# Lecture 6: Color Vision


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Computer Imaging & Graphics

# The Roadmap

Theoretical Preliminaries

**Human Visual Systems**

Display and Camera

Modeling and Rendering

Overview

**Color Vision**

Colorimetry

Visual Adaptation

# Red Without Any Red Pixel

https://www.vice.com/en/article/wnkq5n/this-picture-has-no-red-pixelsso-why-do-the-strawberries-still-look-red

# The Dress

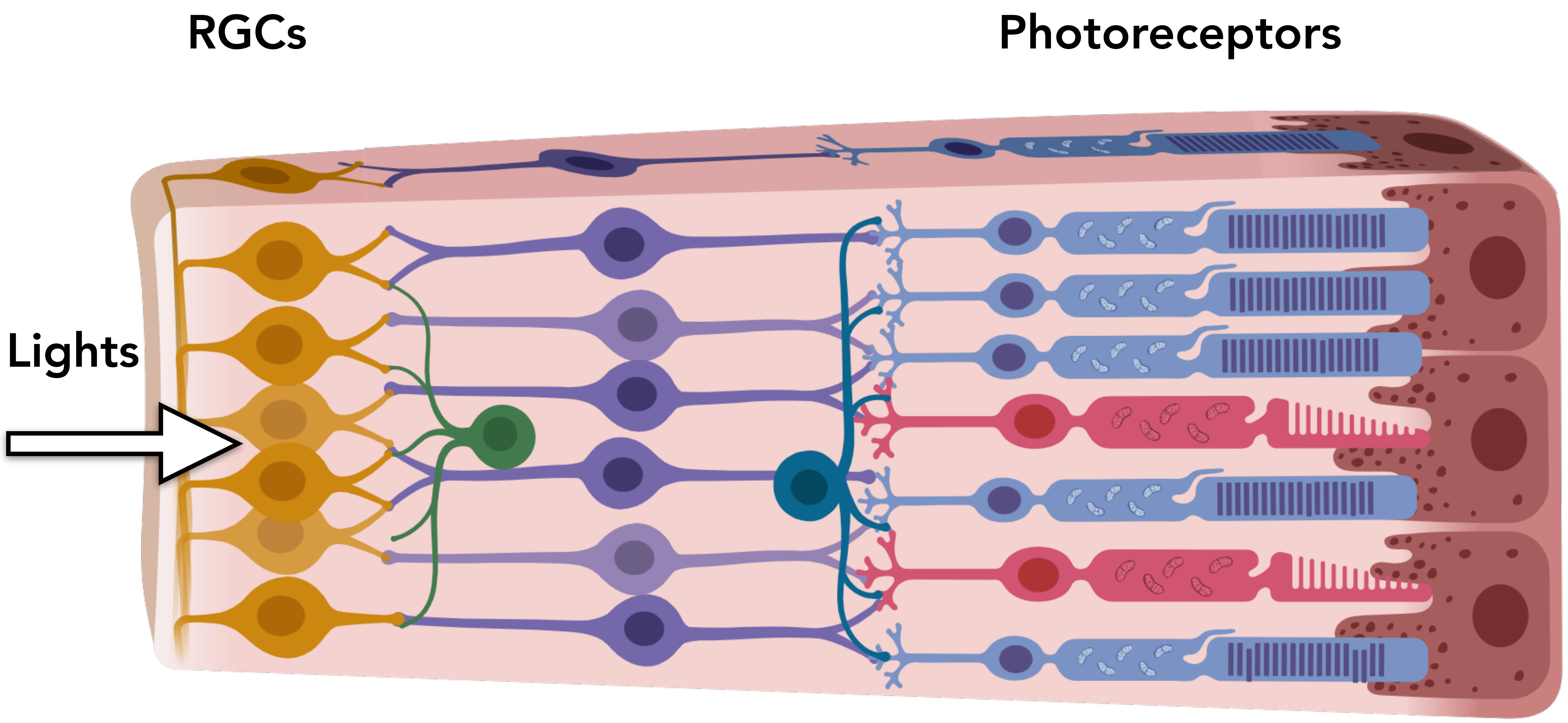
X

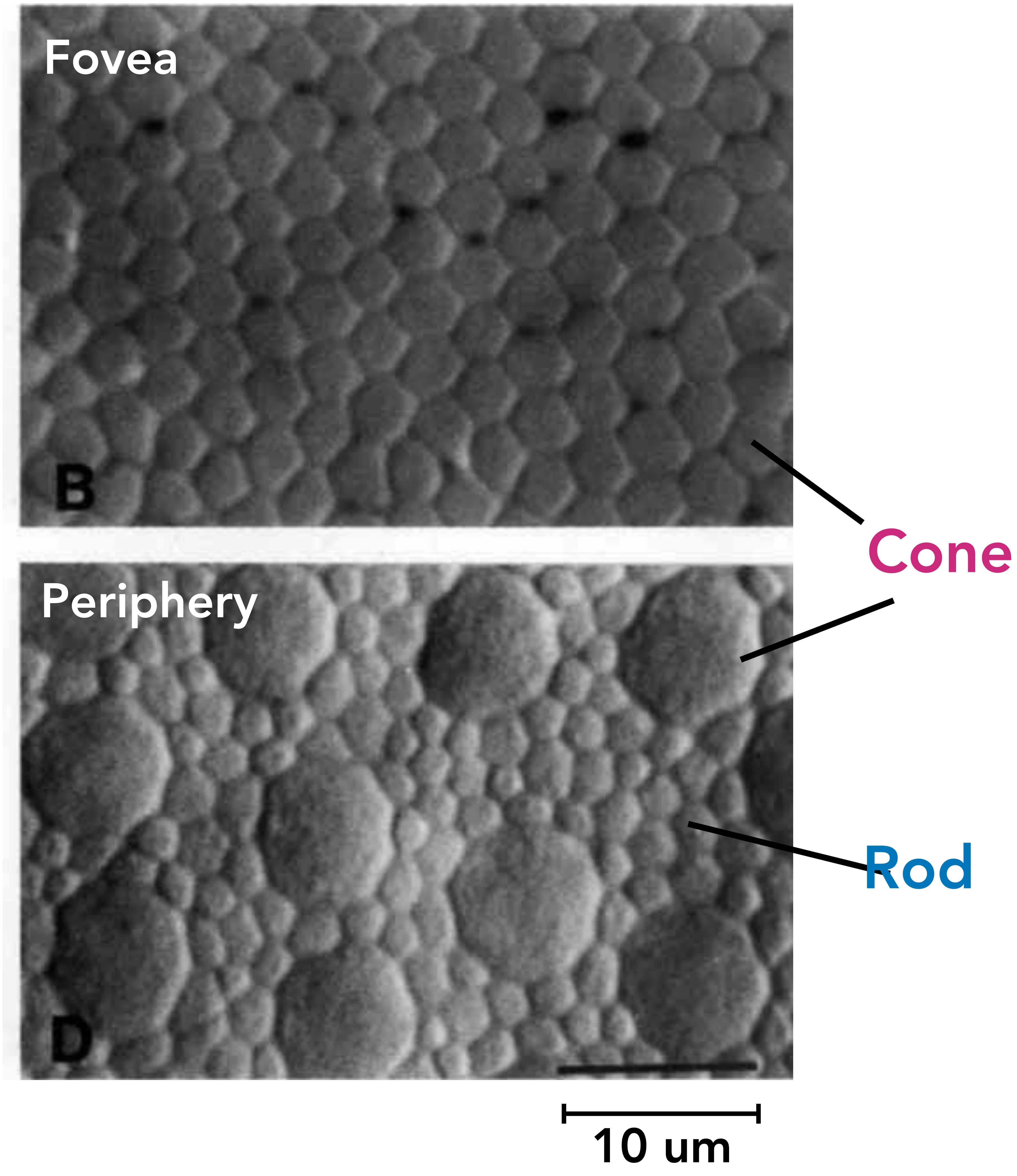
X

# Counting Photons

# (Principle of Univariance)

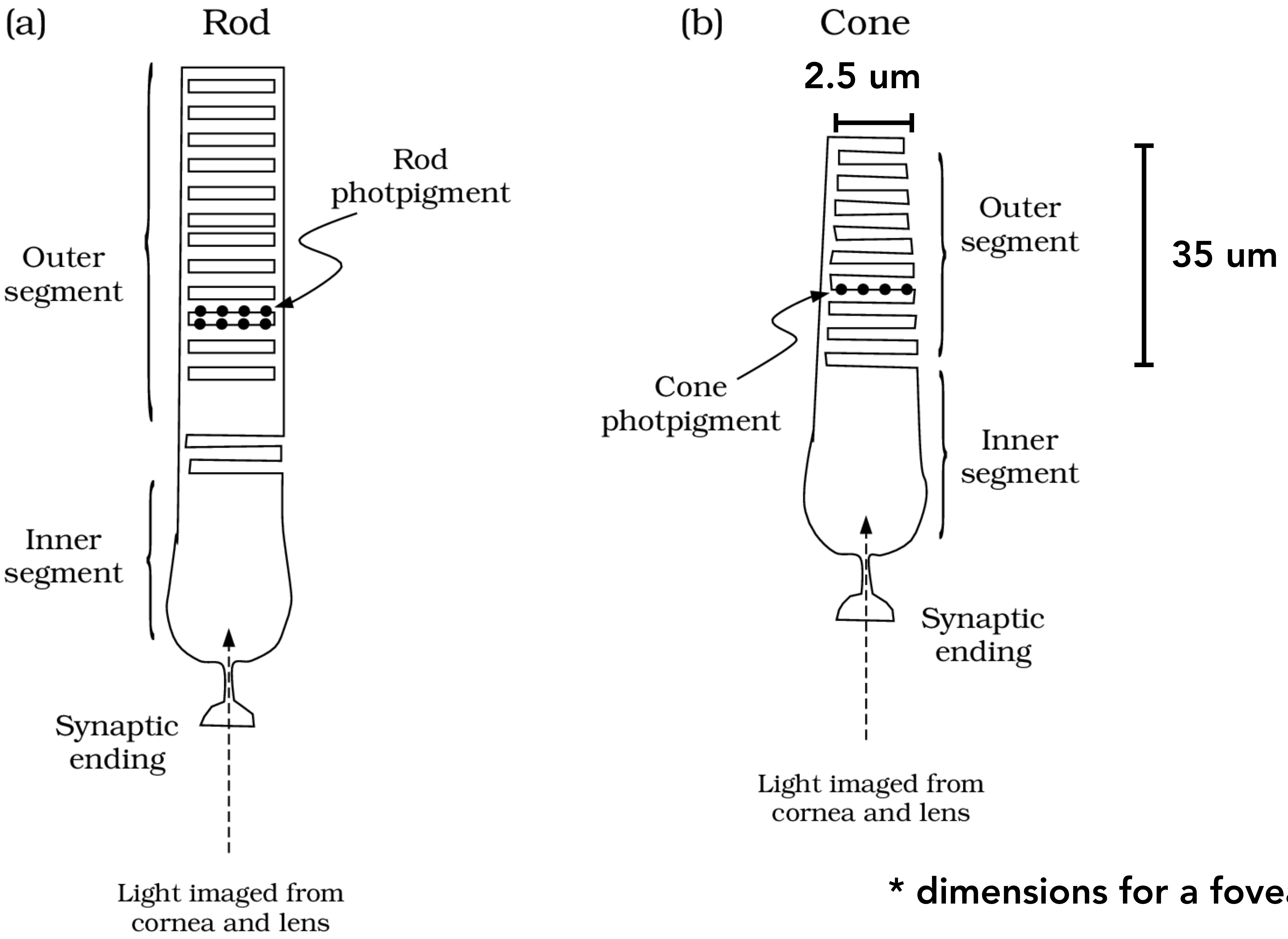
# Recall: Photoreceptors on the Retina




Crucio et al. 1990

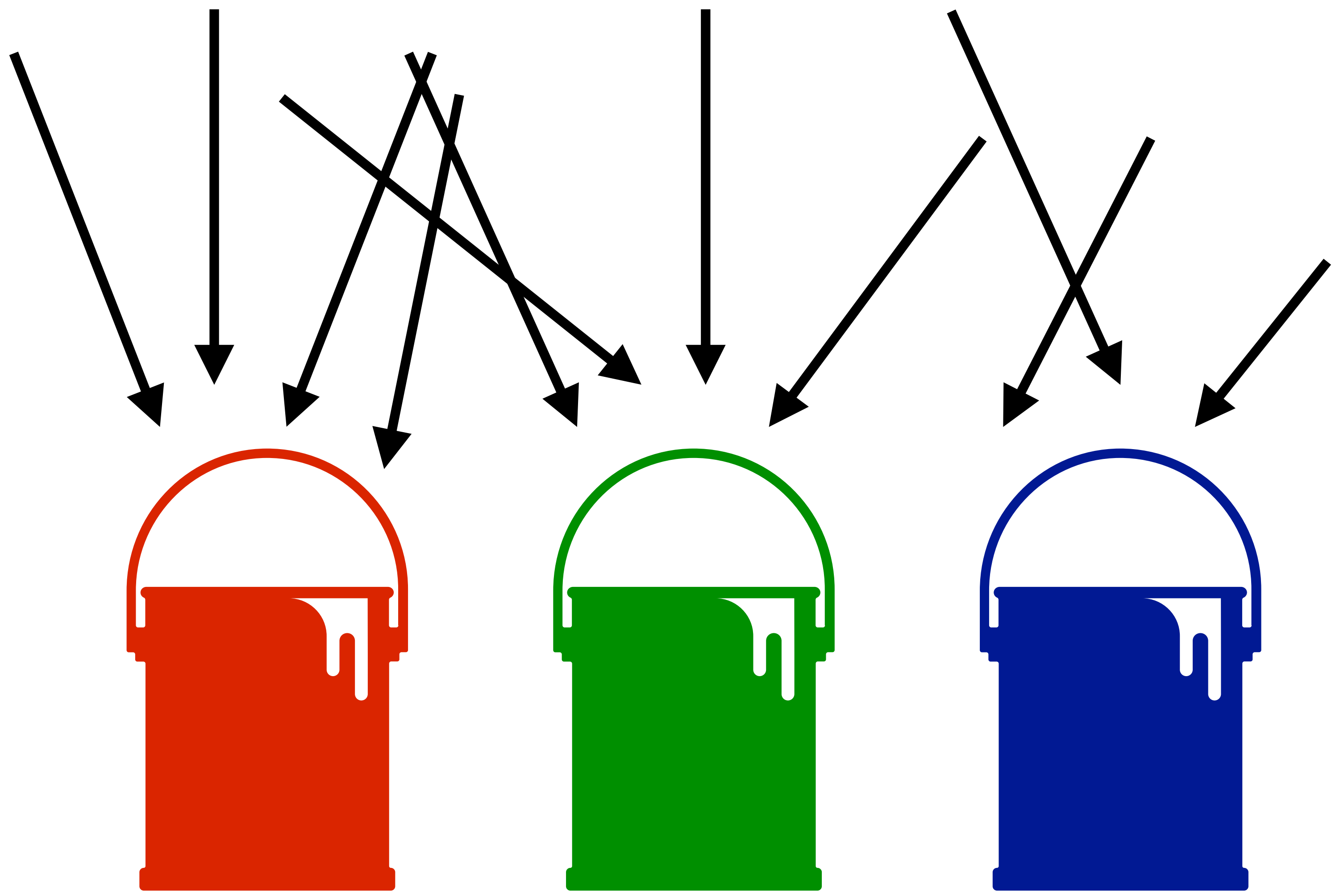
# Anatomical Structure




* dimensions for a fovea cone; from Polyak 1941

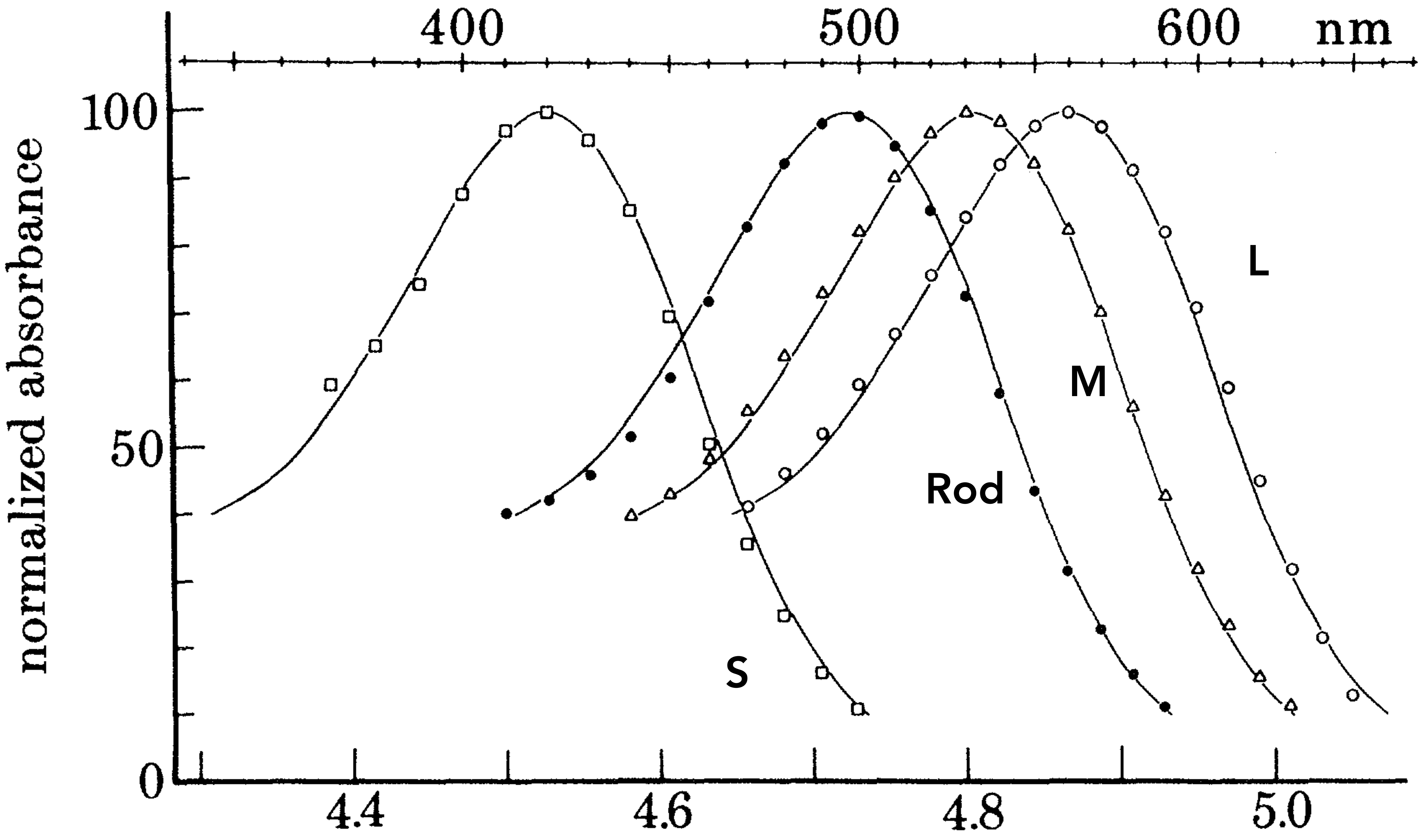
Wandell, 1995

# A Simplified Functional View of Photoreceptors



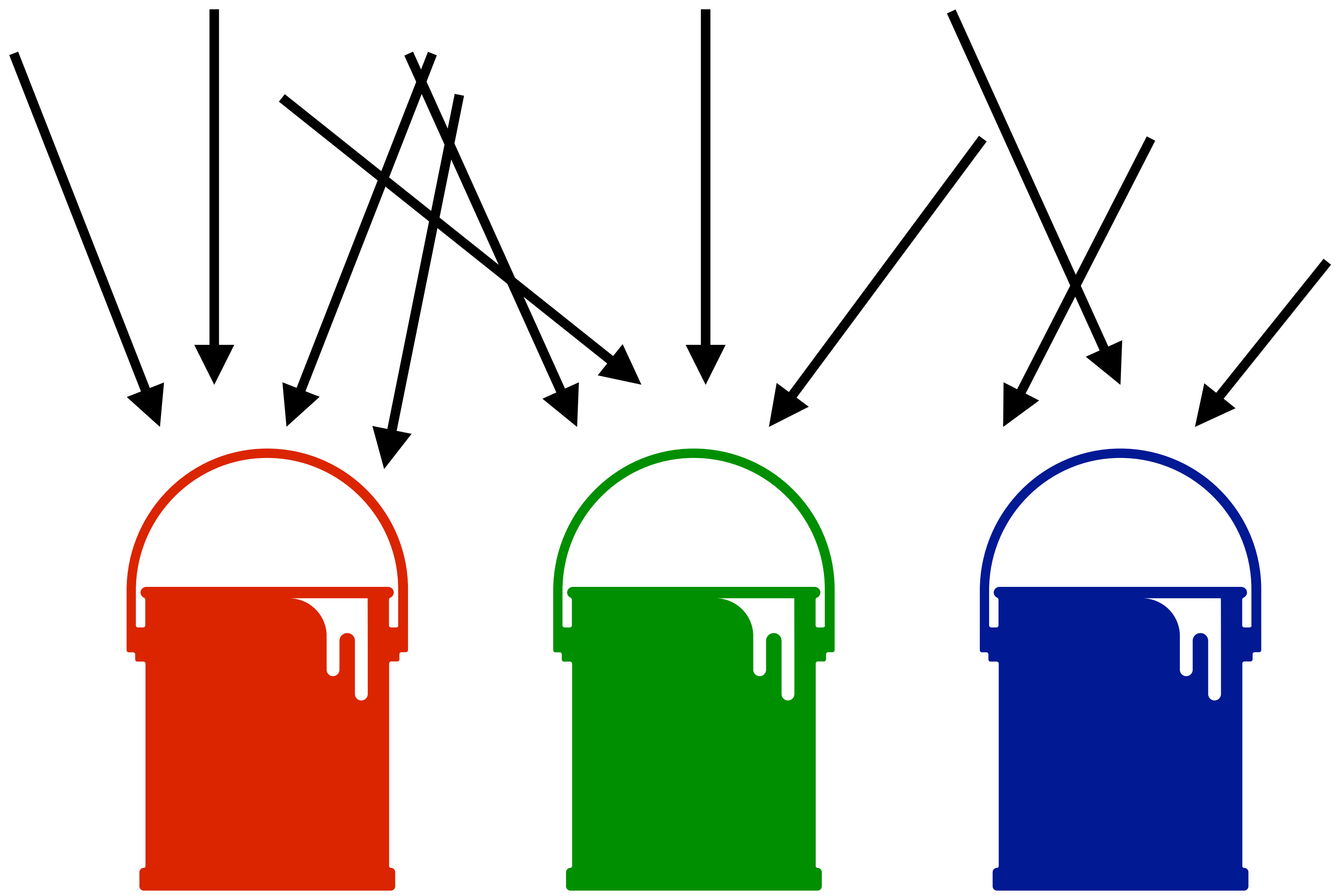
- Each photoreceptor is a bucket that collects photons.
  - Many photons that enter the eye don't hit any bucket (absorbed by lens and macular pigments)
- Any photon that hits a bucket has a probability of being absorbed.
  - The probability p(λ) is wavelength-specific and varies between photoreceptor types
  - Single photon absorption probability ~= percentage for a large population of photons

# Spectral Absorbance



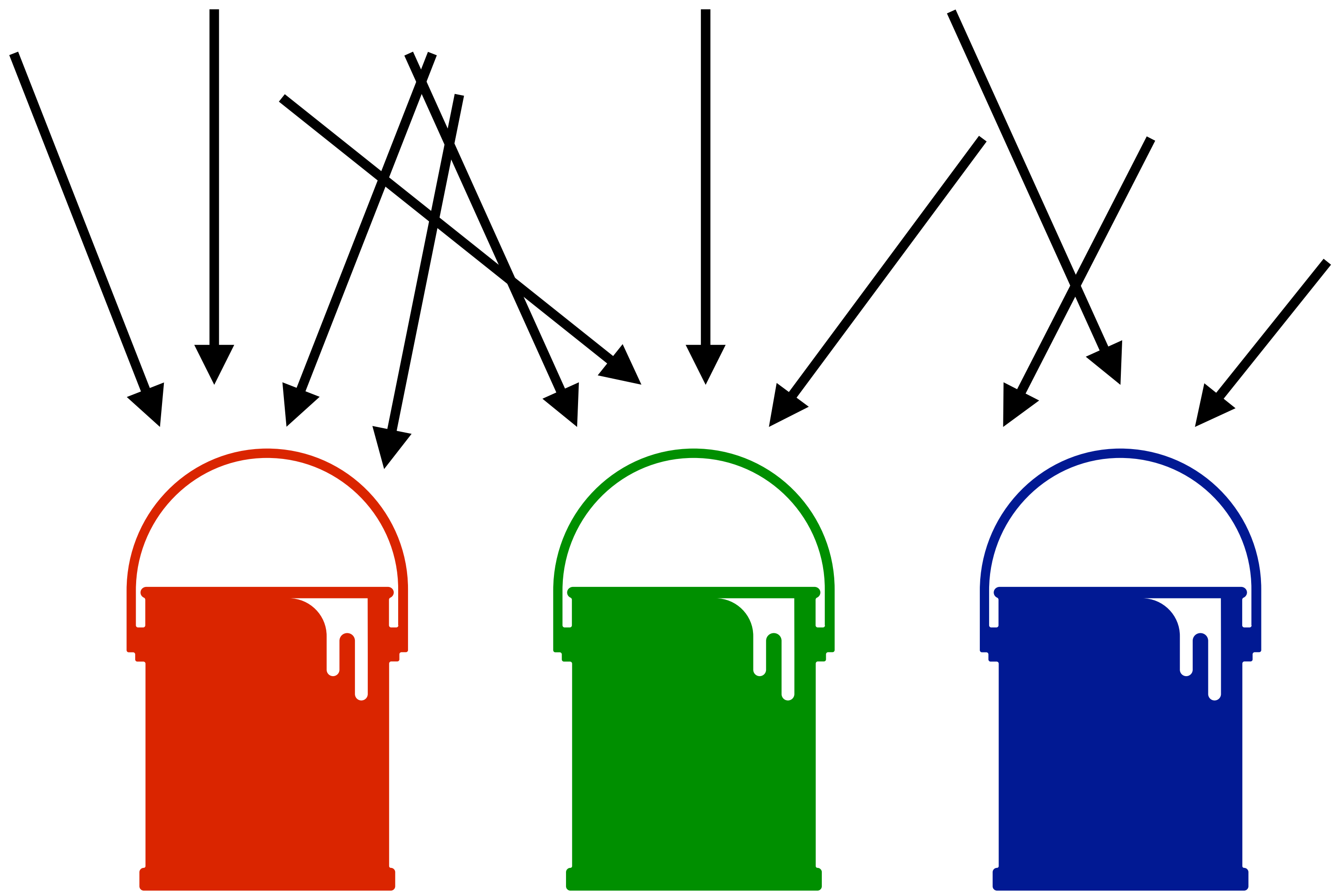

- Absorbance spectra from microspectrophotometry
  - Absorbance is proportional to fraction absorbed under certain simplifications (see lecture notes)
  - Data from humans
- Three cone types
  - Long, Medium, Short

# A Simplified Functional View of Photoreceptors



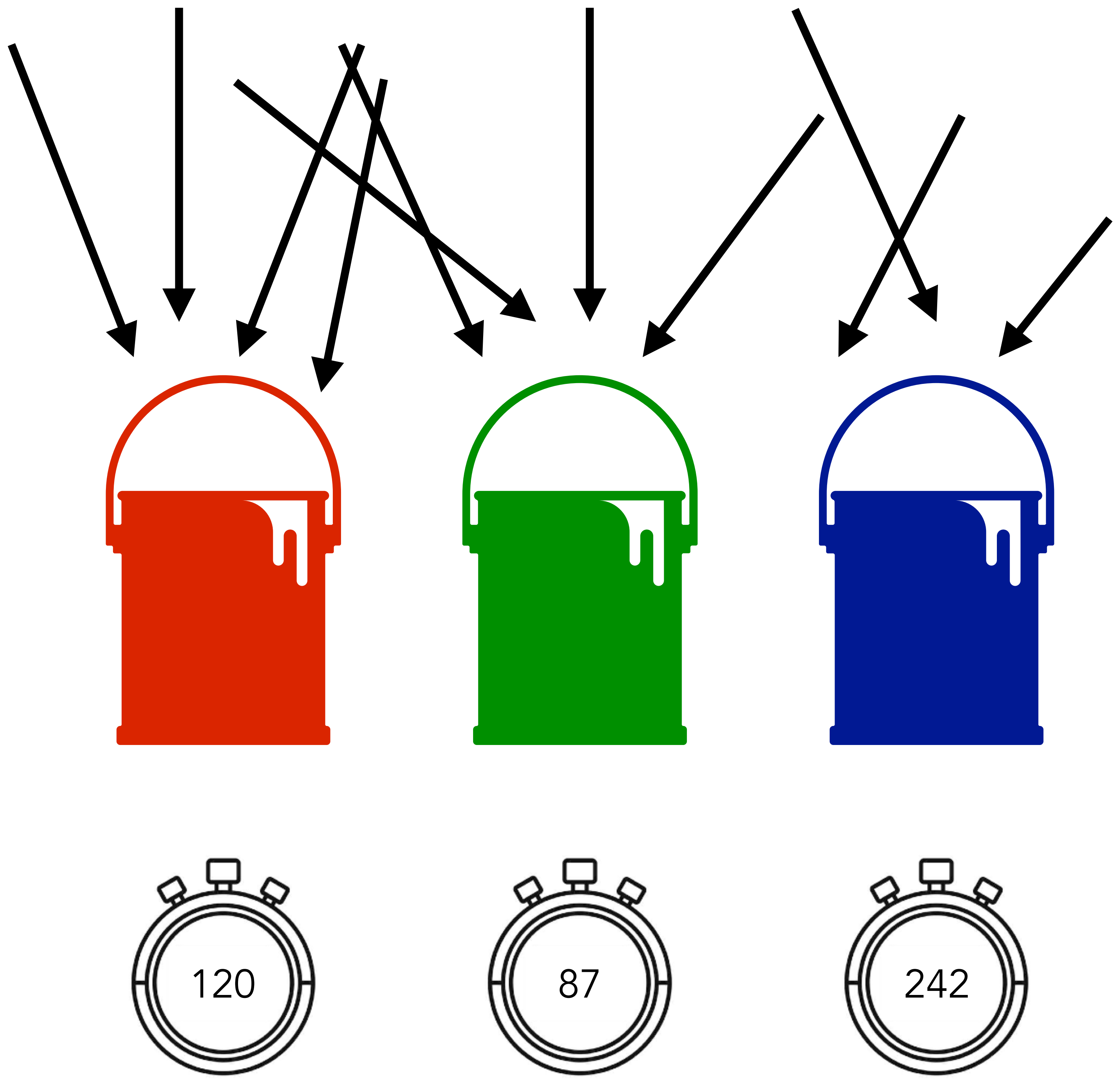
- Once a photon is absorbed, it has a probability of **exciting** the photoreceptor
  - Excitation means generating an electrical response, also called isomerization or pigment bleach
  - This probability is called the **Quantum Efficiency** of the photoreceptor (QE), and is roughly 2/3 invariant to wavelength
- Excitation probability of a single photon:
  - p(λ) QE
  - Usually we ignore QE since it's a constant and just talk about absorption

# Principle of Univariance



- For each photoreceptor type:
  - The electrical response caused by a pigment excitation is constant regardless of the photon's wavelength.
  - Each photon generating an electrical response has the same effect as any other photon that does so.
  - The only effect that wavelength has is to impact the probability a photon gets absorbed; after absorption the wavelength information is lost.

# Principle of Univariance

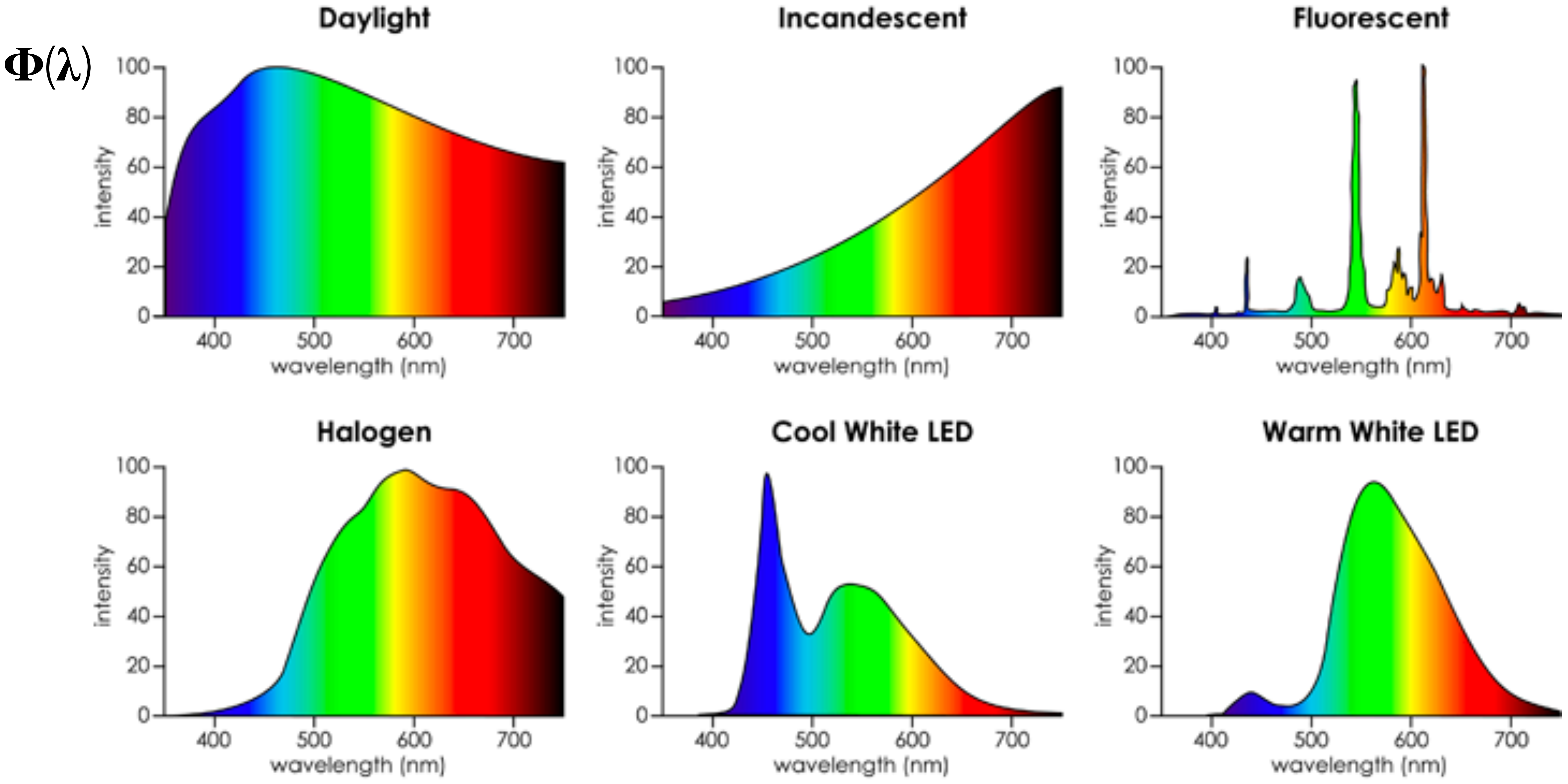



- As if each photoreceptor is counting excitations
- If two photoreceptors (of the same type) have the same "counter value", their electrical responses are the same.
  - It does **not** mean the electrical response is proportional to the number of excitations; more later.

# Wavelength Encoding in Photoreceptors

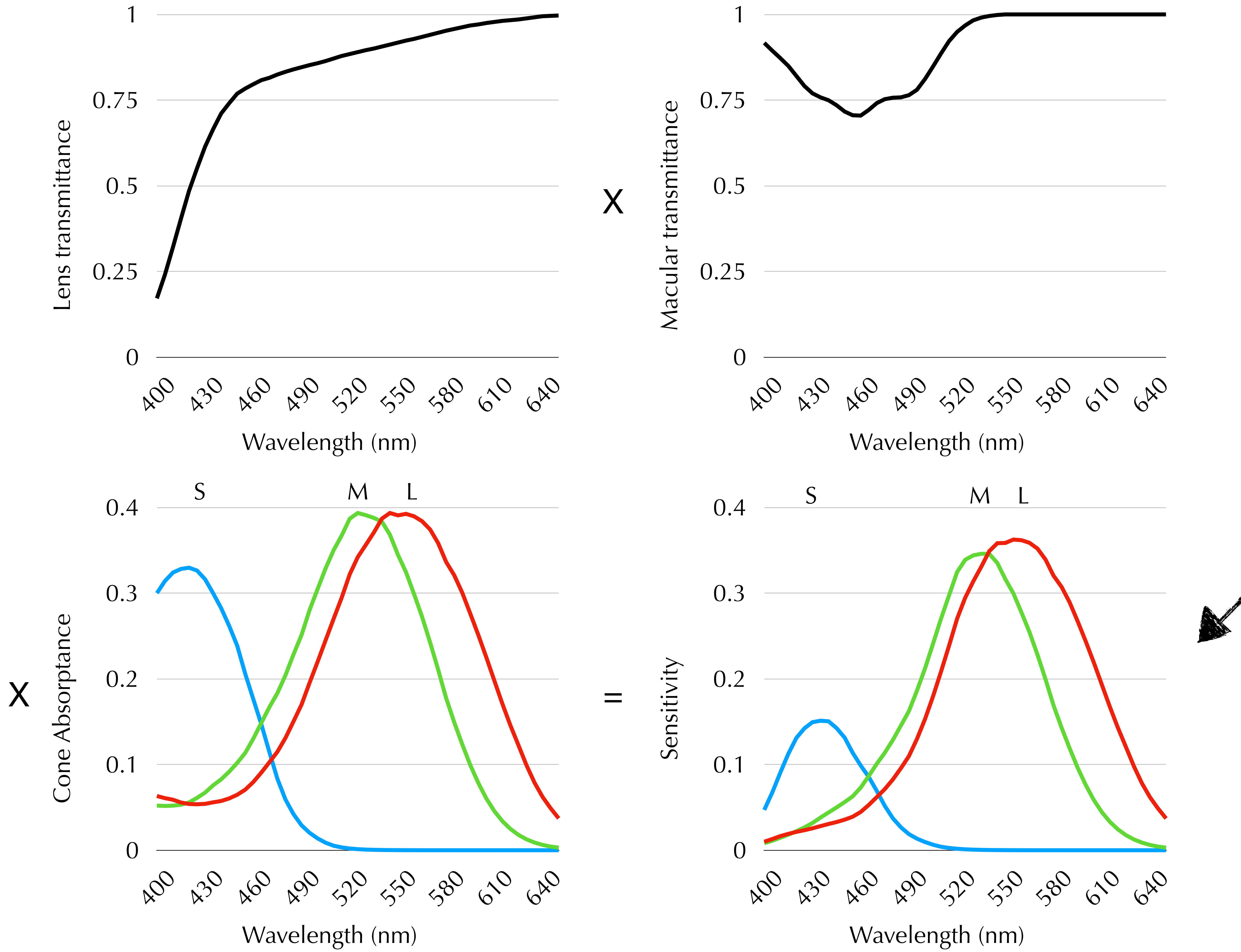
# Spectral Power Distribution

**SPD $\Phi(\lambda)$**: the power distribution by wavelength. Unit is W/nm.




https://www.quora.com/Is-there-a-difference-between-sun-rays-and-normal-bulb-light

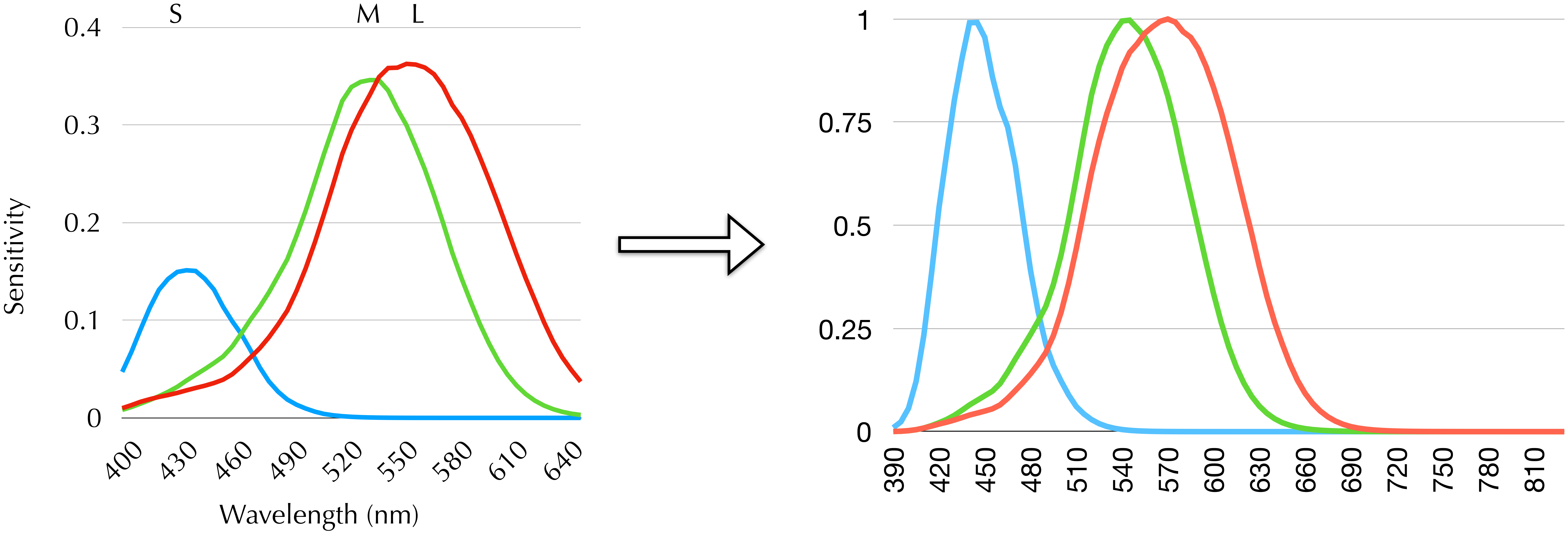
# Cone Fundamentals (Cornea-Referred Spectral Sensitivity)




**Percentage of photons arriving at the cornea that is absorbed by the photoreceptors (at each wavelength)**

# Equal-Energy Cone Fundamentals

Relative percentage of photons absorbed assuming lights at each wavelength have the same power/energy



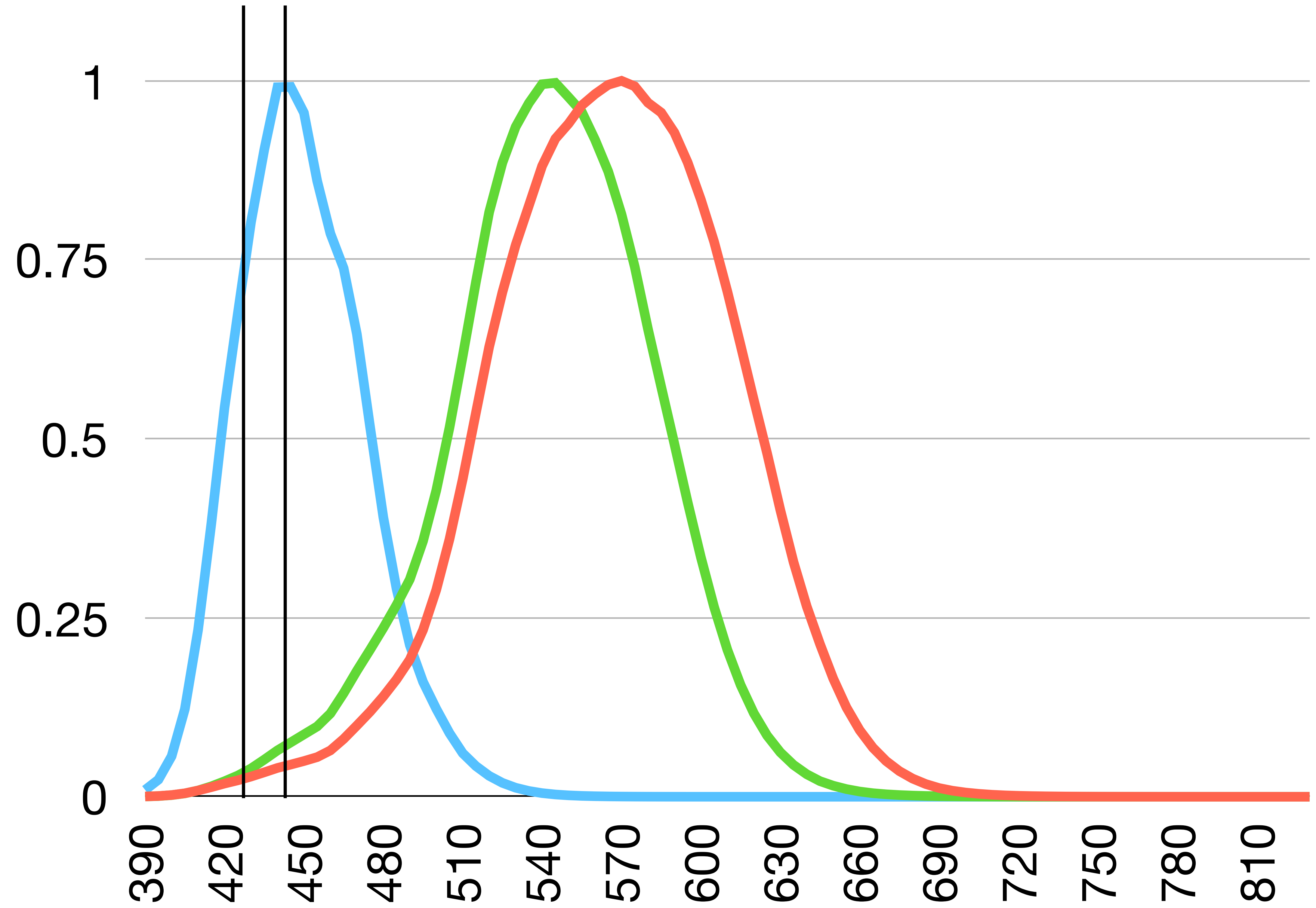

**Normalized each curve to peak at unity, and change equal-quantal to equal-energy**

Stockman & Sharpe (2000); Wandell et al., 2022

# Equal-Energy Cone Fundamentals

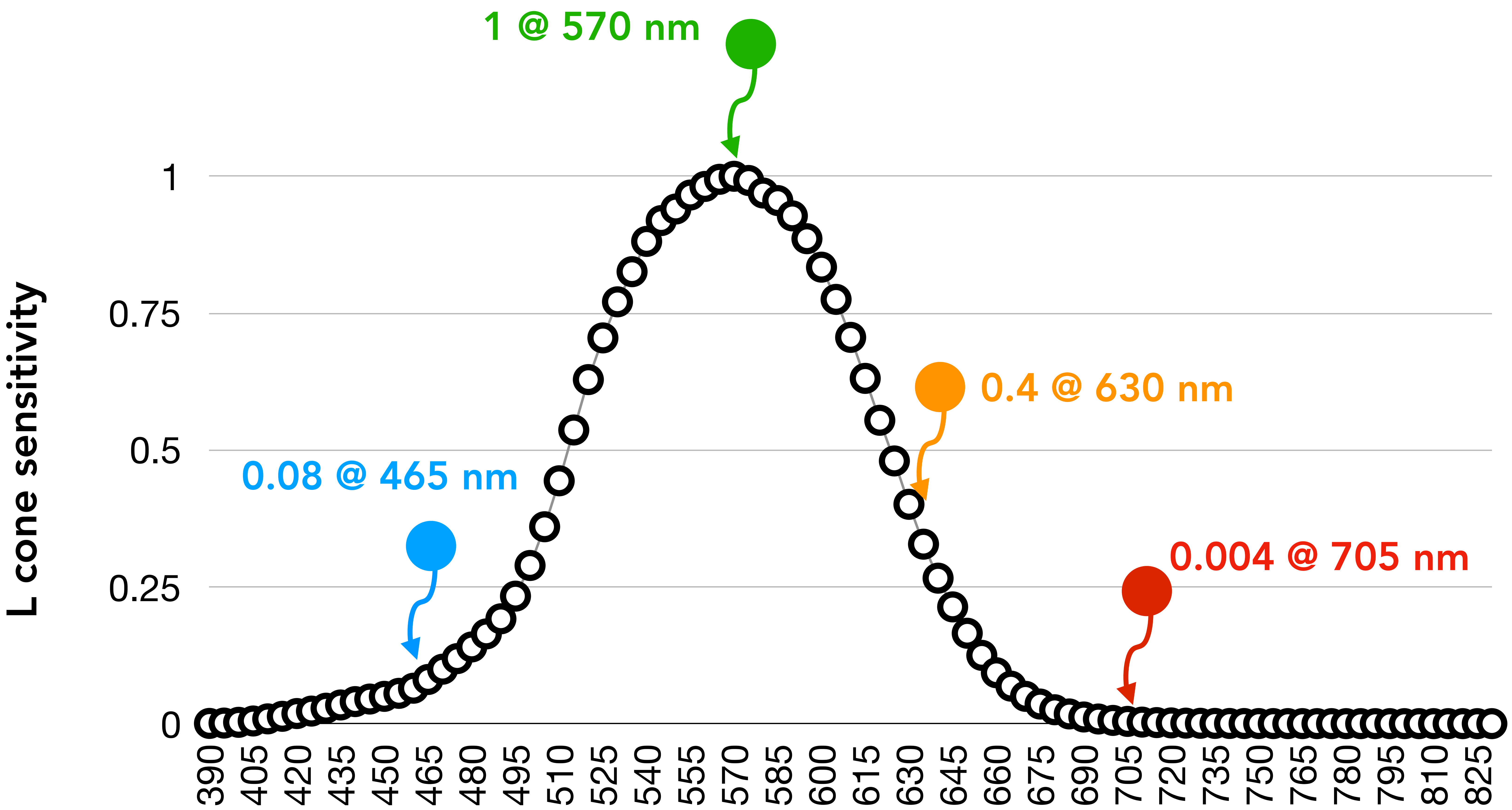
- For S cone, the relative sensitivity at 445nm is 1 and the relative sensitivity at 428 nm is 0.75. This means:
    - Given two lights, one at 445 nm and the other at 428 nm, that have the same power, the relative photon absorption at 428 nm is 75% of that at 445 nm.

**Relative percentage of photons absorbed assuming lights at each wavelength have the same power/energy**

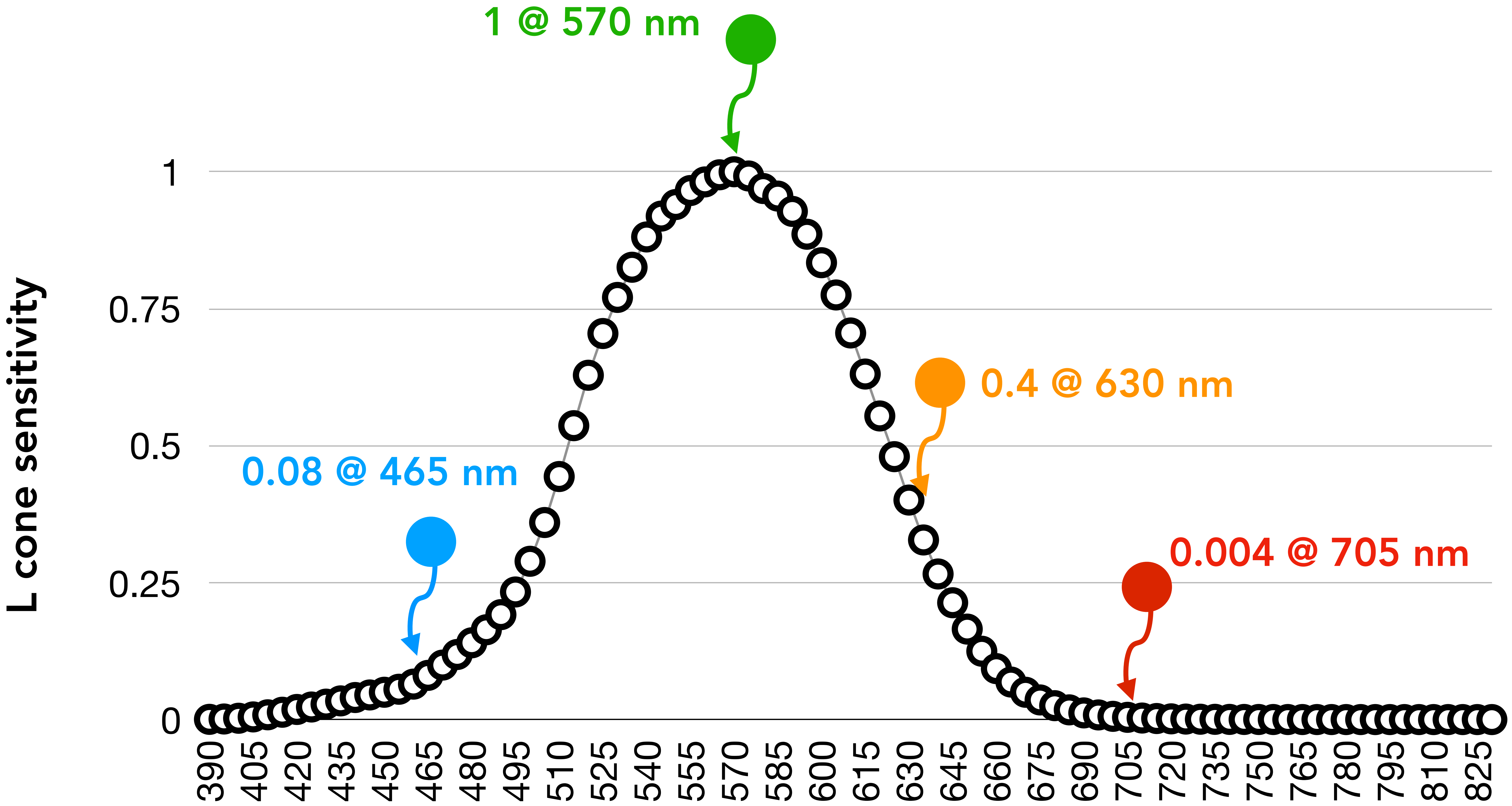



Stockman & Sharpe (2000); Wandell et al., 2022 

# Spectral Sensitivity of L Cones



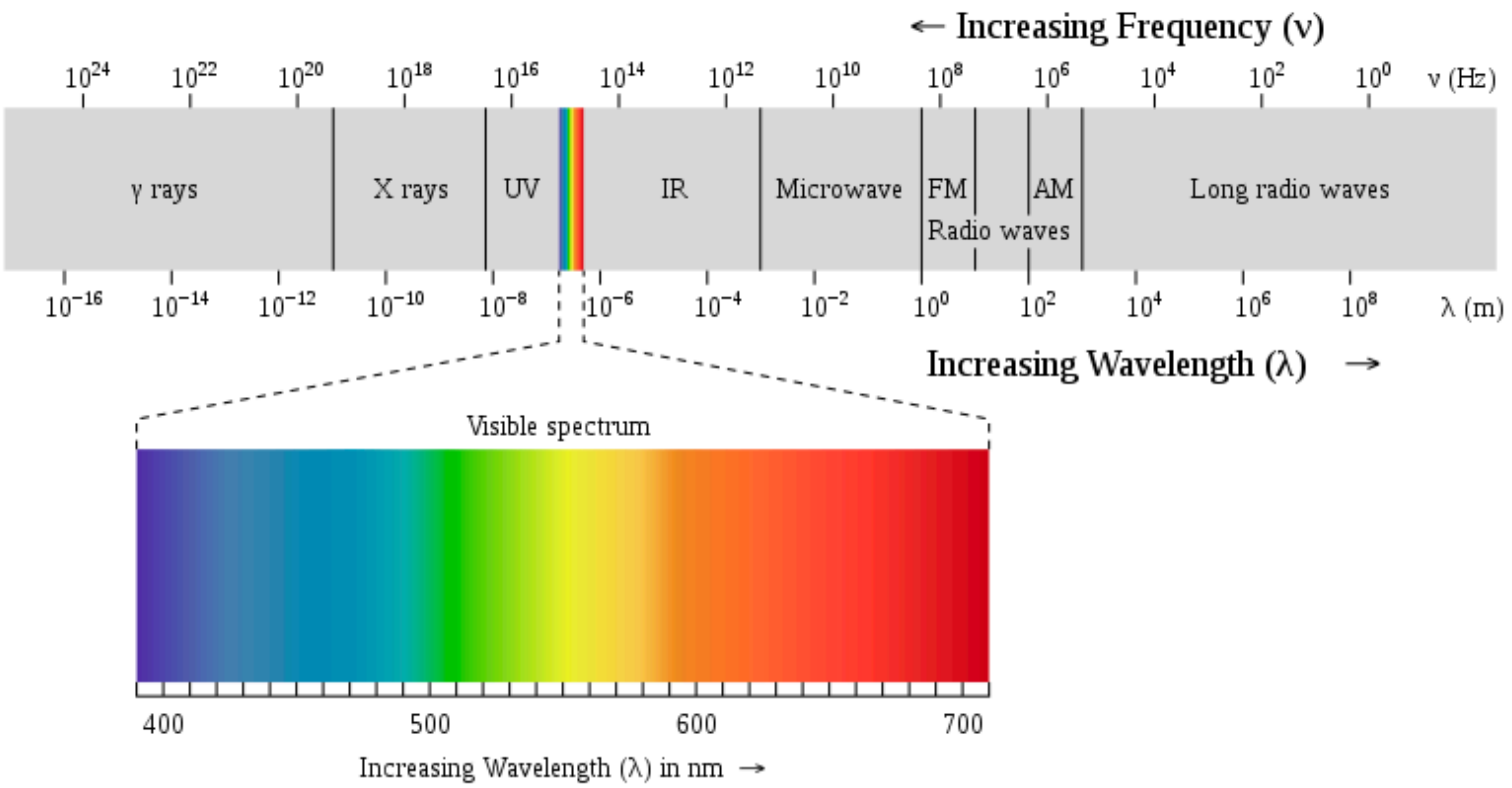
# If We Had Only L Cones

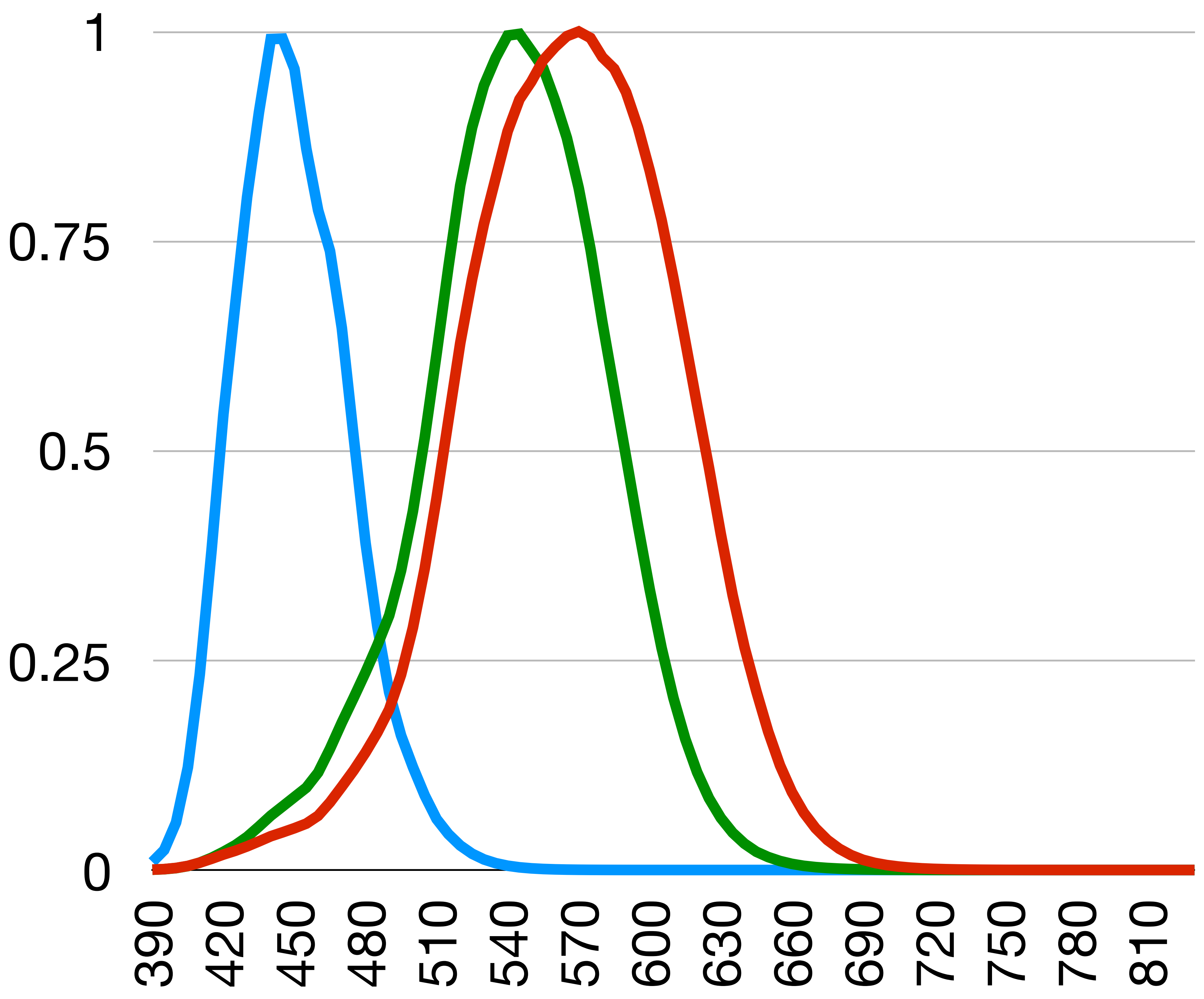



- These lights will be perceived as the same light:
  - 570 nm at 1 W
  - 465 nm at 12.5 W
  - 630 nm at 2.5 W
  - 705 nm at 250 W
- Because the brain will receive exactly the same signal

# Interlude: Invisible Light

- “Invisible”: Lights (electromagnetic waves) not interacting with cones cells.



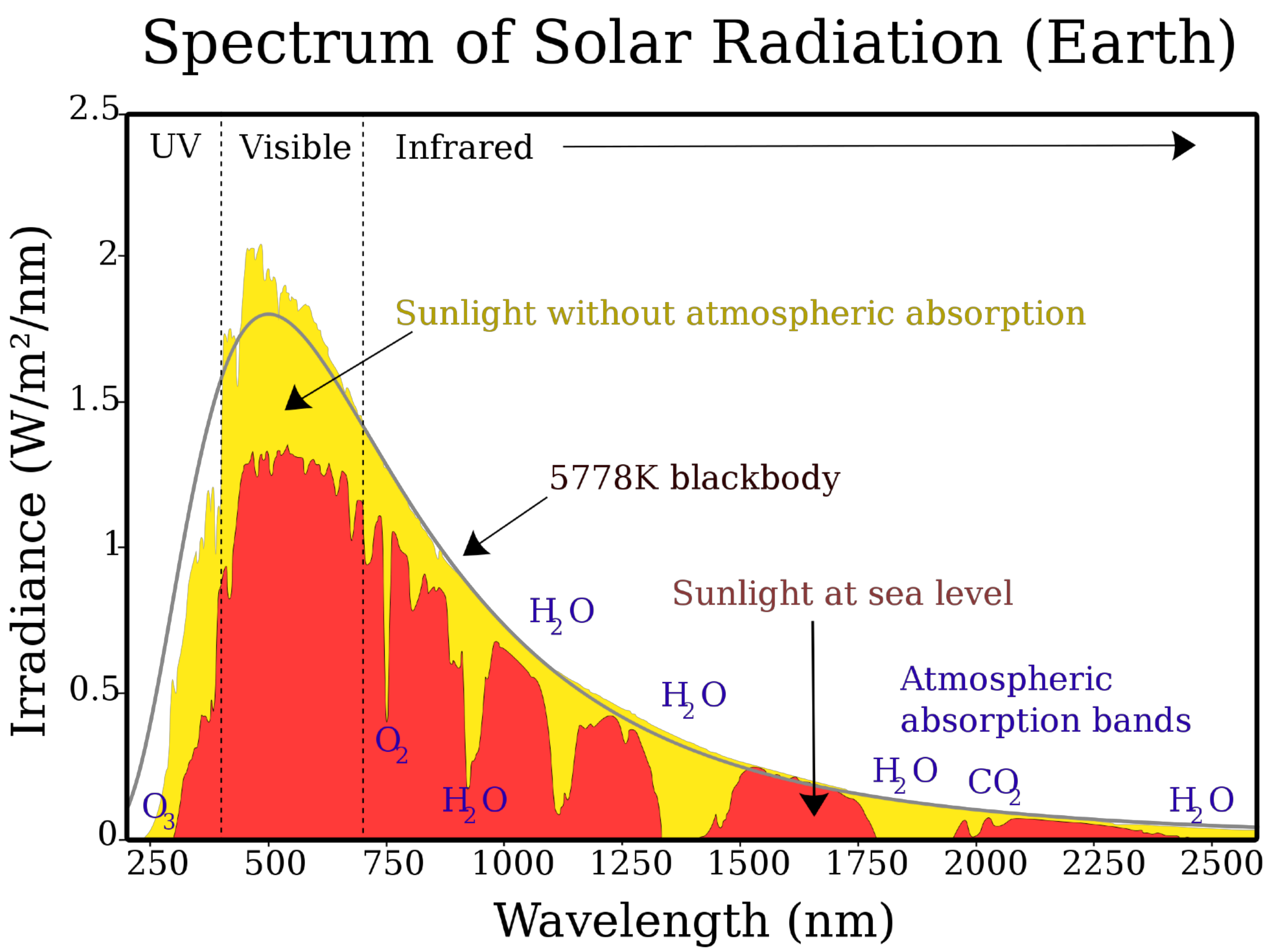




Stockman & Sharpe (2000)

https://commons.wikimedia.org/wiki/File:EM_spectrum.svg

# Interlude: Invisible Light

- Sunlight has invisible components, which don't interact with retina photoreceptors, but interact with other kinds of cells.
  - Case in point: skin cancer, where Ultra-Violet (UV) lights interact with skin cells.
- Another example: Infra-Red (IR) cameras can "see" things we can't.
  - They use materials that do interact with IR lights. More later.




https://en.wikipedia.org/wiki/Sunlight#/media/File:Solar_spectrum_en.svg http://www.psicorp.com/content/psis-applied-research-activities-0

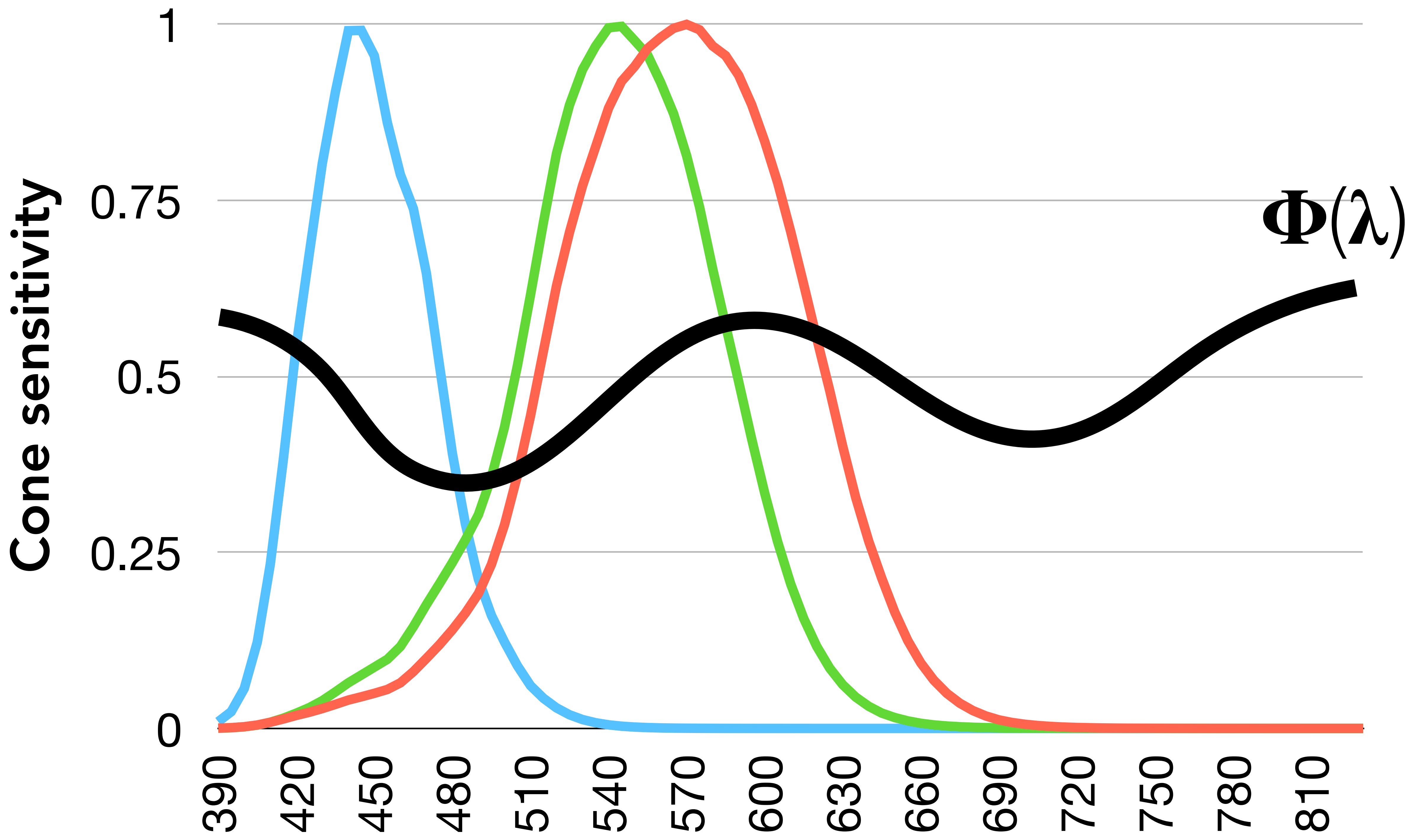
# Total Cone Response

- Given a light $\Phi(\lambda)$, we can calculate the **total responses** of the L, M, and S cone cells. The three signals are called the **tristimulus values** of the light.

$$L = \sum_{\lambda=380}^{780} \Phi(\lambda) L(\lambda)$$

In practice both the SPD and the cone responses are given at discrete intervals.

$$L = \int_{\lambda} \Phi(\lambda) L(\lambda) d\lambda$$




Stockman and Sharpe (2000)

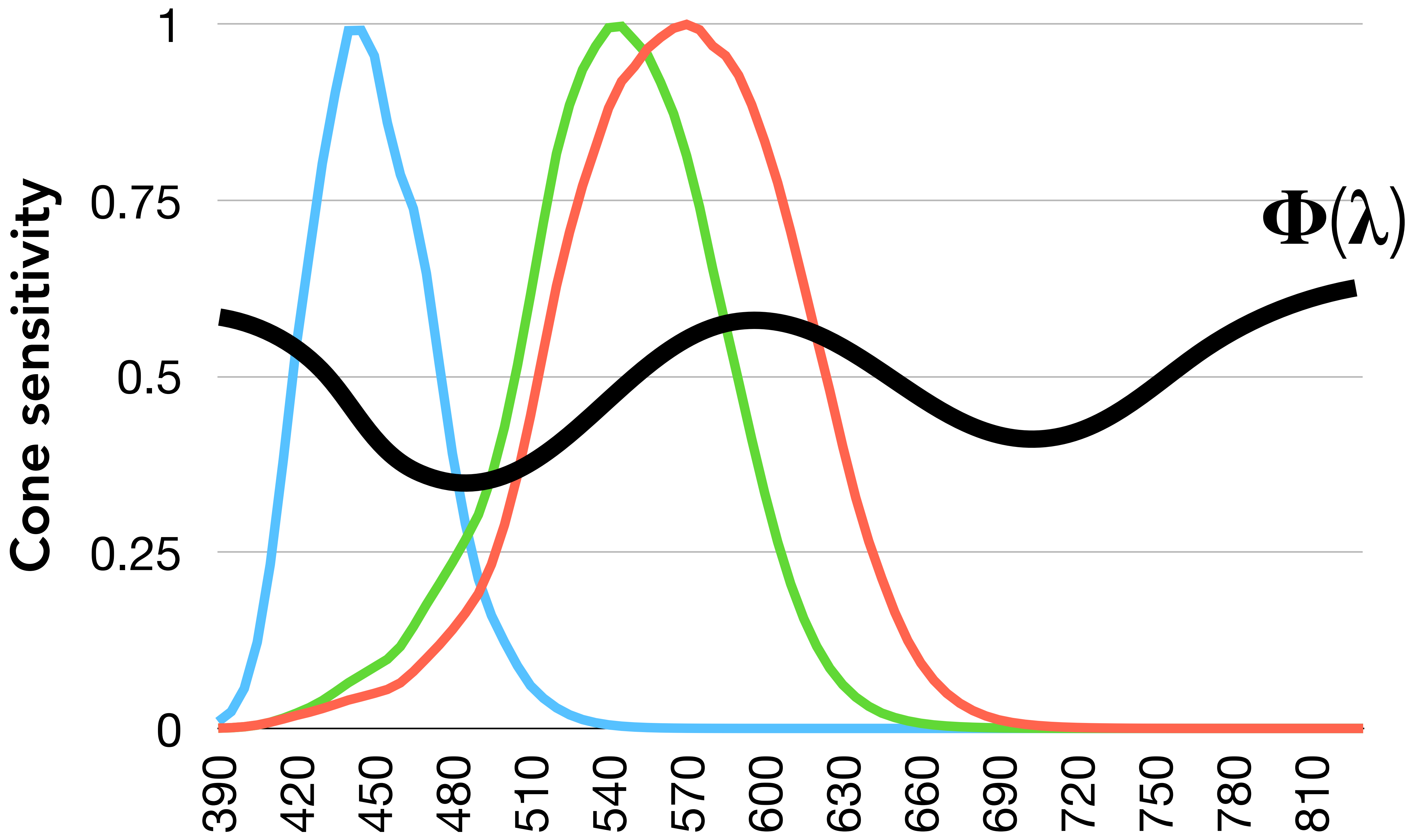
# Total Cone Response

- Any light could be expressed as a <L, M, S> triplet.

$$L = \sum_{\lambda=380}^{780} \Phi(\lambda) L(\lambda)$$

$$M = \sum_{\lambda=380}^{780} \Phi(\lambda) M(\lambda)$$

$$S = \sum_{\lambda=380}^{780} \Phi(\lambda) S(\lambda)$$

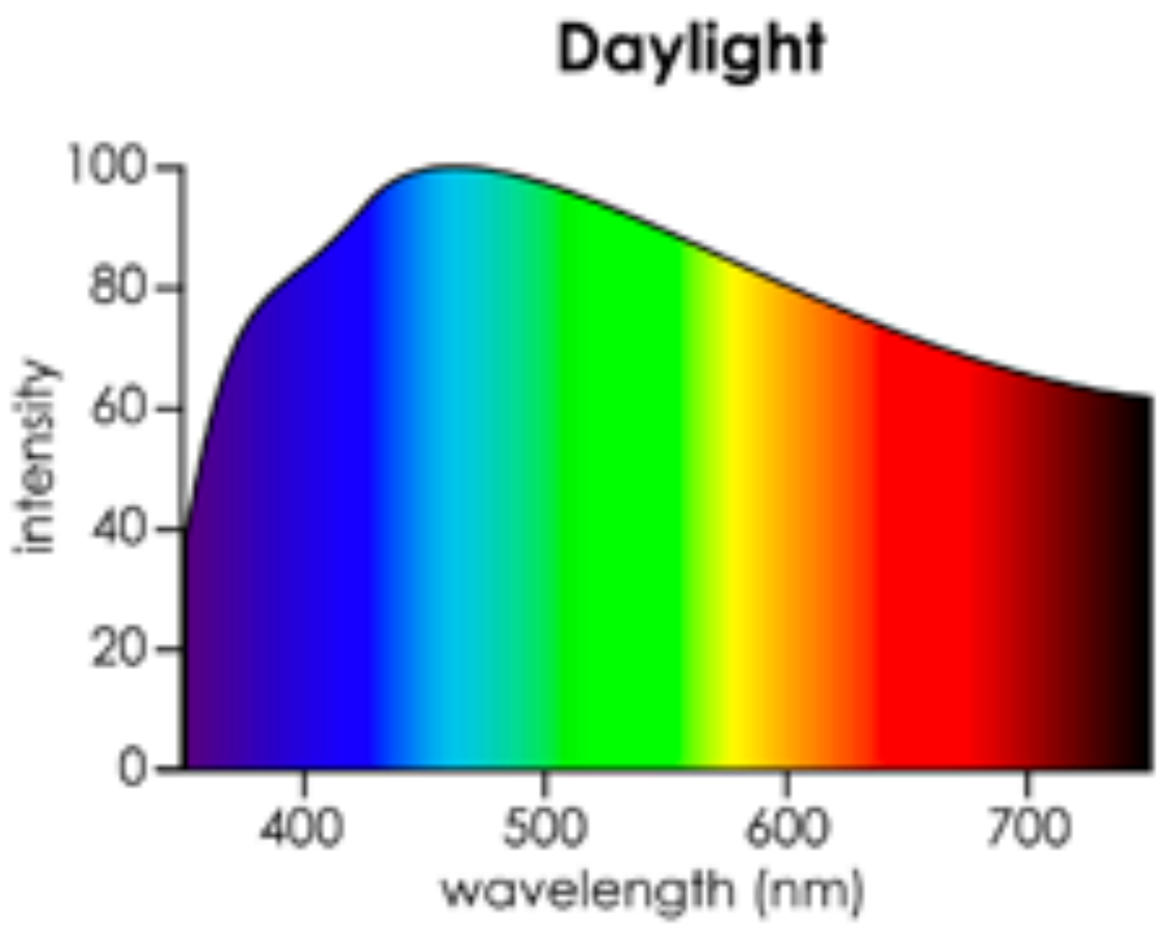



Stockman and Sharpe (2000)

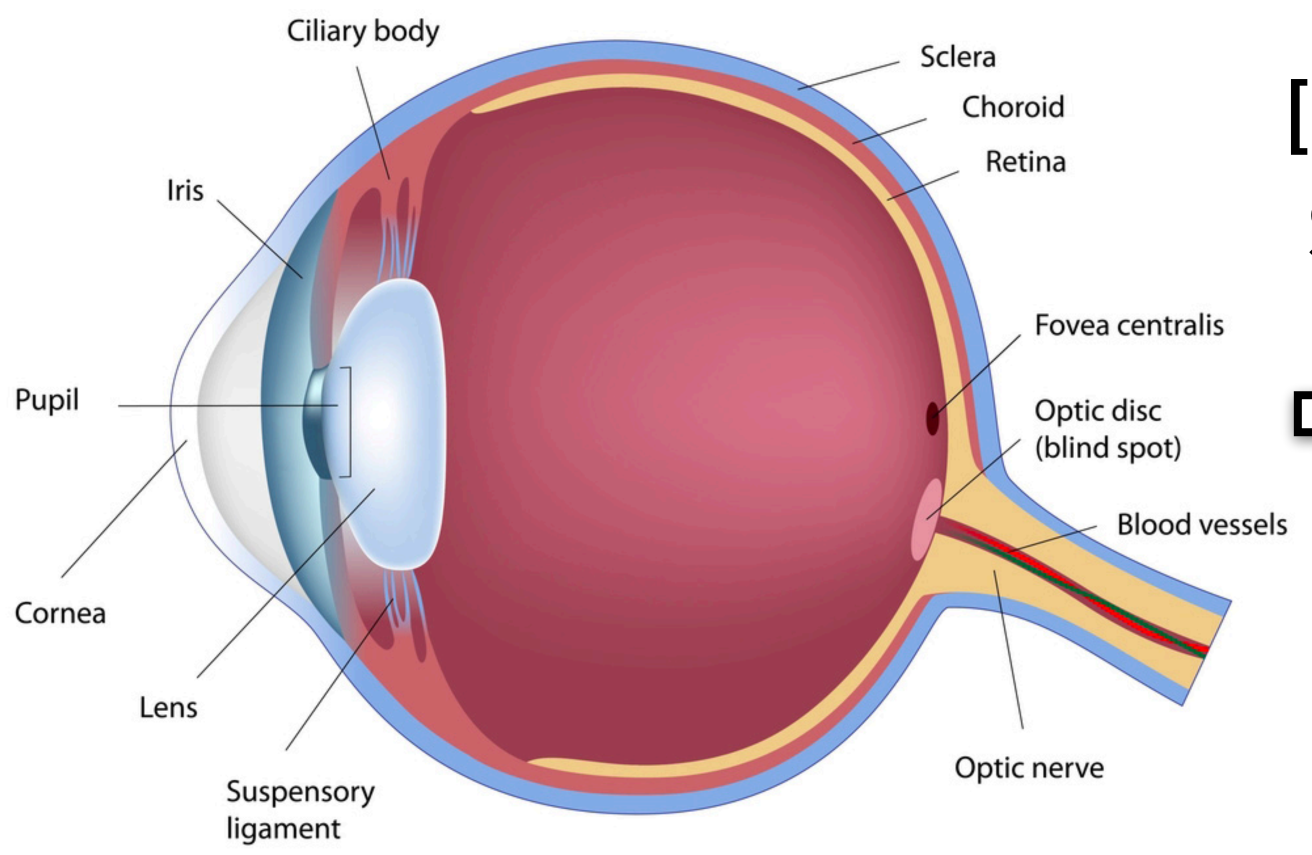
https://en.wikipedia.org/wiki/CIE_1931_color_space

# Brain Receives Only the LMS Signals, not Light!

**Light Spectrum**

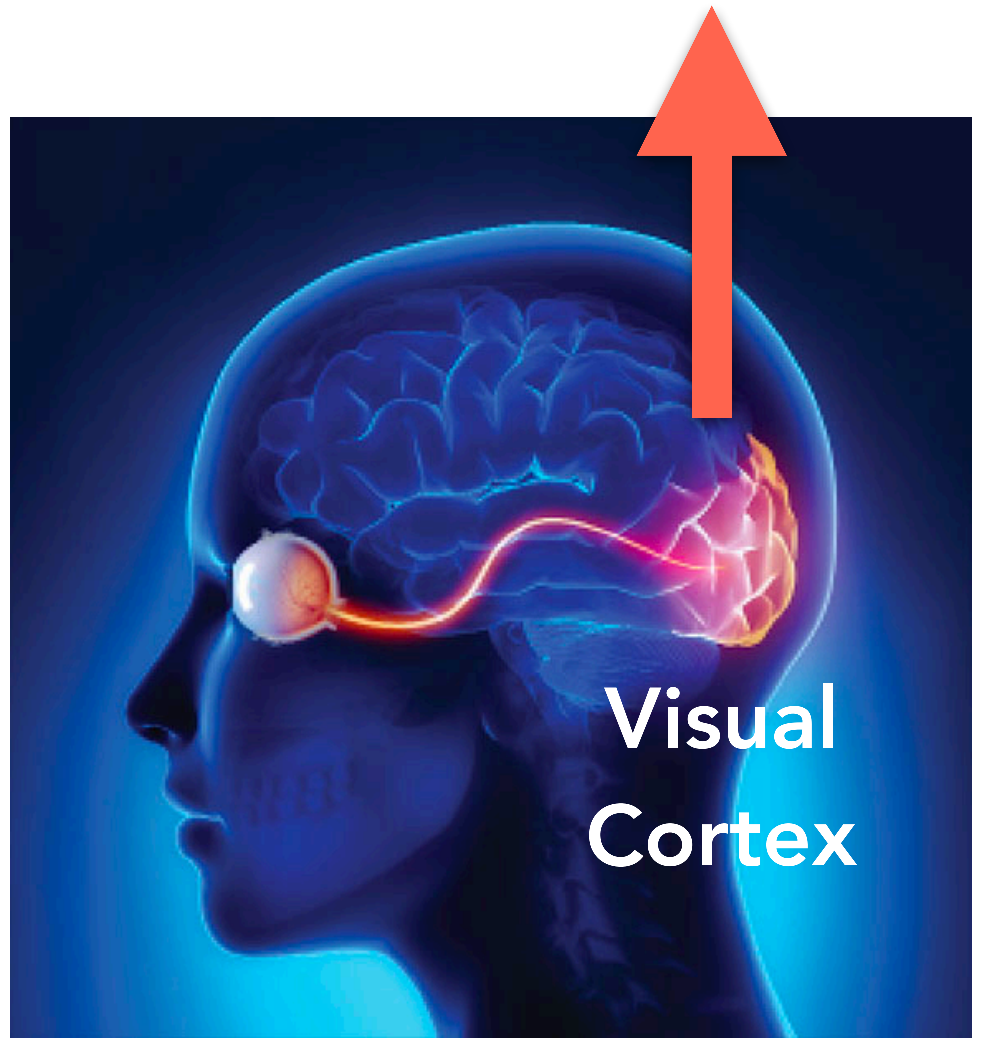



**Eye**




**[L, M, S]**
**Signals**

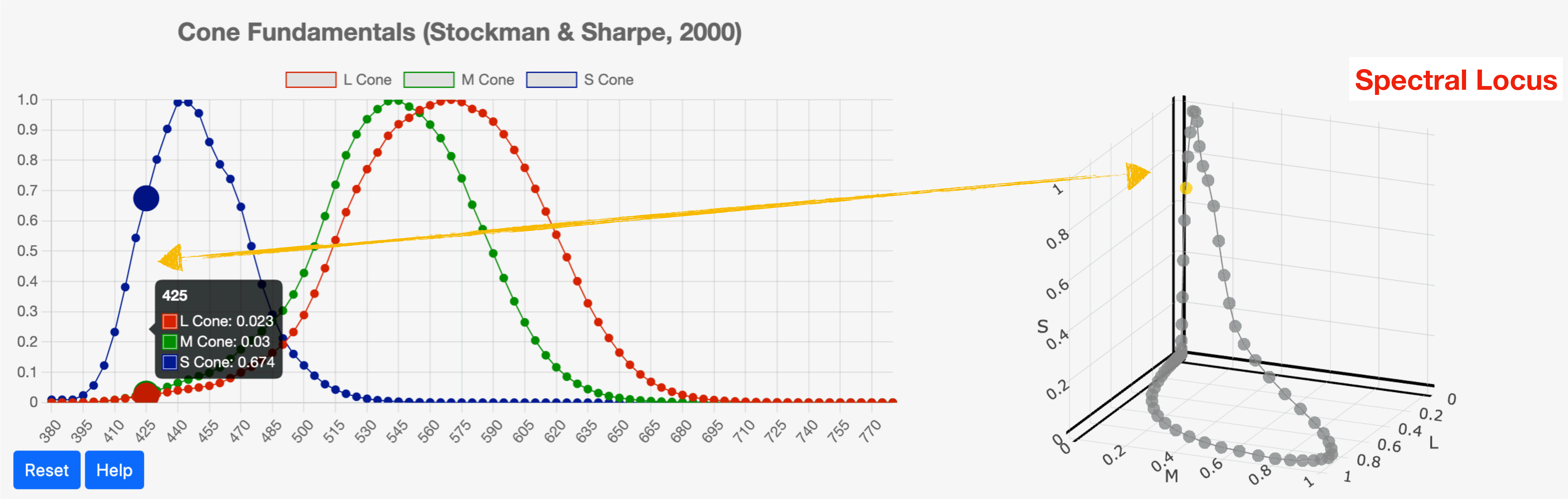
**A color sensation!**




- A particular **LMS combination** leads to a particular **color sensation.**
- This is a huge dimensionality reduction: the light spectrum is a very high dimensional vector (numerous wavelengths), but is transformed by the retina to only a three-dimensional vector!

https://www.eyeworld.org/research-finds-human-visual-cortex-continues-development-mid-30s

# Visualizing Spectral Lights in the LMS Color Space




Play with an interactive app here: https://horizon-lab.org/colorvis/cone2cmf.html

# The LMS Color Space

- By expressing any light's color as a <L, M, S> point, we have inherently defined a 3D color space, in which every color is a point.
- See visualization here: https://horizon-lab.org/colorvis/chromaticity.html.
- What if we uniformly double the power at each wavelength? How would the corresponding <L, M, S> change?

# Two Fascinating Questions

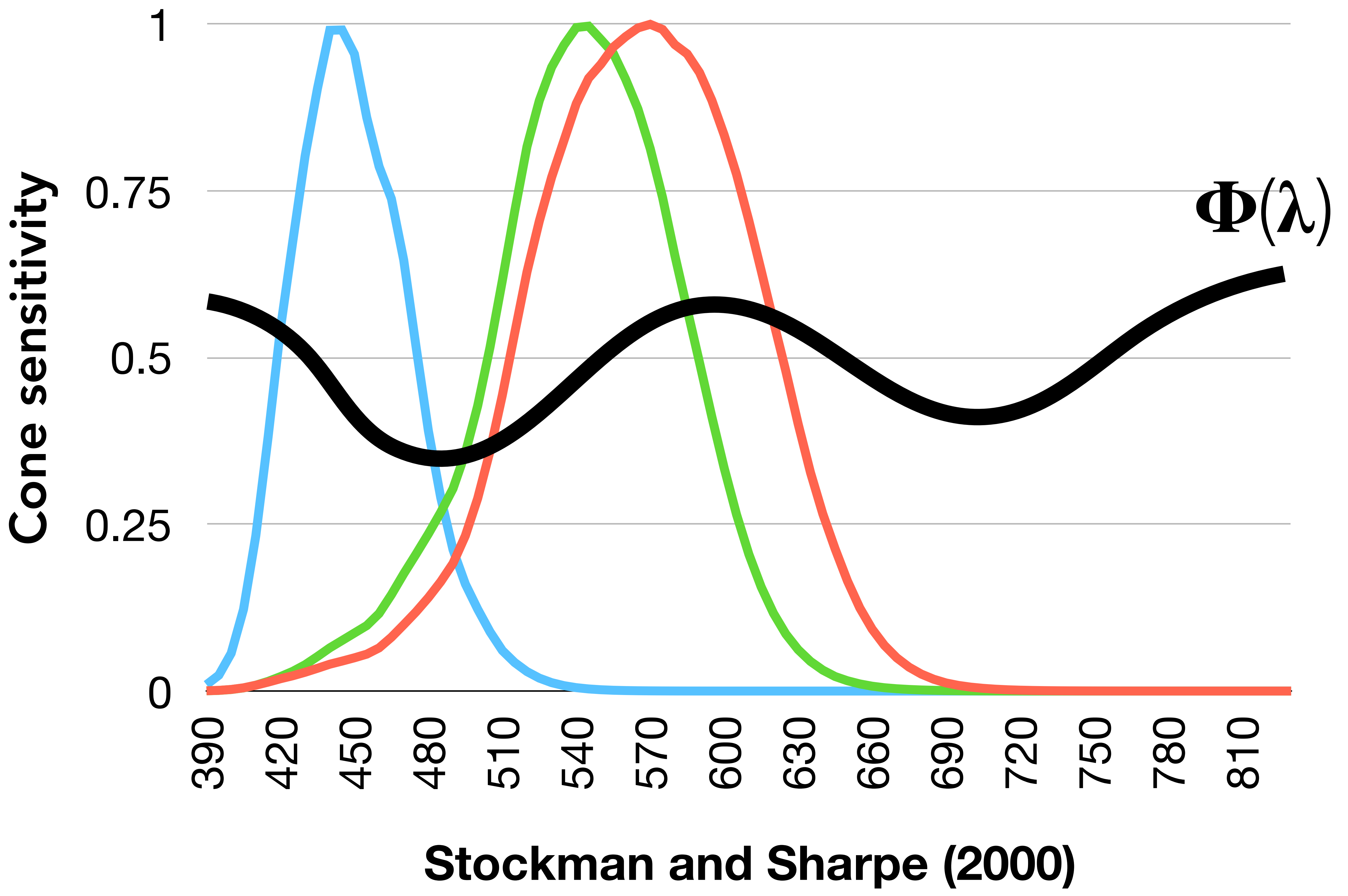
- 1. Does any arbitrary point in the LMS space correspond to a color?
- 2. Will there be multiple lights that correspond to the same point, i.e., color in the LMS space?

# Recall: Calculating Total Cone Responses

$$L = \sum_{\lambda=380}^{780} \Phi(\lambda) L(\lambda)$$

$$M = \sum_{\lambda=380}^{780} \Phi(\lambda) M(\lambda)$$

$$S = \sum_{\lambda=380}^{780} \Phi(\lambda) S(\lambda)$$



# Matrix Perspective

$$L = \sum_{\lambda=380}^{780} \Phi(\lambda)L(\lambda)$$

$$M = \sum_{\lambda=380}^{780} \Phi(\lambda)M(\lambda)$$

$$S = \sum_{\lambda=380}^{780} \Phi(\lambda)S(\lambda)$$

$$\begin{bmatrix} L_{380}, L_{381}, \cdots, L_{780} \\ M_{380}, M_{381}, \cdots, M_{780} \\ S_{380}, S_{381}, \cdots, S_{780} \end{bmatrix} \times \begin{bmatrix} \Phi_{380} \\ \Phi_{381} \\ \vdots \\ \Phi_{780} \end{bmatrix} = \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

# Matrix Perspective

- It's an under-determined system of linear equations. **In general**, it has infinitely many solutions.
- In general, there are infinitely many lights that correspond to the same color. This is called **metamerism**. More on this soon.

**An additional constraint: $\Phi$ must all be non-negative, since there is no negative power!**

$$\begin{bmatrix} L_{380}, L_{381}, \cdots, L_{780} \\ M_{380}, M_{381}, \cdots, M_{780} \\ S_{380}, S_{381}, \cdots, S_{780} \end{bmatrix} \times \begin{bmatrix} \Phi_{380} \\ \Phi_{381} \\ \vdots \\ \Phi_{780} \end{bmatrix} = \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

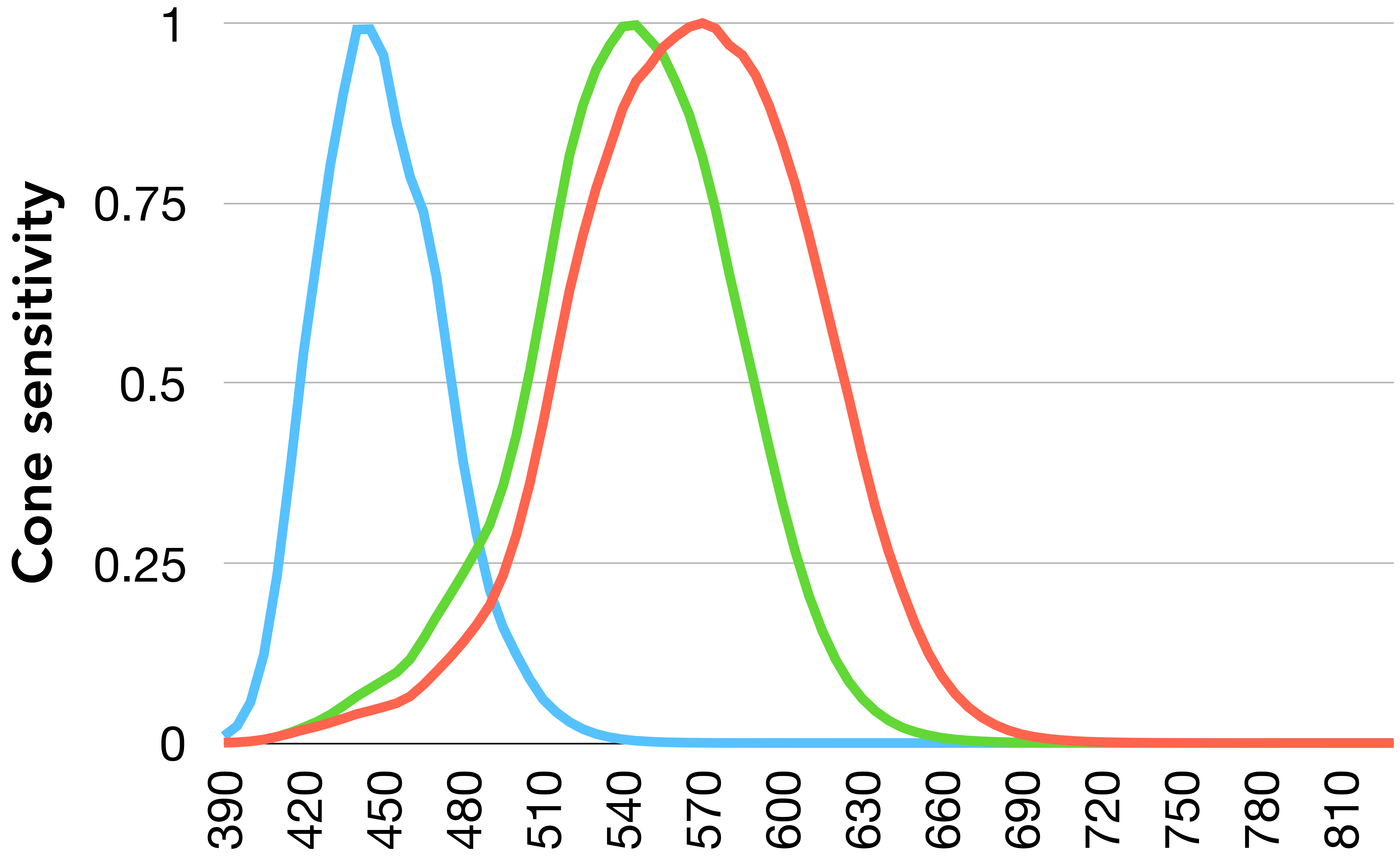
**The first matrix is known. Given the [L, M, S] vector, can you solve for the $\Phi$ vector?**

# But Can There Be No Solution?

- One scenario: negative L, M, or S responses. All cone responses are physical quantities, which must be positive.

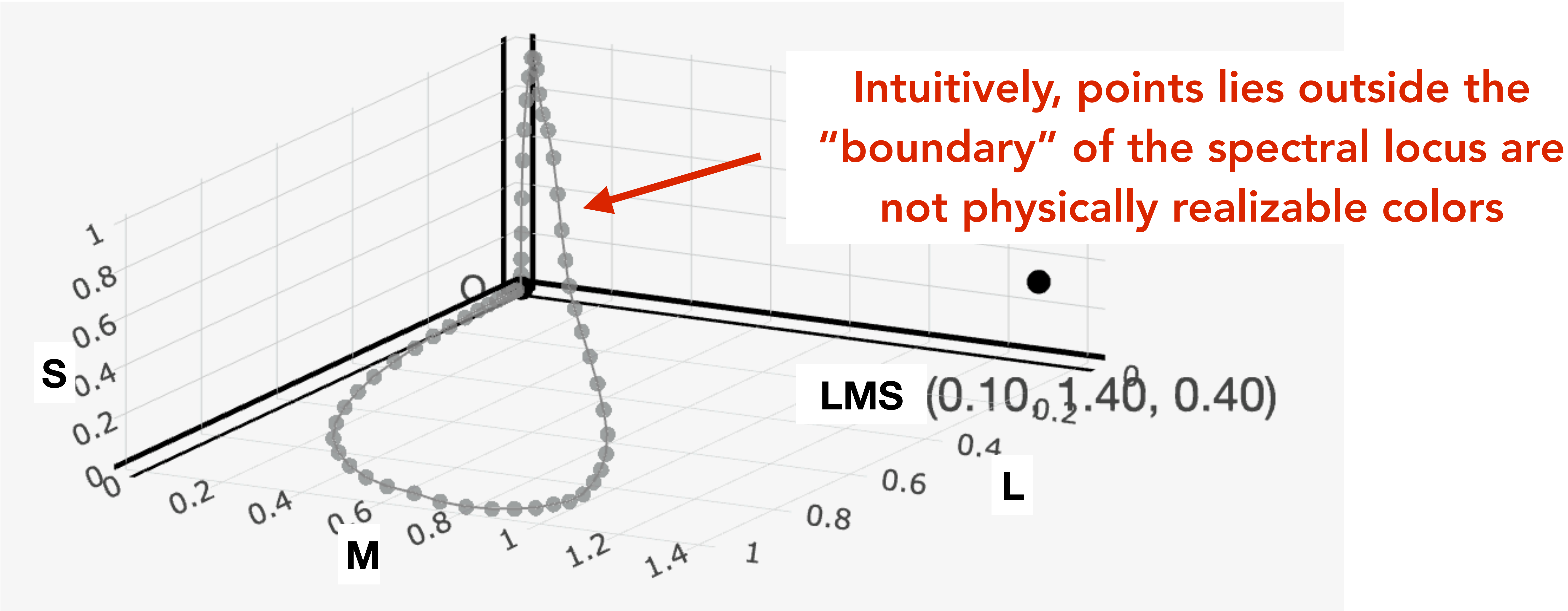
# But Can There Be No Solution?

- Another scenario: impossible cone response combinations.
  - E.g., [1, 0, 0]: there is no real light that can produce only L cone response but no responses from M and S cones.
  - Except when…



# But Can There Be No Solution?

- Another scenario: impossible cone response combinations.
  - An impossible cone combination still corresponds to a point in the LMS space. We call it an imaginary color. It's not a color that can be produced by physical lights.




Play with an interactive app here: https://horizon-lab.org/colorvis/chromaticity.html

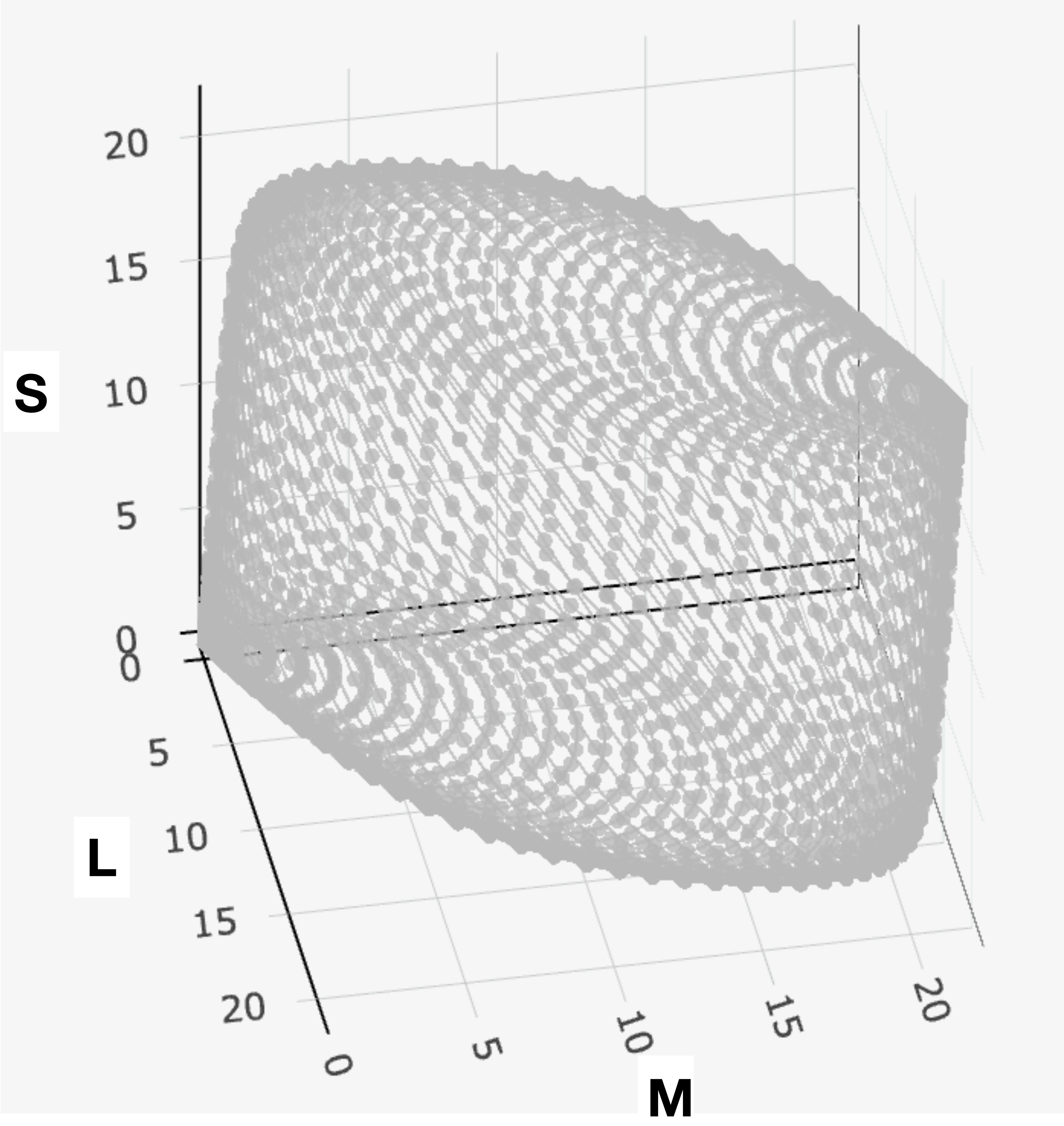
# Color Gamut of Human Visual System (HVS)

- HVS Color Gamut: the set of all the colors that human can see.
- The HVS color gamut encapsulates all the LMS points for which the system of linear equations below has a non-negative solution.

$$\begin{bmatrix} L_{380}, L_{381}, \cdots, L_{780} \\ M_{380}, M_{381}, \cdots, M_{780} \\ S_{380}, S_{381}, \cdots, S_{780} \end{bmatrix} \times \begin{bmatrix} \Phi_{380} \\ \Phi_{381} \\ \vdots \\ \Phi_{780} \end{bmatrix} = \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

# Visualizing HVS Gamut

- See visualization here: https://horizon-lab.org/colorvis/gamutvis.html.
- In reality, the gamut has no boundary, since we can arbitrarily increase the light power and this 3D volume will extend to infinity.



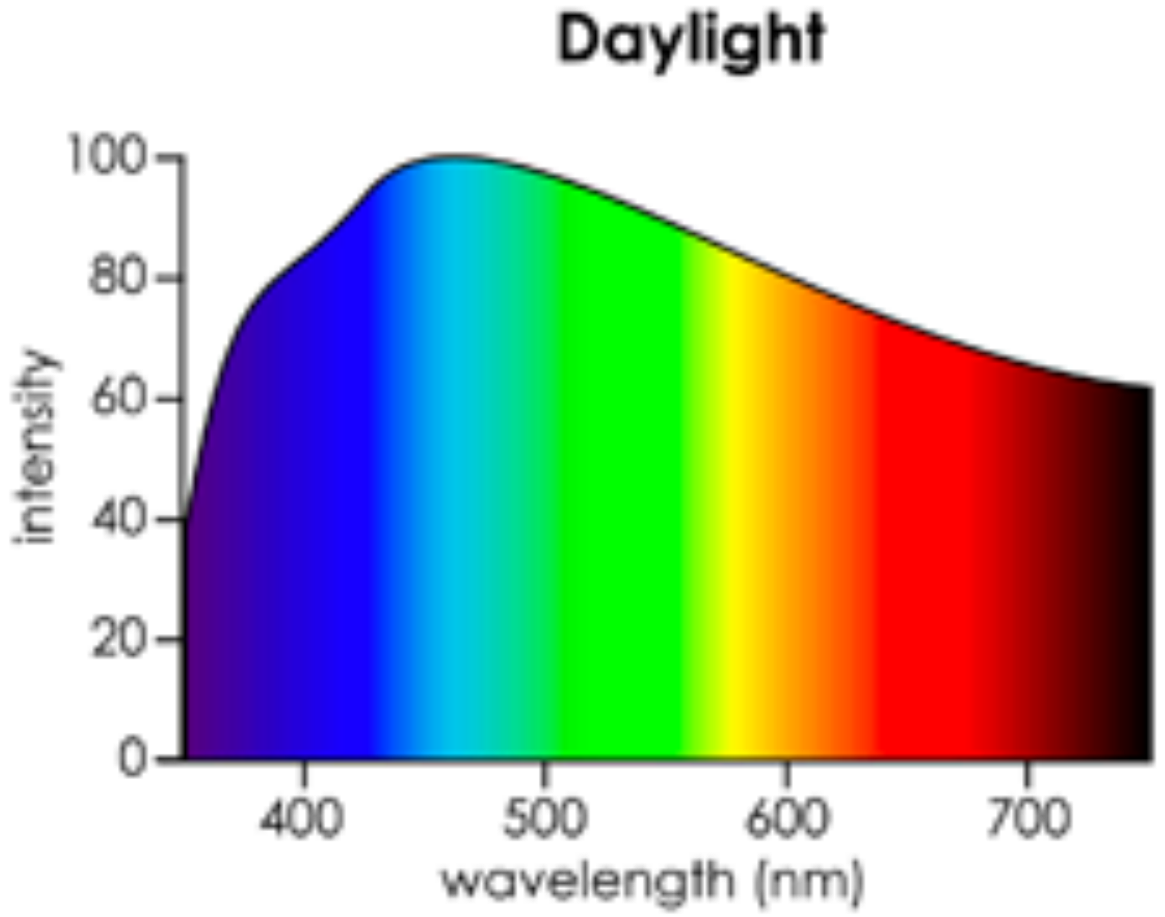
# Two Fascinating Questions

- 1. Does any arbitrary point in the LMS space correspond to a color?
  - No
- 2. Will there be multiple lights that correspond to the same point, i.e., color in the LMS space?
  - Yes. In fact, there are infinitely many.

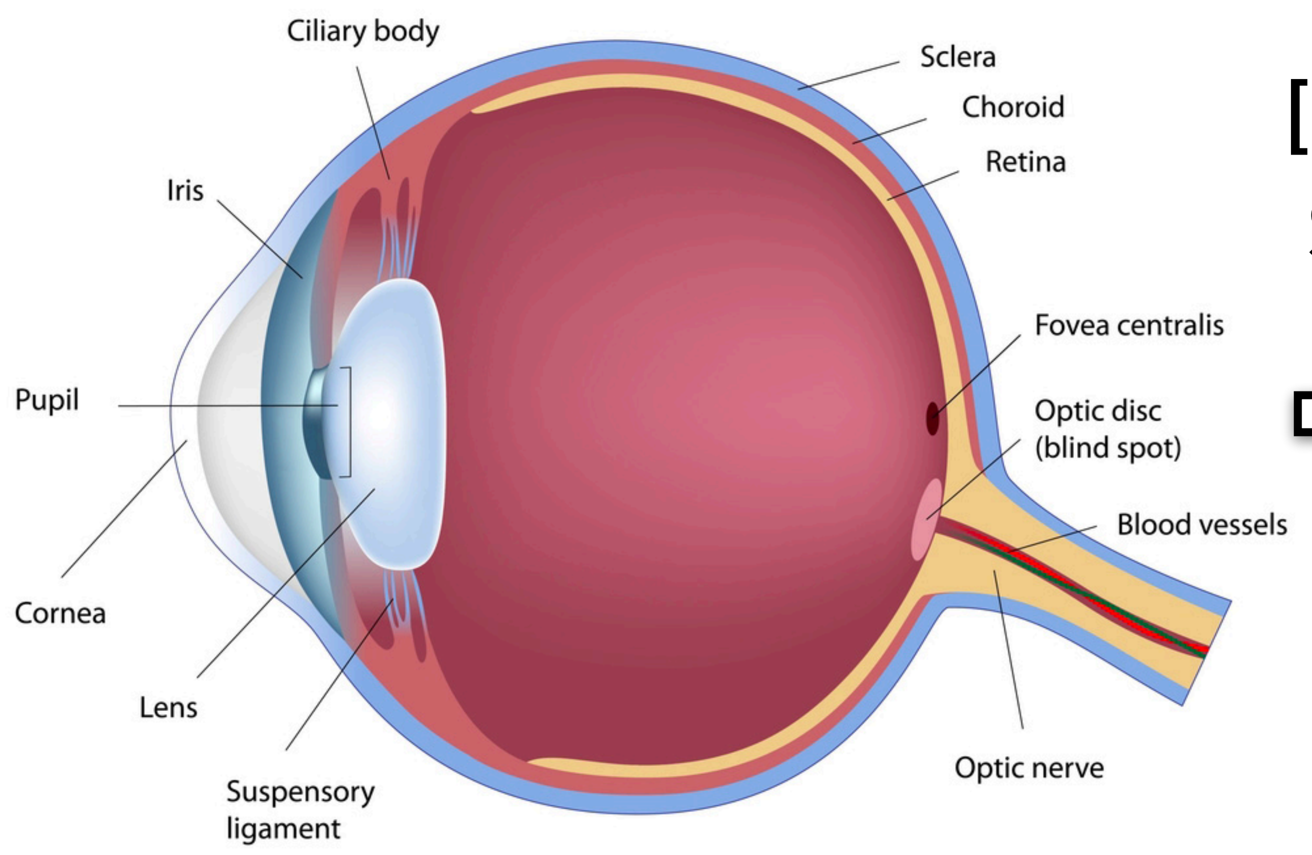
# Color Matching Experiments and Metamerism

# Intuition

**Light Spectrum**

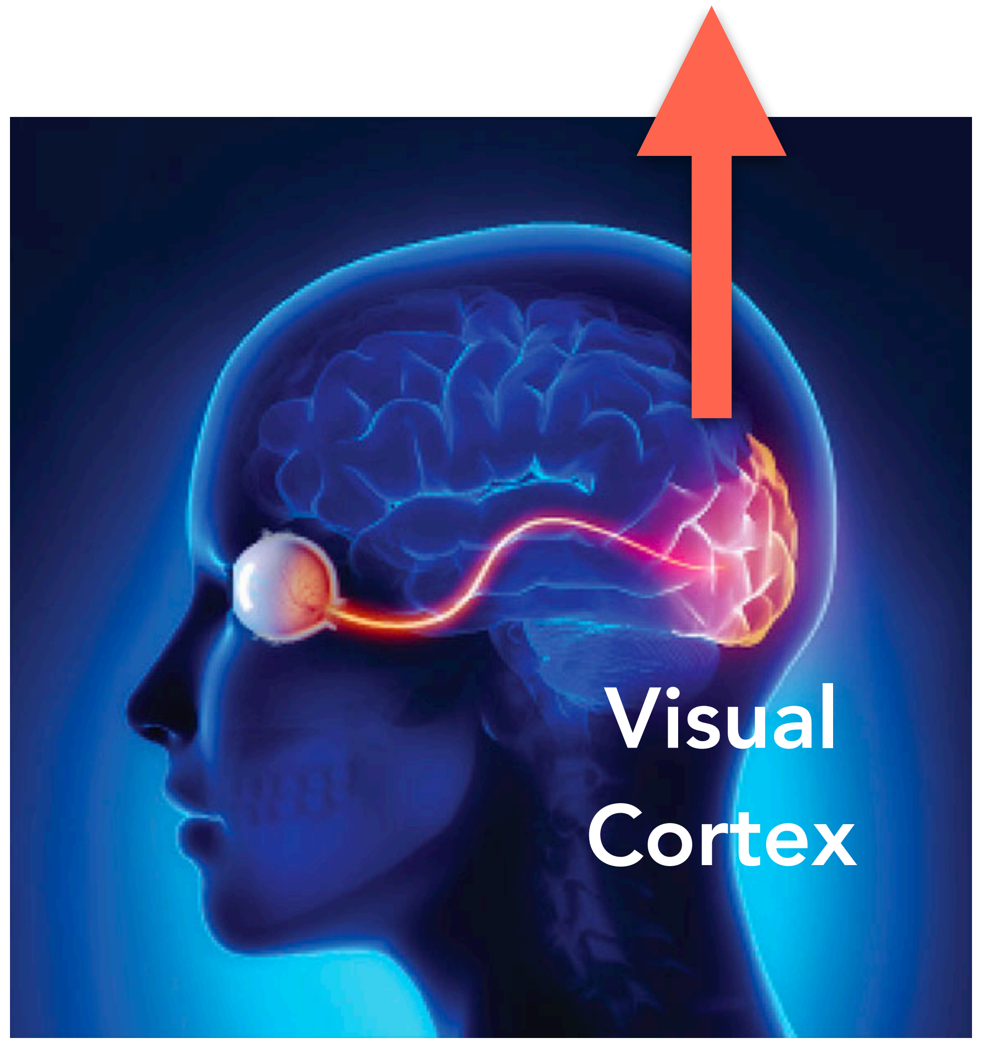



**Eye**




**[L, M, S]**
**Signals**

**A color sensation!**




- Lights that produce the same LMS signals are ***perceived*** by the brain as the same "color", even if their spectra might be totally different!
- …because retinal processing is a dramatic dimensionality reduction.

https://www.eyeworld.org/research-finds-human-visual-cortex-continues-development-mid-30s

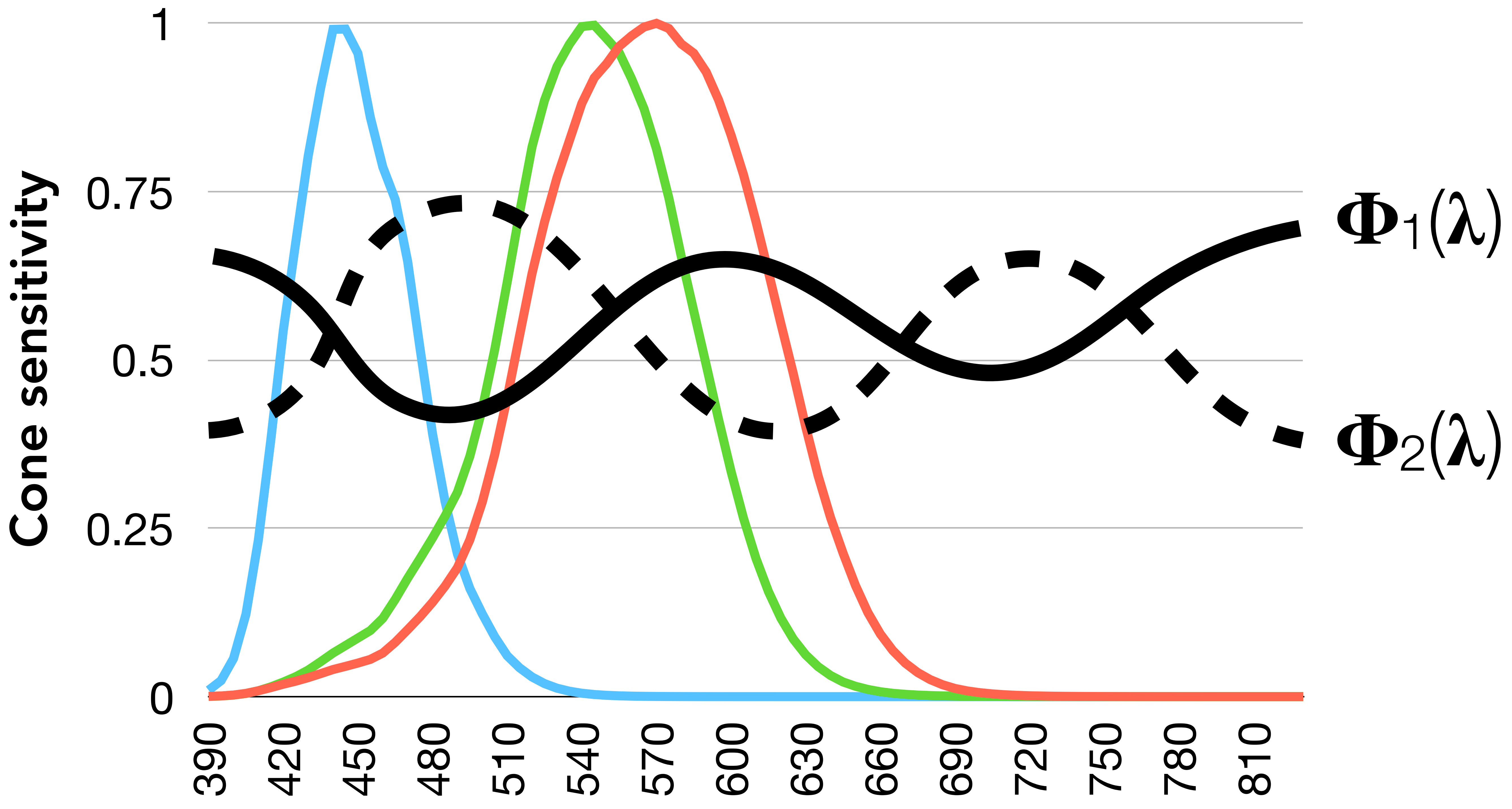
# Metamerism

- The two lights are metamers of each other (i.e., seen as having the same color) if their LMS cone responses match.

$$\sum_{\lambda} \Phi_1(\lambda)L(\lambda) = \sum_{\lambda} \Phi_2(\lambda)L(\lambda)$$

$$\sum_{\lambda} \Phi_1(\lambda)M(\lambda) = \sum_{\lambda} \Phi_2(\lambda)M(\lambda)$$

$$\sum_{\lambda} \Phi_1(\lambda)S(\lambda) = \sum_{\lambda} \Phi_2(\lambda)S(\lambda)$$




Stockman and Sharpe (2000)

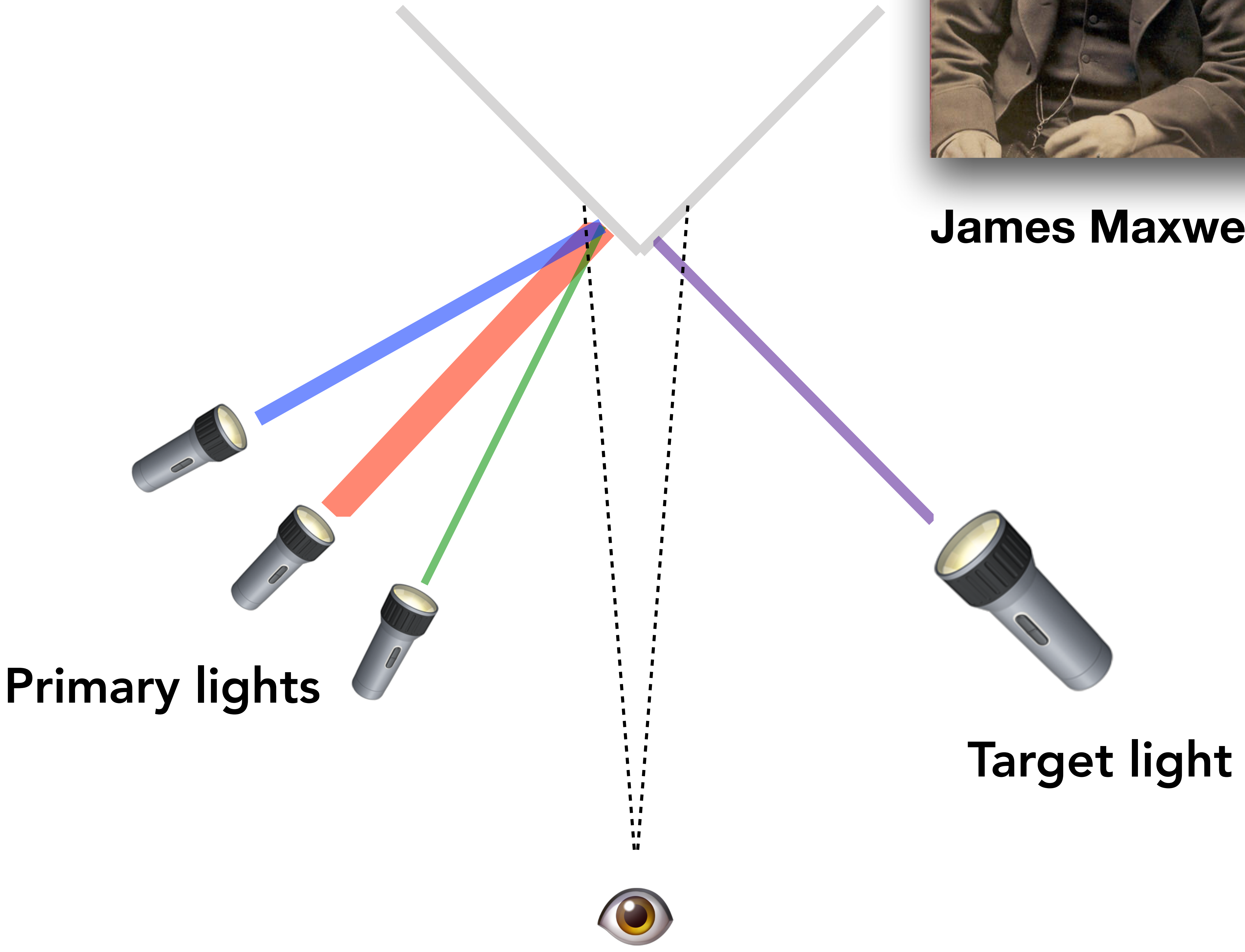
# Metamerism

- Does metamerism exist? That is, given a spectrum $\Phi$, can we always find at least one $\Phi'$ that is the metamer of $\Phi$? Can we find multiple?
- The answer is yes. We showed earlier how to find metamers by solving system of linear equations. But that requires knowing $L(\lambda)$, $M(\lambda)$, and $S(\lambda)$, which were not measured until after 1900s
    - we've talked before about how they are measured
- But people were able to find metamers as early as late 19th century, through a clever ***color matching*** experiment.

# Color Matching Experiments

- You are given three monochromatic (spectral) lights with known spectra pointing to the same point.
  - They are call the *primary lights*.
- You are also given a target light with unknown spectrum.
- Tune each primary light's power so that the color of the primary light mix matches the color of the target light.

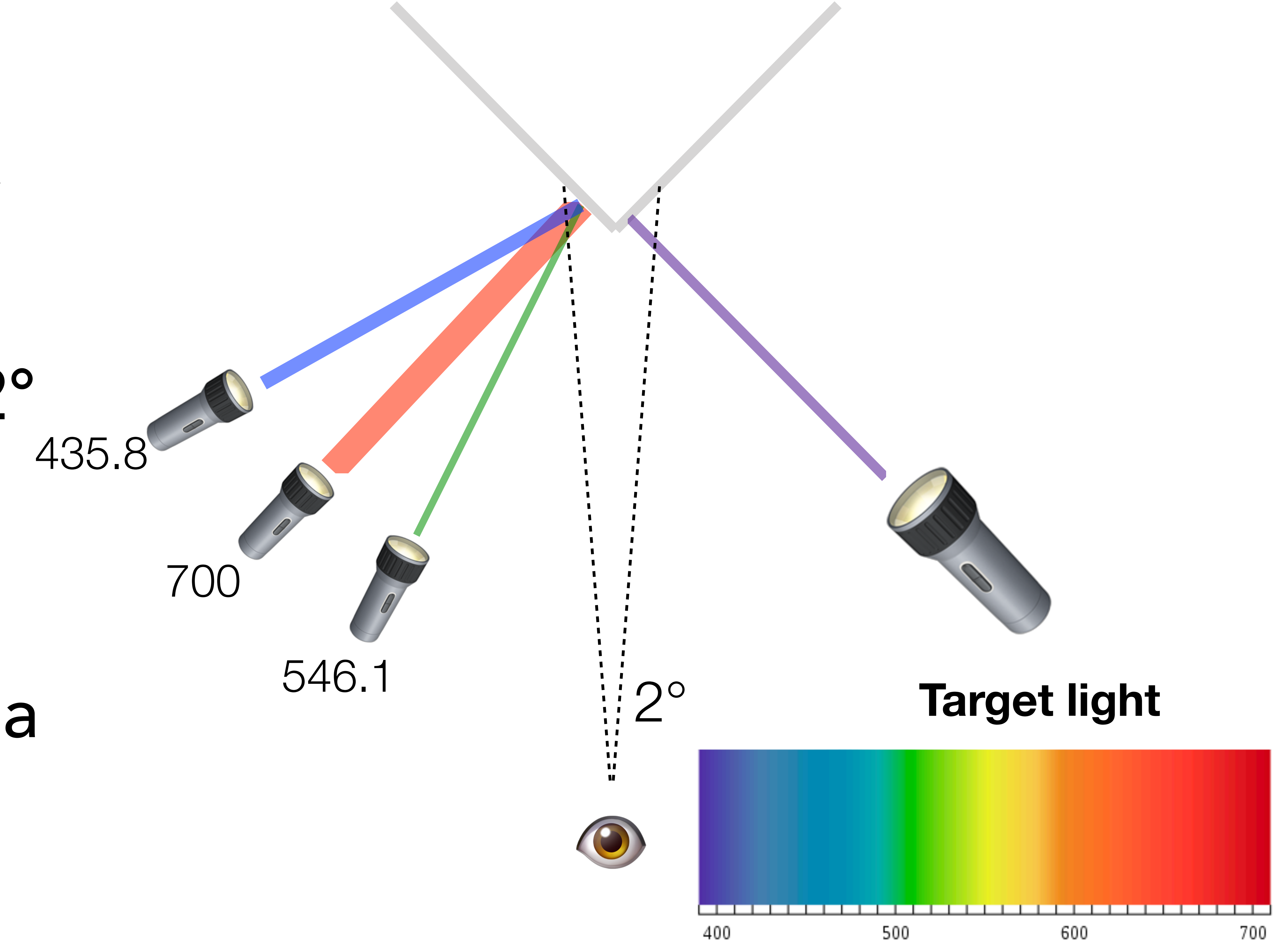
James Maxwell



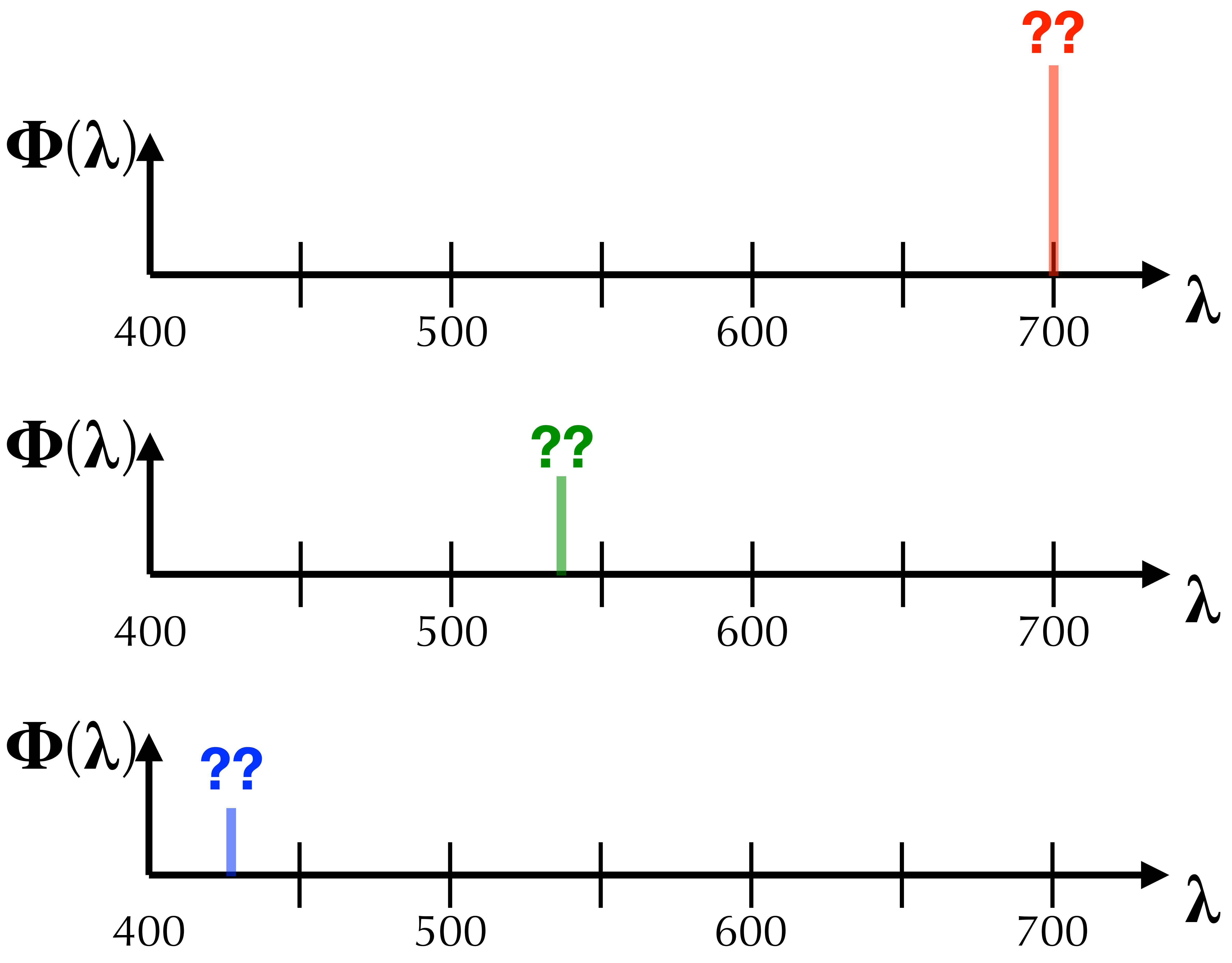

Color matching experiment; circa 1835

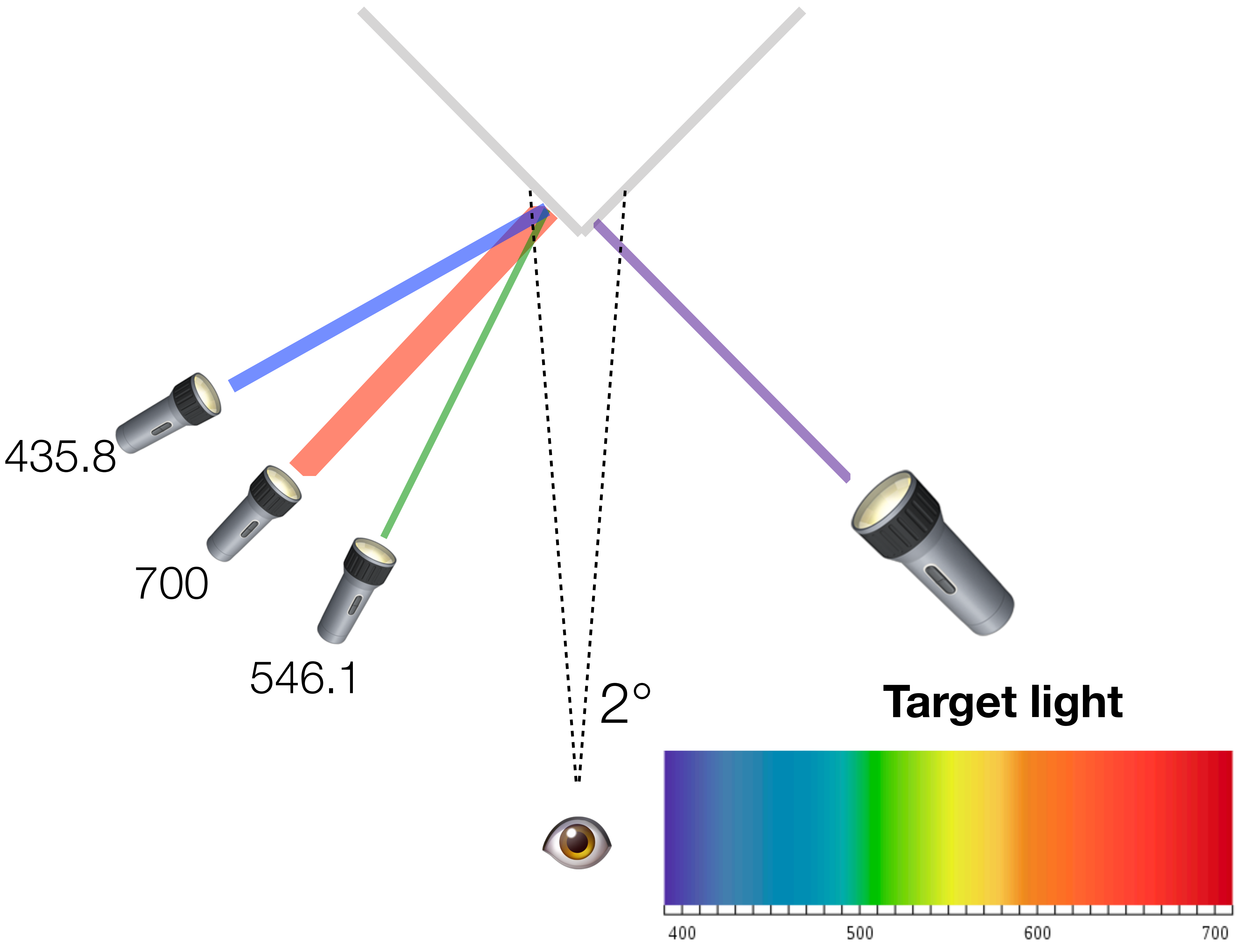
# Color Matching Experiments

- The three primary lights used in CIE 1931 standard: $\lambda$=435.8 nm, 546.1 nm, 700 nm; roughly blue, green, and red.
- CIE 1931 limited viewing angle to be 2° so that only the fovea gets light.
- It swept all the spectral lights in the visible spectrum as the target light (at a 5-nm interval).

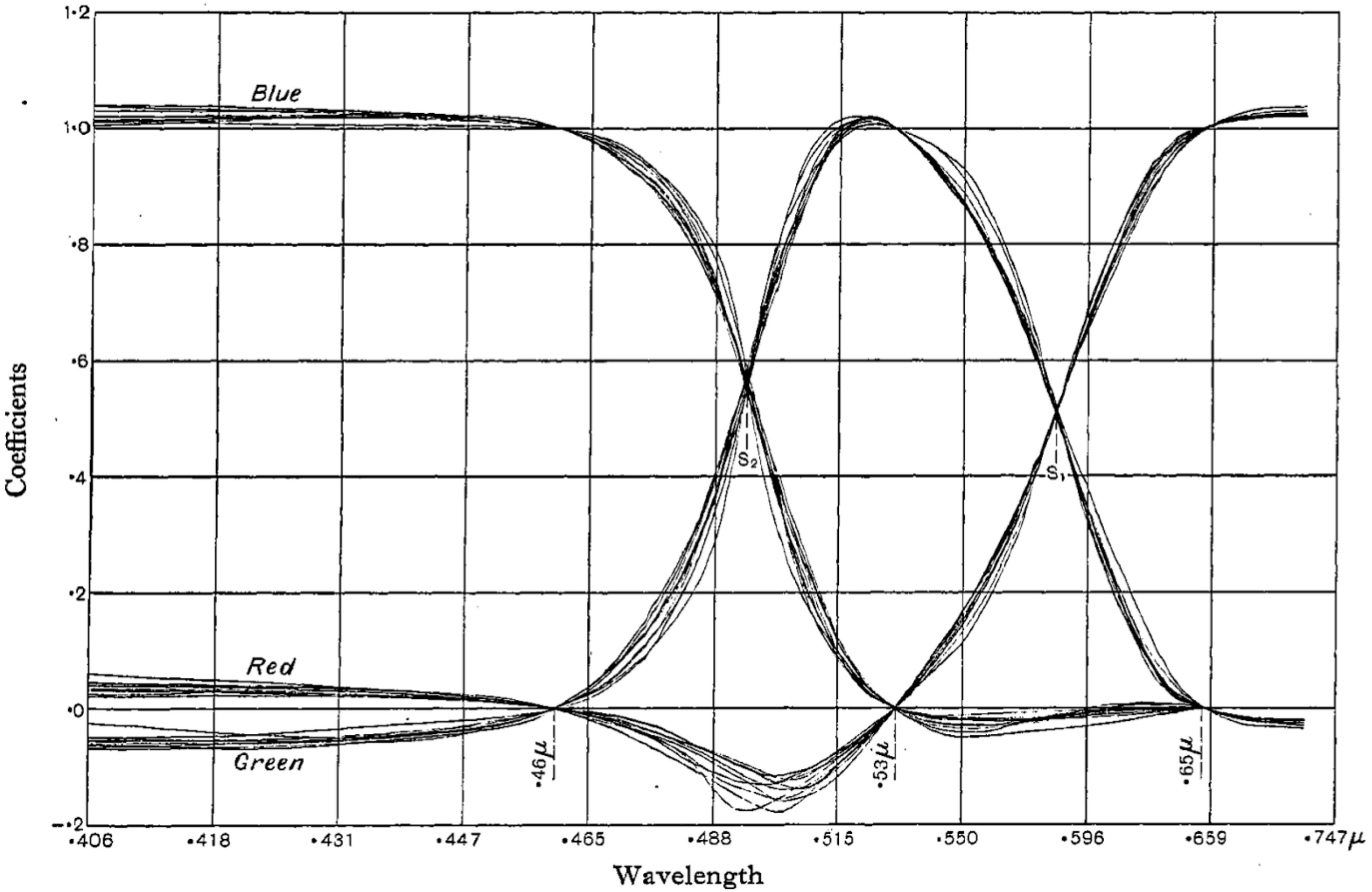


# Color Matching Experiments



# Color Matching Experiments By Wright and Guild

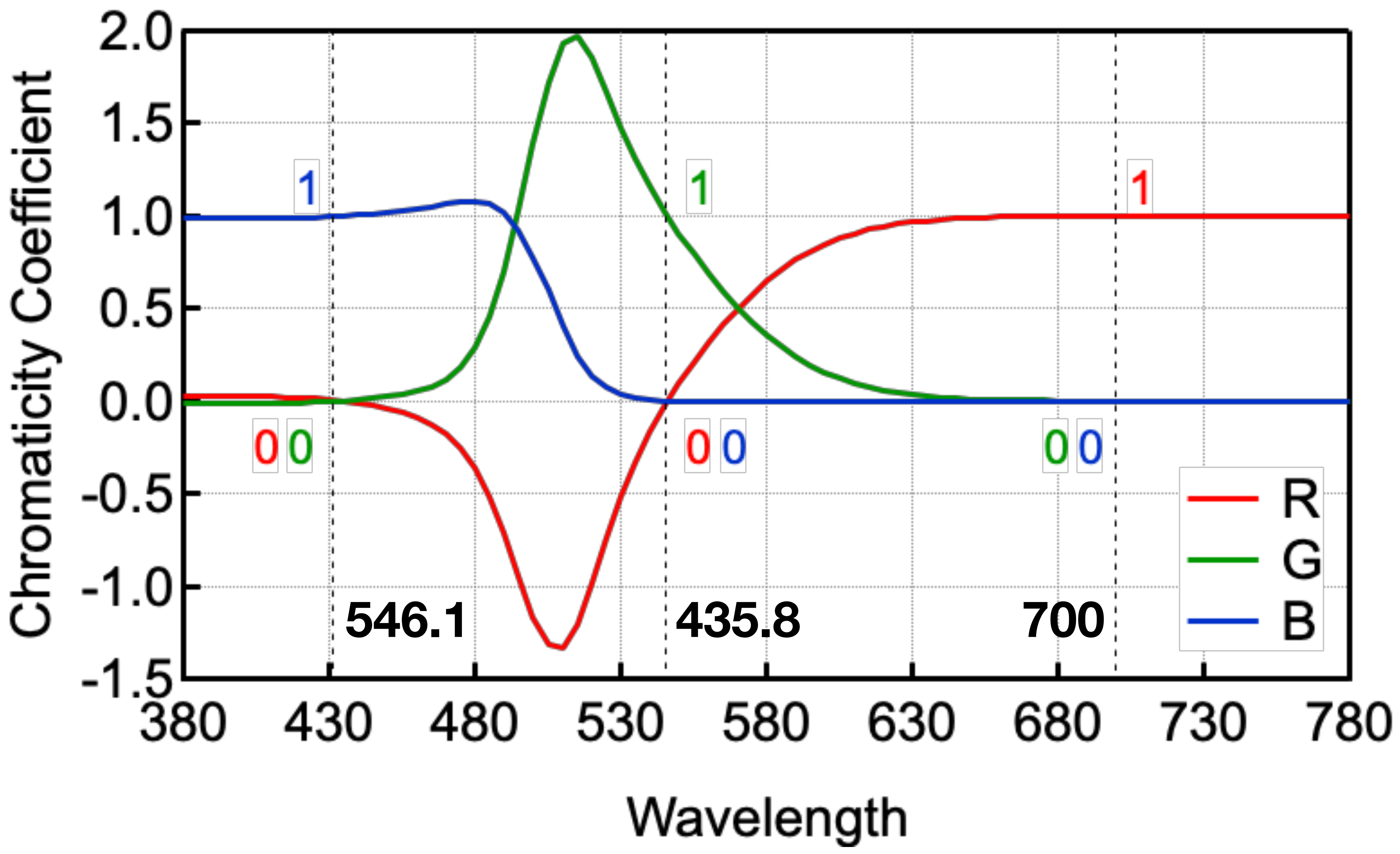



- Wright and Guild did two separate color matching experiments in the late 1920s
  - They used primaries and white point different from the CIE standard.
  - This is the results of Wright's experiment.
  - Their data eventually were turned into CIE data.

https://yuhaozhu.com/blog/cmf.html Wright 1928

# CIE 1931 Color Matching Experiments

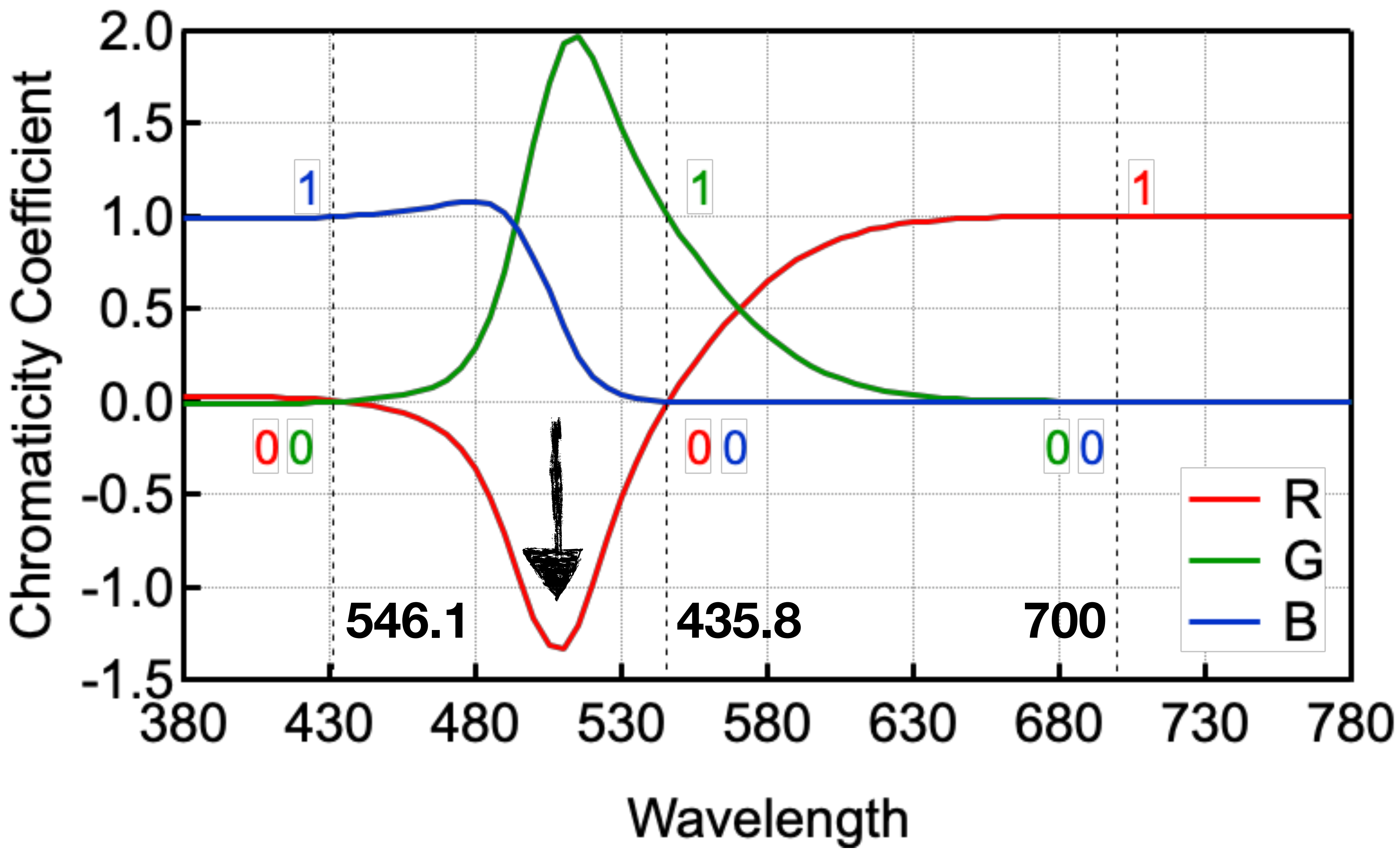
**Record the units of primary lights and normalize them so that they add up to 1.**

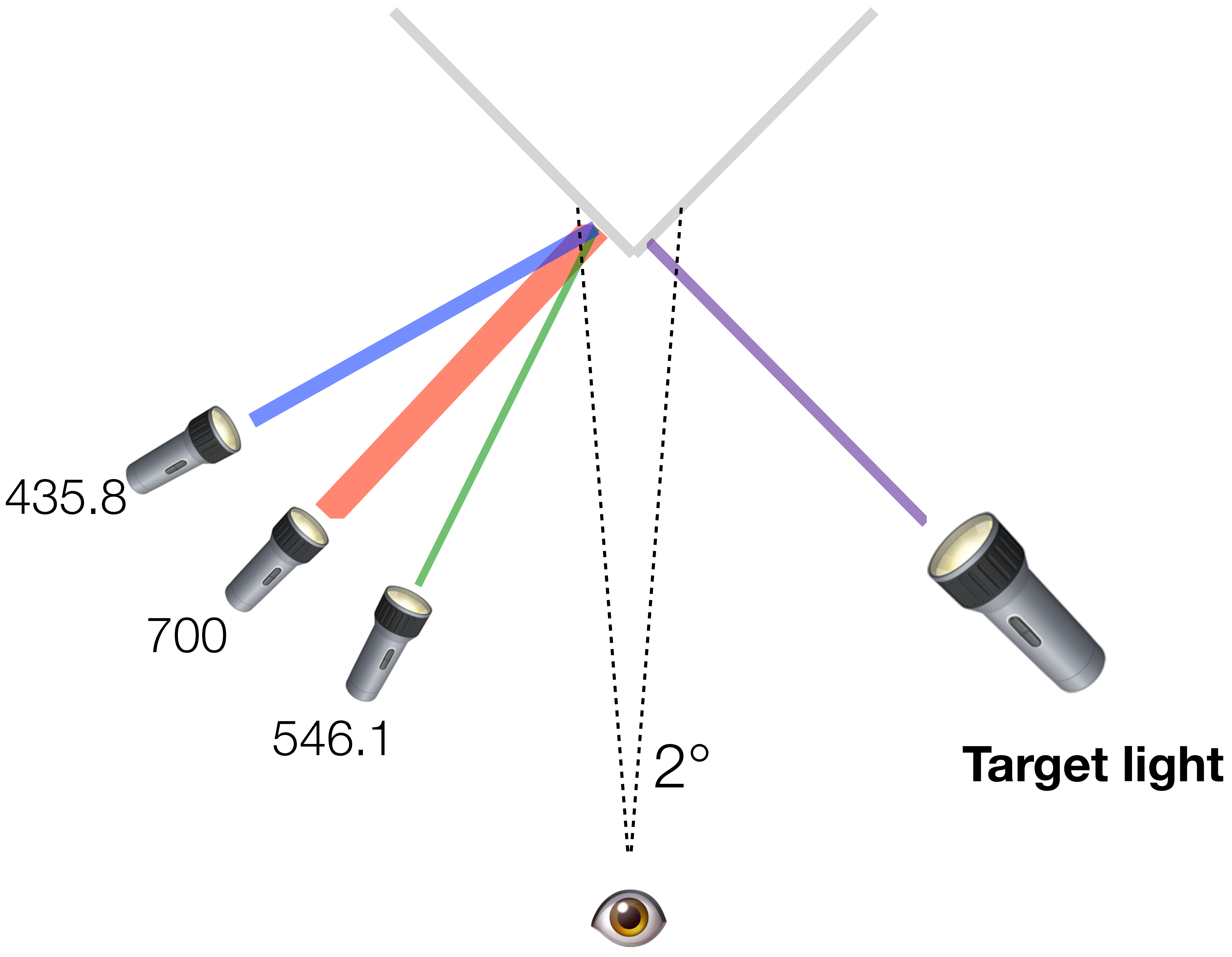



- Y-axis is the "amount" of each of the primary.
- The unit system is so defined that mixing equal amount of primaries produces the color of the "equal-energy white", whose SPD is a constant at all wavelengths.
  - Different color spaces can choose different white points. More later.

# Negative Color?

To match color of λ=500 nm, we need a bit of negative red? What does this mean???

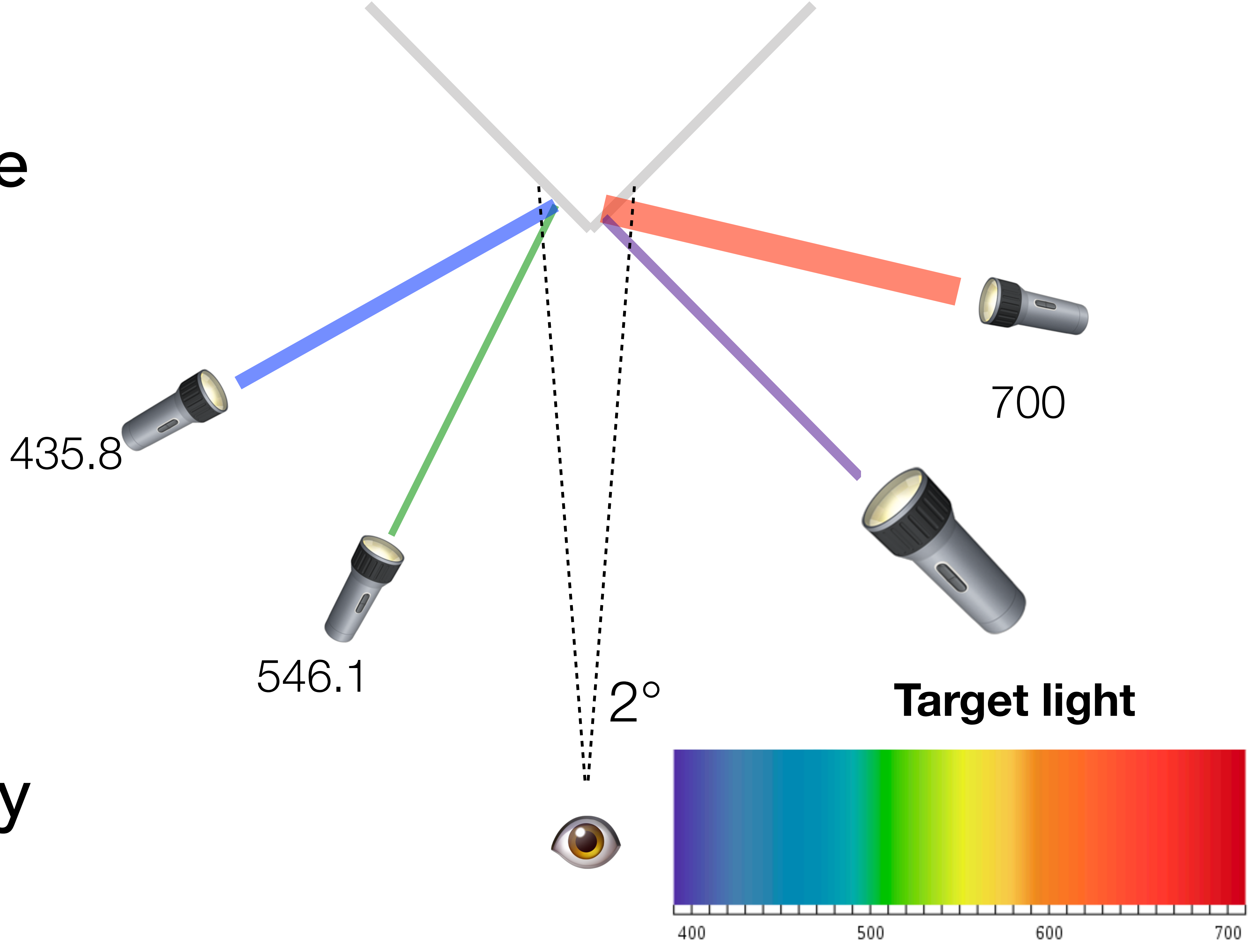


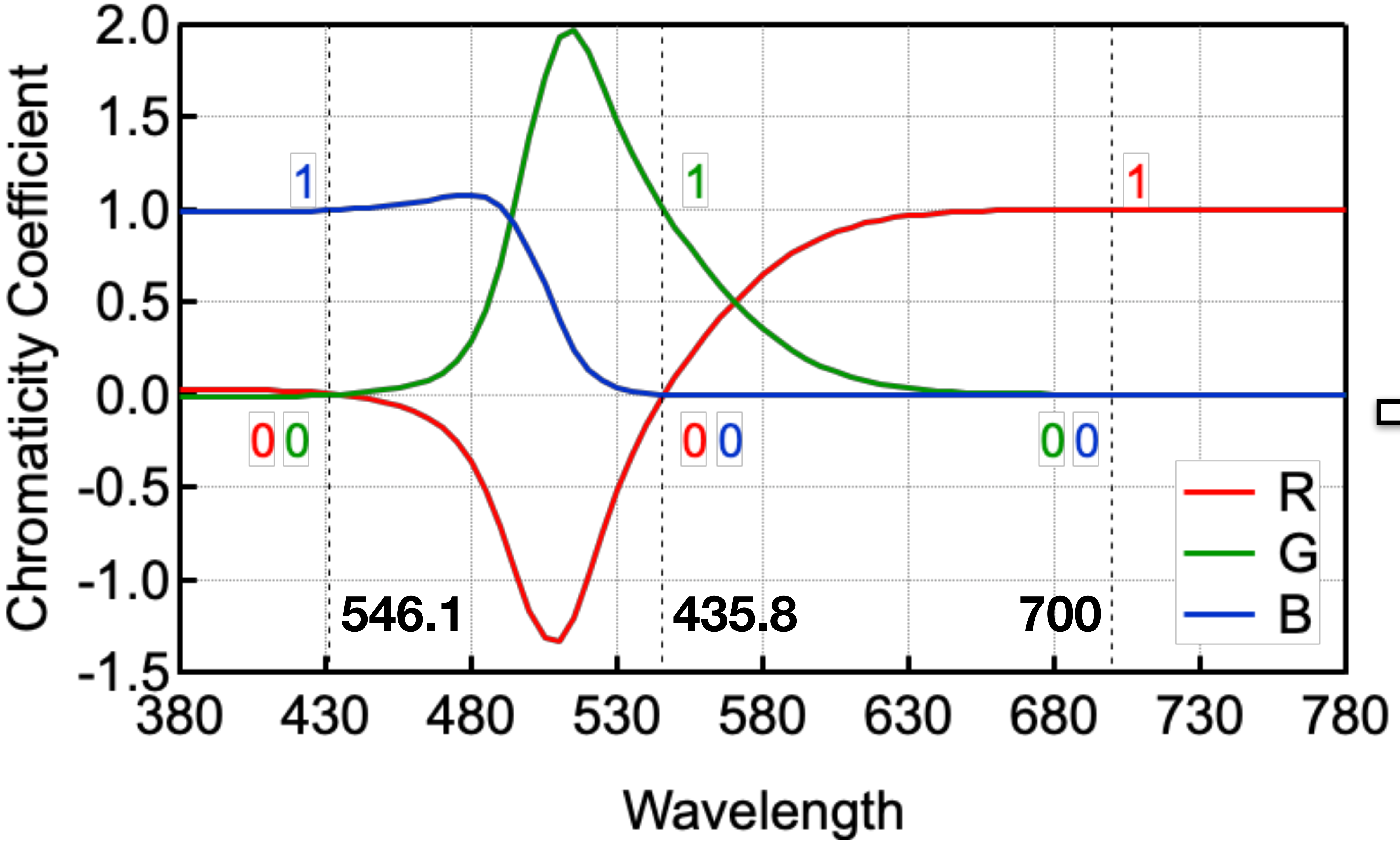

https://en.wikipedia.org/wiki/CIE_1931_color_space

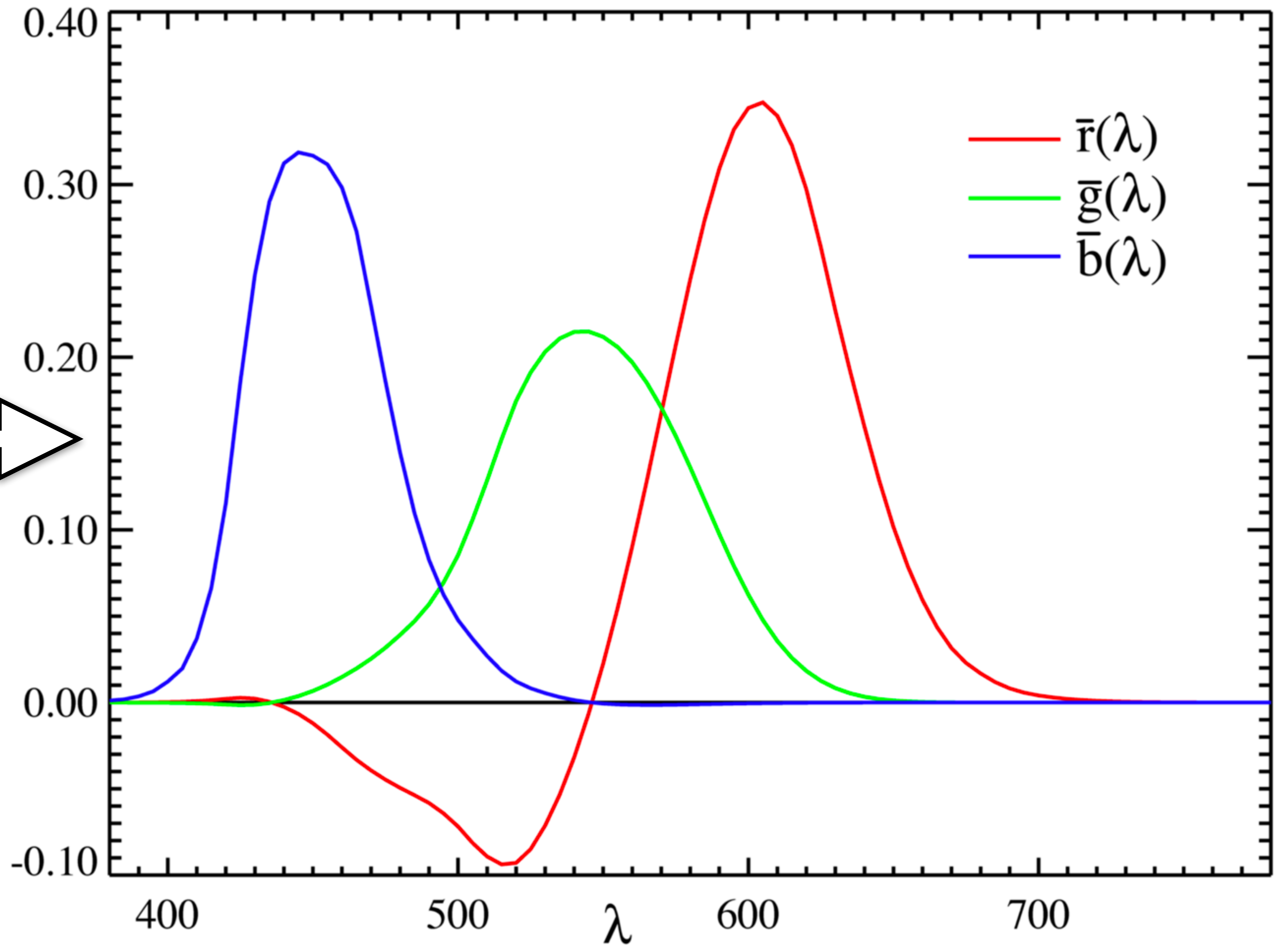
# Negative Values in Color Matching Functions

- No combination of RGB matches the target light, but adding some R with the target light matches the color of some combination of G and B.
- Color of light λ=540 nm can't be reproduced by R, G, B. Doesn't mean λ=540 nm doesn't exist; it just can't be generated from the three chose primary lights. More later (gamut).



# Normalization



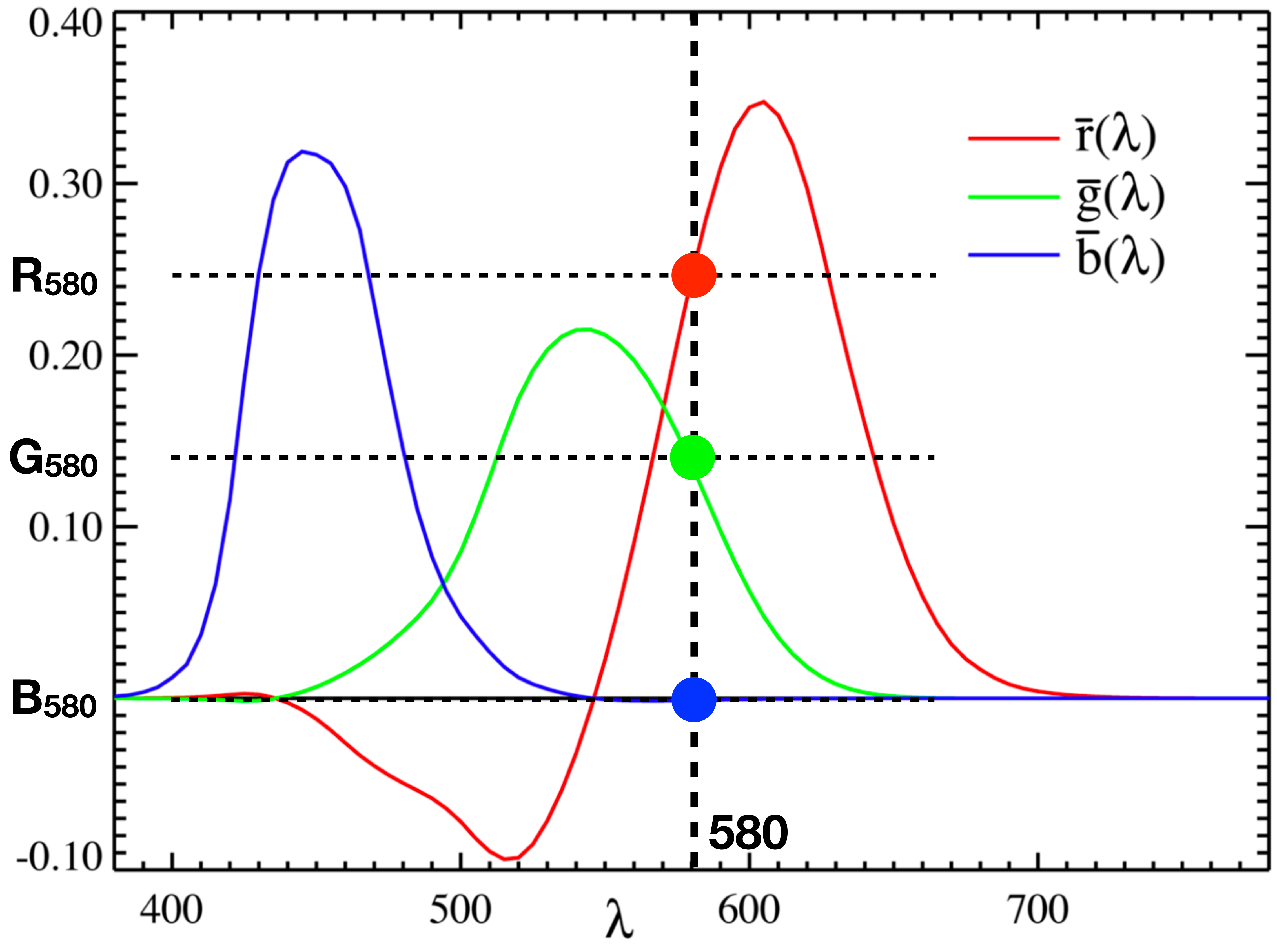

The plot on the left gives the relative ratio of the primaries, the plot on the right gives the absolute amount, scaled from the left to match the luminance at each wavelength.

See: https://yuhaozhu.com/blog/cmf.html.

# The Results are the Color Matching Functions

- For **one unit power** of each target light, how many units of each primary light are needed to match the color.
- RGB values of a target color are called the **tristimulus** values of the color in the CIE 1931 RGB color space.

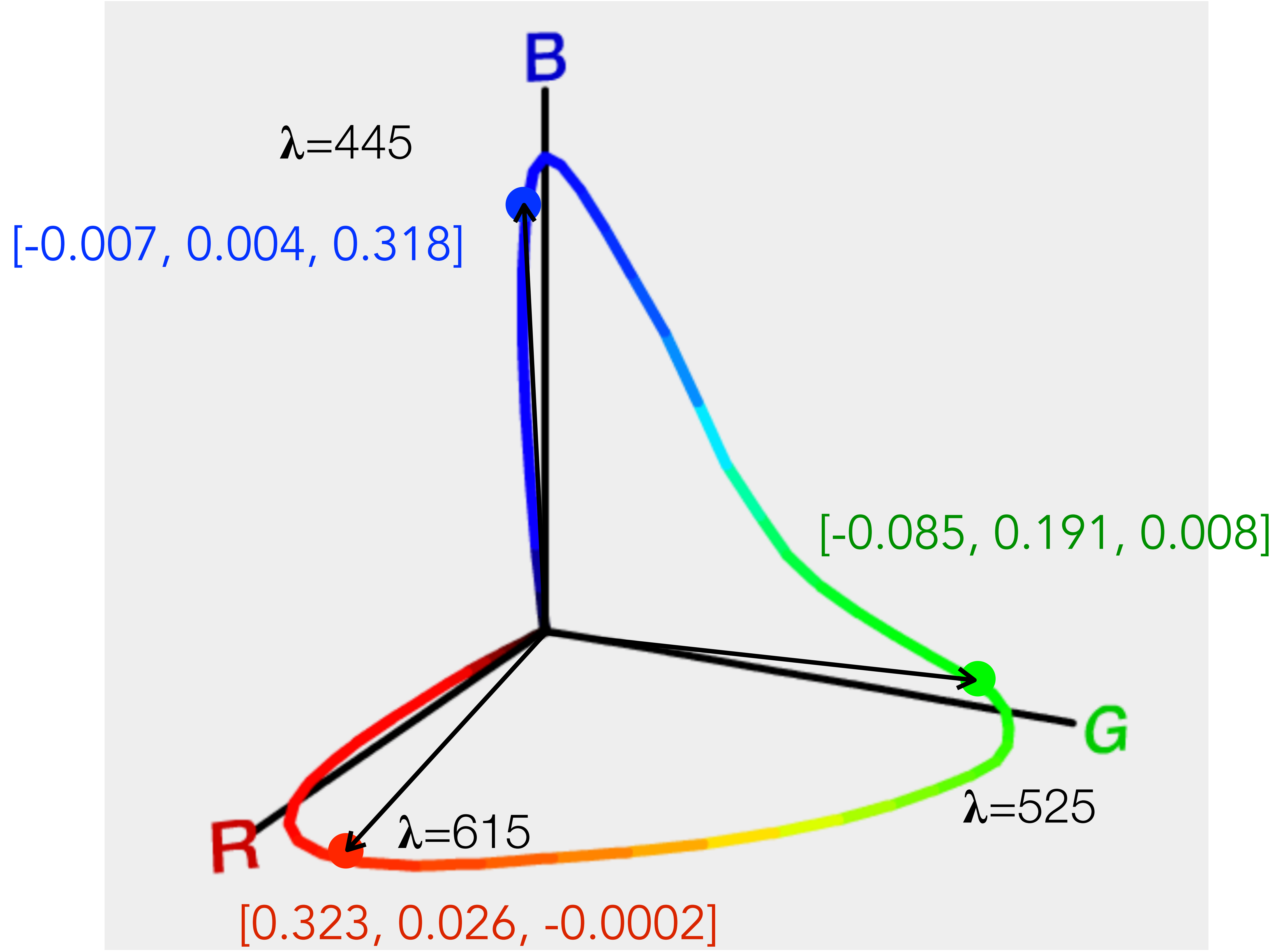
If the target light $\lambda$ has a power of $\Phi_{580}$, the amounts of the primary lights we need are: $\Phi_{580} \times \mathbf{R}_{580}$, $\Phi_{580} \times \mathbf{G}_{580}$, $\Phi_{580} \times \mathbf{B}_{580}$



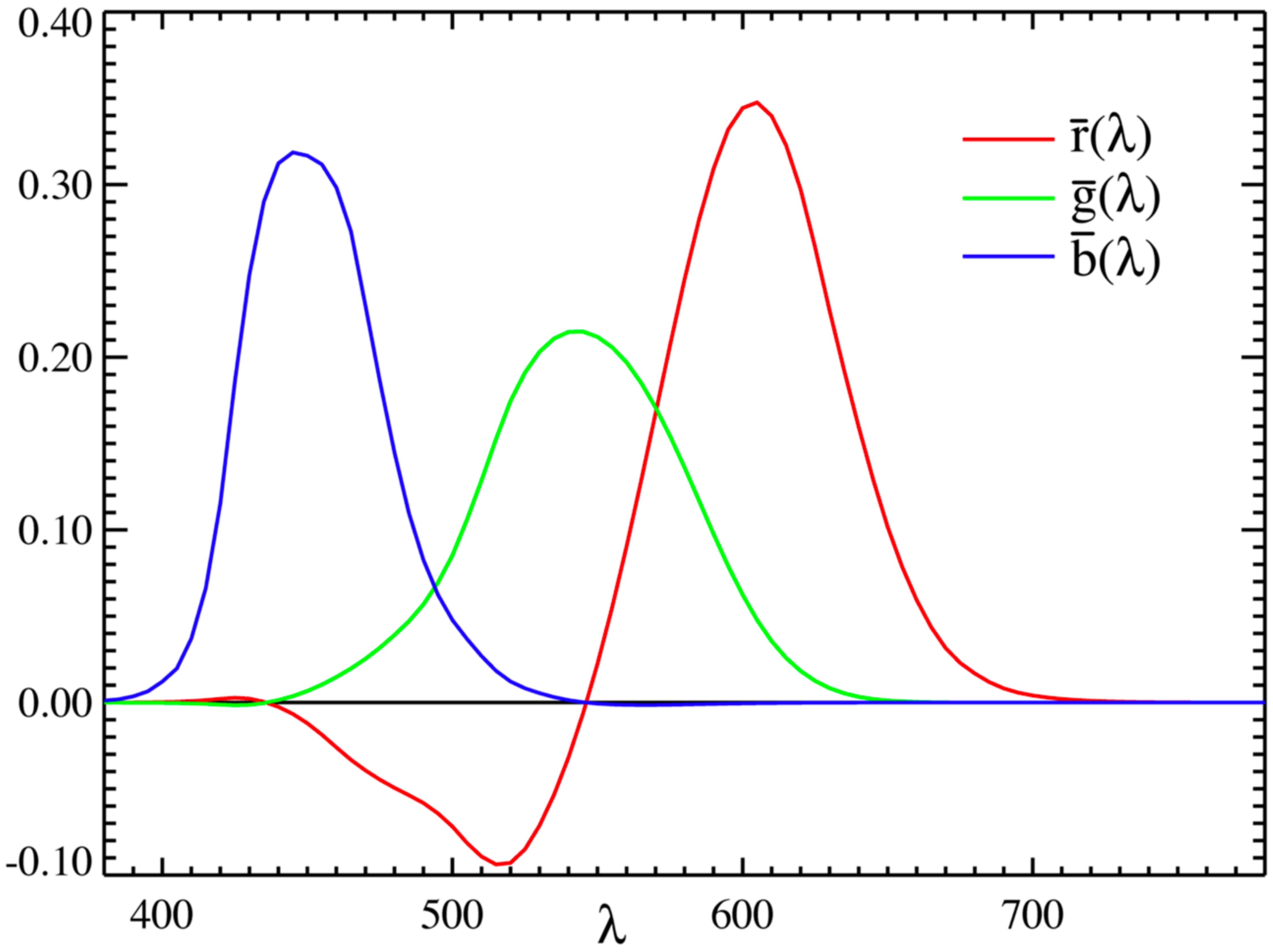

CIE 1931 2° RGB Color Matching Functions

https://en.wikipedia.org/wiki/CIE_1931_color_space

# Spectral Locus in the CIE 1931 RGB Space




Spectral locus in CIE 1931 RGB




CIE 1931 2° RGB Color Matching Functions

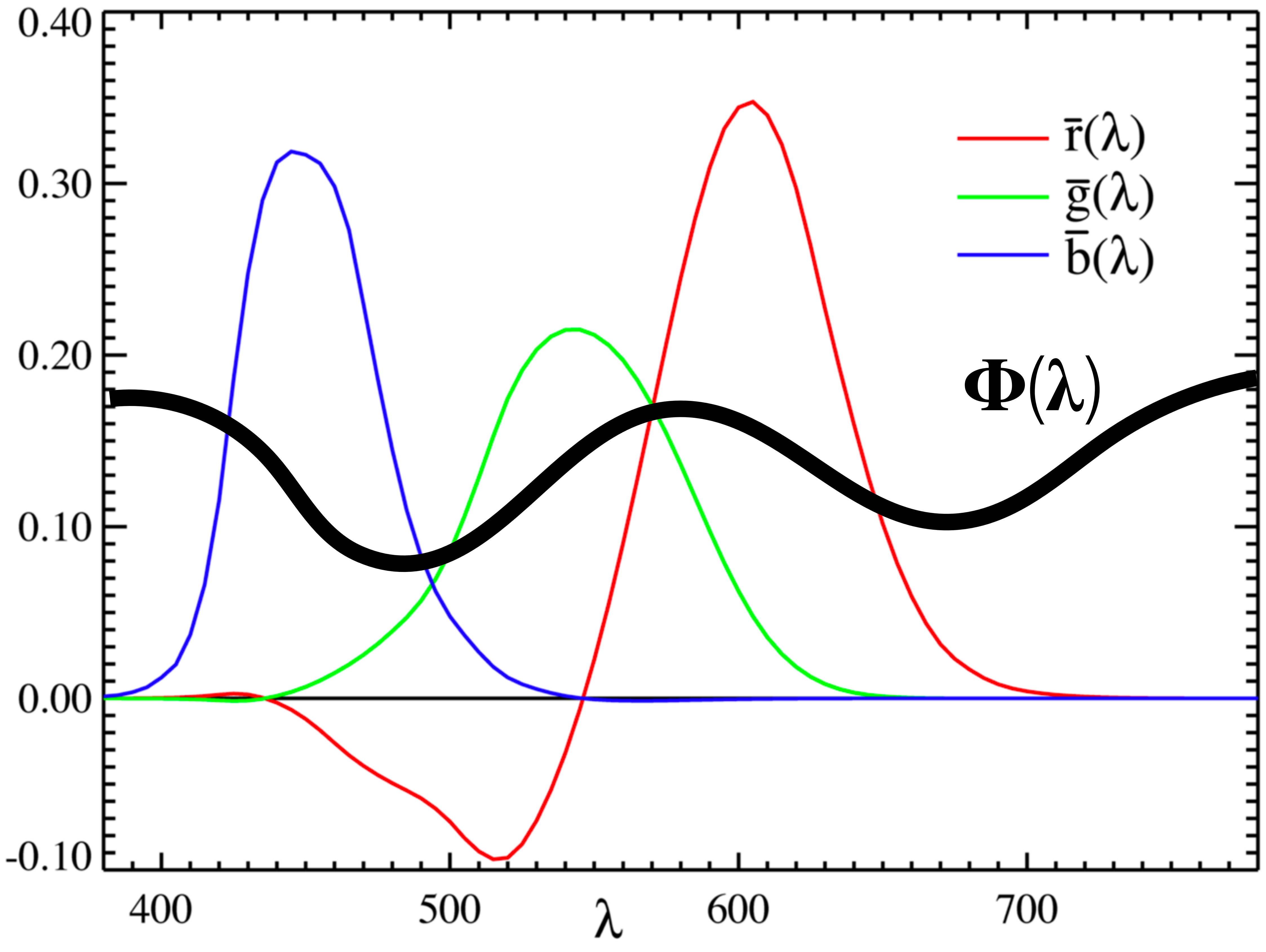
# Expressing Arbitrary Light Using CMFs

- How much total R, G, and B primaries are needed to match the color of an arbitrary light $\mathbf{\Phi}$?
- Calculate it wavelength-by-wavelength and sum it up.

$$R = \sum_{\lambda} \Phi(\lambda)\bar{r}(\lambda)$$

$$G = \sum_{\lambda} \Phi(\lambda)\bar{g}(\lambda)$$

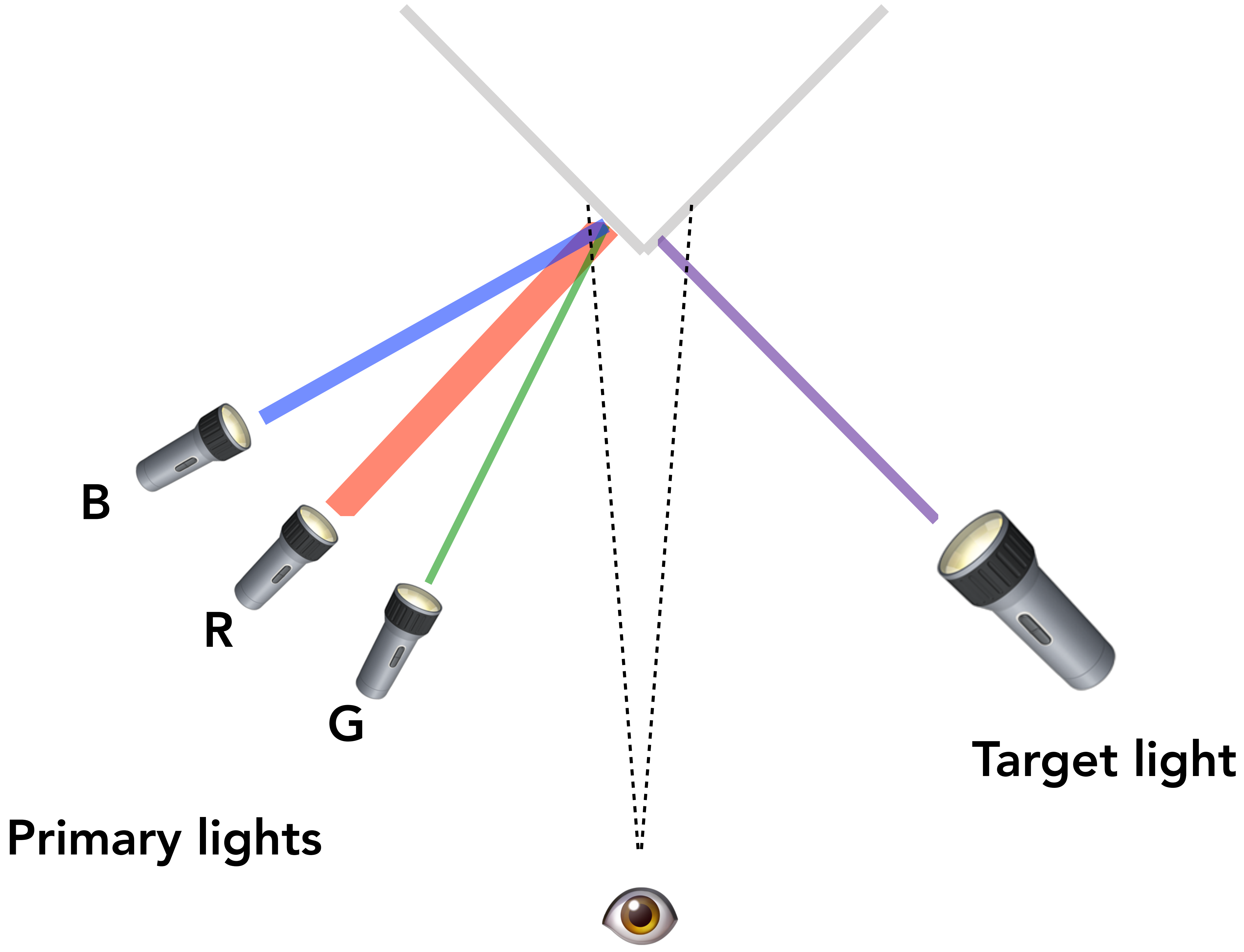
$$B = \sum_{\lambda} \Phi(\lambda)\bar{b}(\lambda)$$




CIE 1931 2° RGB Color Matching Functions

# How To Explain Color Matching Experiments?

- Given a target light $\Phi(\lambda)$ and the primary lights, can we calculate the tristimulus values for $\Phi(\lambda)$ ***without*** the color matching experiment?
- Key: the total L, M and S cone responses of the three primary lights must match the total cone responses of the target light.



# Intuition

L cone response of target light = L cone response of primary R
+ L cone response of primary G
+ L cone response of primary B

M cone response of target light = M cone response of primary R
+ M cone response of primary G
+ M cone response of primary B

S cone response of target light = S cone response of primary R
+ S cone response of primary G
+ S cone response of primary B

# Matching L Cone Responses

Amount of primary lights needed (unknown)

$$= \sum \Phi(\lambda)L(\lambda)$$

Amount of L cone responses of one unit of each primary light

The L cone response of the target light

# Matching All Cone Responses

$$\sum R(\lambda)L(\lambda) \times r + \sum G(\lambda)L(\lambda) \times g + \sum B(\lambda)L(\lambda) \times b = \sum \Phi(\lambda)L(\lambda)$$

$$\sum R(\lambda)M(\lambda) \times r + \sum G(\lambda)M(\lambda) \times g + \sum B(\lambda)M(\lambda) \times b = \sum \Phi(\lambda)M(\lambda)$$

$$\sum R(\lambda)S(\lambda) \times r + \sum G(\lambda)S(\lambda) \times g + \sum B(\lambda)S(\lambda) \times b = \sum \Phi(\lambda)S(\lambda)$$

# Equivalent to Solving a System of Linear Equations

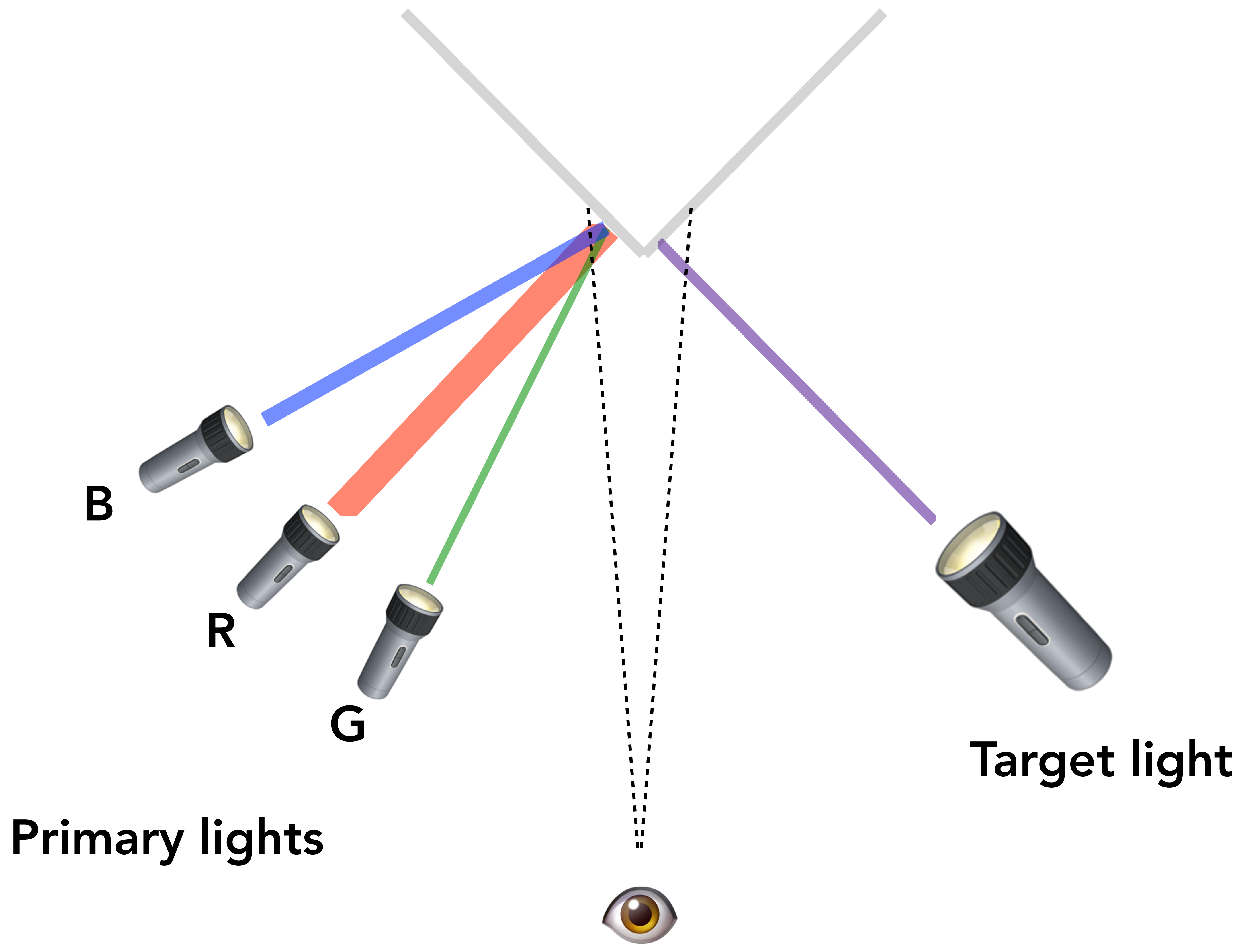
- Equivalent to solving the system of linear equations with 3 equations and 3 variables (**r**, **g**, **b**). The system has a unique solution (generally).

$$\begin{bmatrix} \sum R(\lambda)L(\lambda) & \sum G(\lambda)L(\lambda) & \sum B(\lambda)L(\lambda) \\ \sum R(\lambda)M(\lambda) & \sum G(\lambda)M(\lambda) & \sum B(\lambda)M(\lambda) \\ \sum R(\lambda)S(\lambda) & \sum G(\lambda)S(\lambda) & \sum B(\lambda)S(\lambda) \end{bmatrix} \times \begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} \sum \Phi(\lambda)L(\lambda) \\ \sum \Phi(\lambda)M(\lambda) \\ \sum \Phi(\lambda)S(\lambda) \end{bmatrix}$$

# Why Three Primary Colors?

- If only two primary colors, the system of equations has **two** variables with **three** equations, which generally doesn't have a solution, i.e., no way of finding a match, generally. Over-determined system.
- More than three primaries: more variables than equations, so there are infinitely many ways of combining the primary lights to match the target color. Under-determined system.

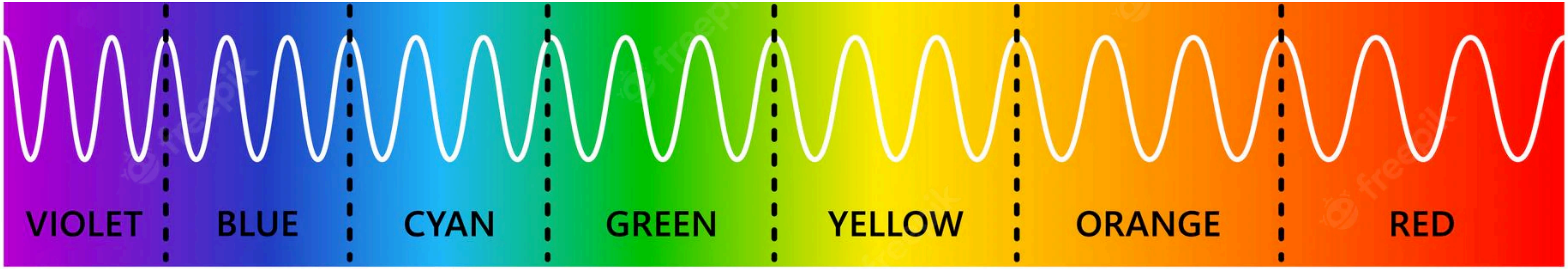
# Trichromatic Theory of Color




- Conjectured by Thomas Young as early as 1801, and expanded by Helmholtz and Maxwell in 1860.
  - Color matching experiments empirically verified it.
  - Experimental measurement of cone sensitivities much later.
- Pervasive applications:
  - Color reproduction chain: display, print
  - Capture color: camera

# A Note on "RGB"

- There is no one single definition of "red", "green", and "blue".
- Usually "RGB" is just synonyms for "primary lights", which usually are so chosen that they *look* red, green, and blue.
- Primary lights **don't** have to be monochromatic spectral lights.




380nm 780nm

https://www.freepik.com/vectors/visible-spectrum

# "Amount of Primaries"

- Imagine two color spaces that use the same primary lights, but
  - one where 1 unit of R means 1 W of red light
  - the other where 1 unit of R means 10 W of red light
  - The RGB value of a given light in these two spaces will be different
- A color space must define what "one unit" of each primary light is.
- In CIE 1931 RGB color space, the "unit system" is that 1 unit of R, G, and B provides the same color as a special light called "Equal-Energy White", which has the same power over all wavelengths.
  - If you are building your own color space, you can choose whatever "white" you want.

# A RGB Color Space

- Given the exact spectra of the "RGB" primary lights R(λ), G(λ), and B(λ), we have a color space.
  - What this means is that the color of any arbitrary light $\Phi(\lambda)$ can be expressed as a point in that 3D coordinate system.
- That is, we can mathematically study colors now — colorimetry.

$$\begin{bmatrix} \sum R(\lambda)L(\lambda) & \sum G(\lambda)L(\lambda) & \sum B(\lambda)L(\lambda) \\ \sum R(\lambda)M(\lambda) & \sum G(\lambda)M(\lambda) & \sum B(\lambda)M(\lambda) \\ \sum R(\lambda)S(\lambda) & \sum G(\lambda)S(\lambda) & \sum B(\lambda)S(\lambda) \end{bmatrix} \times \begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} \sum \Phi(\lambda)L(\lambda) \\ \sum \Phi(\lambda)M(\lambda) \\ \sum \Phi(\lambda)S(\lambda) \end{bmatrix}$$

# Geometric Transformation Perspective

- So far we've seen two ways to express a color in a 3D space:
  - A color can be expressed as its three cone responses (l, m, s)
  - A color can also be expressed as the amount of primary lights (r, g, b) needed
- How are the two expressions related?

$$\begin{bmatrix} \sum \Phi(\lambda)L(\lambda) \\ \sum \Phi(\lambda)M(\lambda) \\ \sum \Phi(\lambda)S(\lambda) \end{bmatrix} = \begin{bmatrix} l \\ m \\ s \end{bmatrix}$$

# Geometric Transformation Perspective

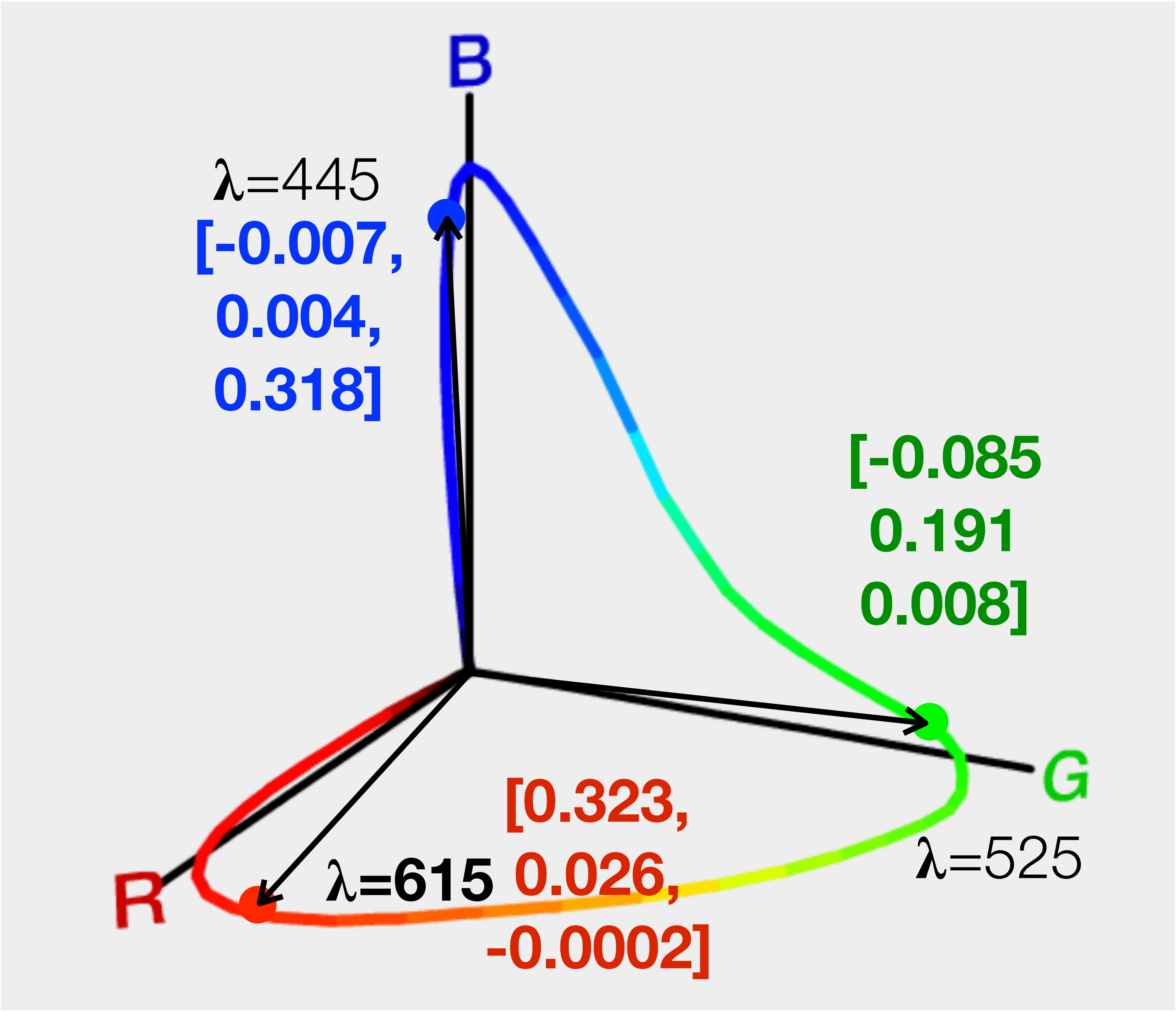
- Expressing a color from the [l, m, s] coordinates to the [r, g, b] coordinates is a geometric transformation.
- Equivalently, it's a transformation of the coordinate systems (color spaces)!
  - Because the transformation matrix is constant, metamers in RGB space are metamers in LMS space too.

Transformation matrix from the RGB space to the LMS space

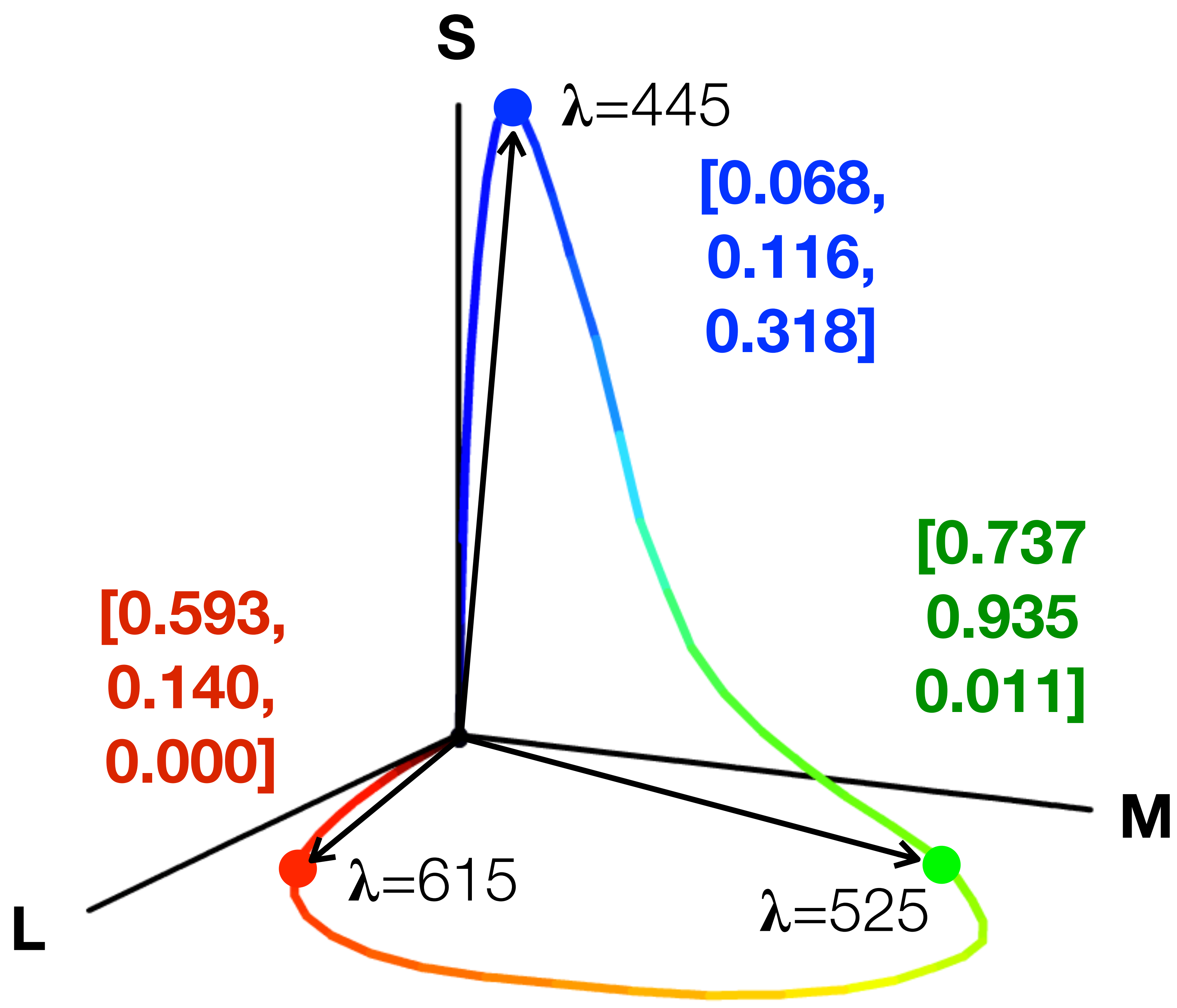
$$\begin{bmatrix} \sum R(\lambda)L(\lambda) & \sum G(\lambda)L(\lambda) & \sum B(\lambda)L(\lambda) \\ \sum R(\lambda)M(\lambda) & \sum G(\lambda)M(\lambda) & \sum B(\lambda)M(\lambda) \\ \sum R(\lambda)S(\lambda) & \sum G(\lambda)S(\lambda) & \sum B(\lambda)S(\lambda) \end{bmatrix} \times \begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} \sum \Phi(\lambda)L(\lambda) \\ \sum \Phi(\lambda)M(\lambda) \\ \sum \Phi(\lambda)S(\lambda) \end{bmatrix} = \begin{bmatrix} l \\ m \\ s \end{bmatrix}$$

# CIE 1931 RGB ⟷ LMS Spaces

Spectral lights in the CIE 1931 RGB space




Spectral lights in the LMS space




Data from: http://www.cvrl.org/database/data/cones/linss10e_5.htm

# Key Things to Take Away

- The cone cells are the first stage of color encoding, where they turn spectral power distribution in light to cone responses
- Since there are three cone classes, color vision is trichromatic
  - Light spectrum is encoded into a three-dimensional space, so any color can *in theory* be produced from mixing three other colors (subject to gamut limit)
- Metamerism: lights having different spectra but the same color.
  - This is because color encoding by photoreceptors is a huge dimensionality reduction
  - Metamerism explains color matching experiments, which is an experimental demonstration of trichromatic theory of color