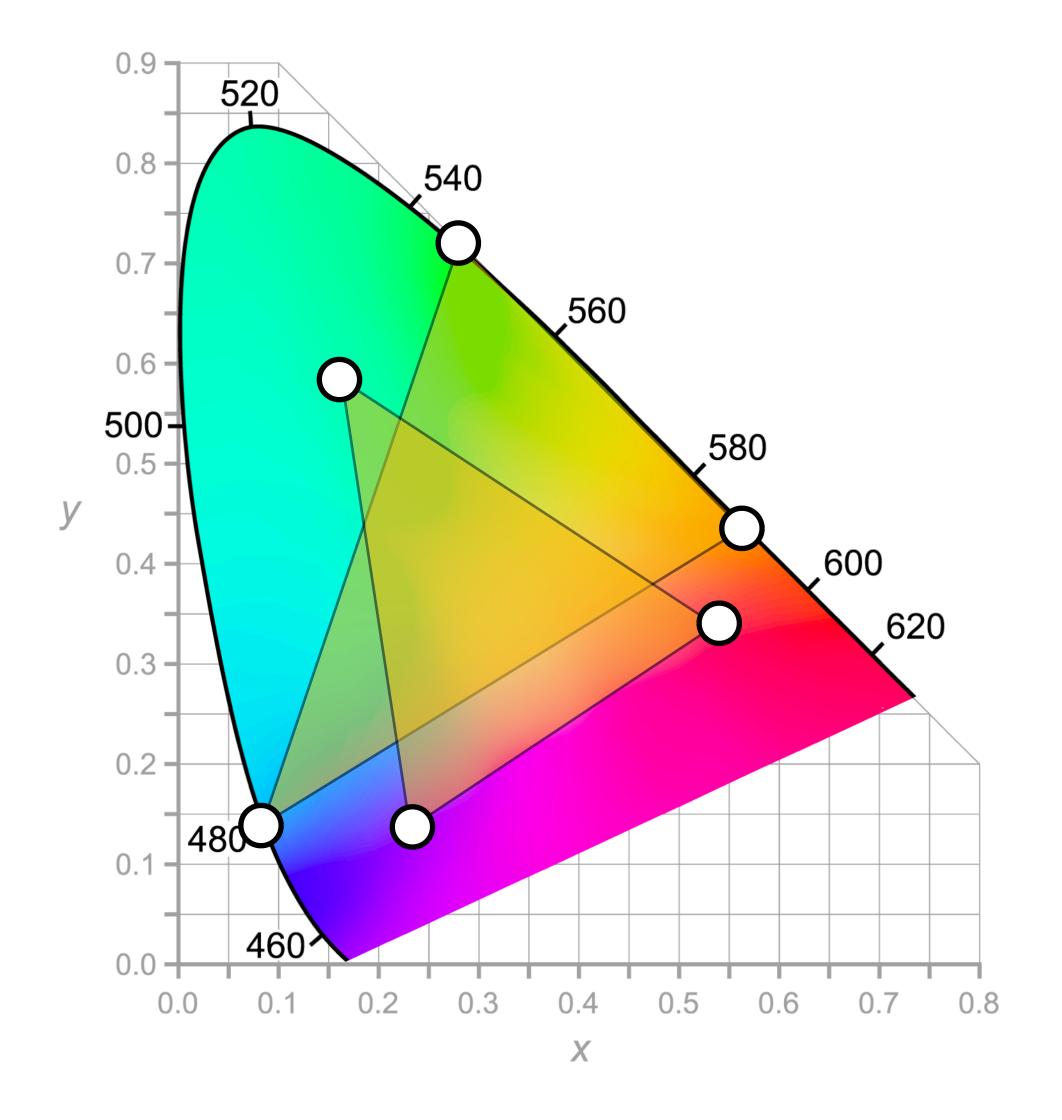
Representing a Color Space in xy-Chromaticity Plot

- Pick three points on or inside the locus as the primary lights.
- A point inside the triangle is a color that can be physically produced by mixing the chosen primary lights.
 - Because every point inside a triangle can be expressed as a **positive** linear combination of the vertices.



Representing a Color Space in xy-Chromaticity Plot

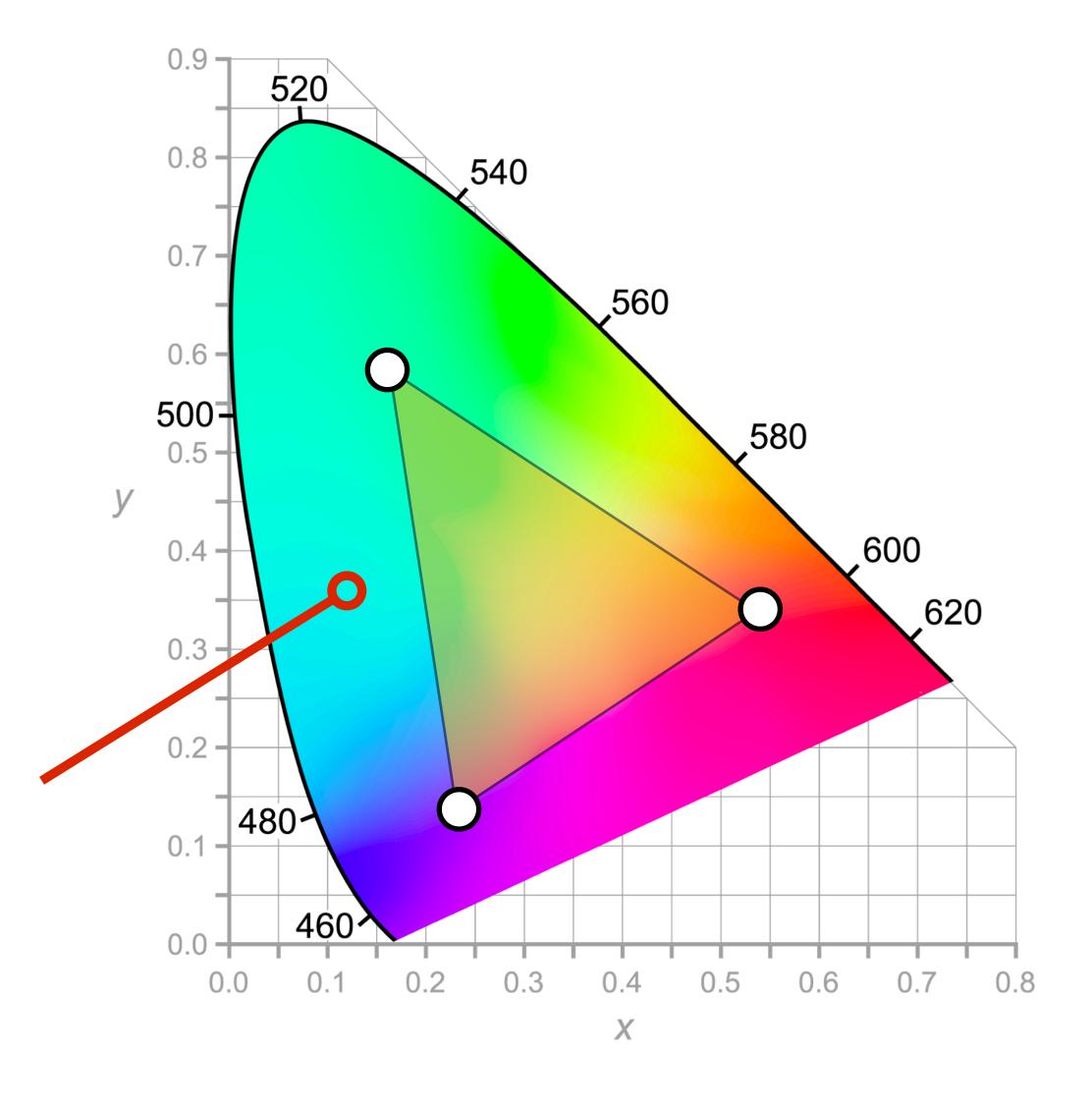
- Another way to put it: the three chosen primaries form a color space, and the triangle represents the color gamut of the color space.
 - The actual gamut of a color space is 3D (considering the luminance dimension)
- Primary lights need not be spectral lights.



Gamut of a Color Space

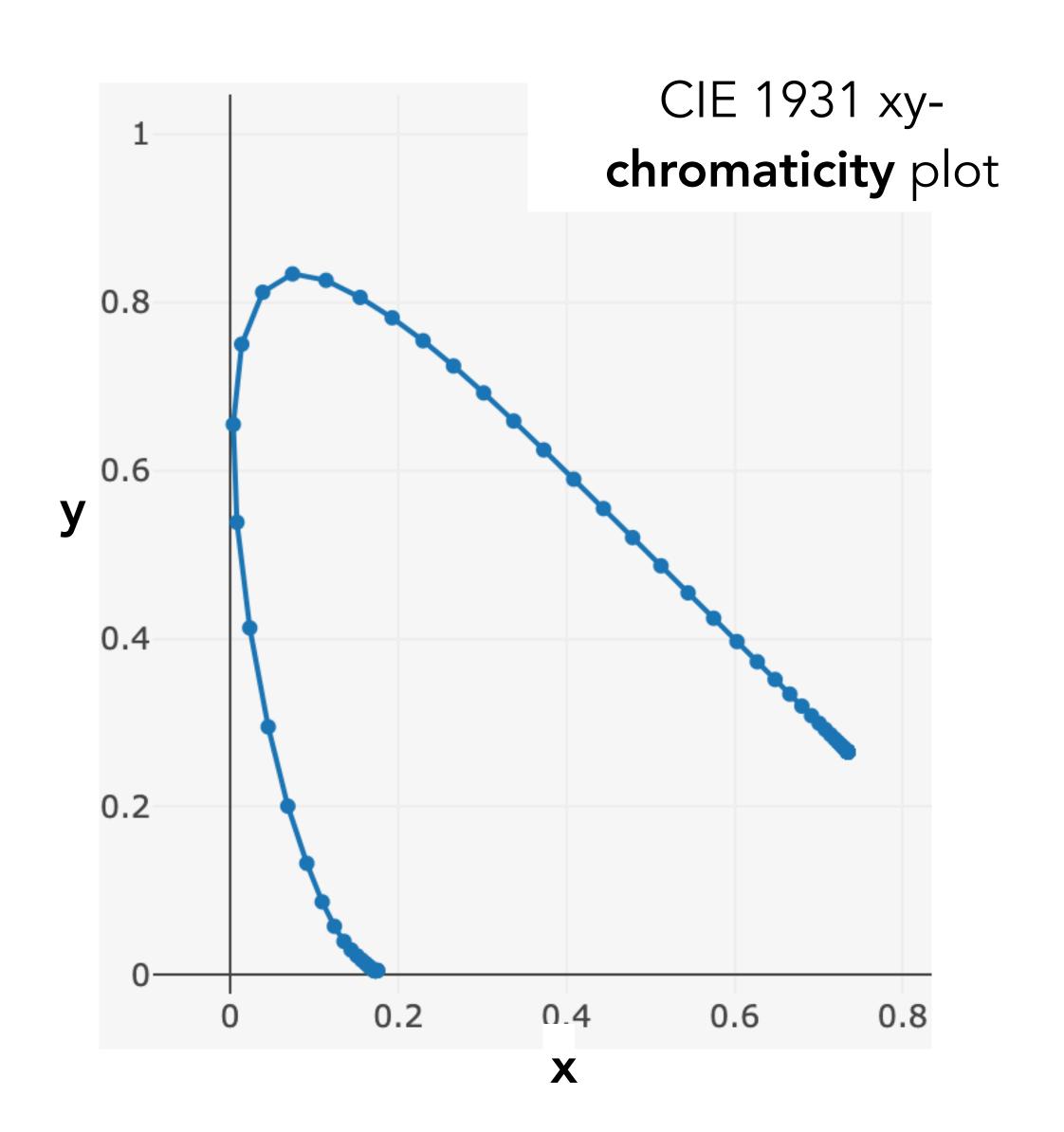
• Colors outside the triangle but inside the spectral locus are real colors, but they can't be physically produced by mixing the primary lights of this color space.

Color that can't be produced from this RGB color space



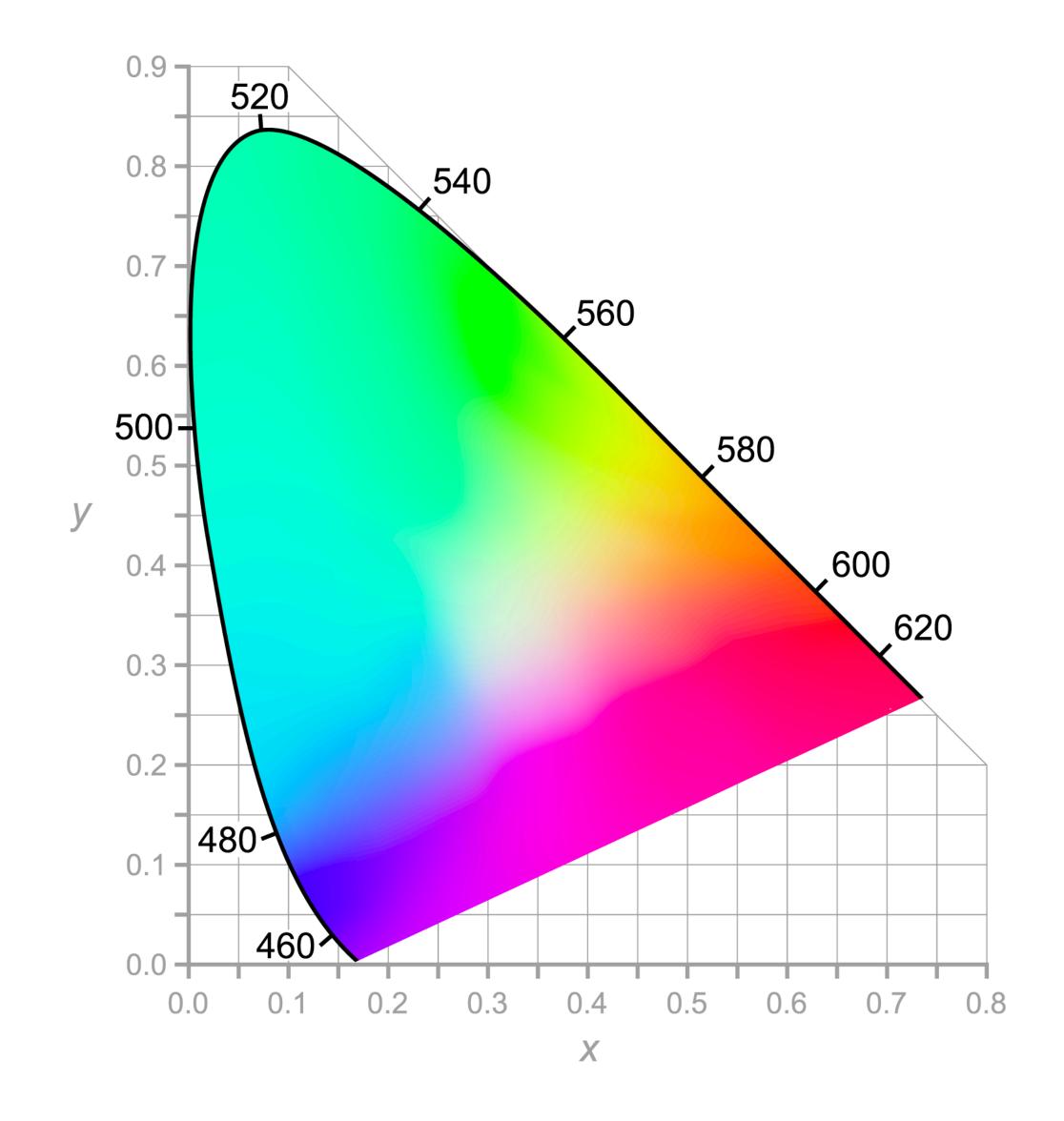
A Fascinating Fact

- No color space made of physically real lights can reproduce all visible colors.
 - No matter how many primaries are used in the color space.
 - Why? Can you give the intuition?
- The locus is **convex**. No polygon whose vertices are on or inside the spectral locus can cover the interior of the locus.
- No display can display all the colors.



Typical Visualization is a Lie

- Technically we can't color the entire gamut, since no display can display all the colors.
- Usually people find the closest displayable colors to show there (called gamut mapping; later)

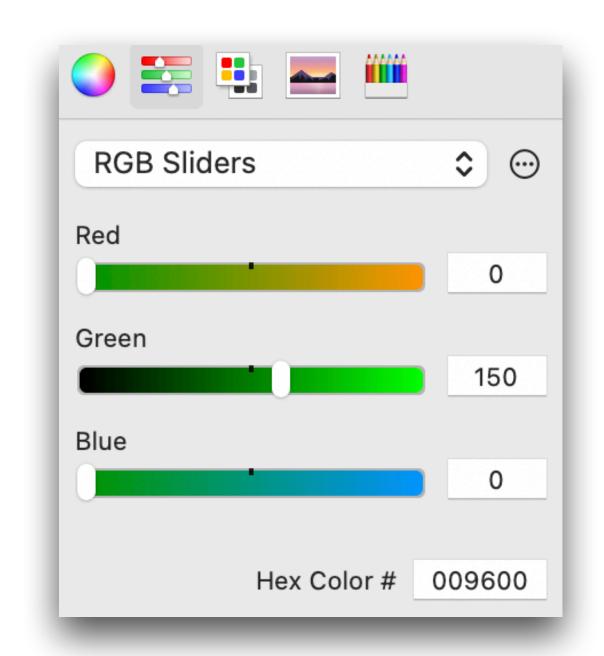


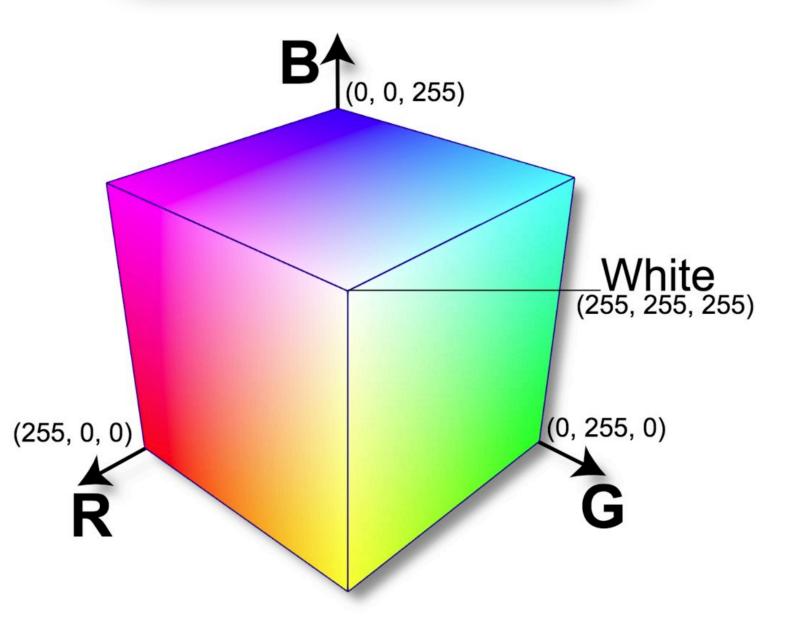
Color Cube (a.k.a., Build Your Own Color Space)

Visualization and math: https://horizon-lab.org/colorvis/colorcube.html

What is a Color Cube?

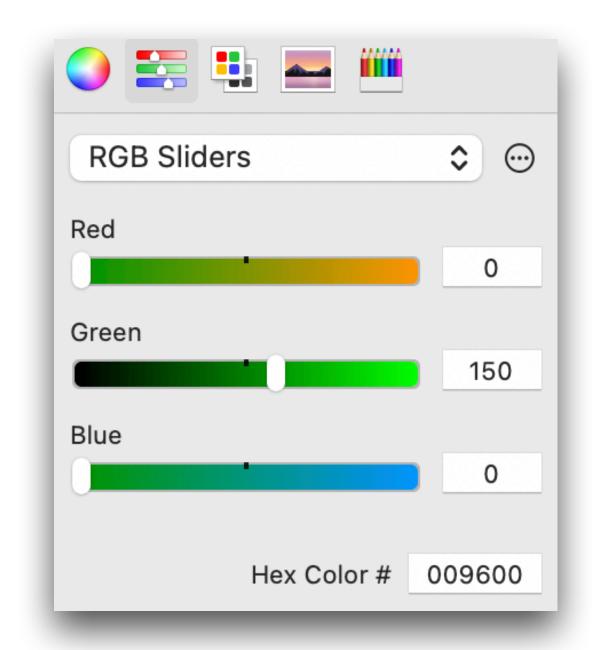
- In usual graphing applications we specify color by three integers, each between 0 and 255, or as a six-digit HEX value. For instance:
- Black [0, 0, 0] / 0x000000
- White [255, 255, 255] / 0xFFFFFF
- Red [255, 0, 0] / 0xFF0000
 - Some sort of gray [146, 146, 146] / 0x929292
 - A color cube let us specify a color as a mixture of three primary colors. The numerical value represents how much of a primary color we want.

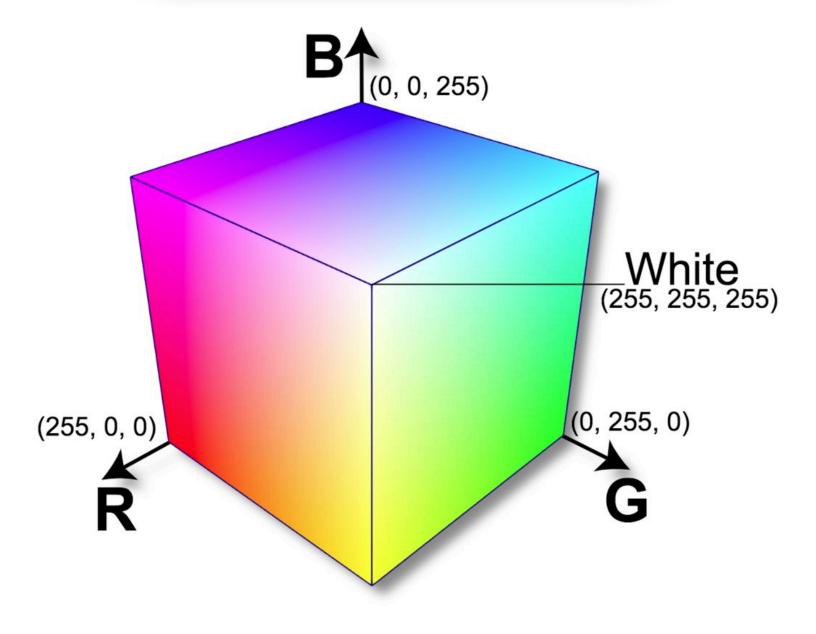




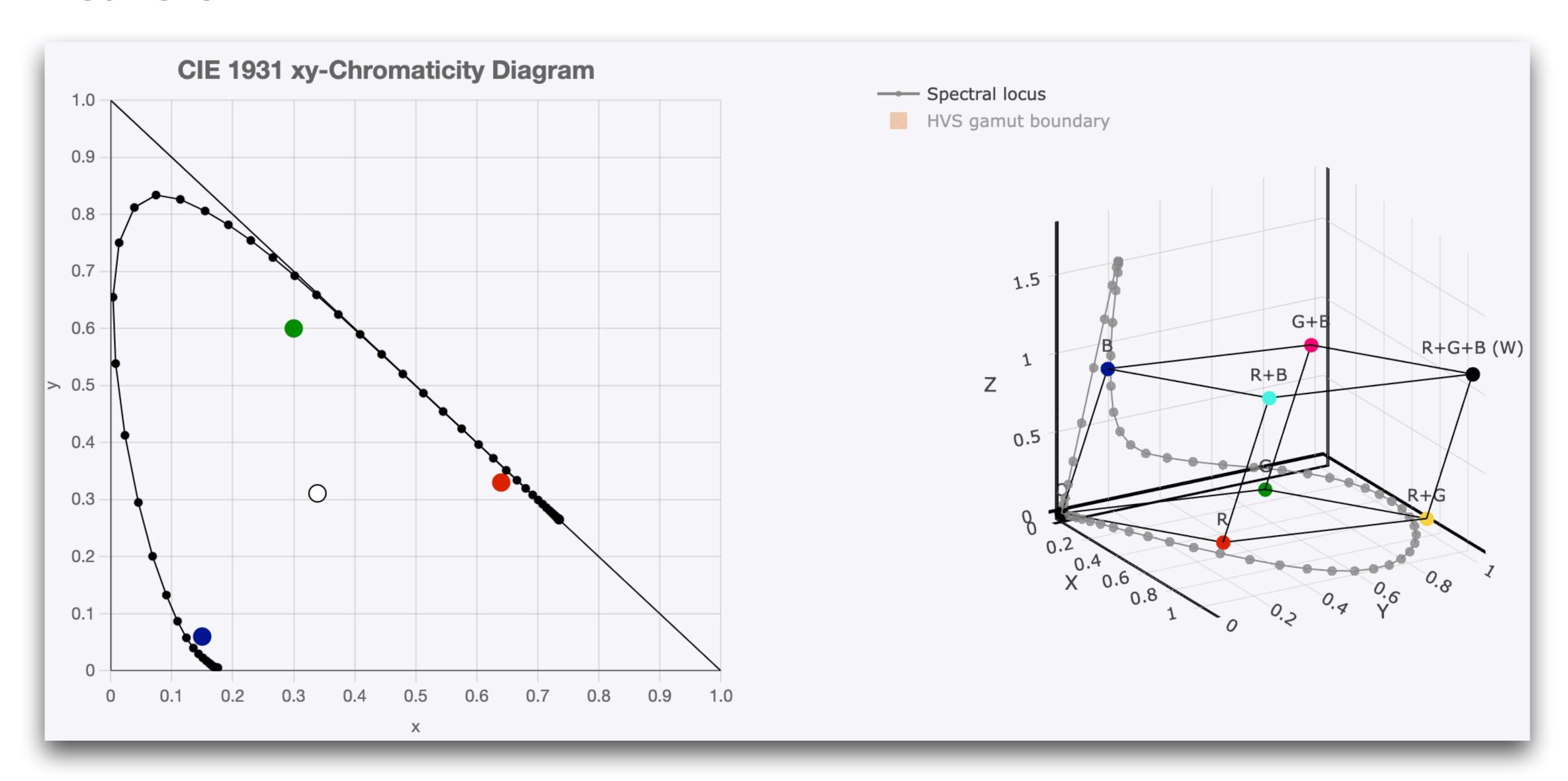
How to Build a Color Cube?

- We need to define the color space first.
 - Surprisingly, we need not only the **three primaries**, but also the **white point** (the "white" color in your color space).
- We will then figure out how to quantize the color space: there are infinitely many colors in a color space, but we can afford to represent only 2²⁴ colors.
 - Assuming 8 bits per primary (color channel).
 - There are encodings that use more bits per channel.





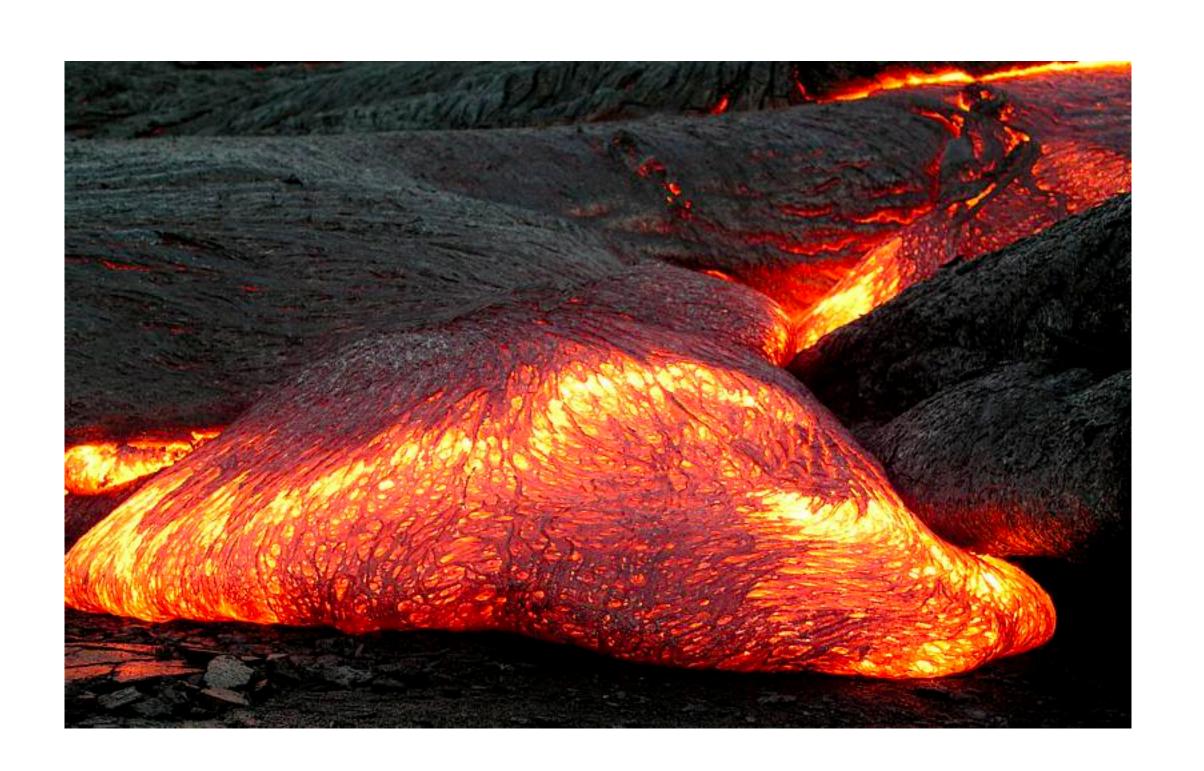
Intuition

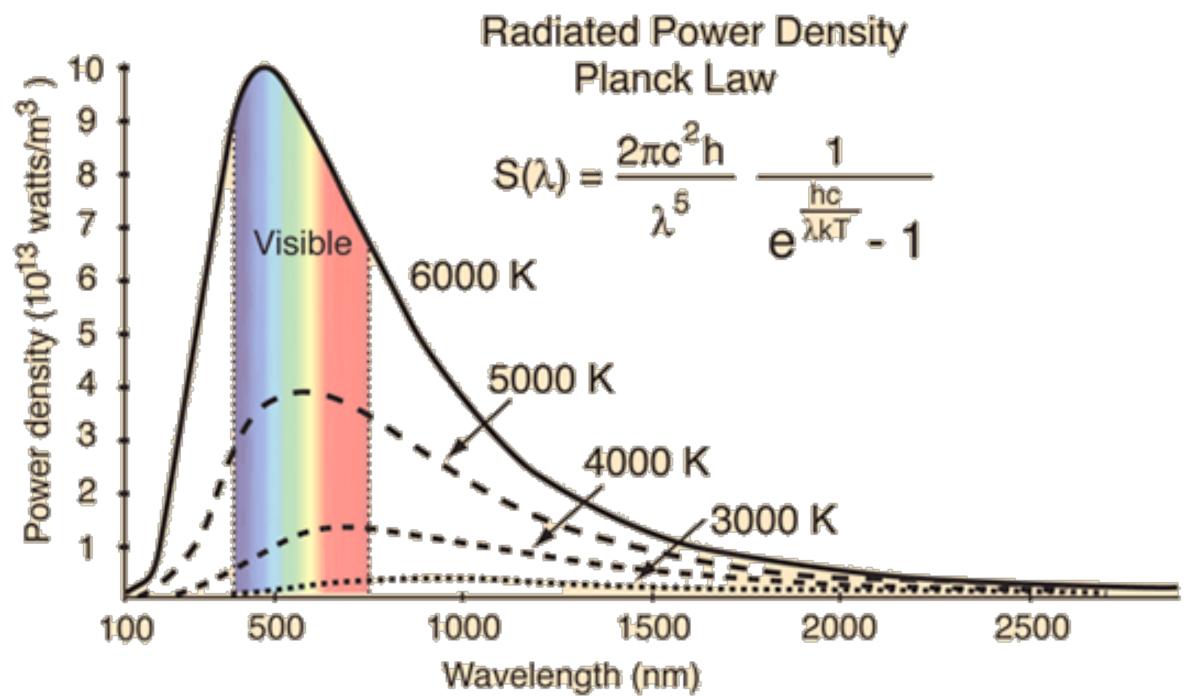


What is White?

- Intuition: Many colors are perceptually "white"
 - So you get to define what white is in your color space.
- Scientifically: white refers to colors from black-body radiation under different temperatures.
 - Many colors closer to colors of black-body radiation can be chosen as the white point of a color space.
- "Requirement": equal amount of maximum primary lights in a color space is white.
 - You don't have to follow this, but this makes intuitive sense (virtually all color spaces do)

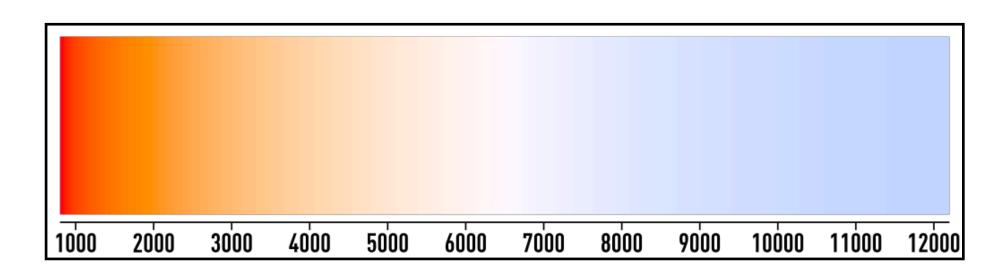
Black Body Radiation

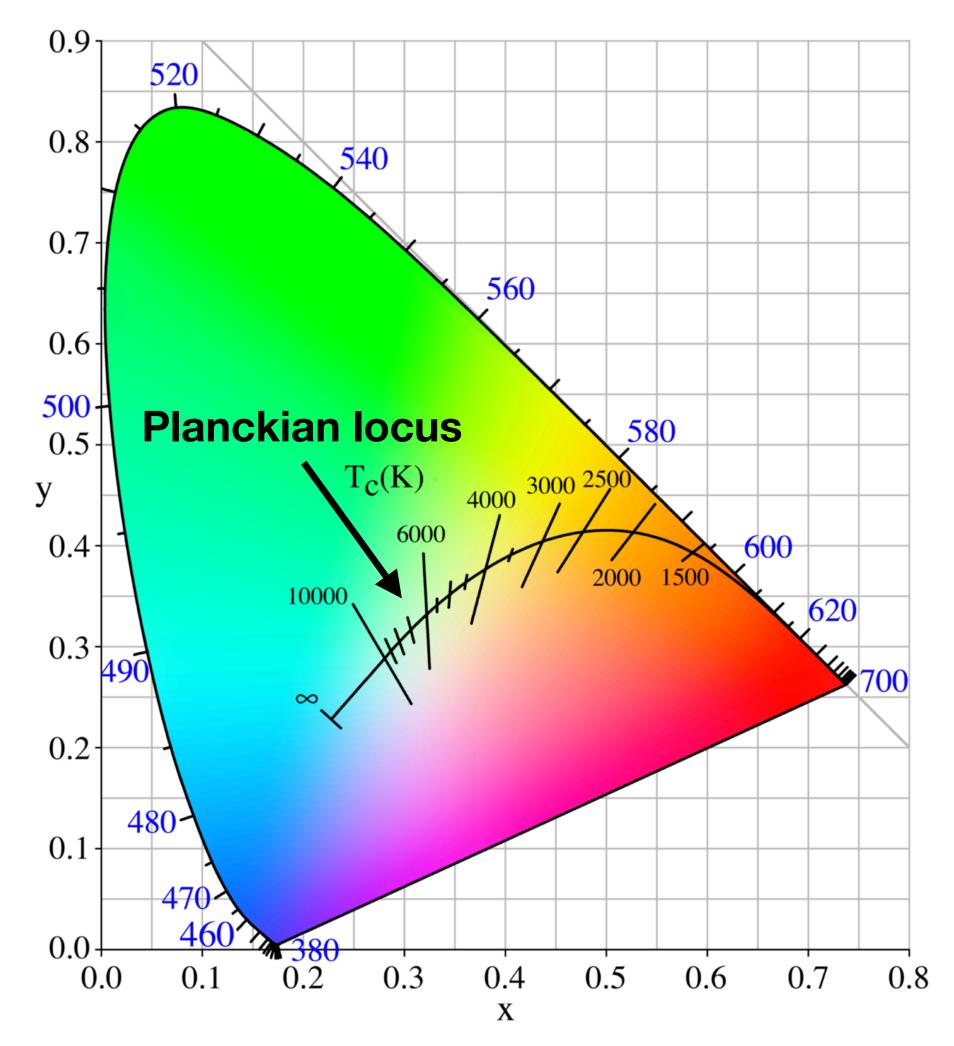




Color Temperature

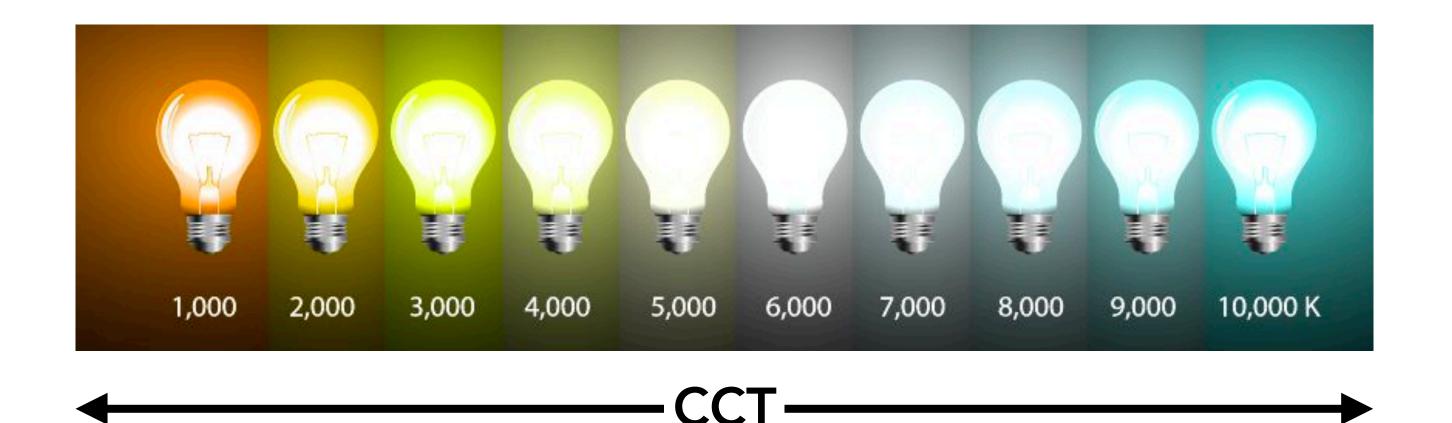
- The colors of black body radiation under different temperatures form a curve when plotted in the xychromaticity diagram. It's called the Planckian locus.
- A color on the locus has a color temperature.





Correlated Color Temperature (CCT)

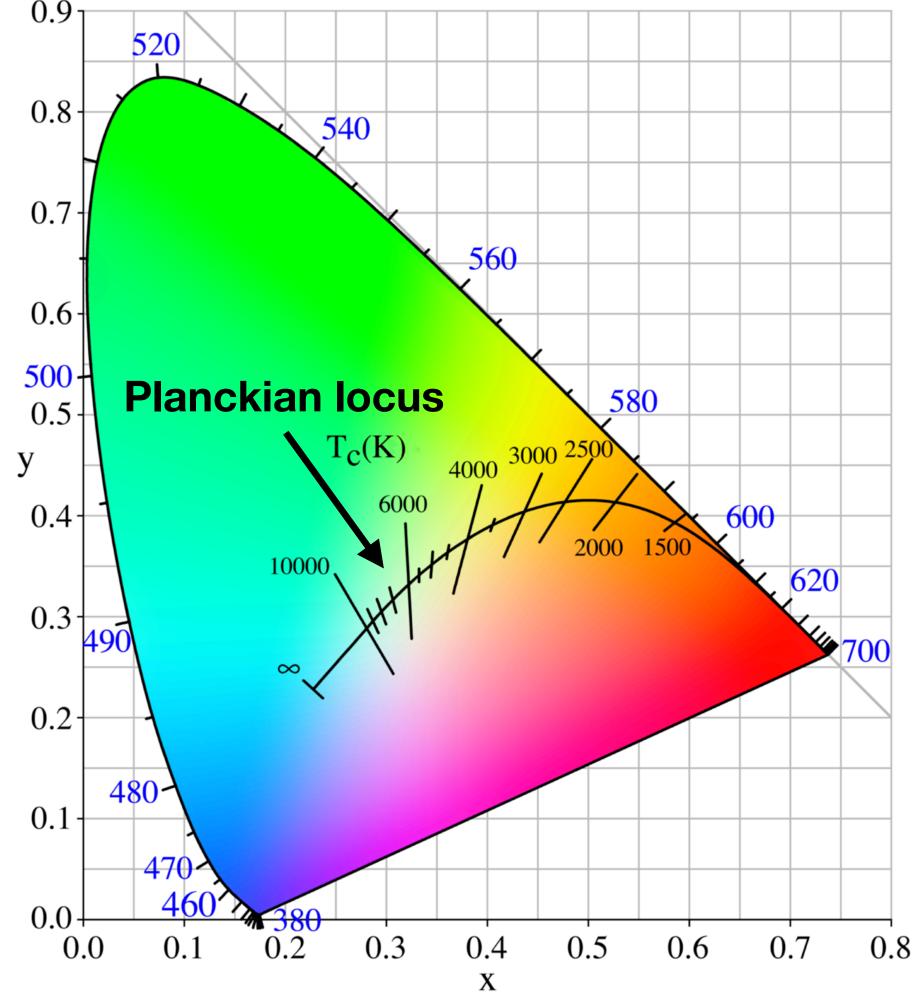
"Cool"



 Colors of actual lights closer to a color of black body radiation can be assigned a correlated color temperature.

"Warm"

The two are (mis)used interchangeably.



https://www.energystar.gov/products/lighting_fans/light_bulbs/color_mood

35

CHOOSING THE RIGHT COLOR







Soft White, Warm White

The standard color of incandescent bulbs.

Neutral White, Cool White

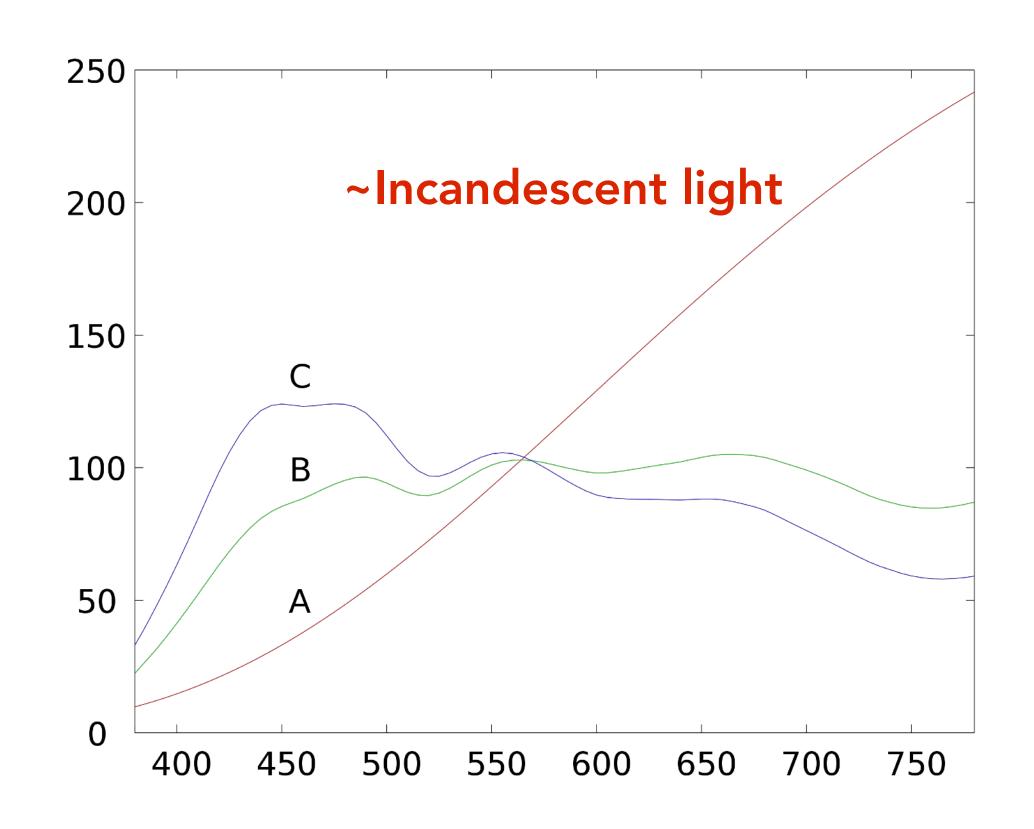
Good for kitchens and work spaces.

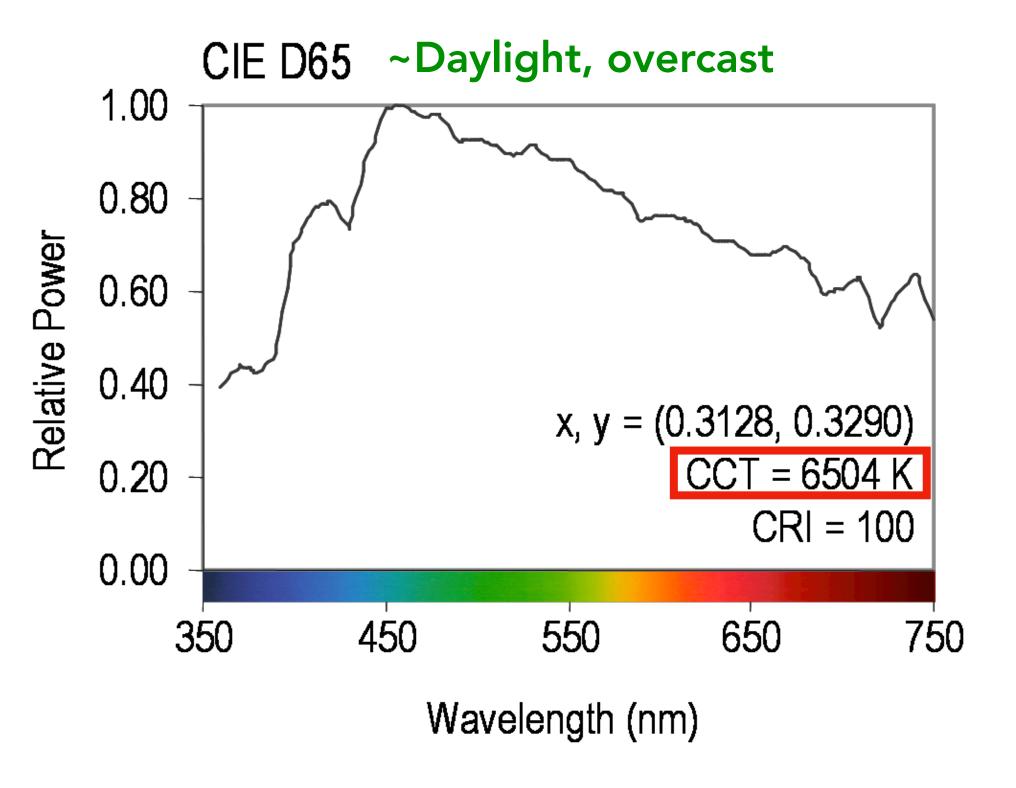
Daylight

Good for reading.

2200K 3000K 3500K 4100K 5000K 6500K

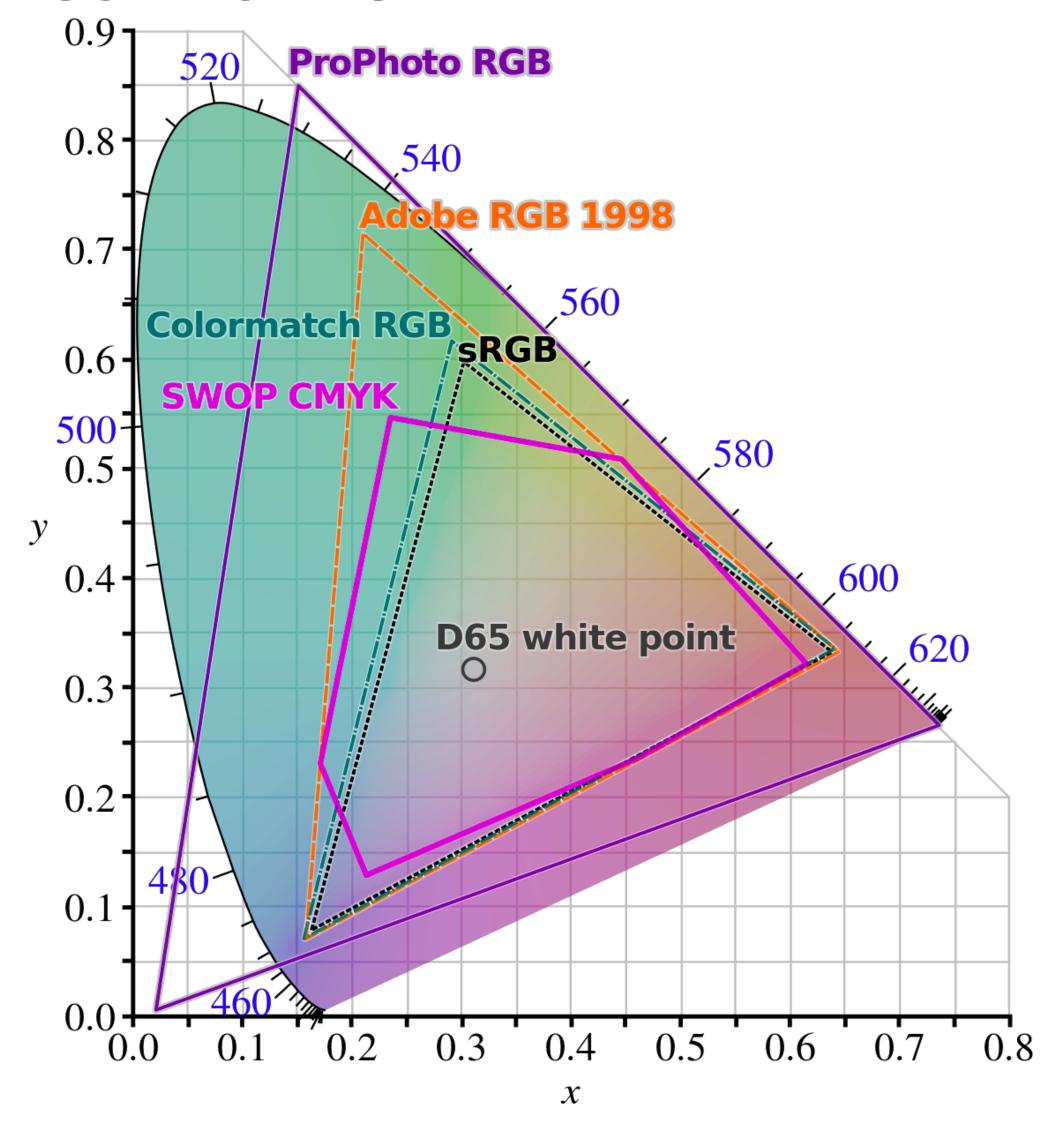
CIE Standard Illuminants



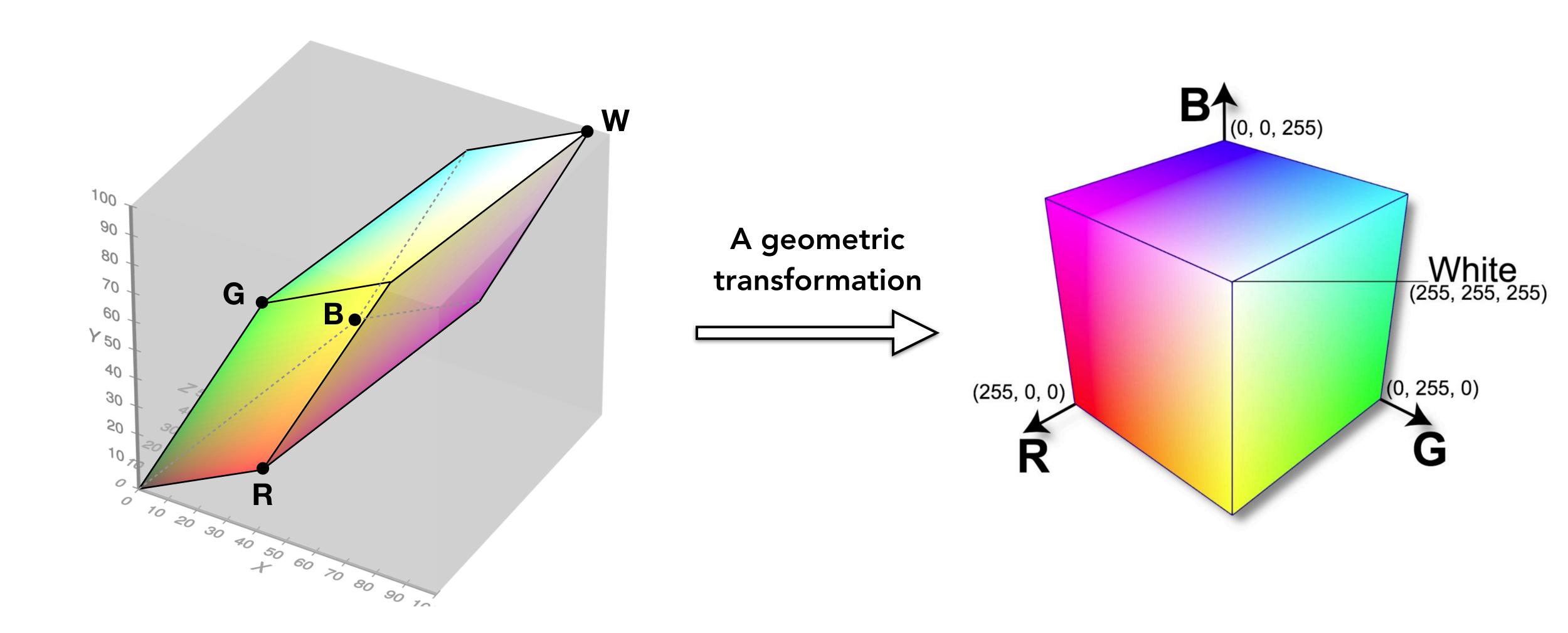


D65 is the Most Common White Point

- D65 is the most commonly used white.
- sRGB is the most common color space, but its gamut is very small.
- CMYK is used in printers. More later.
- ProPhoto RGB is a conceptual color space; its primaries are not real colors.
- All RGB color spaces are just one linear transformation away from each other.

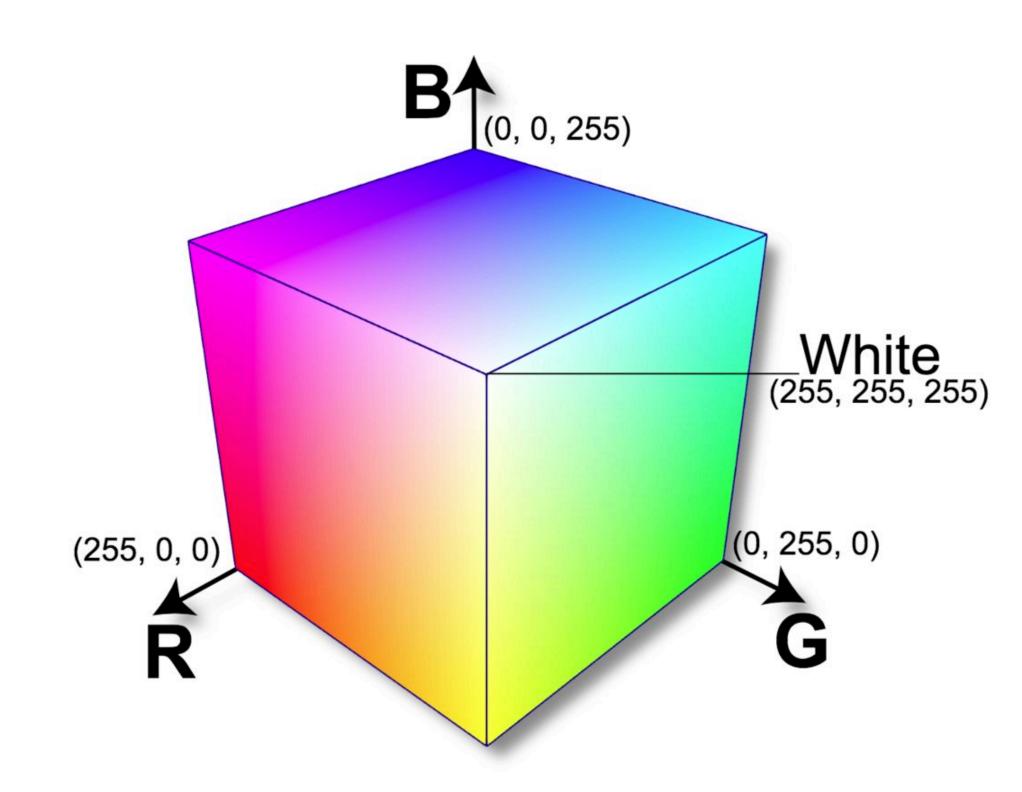


Map the Gamut to a Cube

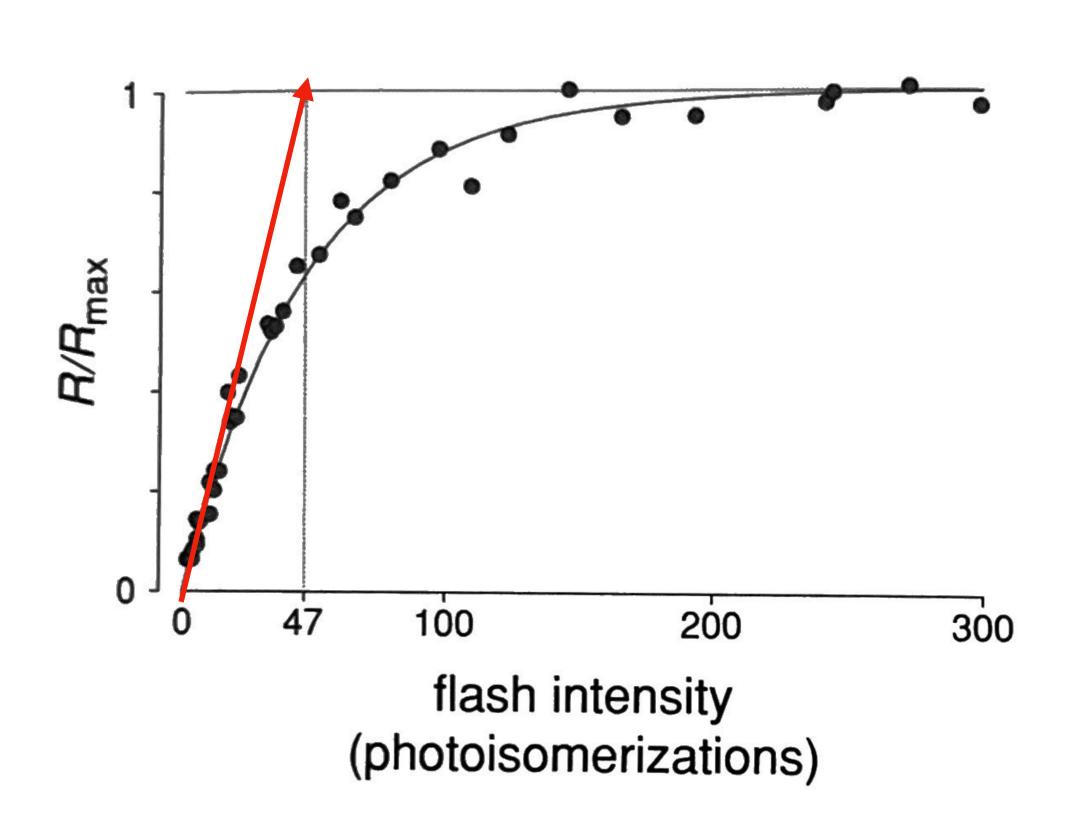


Quantize the Color Cube

- The color space so far is continuous, but we have to discretize/quantize the space since we have limited bits. Popular sRGB color space uses 8 bits per primary. That is, we can represent at most 2^24 colors.
- Ideally, we want to evenly divide the brightness.
 - Increasing R from 0 to 1 should add the same amount of perceived redness as increasing R from 240 to 241.

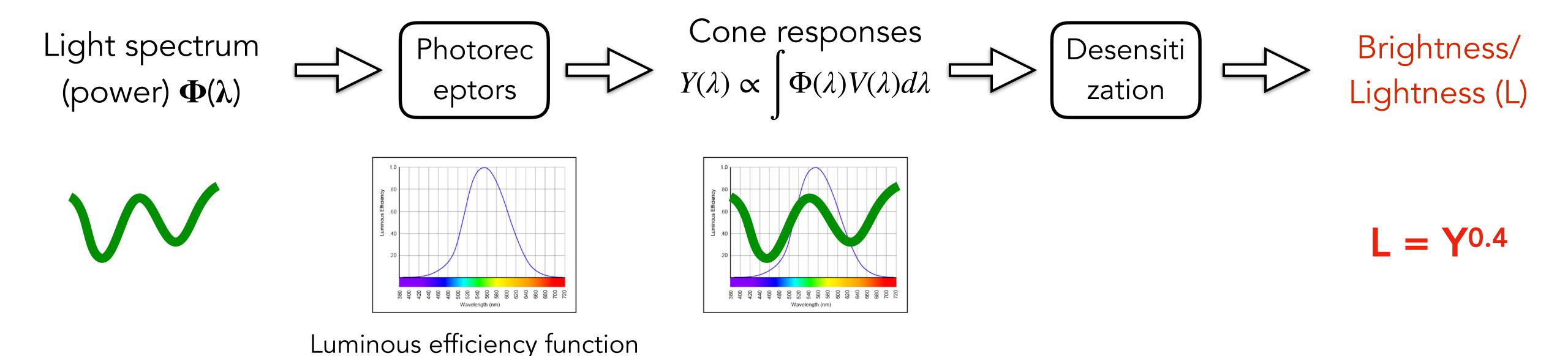


Recall: Electrical Response vs. Light Power



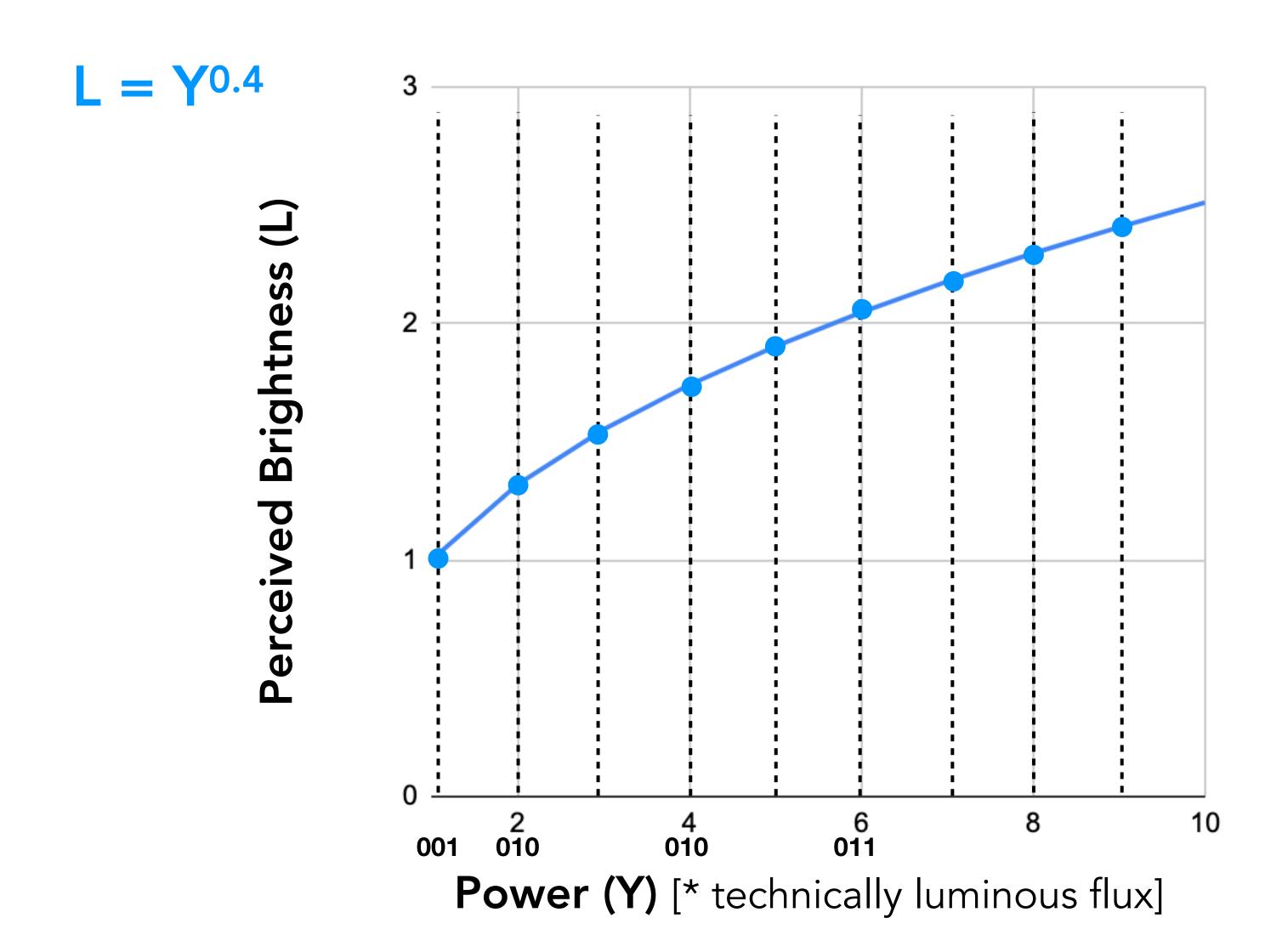
- Photoreceptor response vs. light intensity isn't linear.
- Luminance is a positive linear combination of L/M cone responses.
- Naturally, perceived brightness vs. light power isn't linear.

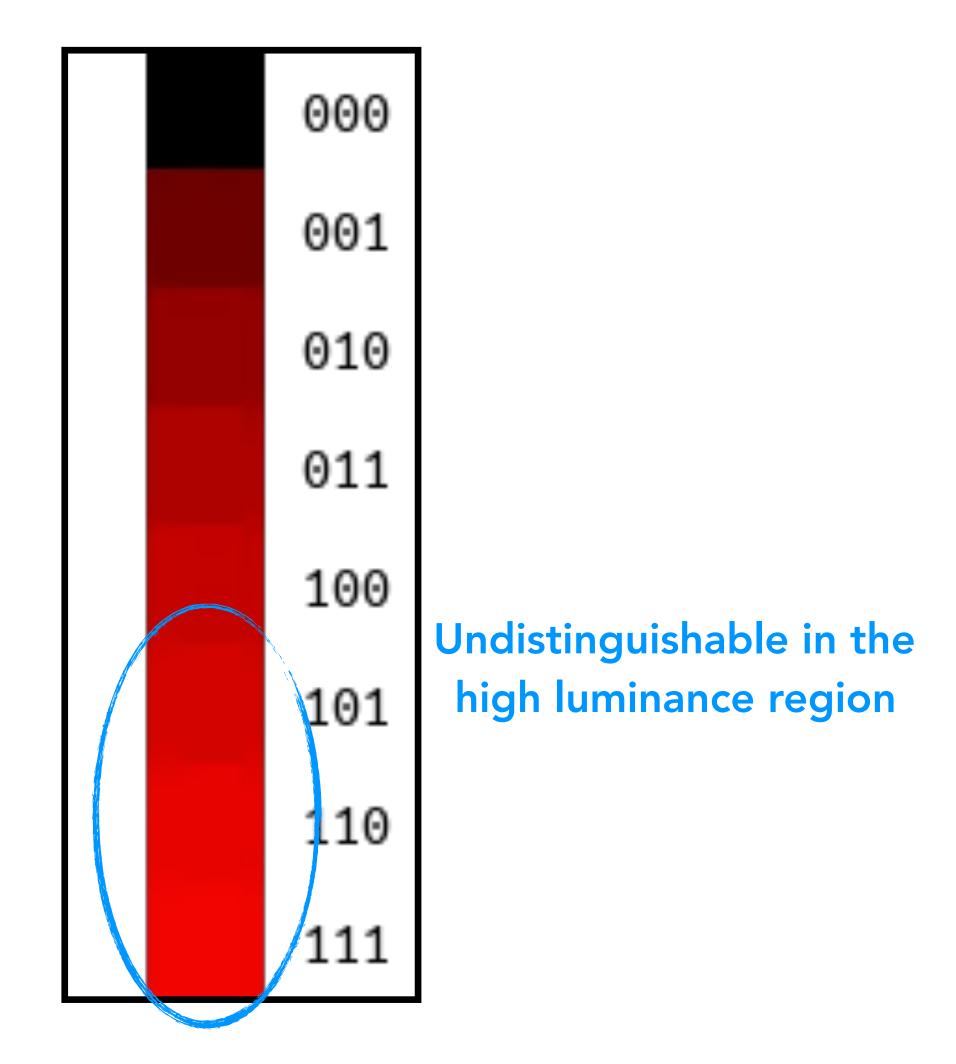
Power vs. Perceived Brightness



- In color science, people empirically define the relationship between the perceived brightness and the light power using a power law.
 - It doesn't mean the actual human brightness perception follows this exactly; it's more of an engineering convenience.

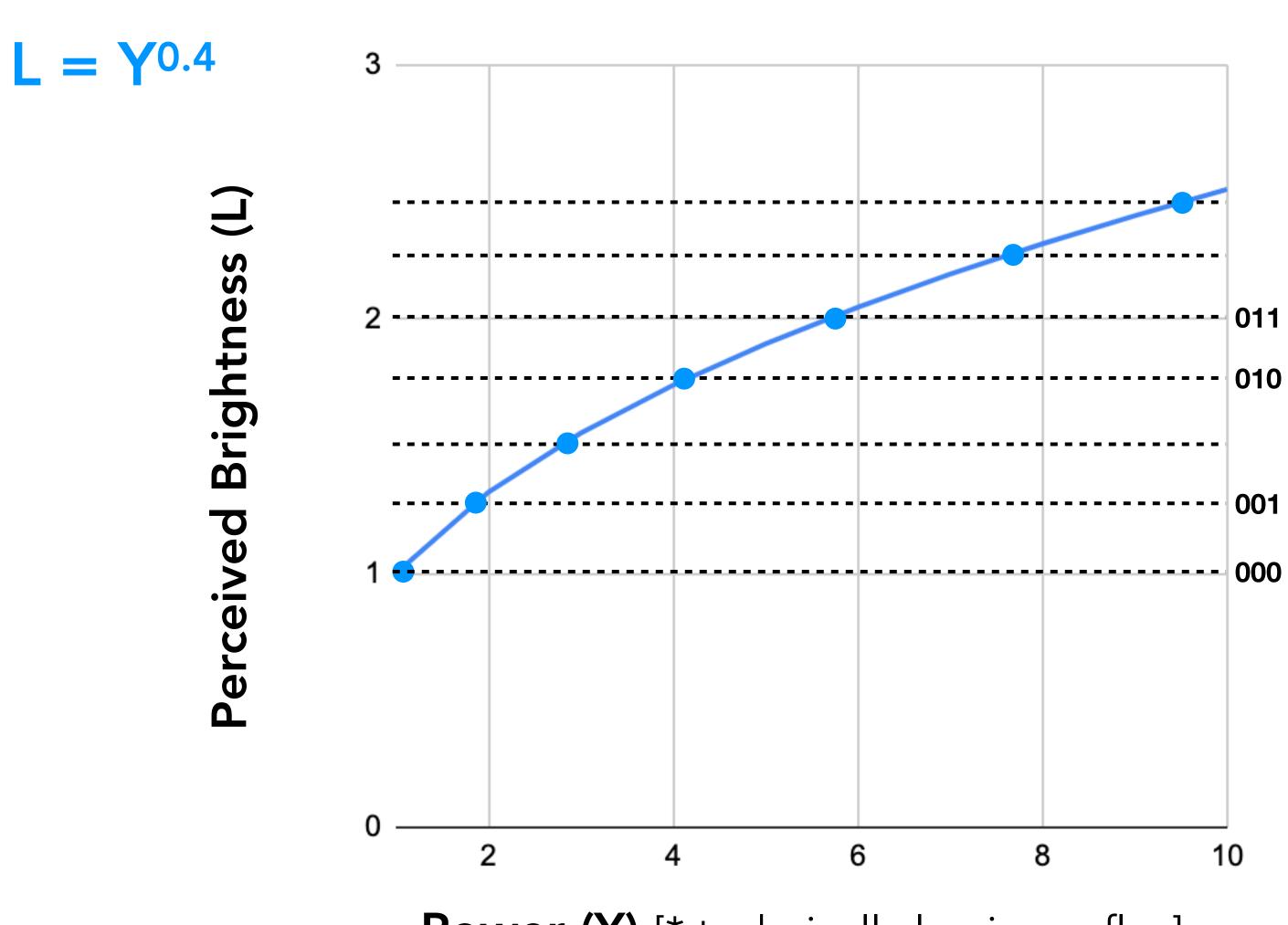
Encoding Power Evenly





Encoding Brightness Evenly

Perceptually uniform



Power (Y) [* technically luminous flux]

Gamma Color Encoding

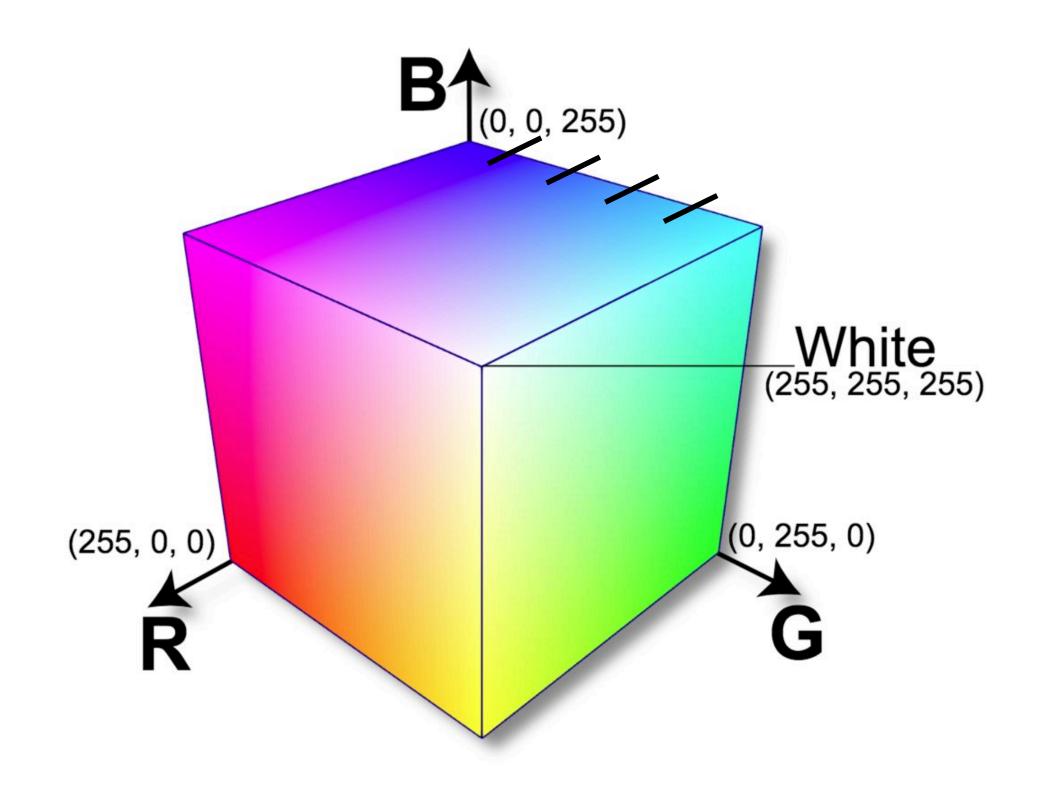
Assuming after linear transformation, R/G/B are in the [0, 1] range

$$R_{8bit} = [R^{\frac{1}{2.2}} \times 255]$$

$$G_{8bit} = [G^{\frac{1}{2.2}} \times 255]$$

$$B_{8bit} = \lfloor R^{\frac{1}{2.2}} \times 255 \rfloor$$

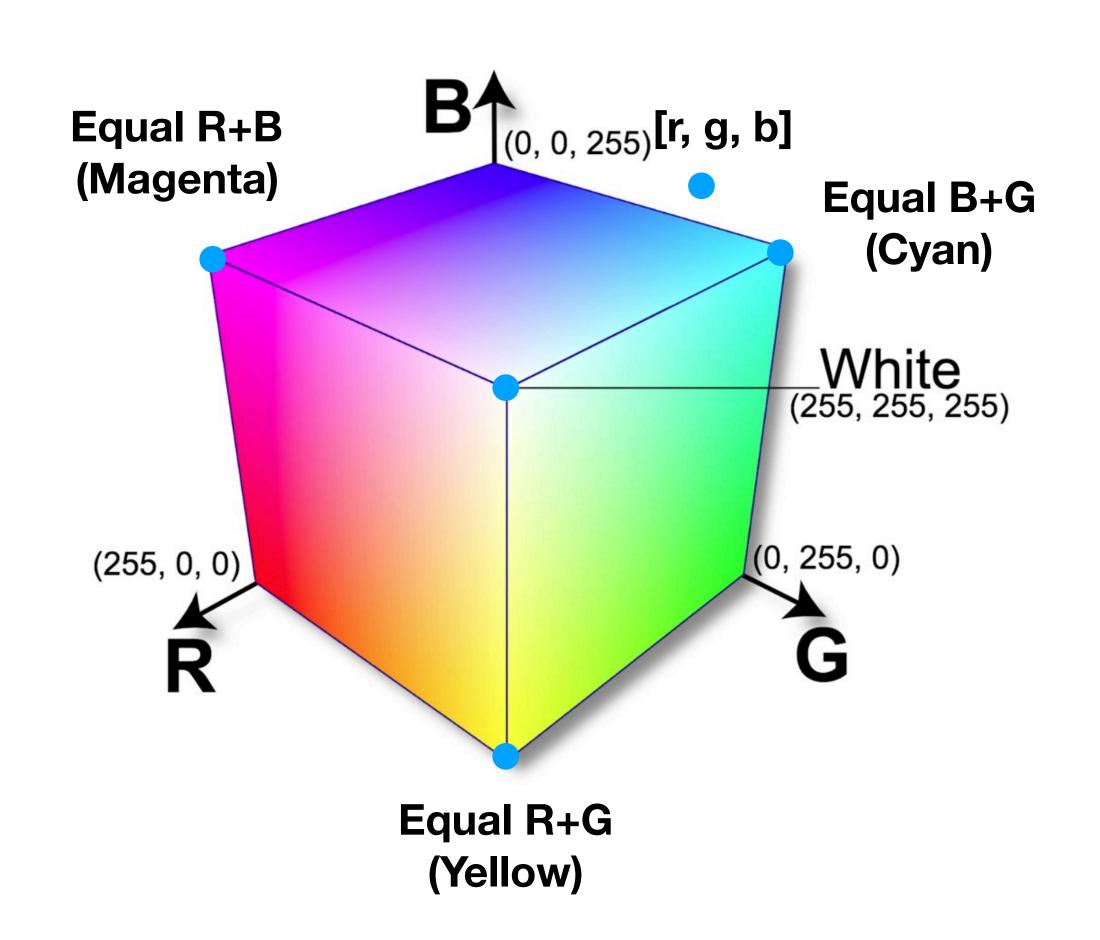
Gamma used by sRGB



^{*} sRGB actually uses a piece-wise gamma

Color Cube

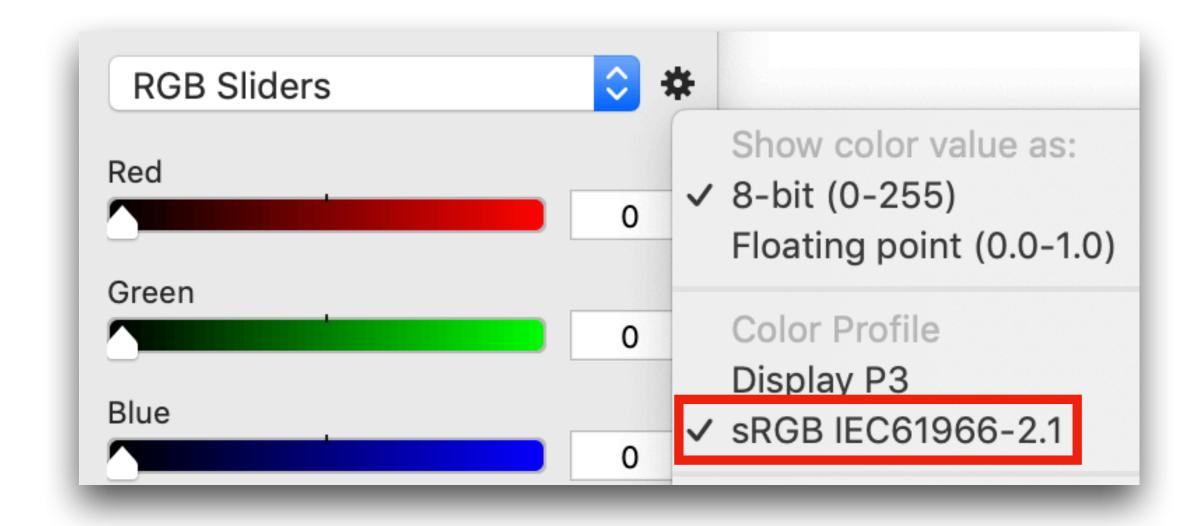
- Colors on vertices have equal mix of 2 or 3 primaries with maximum quantity.
- Colors outside of the cube are either:
 - physically unrealizable or
 - realizable but can't be produced from the chosen primaries.

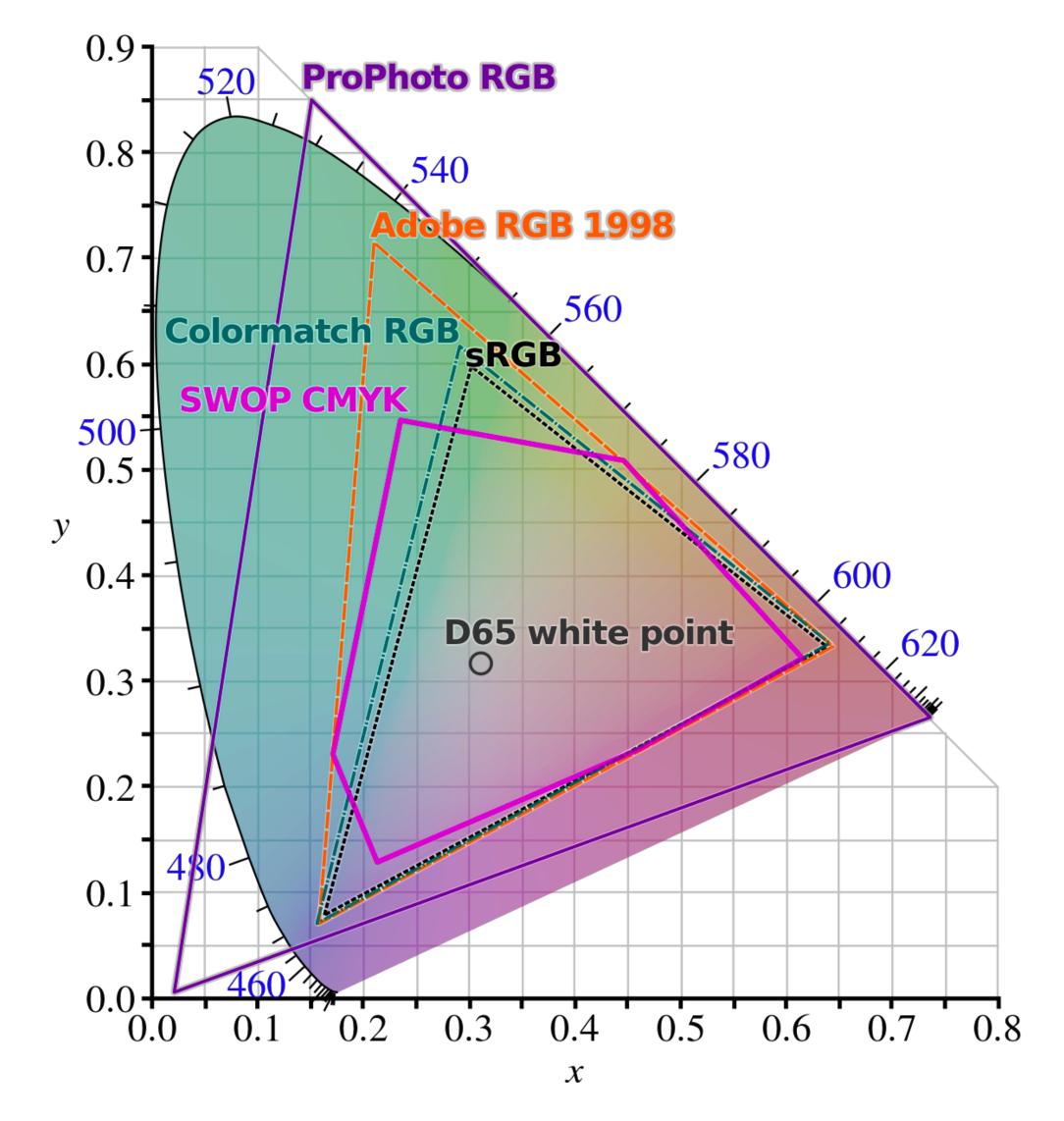


A Tour of Color Spaces

RGB-based Color Spaces

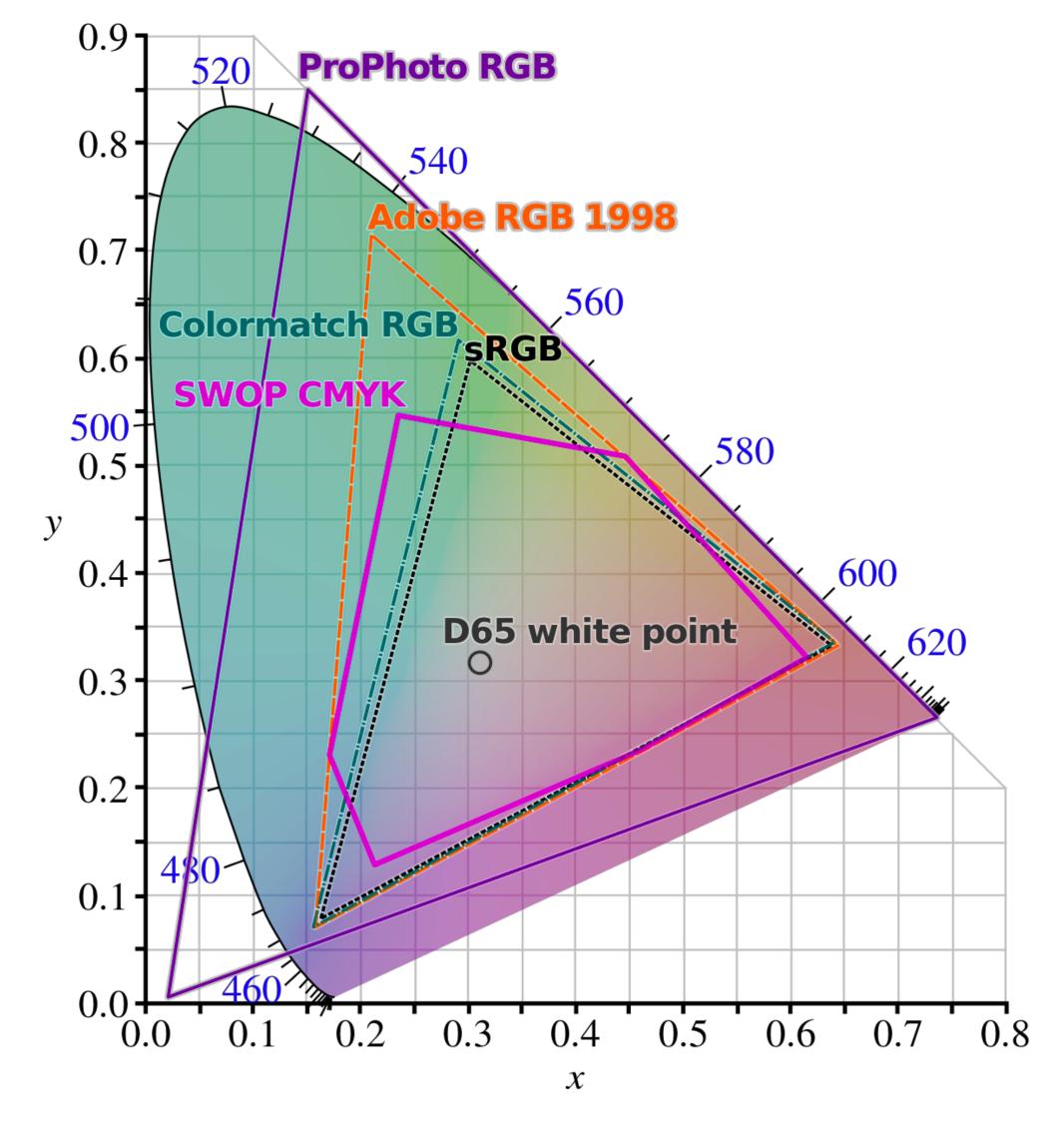
- sRGB is what we commonly refer to as the RGB space in the real world.
- Adobe RGB has a wider gamut.





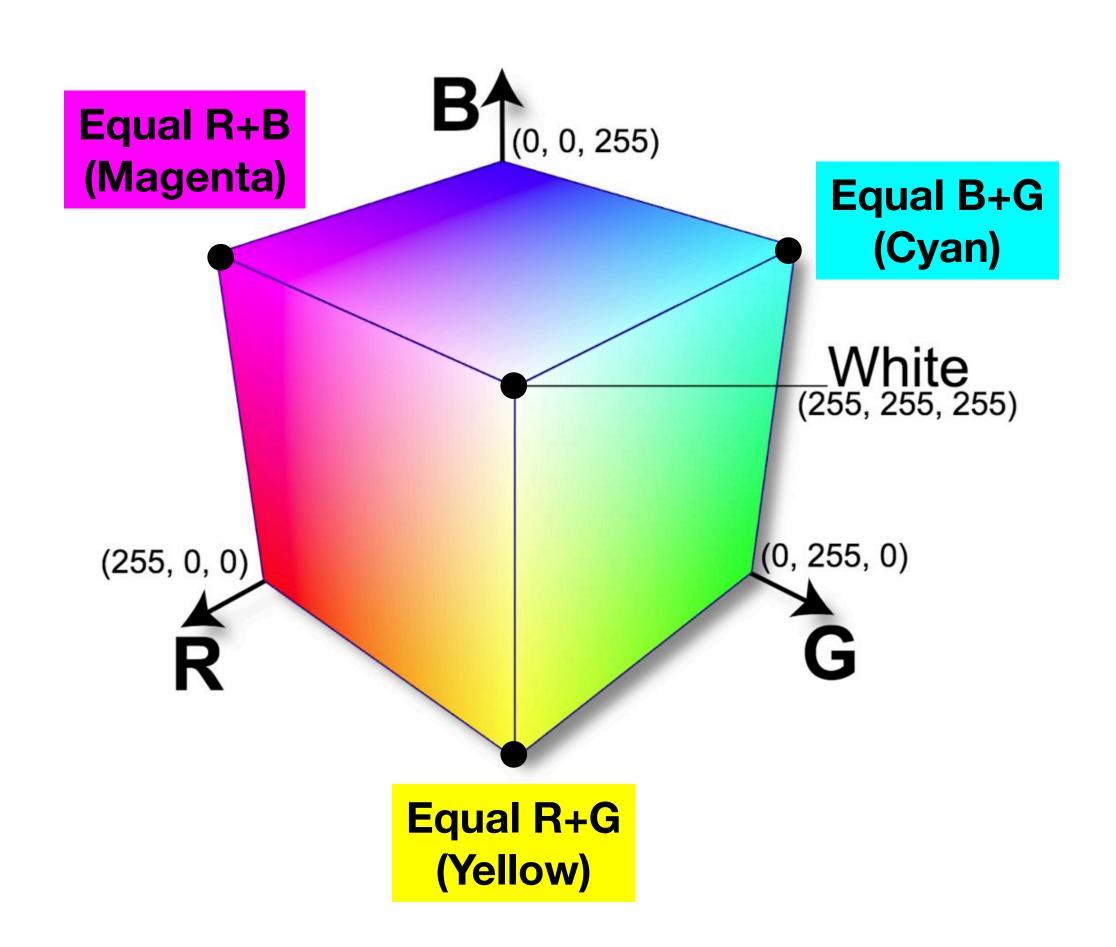
ProPhoto RGB

- ProPhoto RGB has a wide gamut, but ~13% of is imaginary. Two of the three primaries are not physically realizable.
- Q: Can you build a display that shows all physically real colors in ProPhoto RGB?
 - Use >3 primaries in the display



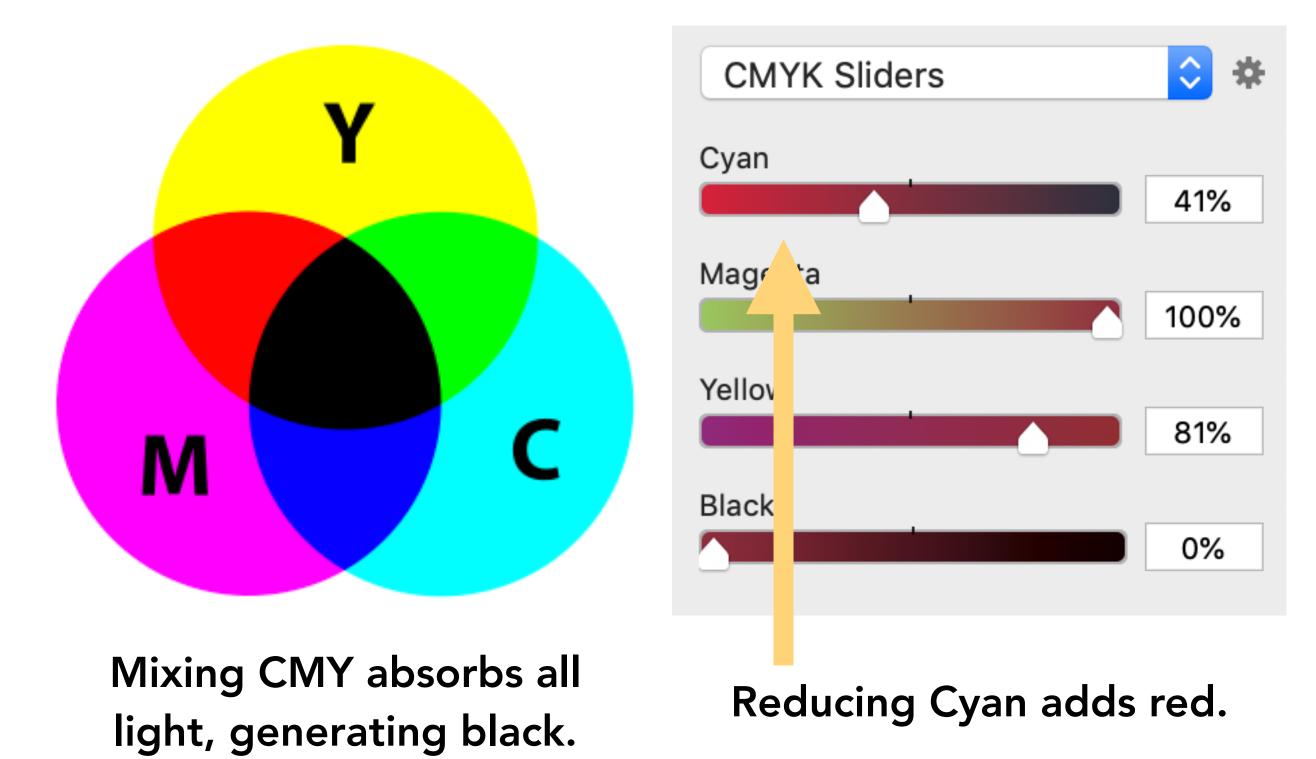
Additive vs. Subtractive Color Space

- RGB color spaces are additive: adding multiple colors to form a new color.
 - Good for generated lights (e.g., display)
- Cyan material looks cyan because it absorbs red light.
 - Cyan = White Red = Blue + Green
 - Yellow = White Blue = Red + Green
 - Magenta = White Green = Red + Blue

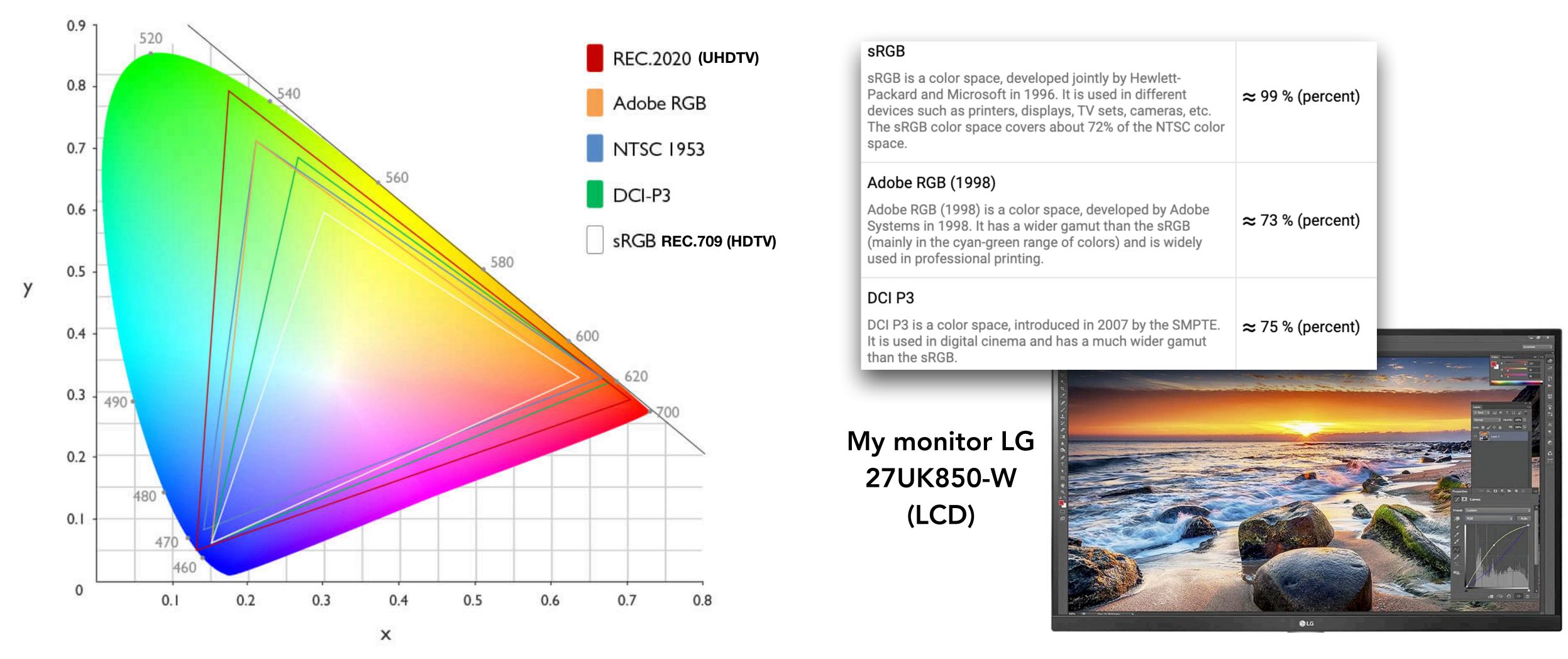


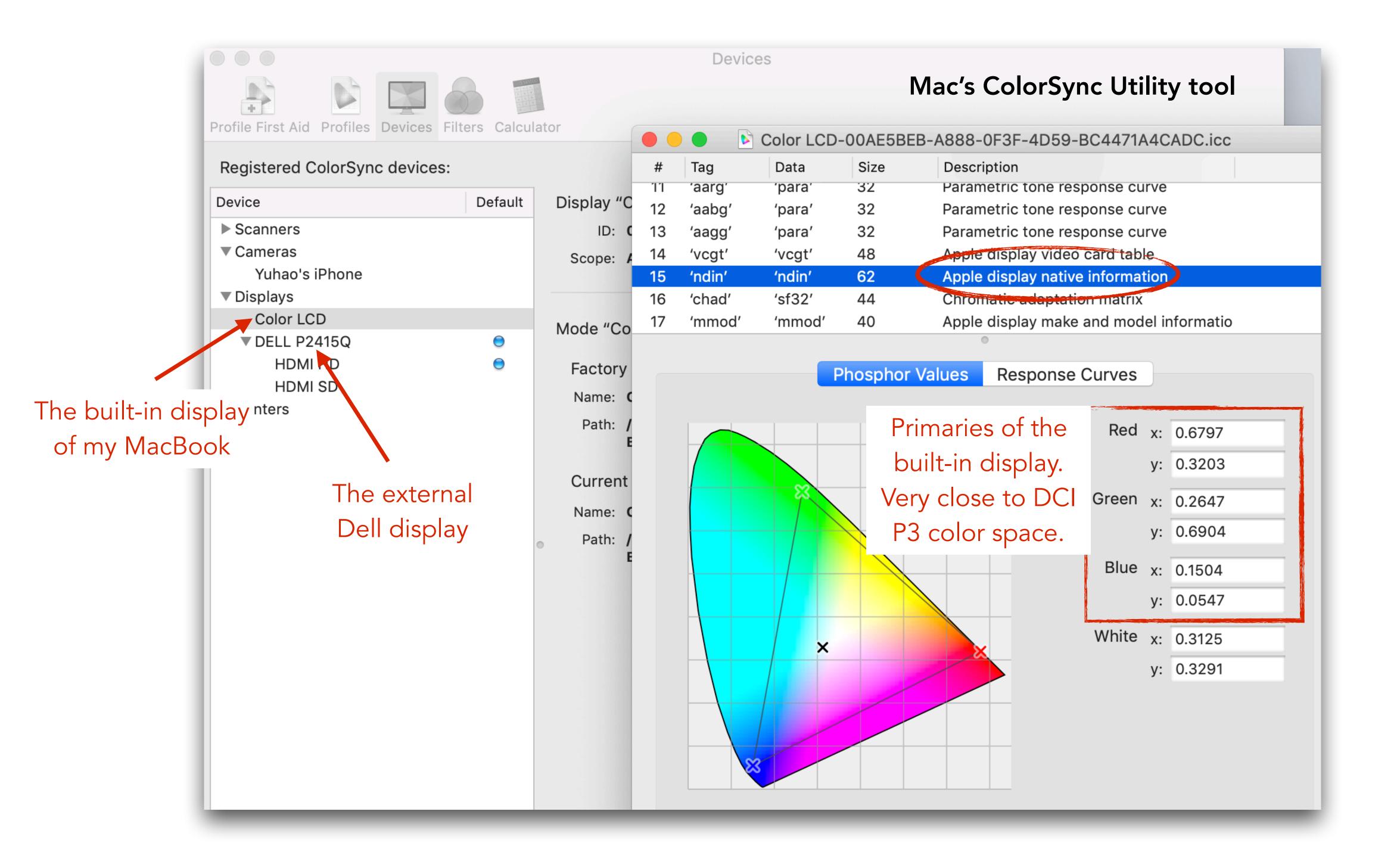
Subtractive Color Space: CMYK

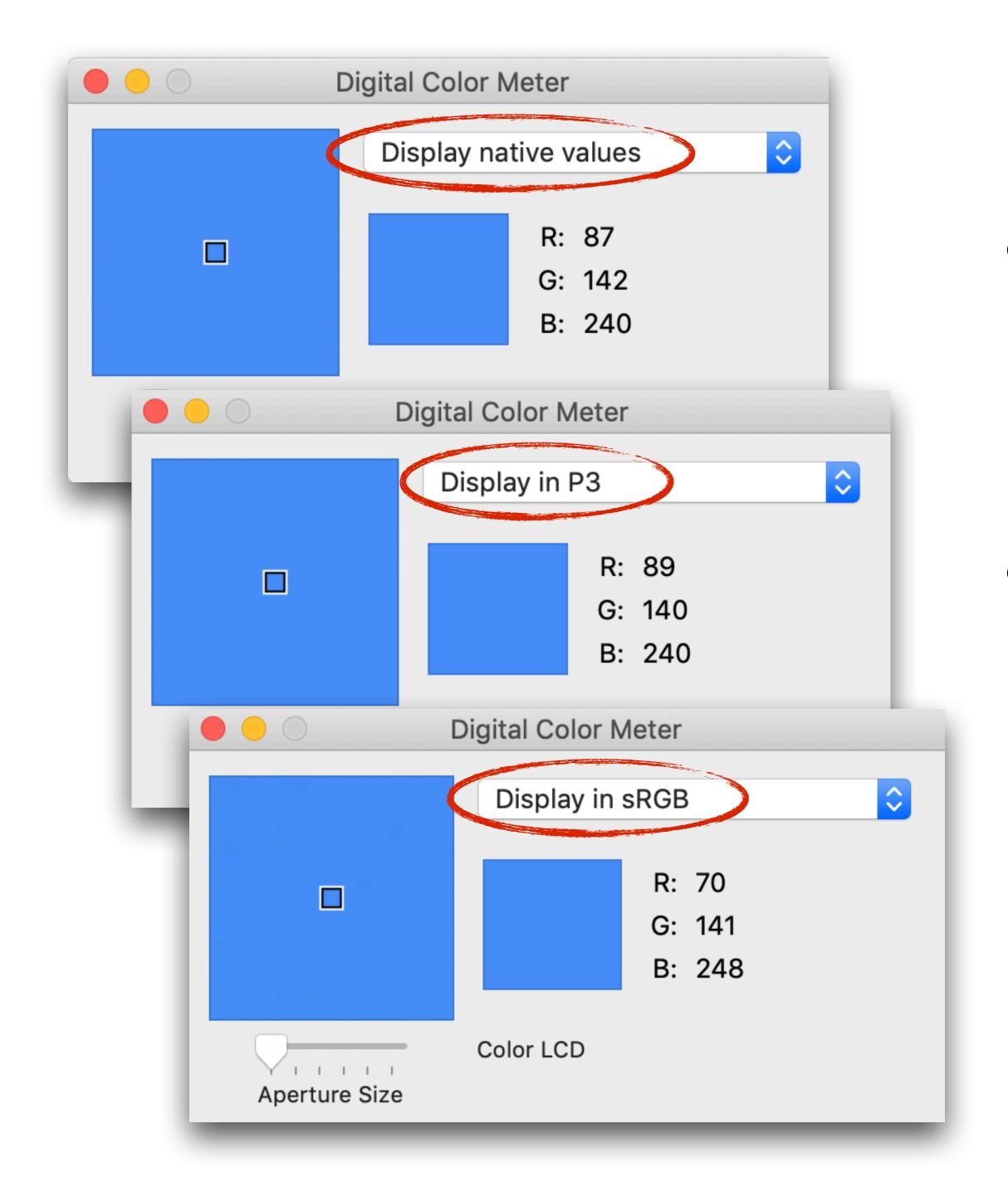
- Subtractive color space is better at describing reflected light, e.g., painting, printing.
 - Paper has color because it absorbs lights of certain wavelengths.
- Pure subtractive color space rarely reflects reality, because it assumes pure absorption. Actual materials scatter lights too. More later.



Color Spaces Designed For Displays



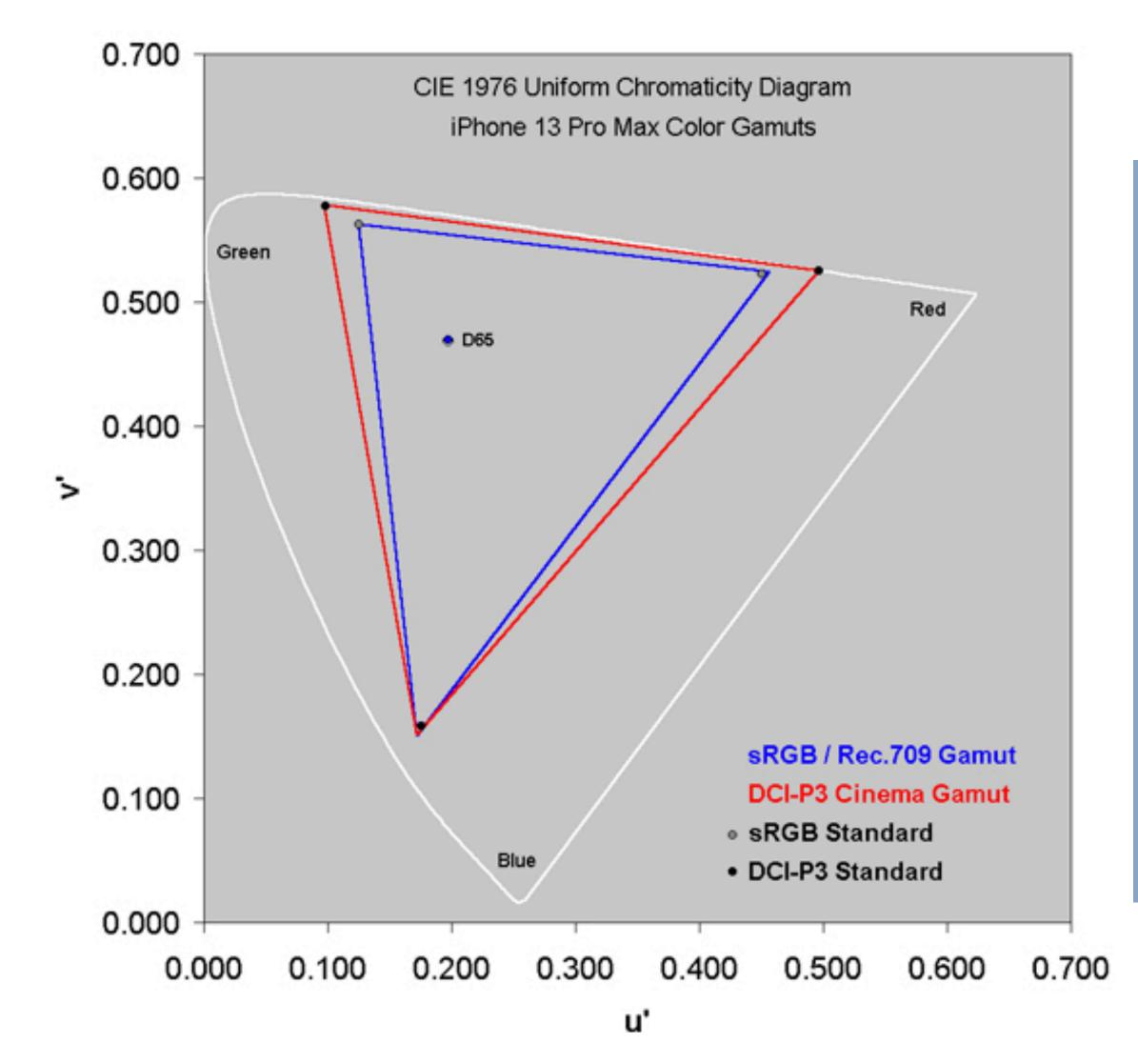


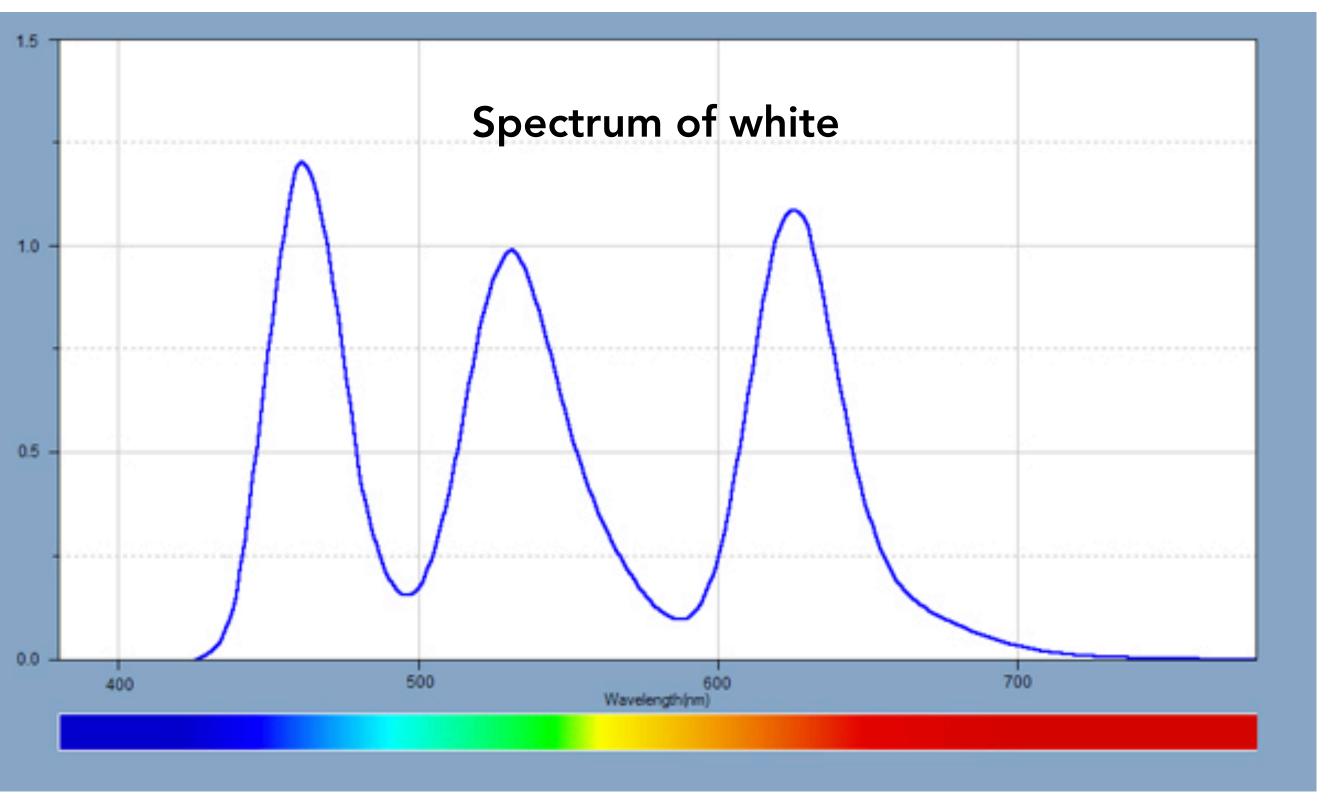


- Display native color space rarely matches exactly with an existing color space (manufacturing constraints).
- The same pixel shown in three different color spaces. The RGB values in the display native space roughly match the values in the DCI P3 color space, but are very different from the values in the sRGB color space.

iPhone13 Pro Max measurement data

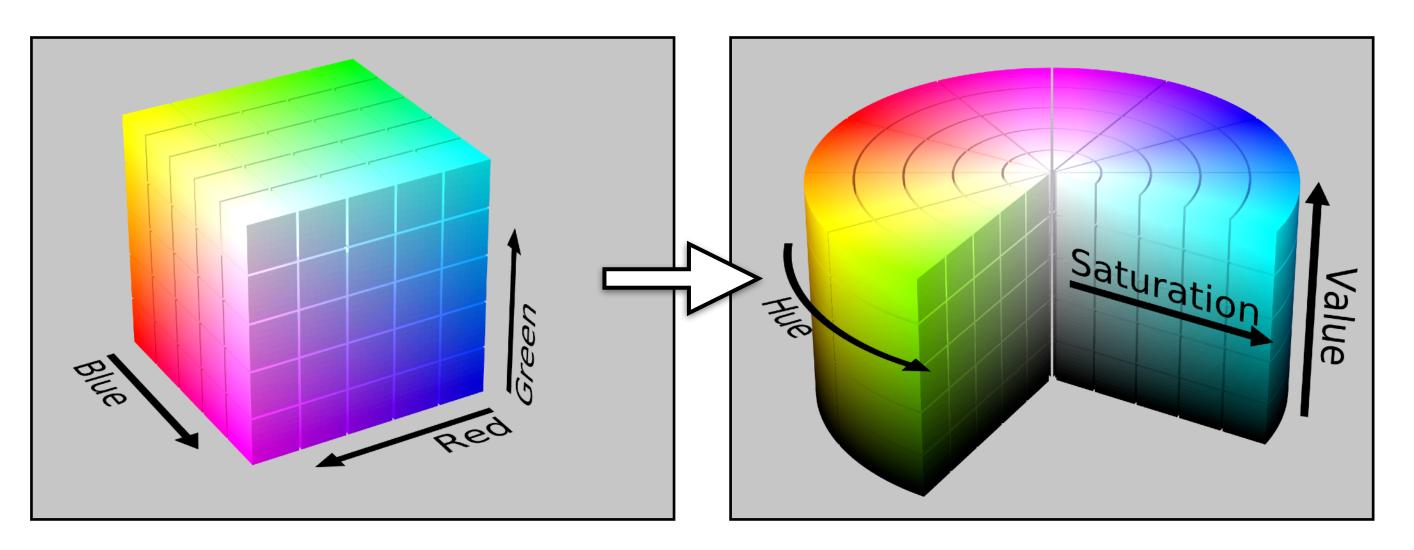
Matches the sRGB and DCI P3 gamuts pretty well, but not perfectly.

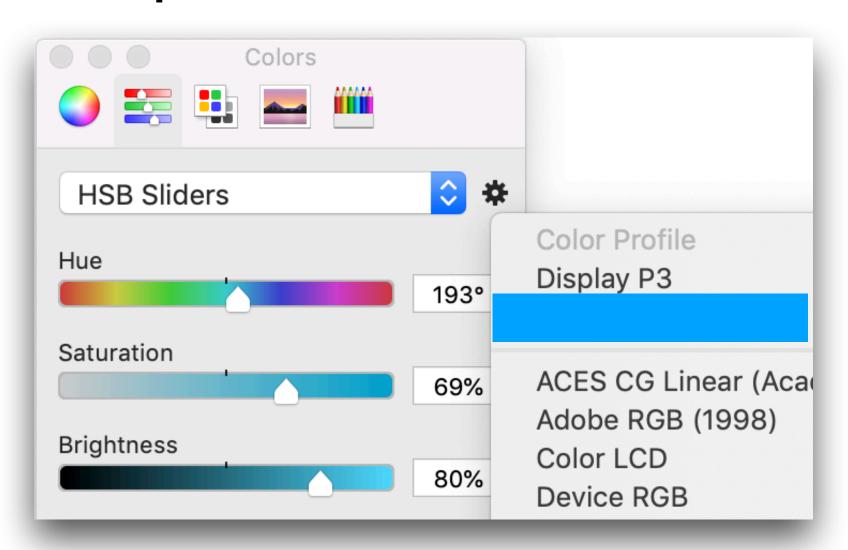




Perceptual Color Space

- HSB/HSV/HSL: intuitive representation of RGB space
 - Hue: "nuances" of color
 - Saturation: purity (how much white there is in color?)
 - Value/brightness/lightness: amount of color
- HSB/HSV/HSL don't change the gamut; they just map a cube to a cylinder.





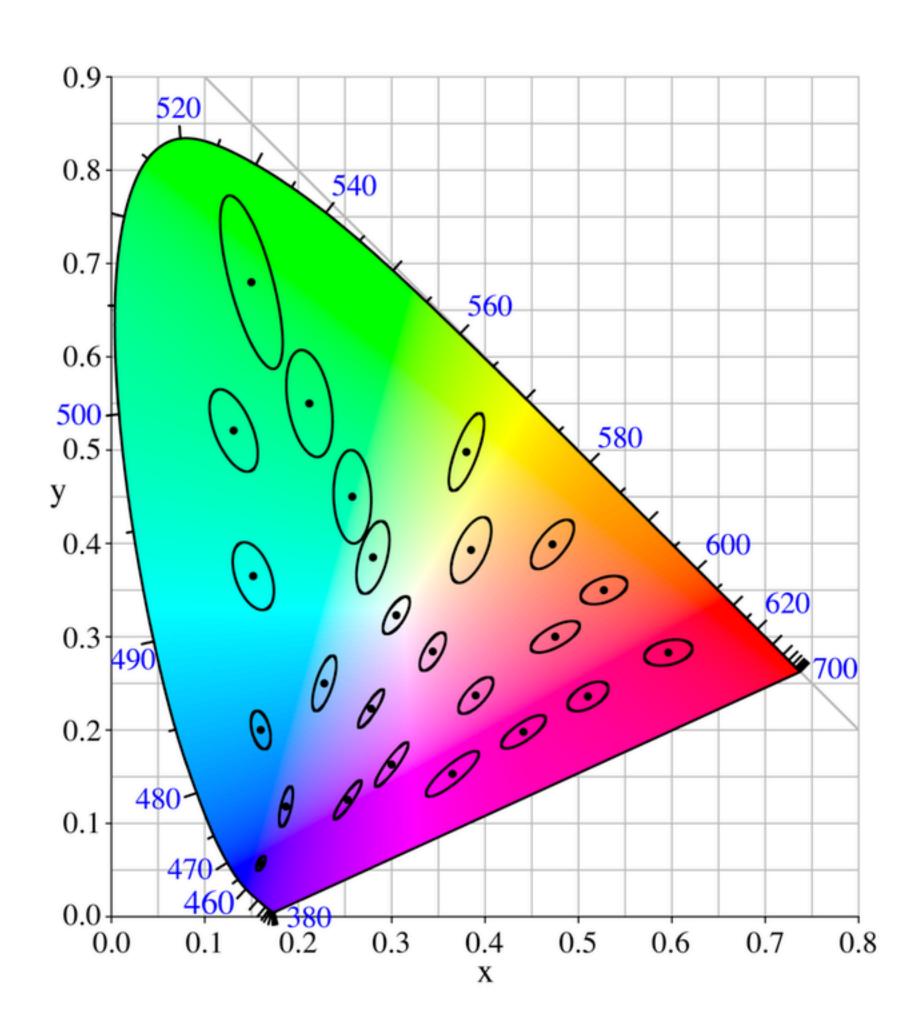
Color Difference

- How do we quantify the difference between two colors?
- Euclidean distance in some color space? Seems like a natural choice.
- What are the requirements for a color difference metric?
 - Perceptually uniform: distance is proportional to perceptual difference
- Is CIE XYZ a "perceptually uniform" color space?
- To answer this question, we can measure the JND threshold of each color:
 - Given a color, we find all the colors that are just noticeably different from it
 - Ideally, these colors should form a perfect circle

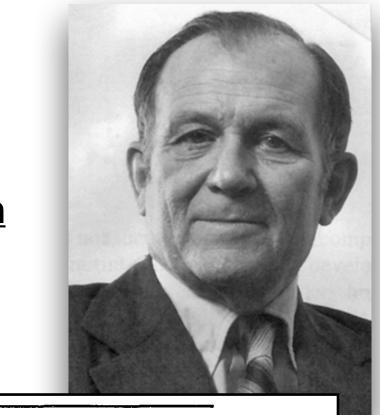
Psychophysics to Find the Threshold Colors



MacAdam Ellipses



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Visual Sensitivities to Color Differences in Daylight*

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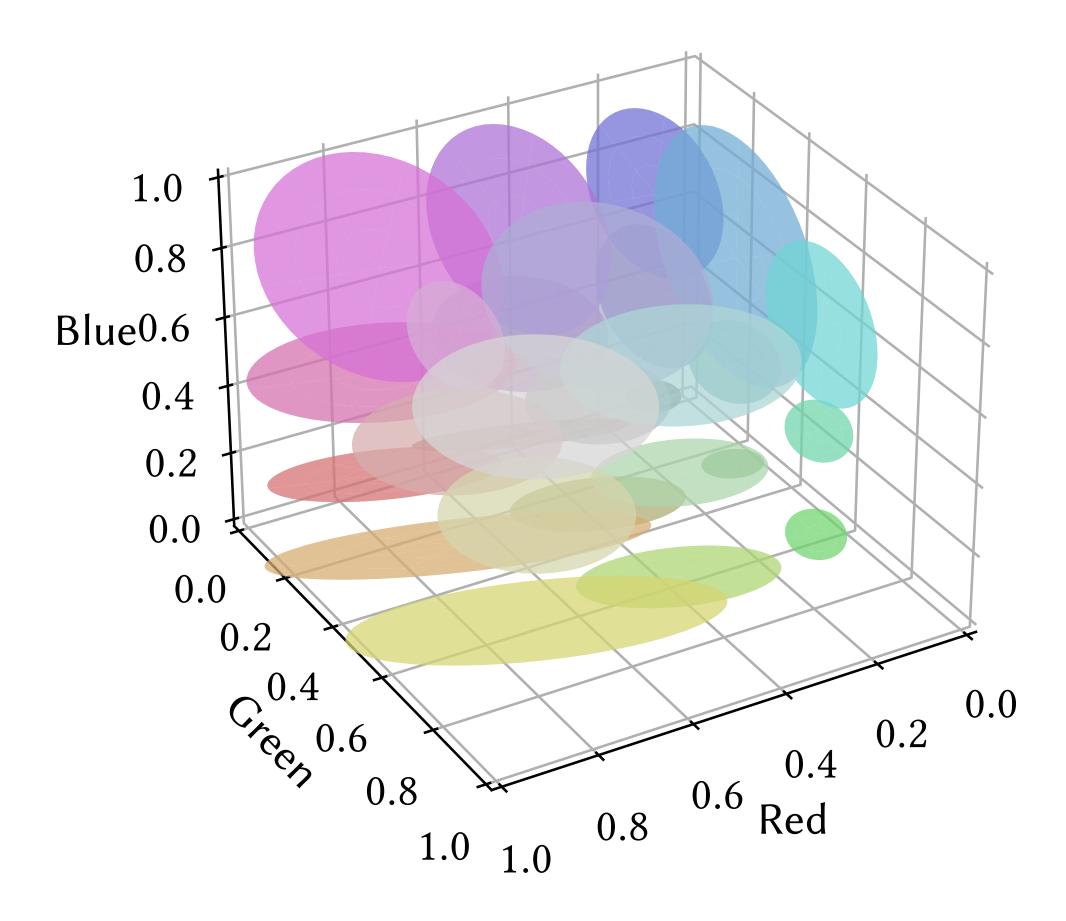
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An apparatus is described which facilitates the presentation of pairs of variable colors without variation of luminance. With this instrument, various criteria of visual sensitivity to color difference have been investigated. The standard deviation of color matching was finally adopted as the most reproducible criterion. The test field was two degrees in diameter, divided by a vertical biprism edge, and was viewed centrally with a surrounding field of fortytwo degrees diameter uniformly illuminated so as to have a chromaticity similar to that of the I.C.I. Standard Illuminant C (average daylight). The luminance of the test field was maintained constant at 15 millilamberts, and the surrounding field was 7.5 millilamberts. These fields were viewed monocularly through an artificial pupil, 2.6 mm in diameter. Over twenty-five thousand trials at color matching have been recorded for a single observer, and the readings are analyzed in detail and compared with previously available data. The standard deviations of the trials are represented in terms of distance in the standard 1931 I.C.I. chromaticity diagram. These increments of distance are represented as functions of position along

straight lines in the chromaticity diagram, and also as functions of direction of departure from points representing certain standard chromaticities. Such representations are simpler than the traditional representations of wavelength thresholds and purity thresholds as functions of wave-length, and the accuracy of the representations is improved by this simplicity. Chromaticity discrimination for non-spectral colors is represented simultaneously and on the same basis as for spectral colors. Small, equally noticeable chromaticity differences are represented for all chromaticities and for all kinds of variations by the lengths of the radii of a family of ellipses drawn on the standard chromaticity diagram. These ellipses cannot be transformed into equal-sized circles by any projective transformation of the standard chromaticity diagram. The consistency of these data with the results of other investigators is exhibited in terms of the noticeabilities of wave-length differences in the spectrum and of the noticeabilities of purity differences from a neutral stimulus, as functions of dominant wave-length.

MacAdam Ellipses

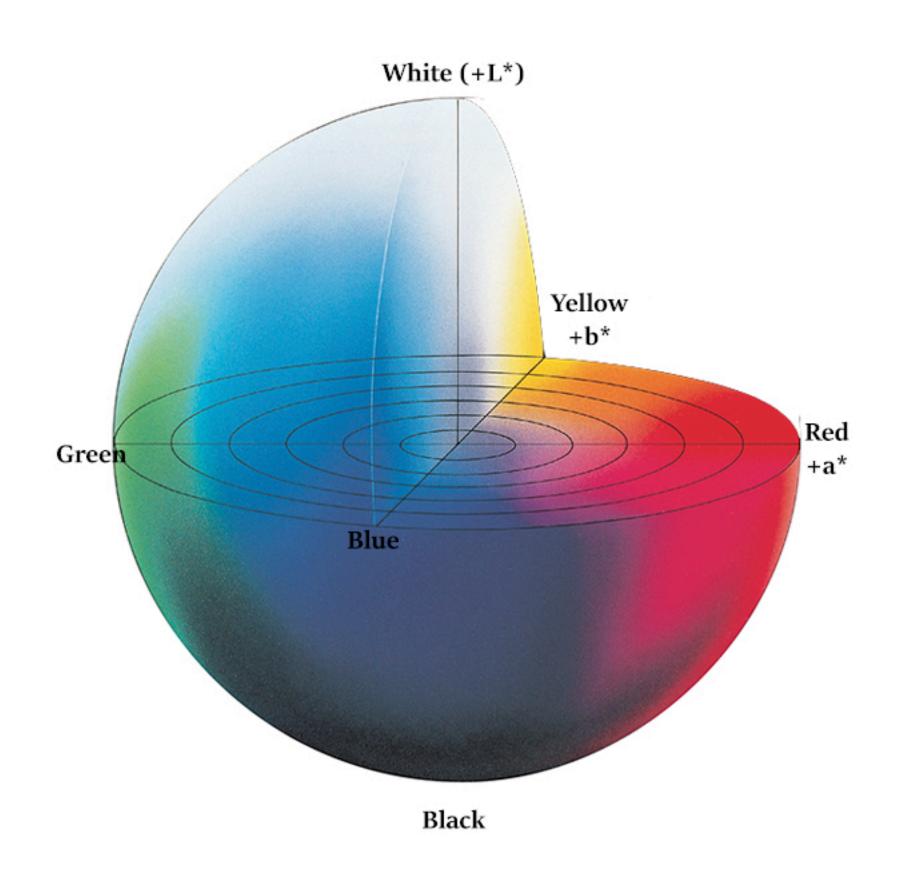
The ellipsoids in the linear sRGB space from our study



- Distances in the XYZ color space (and any RGB color space) are not proportional to the actual color difference.
- Colors in the same MacAdam ellipses (in chromaticity plot) are perceptually the same.
- Ellipses are ellipsoids in 3D color space.

"Perceptually Uniform" Color Spaces

CIELAB (CIE 1976 L*a*b*)



$$X = X_n f^{-1} \left(\frac{L^* + 16}{116} + \frac{a^*}{500} \right)$$

$$Y = Y_n f^{-1} \left(\frac{L^* + 16}{116} \right)$$

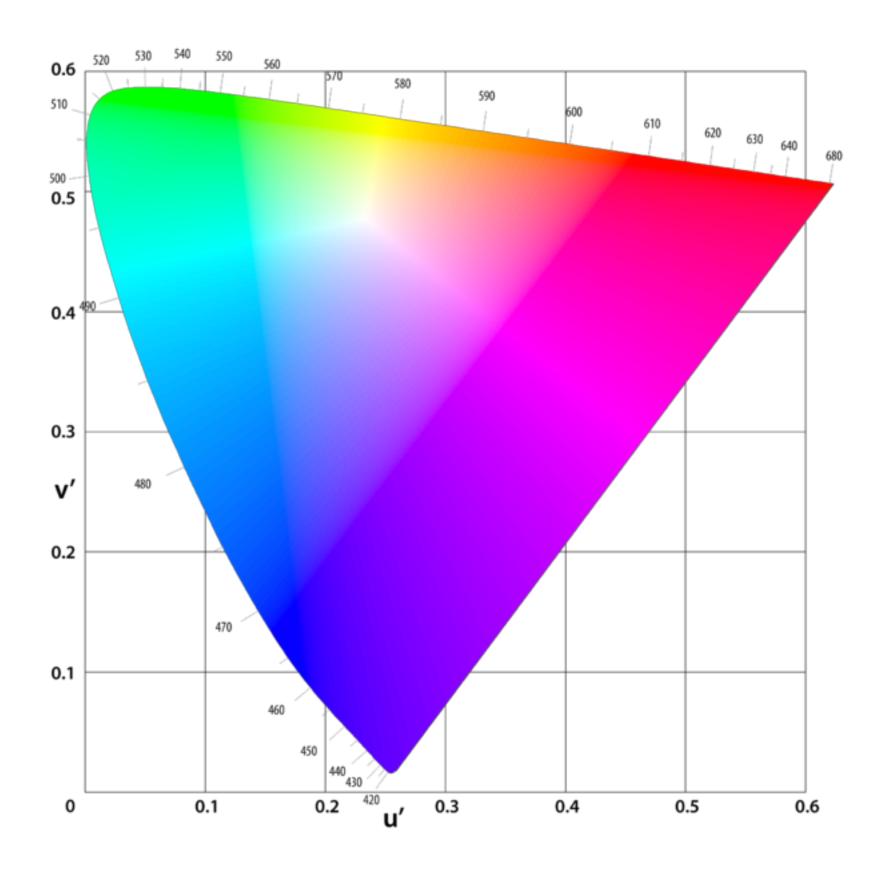
$$Z = Z_n f^{-1} \left(\frac{L^* + 16}{116} - \frac{b^*}{200} \right)$$

$$f^{-1}(t) = \begin{cases} t^3, & t > \delta \\ 3\delta^2(t - \frac{4}{29}) \end{cases}$$
[X_n, Y_n, Z_n] is the brightest

point in the scene

"Perceptually Uniform" Color Spaces

CIELUV (CIE 1976 L*u*v*)



$$L^* = 116 f\left(\frac{Y}{Y_n}\right) - 16 \tag{1.23}$$

$$u^* = 13L^*(u' - u'_n) \tag{1.24}$$

$$v^* = 13L^*(v' - v'_n) \tag{1.25}$$

$$f(x) = \begin{cases} x^{\frac{1}{3}} & x > 0.008856 \\ 7.787x + \frac{16}{116} & x \le 0.008856 \end{cases}$$
 (1.26)

$$u' = \frac{4X}{X + 15Y + 3Z} \tag{1.27}$$

$$v' = \frac{9Y}{X + 15Y + 3Z} \tag{1.28}$$

$$u_n' = \frac{4X_n}{X_n + 15Y_n + 3Z_n} \tag{1.29}$$

$$v_n' = \frac{9Y_n}{X_n + 15Y_n + 3Z_n} \tag{1.30}$$

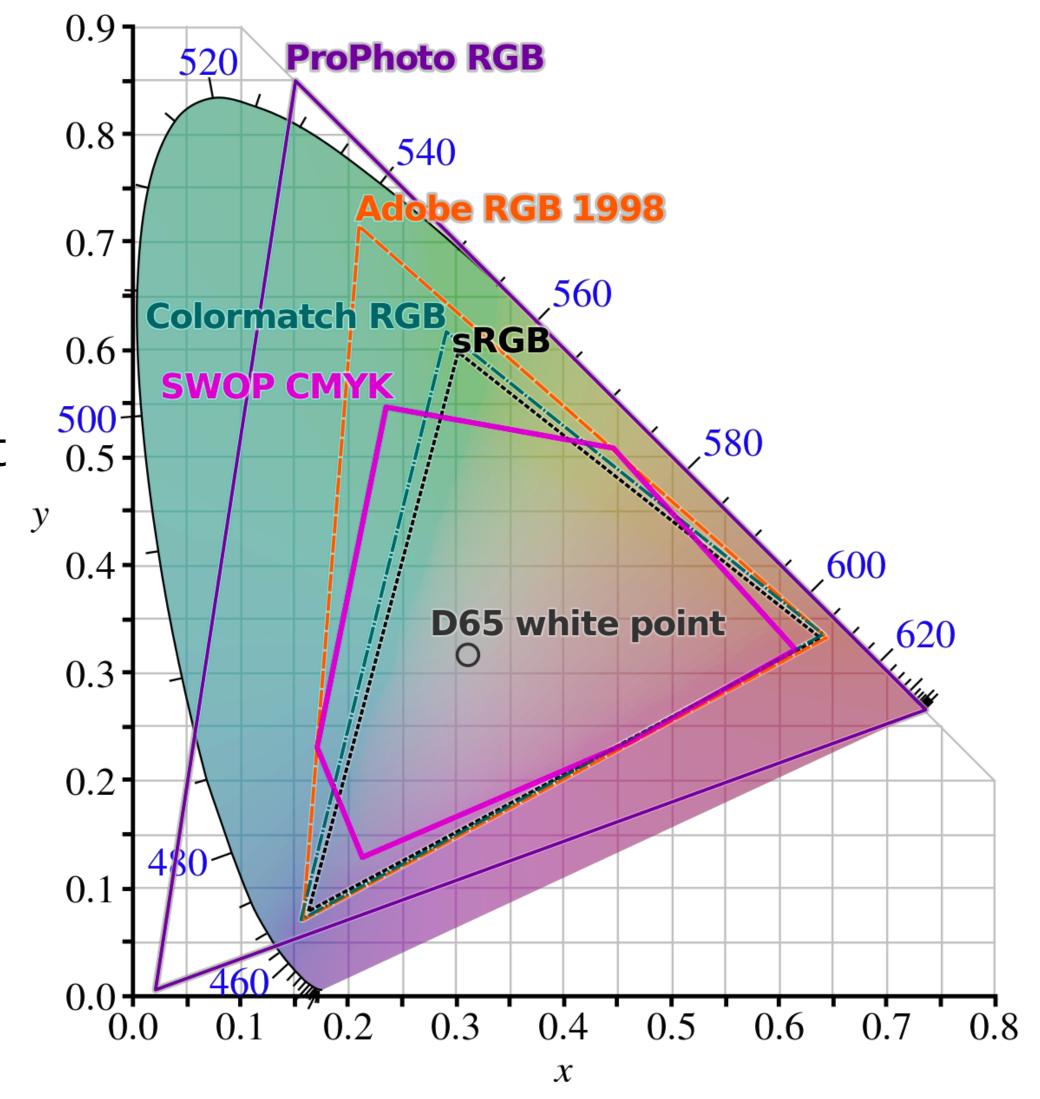
Gamut Mapping and Color Management

Issue: Too ManyColor Spaces

- An image that looks good on display might not look great when printed.
- A typical workflow:
 - Edit a photo using Photoshop on a display that uses AdobeRGB color space.
 - Print the photo using a printer that uses the CMKY color space.

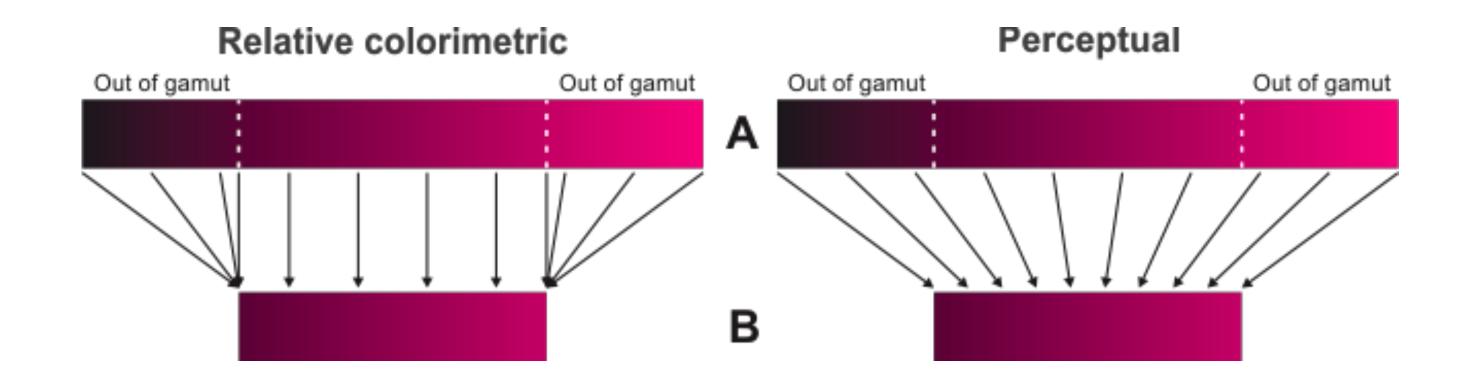
and so forth), we strongly suggest you avoid placing color figures in your document. If you do include color figures, you must (1) use the CMYK (not RGB) colorspace and (2)

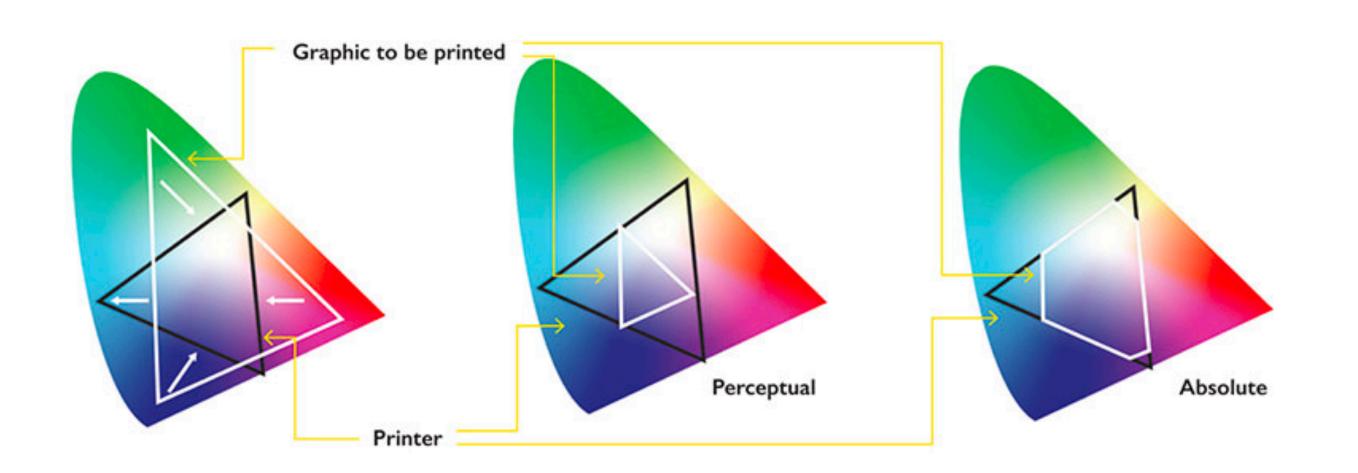
Paper formatting instructions of AAAI 2020.



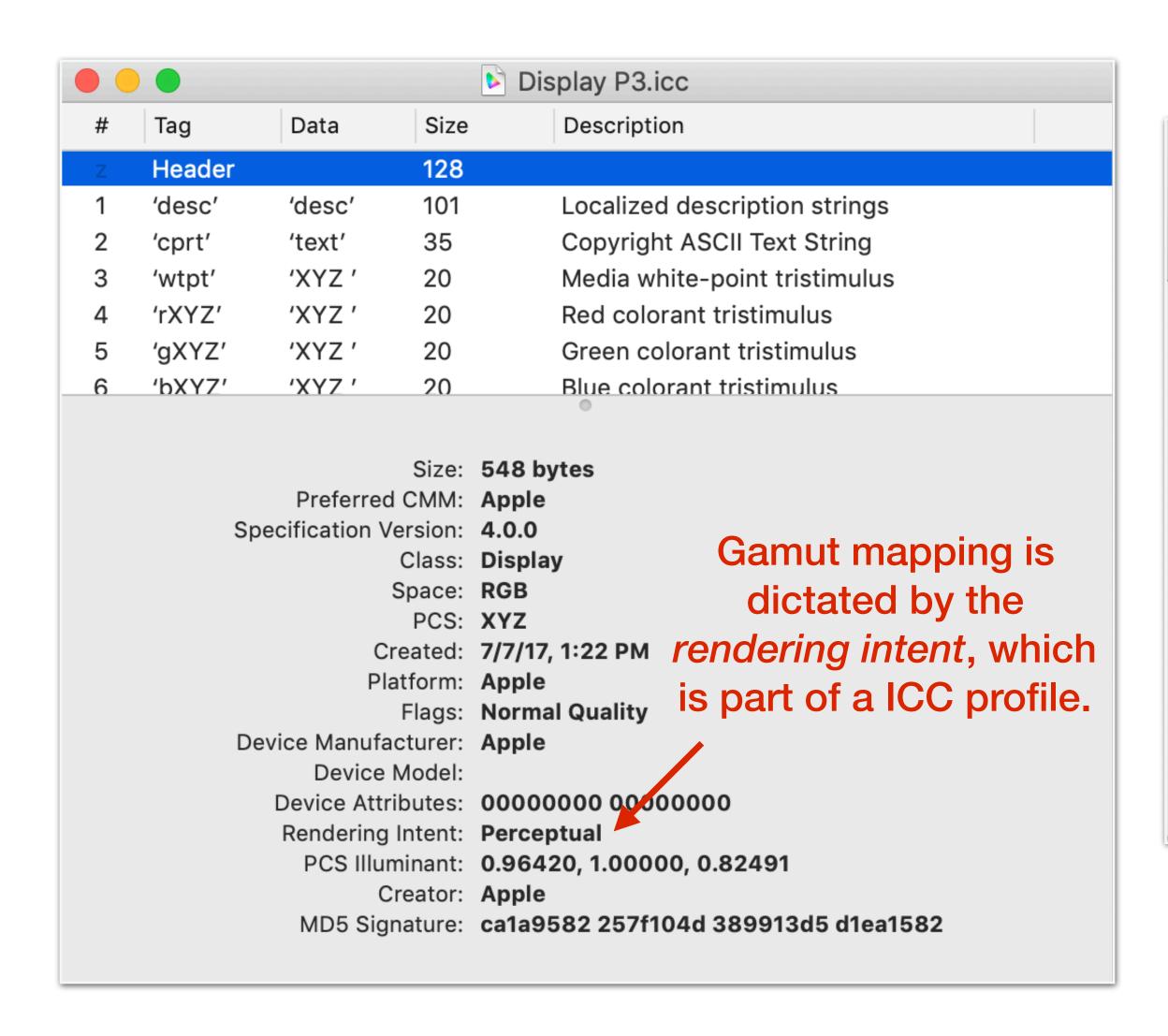
Gamut Mapping

- Modify input color so that it's reproducible on an output device — while maintaining visual quality.
- Many algorithms and strategies.
 International Color Consortium
 (ICC) standards four, called
 rendering intent.
- Part of "color management".

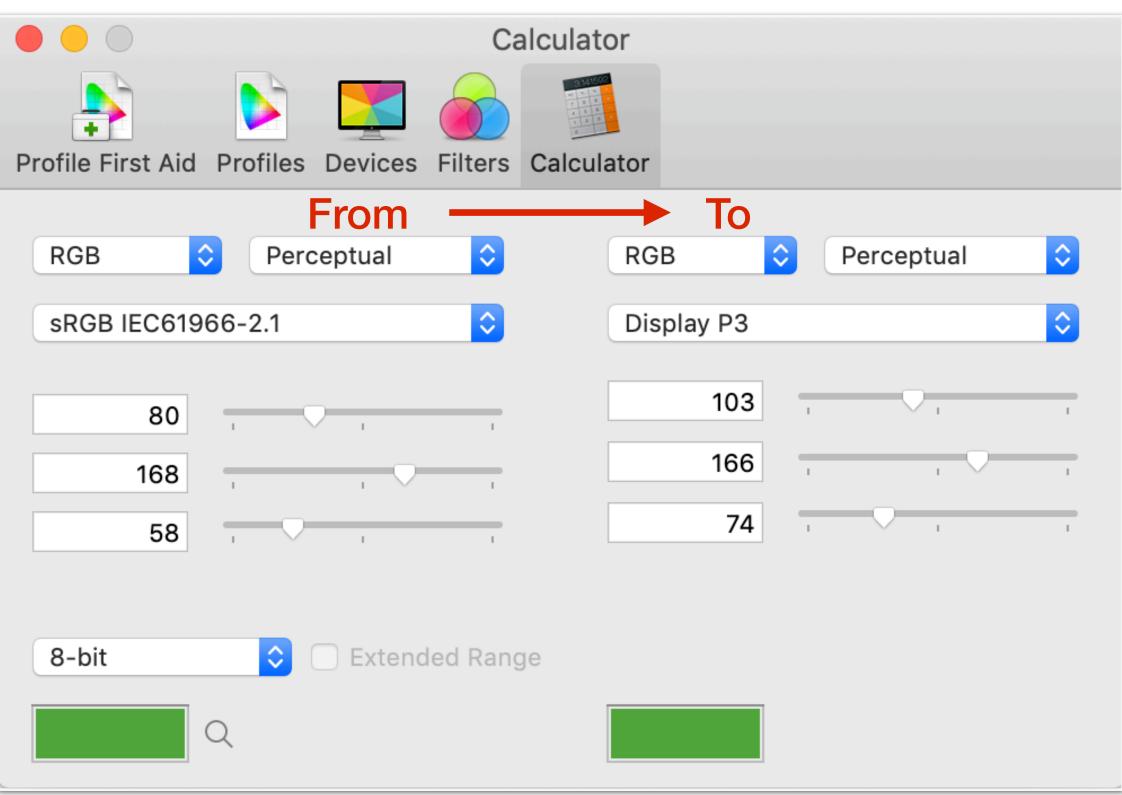




Color Management Tool in Modern OS



Gamut mapping in ColorSync Utility



Key Things to Take Away

- CIE XYZ space has the advantage that its primaries are not dependent on input/output devices and is commonly used to specify colors.
 - XYZ color space is a linear transformation away from the CIE RGB color space, which is a linear transformation away from the LMS space.
- xy-chromaticity plot is a perspective projection from the XYZ space. It decouples the chromaticity from the intensity, and allows us to visualize colors, a 3D concept, in a 2D space.
- Many concepts in color have intuitive explanations in the xy space.
 - E.g., no color space can represent all colors humans can see.

Key Things to Take Away

- To build a color space we will need to specify not only the three primary colors but also the white point.
 - The unit system of any color space is such as mixing equal units of the primaries produces white.
 - There is no single white, but colors of blackbody radiation, which happen to be similar to daylight colors at different times, are usually used as white points.
 - A color space also needs to specify how to quantize colors. We quantify perceived lightness uniformly, rather than the light intensity uniformly.
- CMYK is a subtractive color space, and is useful only when describing colors of materials that are purely absorptive (no scattering).

Key Things to Take Away

- There are attempts to come up with perceptually uniform color spaces; they are better than CIE XYZ, but still not perfect (far from it).
- Gamut mapping deals with the issue of reproducing a color outside the gamut of the displaying device.