

Lecture 25: Sub-Surface and Volume Scattering

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Computer Imaging & Graphics

The Roadmap

Theoretical Preliminaries

Human Visual Systems

Display and Camera

Modeling and Rendering



3D Modeling

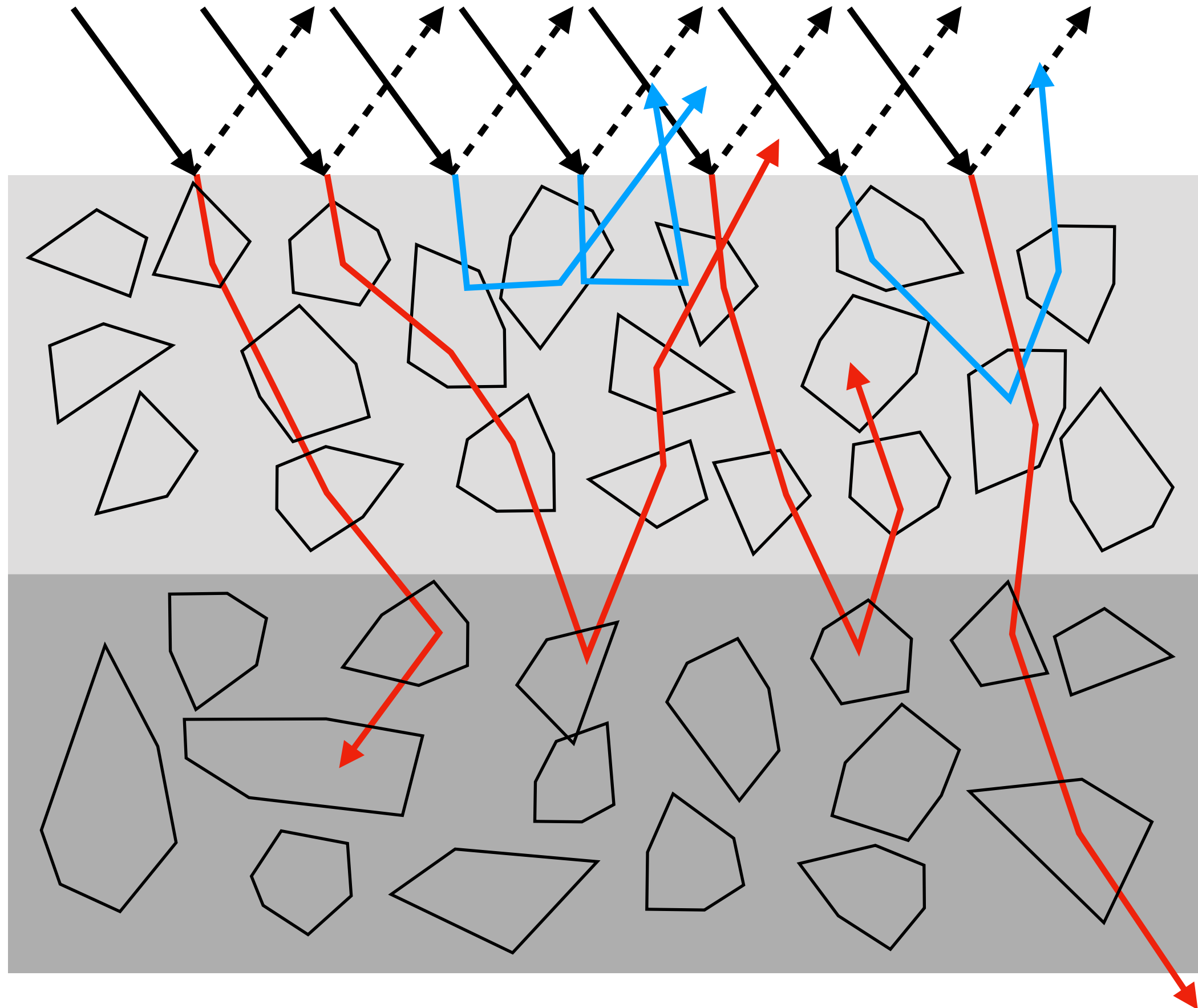
Rasterization and GPU

Ray Tracing

Shading

Rendering in AR/VR

Subsurface/Volume Scattering

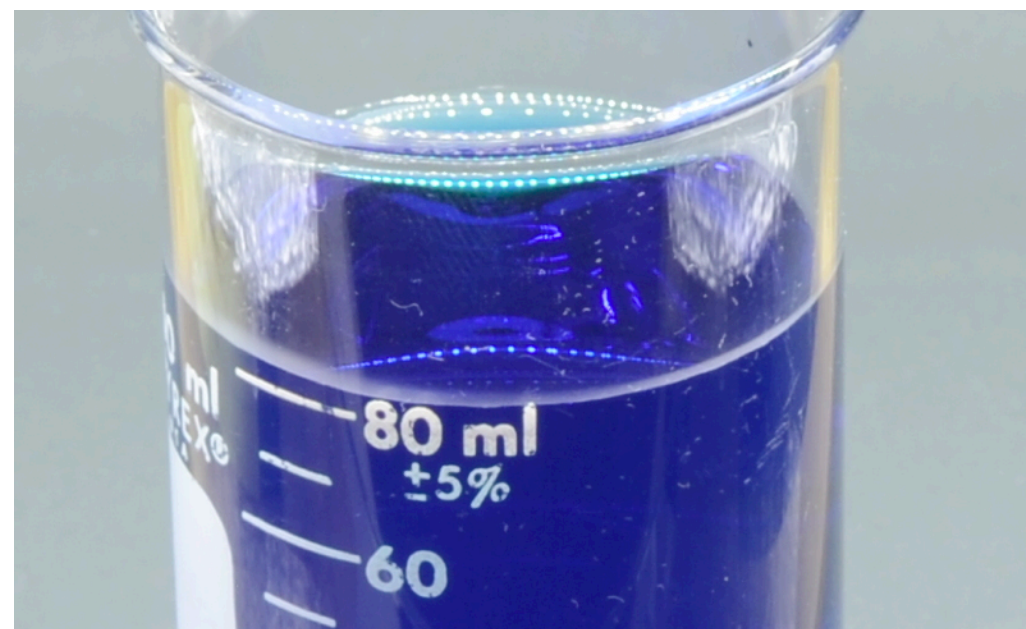
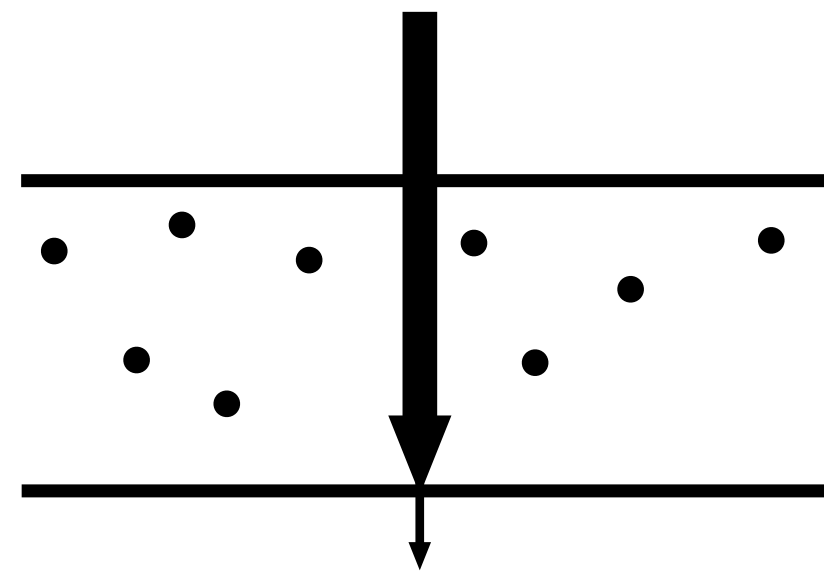


So far we consider only surface reflection, but not how photons interact with particles under surface.

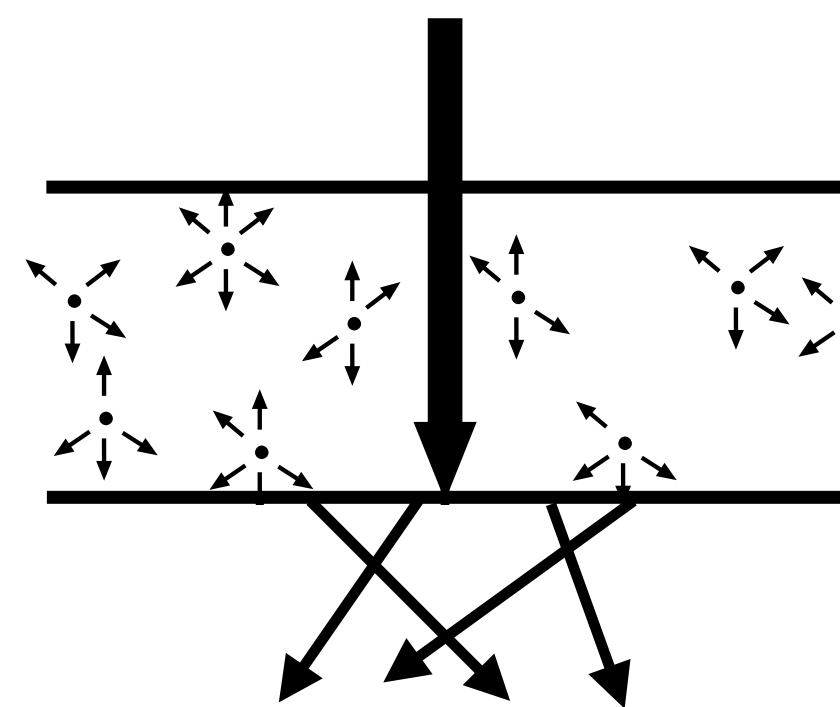
Particles of translucent materials under the material surface scatter lights like crazy and some scattered photons make their way out of the surface.

Material Types

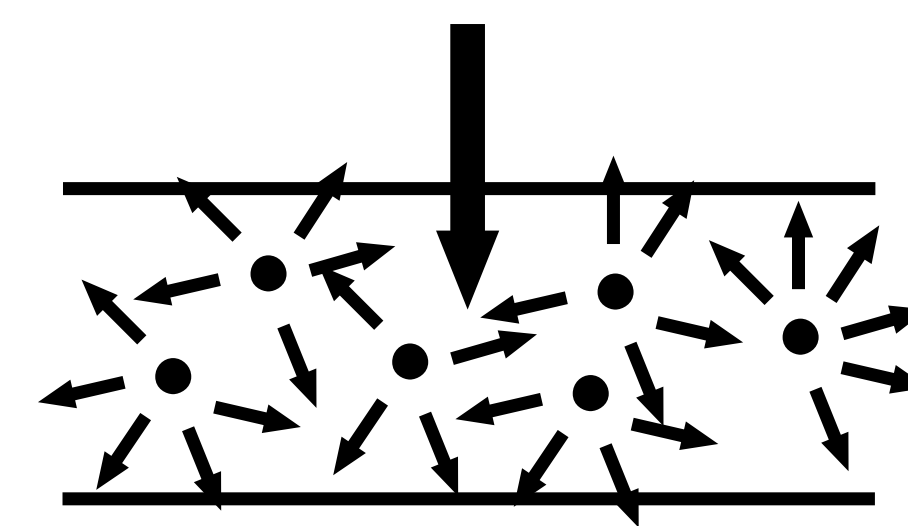
Pure absorption
(Transparent)



A little scattering + absorption
(Translucent)



A lot of scattering + absorption
(Opaque)

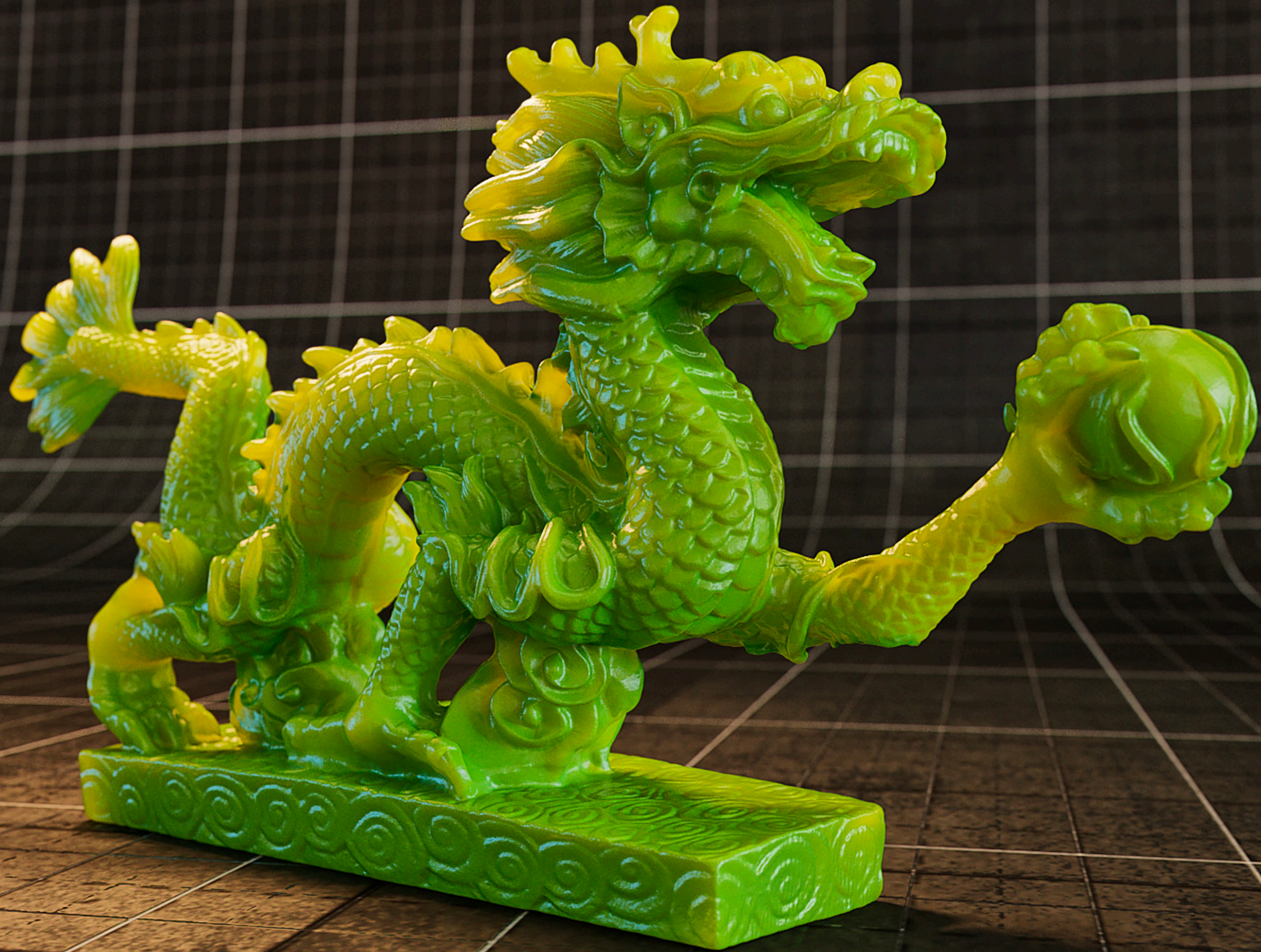


Volume Scattering

Particles scatter and absorb photons, and it's important to model in:

- Translucent materials (e.g., jade, skin), which have strong subsurface scattering (SSS) in addition to surface scattering. SSS can be ignored in cases where refraction is weak or particle absorption is strong so that the back-scattering doesn't contribute much to the overall material appearance.
- "Participating media" (e.g., cloud, smoke), which have no surface and everything in volume scattering.









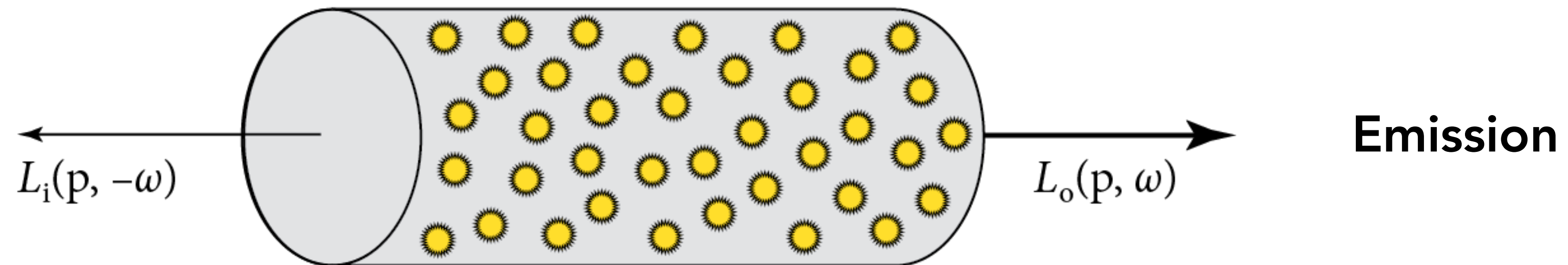
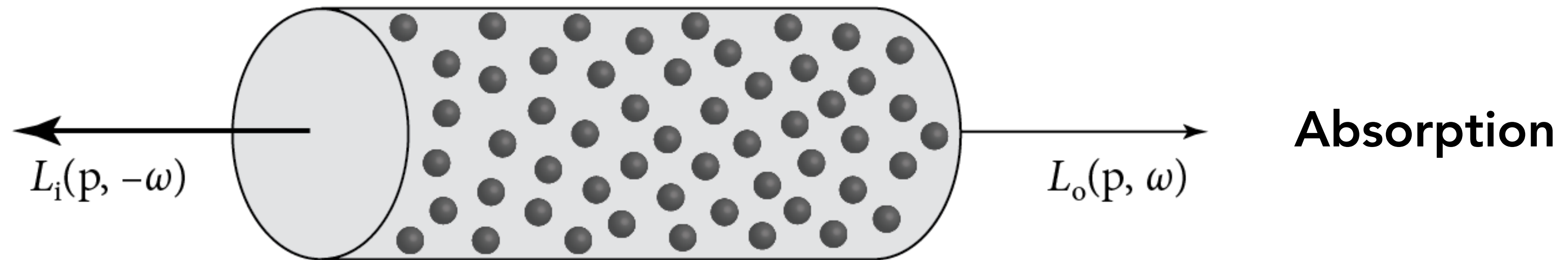
General Idea

The principle way to model volume scattering is the Radiative Transfer theory, which is an integro-differential equation that generally have no analytical solution.

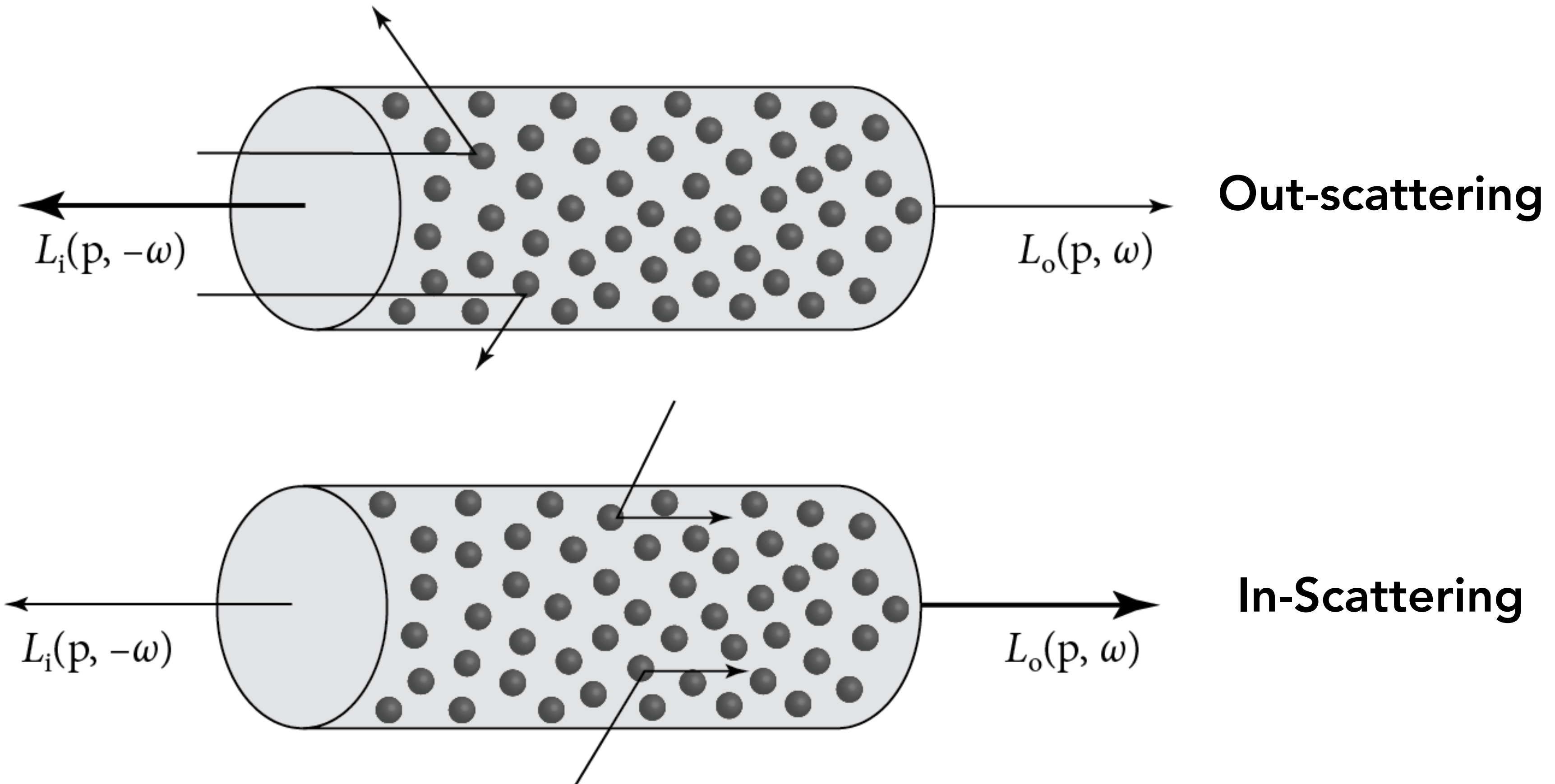
We solve it by:

- make simplified assumptions (e.g., Kubelka-Munk model for diffuse illumination and isotropic scattering, diffusion approximation applied to BSDF [[Jensen et al. 2000](#)])
- plus: using numerical methods to approximate the solution (e.g, monte-carlo estimator)

Volume Scattering Processes



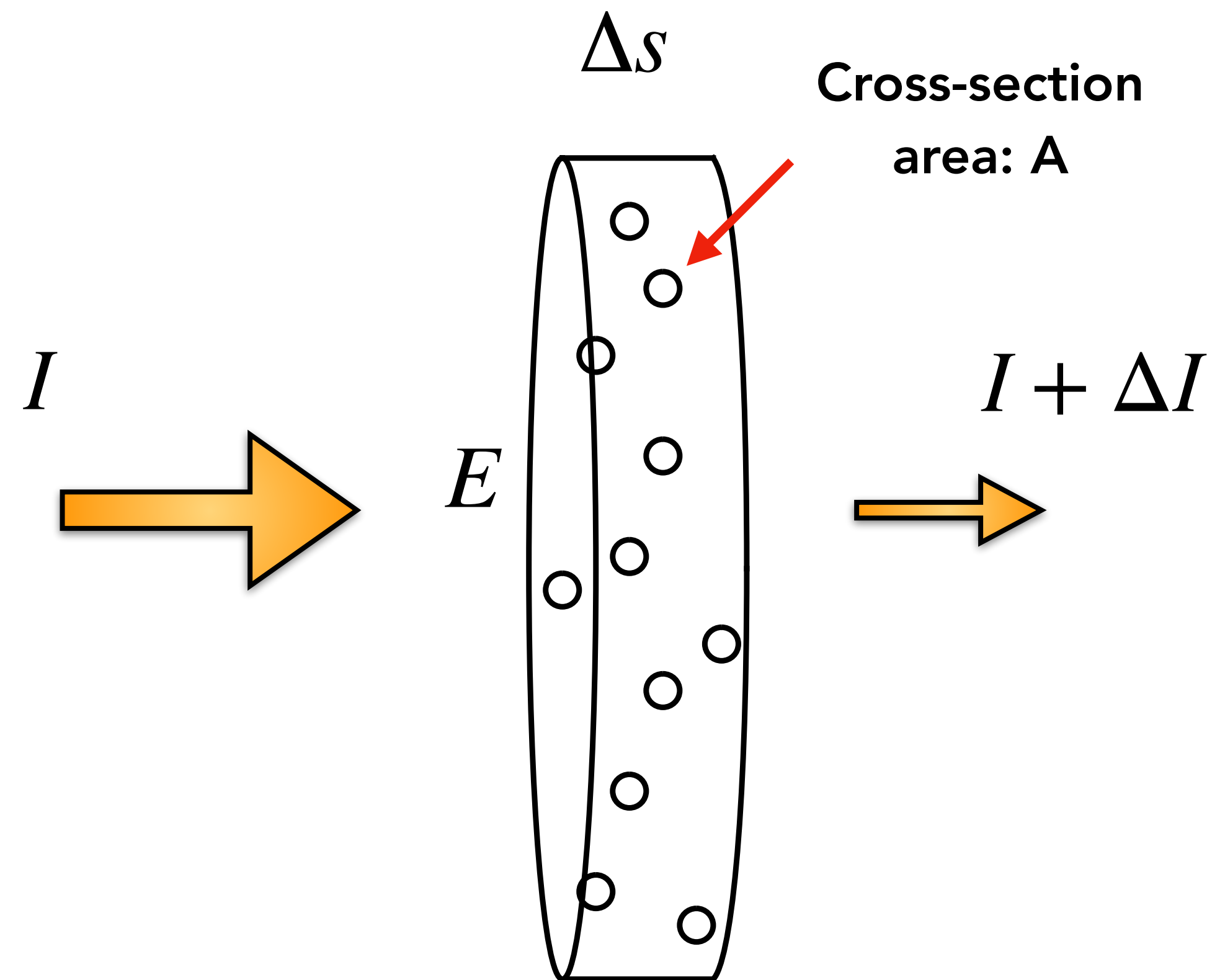
Volume Scattering Processes



Pure Absorption

Modeling Pure Absorption

Consider a very thin slice (so thin that no particles are covering each other)



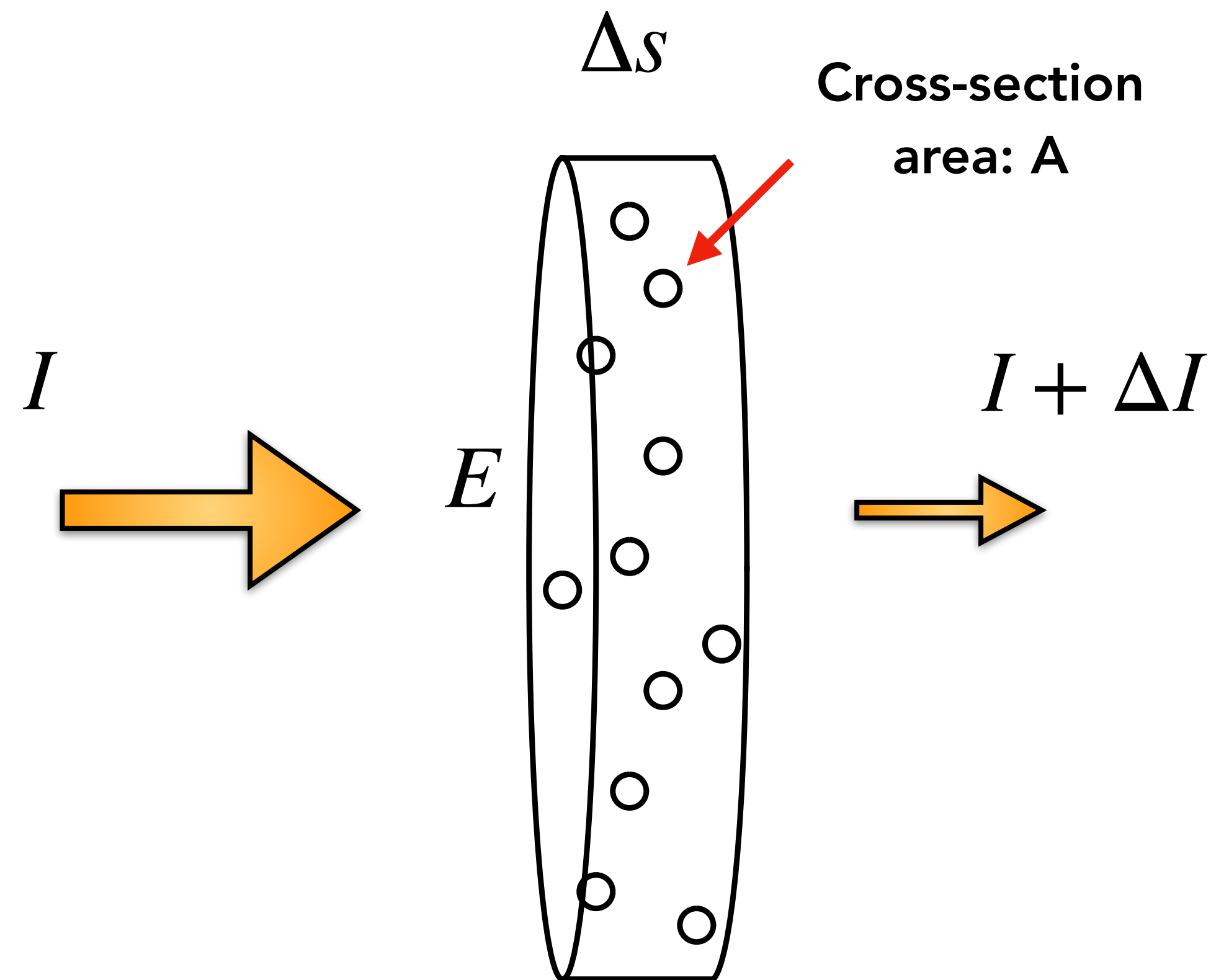
Concentration (# of particles / unit volume)

Total area covered by the particles

A diagram showing the equation for the change in intensity ΔI through a thin slice. The equation is
$$\Delta I = \frac{\rho E \Delta s A}{E} E$$
 The equation is enclosed in a blue box. A pink box highlights the term $\rho E \Delta s A$. A teal box highlights the term ρ . A blue line points from the text "Percentage of area occluded by particles" to the teal box. A pink line points from the text "Total area covered by the particles" to the pink box. A teal line points from the text "Concentration (# of particles / unit volume)" to the teal box.

Modeling Pure Absorption

Consider a very thin slice (so thin that no particles are covering each other)

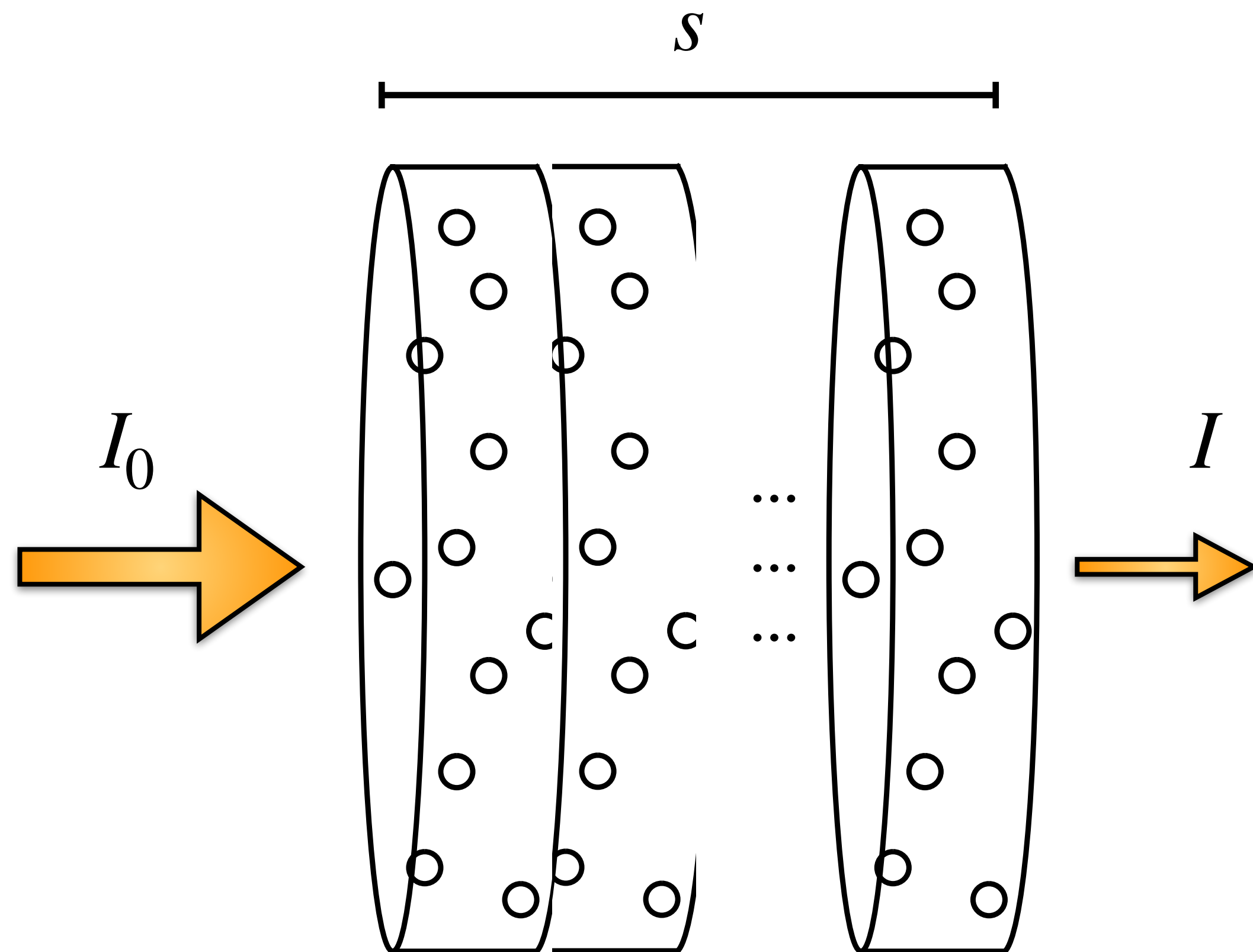


Percentage of
absorbed photons

$$\frac{\rho E \Delta s A}{E} = - \frac{\Delta I}{I}$$

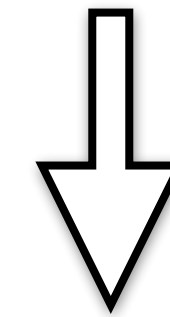
$$\frac{\Delta I}{\Delta s} = -\rho I A$$

Beer-Lambert Law



As s increases (light travel farther), the intensity decreases **exponentially**!

$$\frac{dI}{ds} = -\rho I A$$



$$I = I_0 e^{-\rho A s}$$

If density is uniform

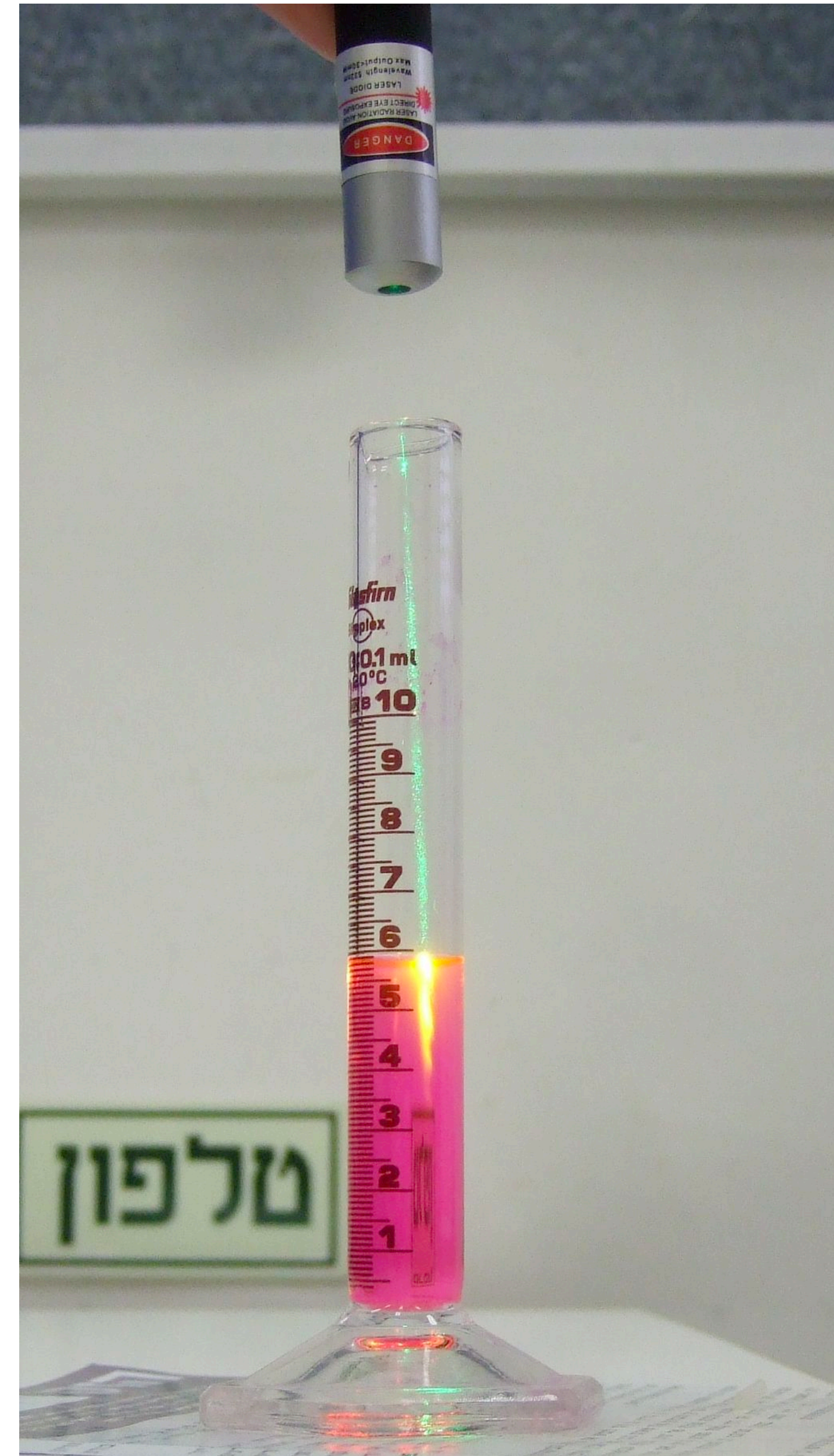
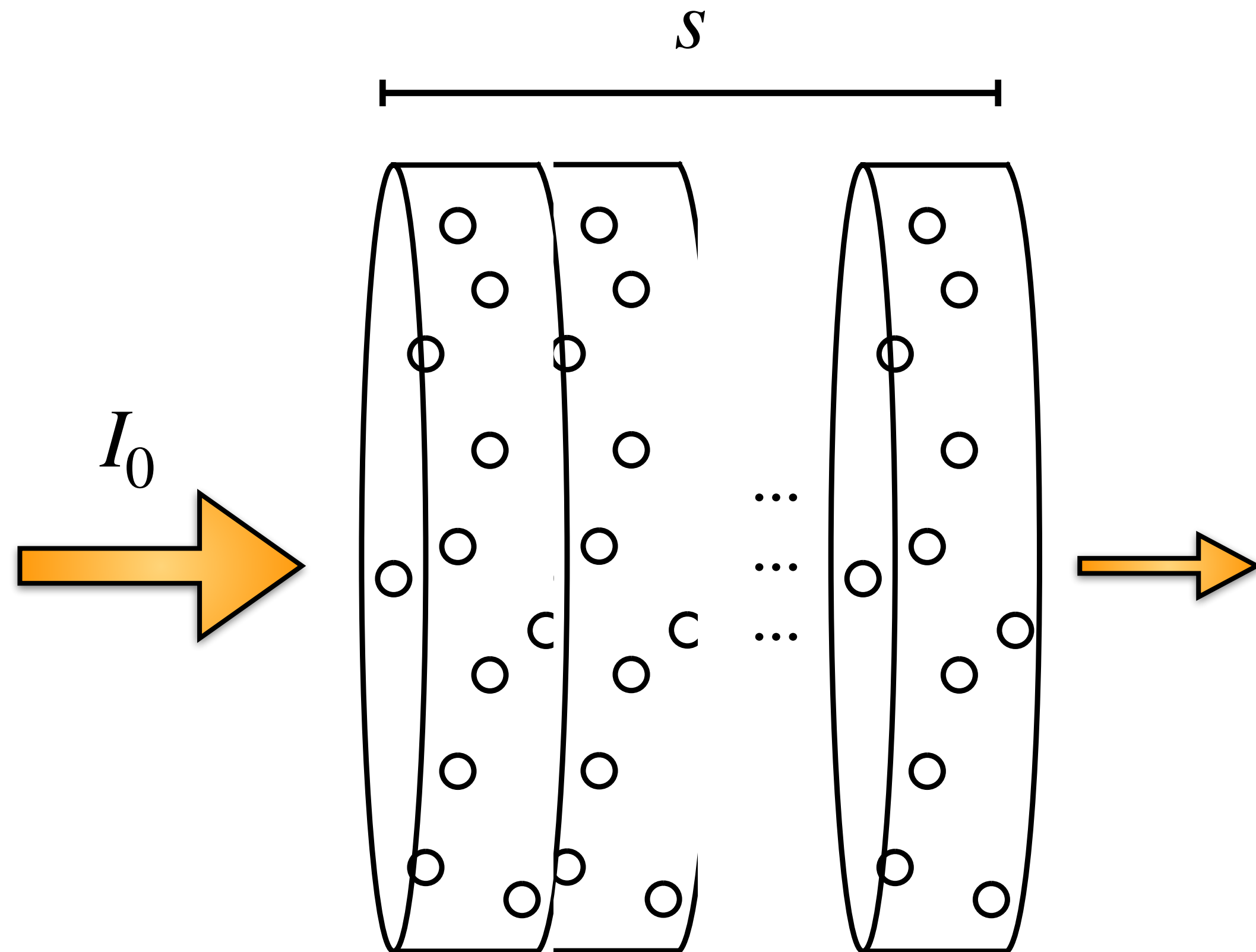
$$I = I_0 e^{-\int_0^s \rho(s) A}$$

If density is non-uniform

"The person credited with a scientific law is almost certainly not the one who discovered it."

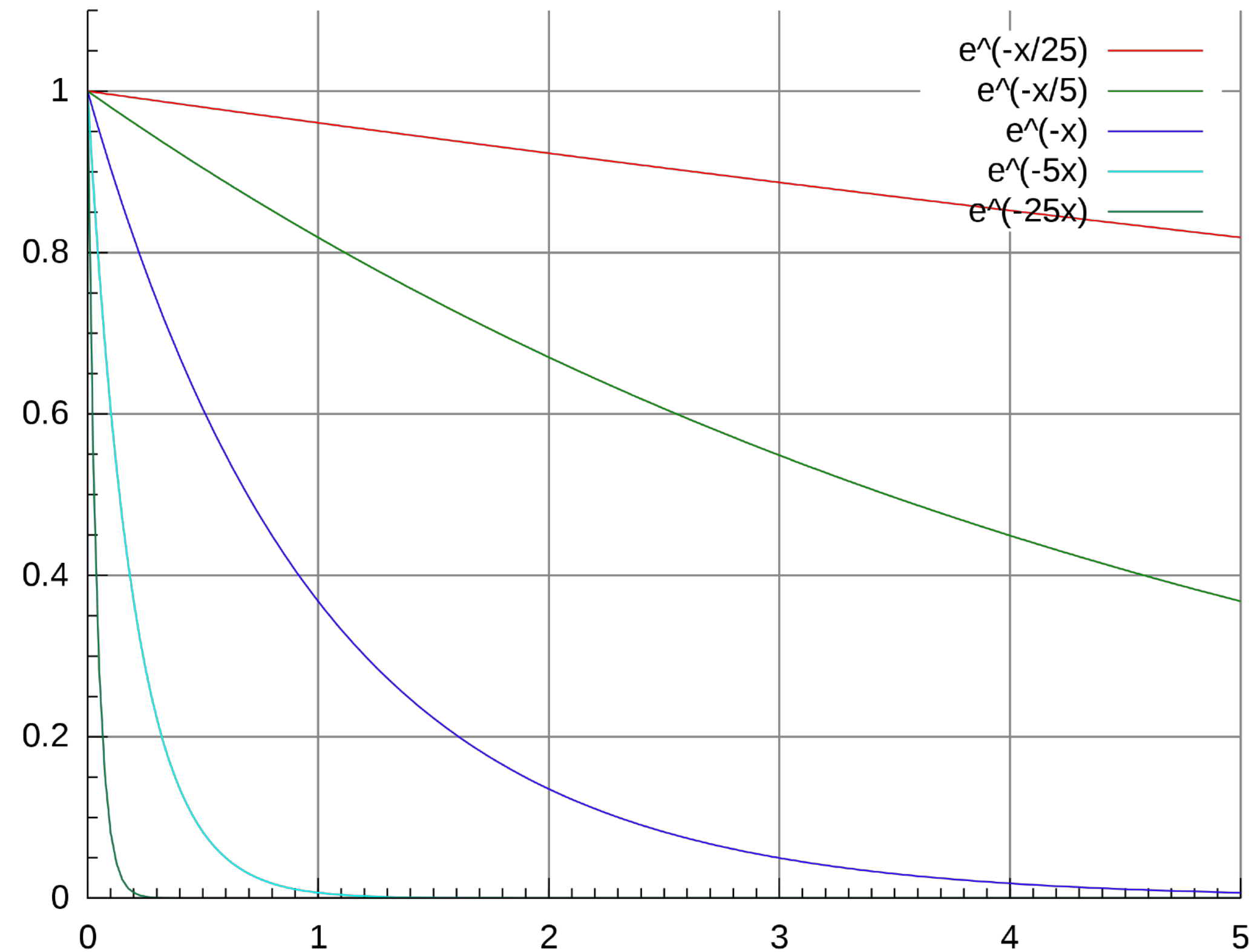
Bouguer was the first to discover this law.

Beer-Lambert Law



Exponential Decay

$$I = I_0 e^{-\rho A s}$$



Exponential decay happens when a quantity decreases at a rate proportional to its current value.

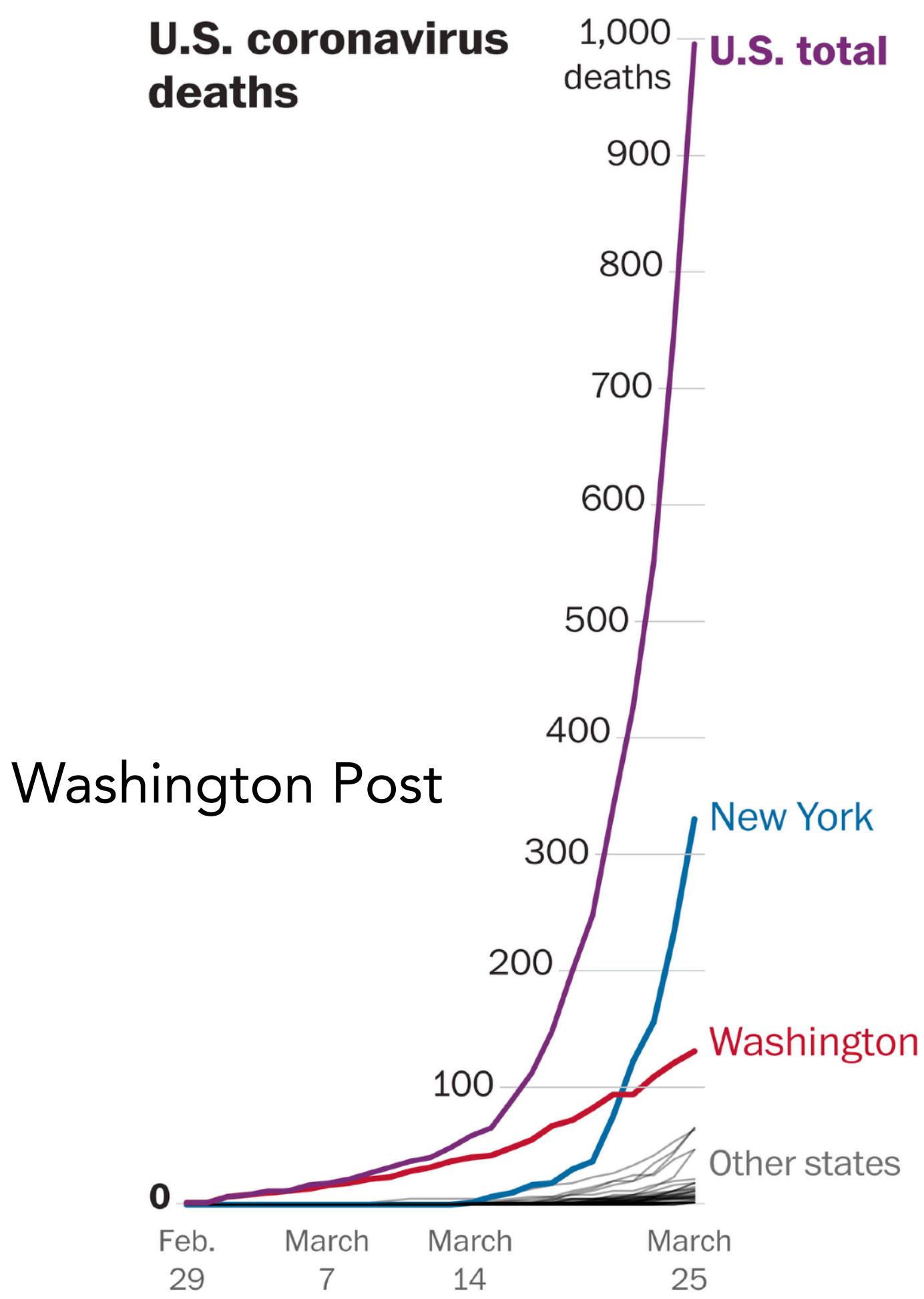
- Opposite of exponential growth.

Exponential decay means lights are hard to be completely absorbed.

- 10% of light @ 70 m deep under ocean, but you can still see lights at 850 m deep.

Many real-world scenarios exhibit exponential growth/decay.

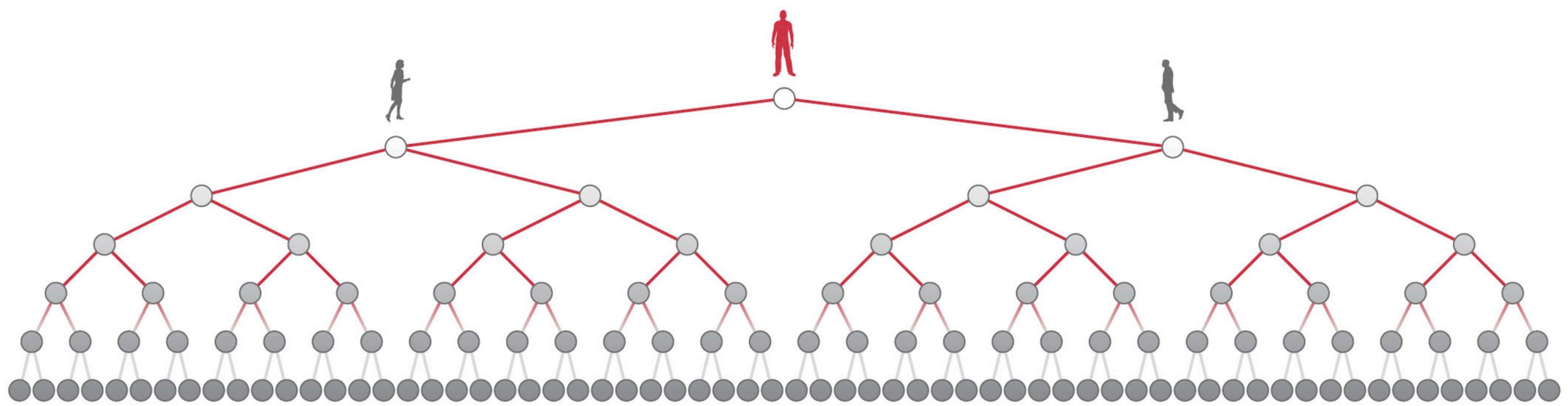
Exponential Growth



Source: Post reporting from state health department data

Every exponential growth will come to an end when...

Coronavirus Chain of Transmission



New York Times

Absorption Efficiency and Coefficient

Density/concentration
(Unit: 1/m³) Cross-section
area (Unit: m²)

$$I = I_0 e^{-\rho A s}$$

More generally

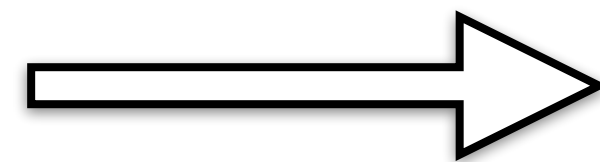


Absorption efficiency (unitless): the probability that a photon is actually absorbed when it hits a particle.

$$I = I_0 e^{-\rho Q_a A s}$$

Absorption coefficient:
Unit: 1/m

$$\mu = \rho Q_a A$$



Percentage of light left

$$I = I_0 e^{-\mu s} \qquad \frac{I}{I_0} = e^{-\mu s}$$

* As always, absorption efficiency and coefficient are **wavelength-dependent**.

$C_a = Q_a A$ is sometimes called **effective absorption cross section**

Absorption Efficiency and Coefficient

Informally, μ quantifies how much light is absorbed per unit distance.

- $\mu = 1$ means after passing through 1 m the radiance is reduced by a factor of e .
- Another way to understand μ is that $\mu = 0.1$ means a photon can travel about 10 m before being absorbed. ($1/\mu$ is the mean lifetime in an exponential decay).

Absorption coefficient:

Unit: 1/m

$$\mu = \rho Q_a A$$


$$I = I_0 e^{-\mu s}$$

Photoreceptor Absorption

Percentage of light left $\frac{I}{I_0} = e^{-\mu s}$

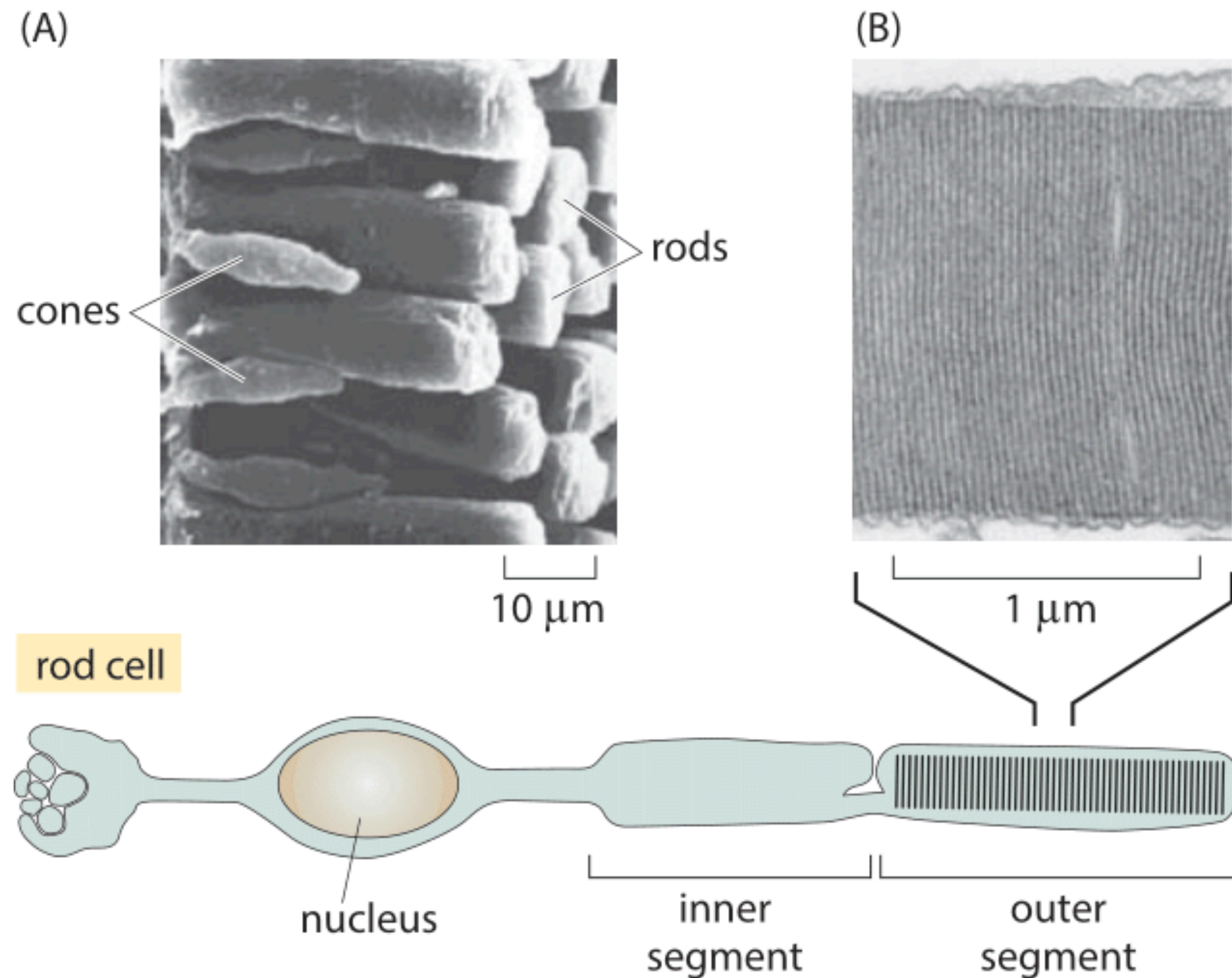
Outer segment is light-sensitive part of rods. It's about 25 μm long.

- It absorbs about 3% of the incident light for every μm .
- How much light is absorbed by a rod cell? (Answer 53%)

$$0.97 = e^{-\mu s}, s = 1\mu\text{m}$$

$$\Rightarrow \mu \approx 0.03/\mu\text{m}$$

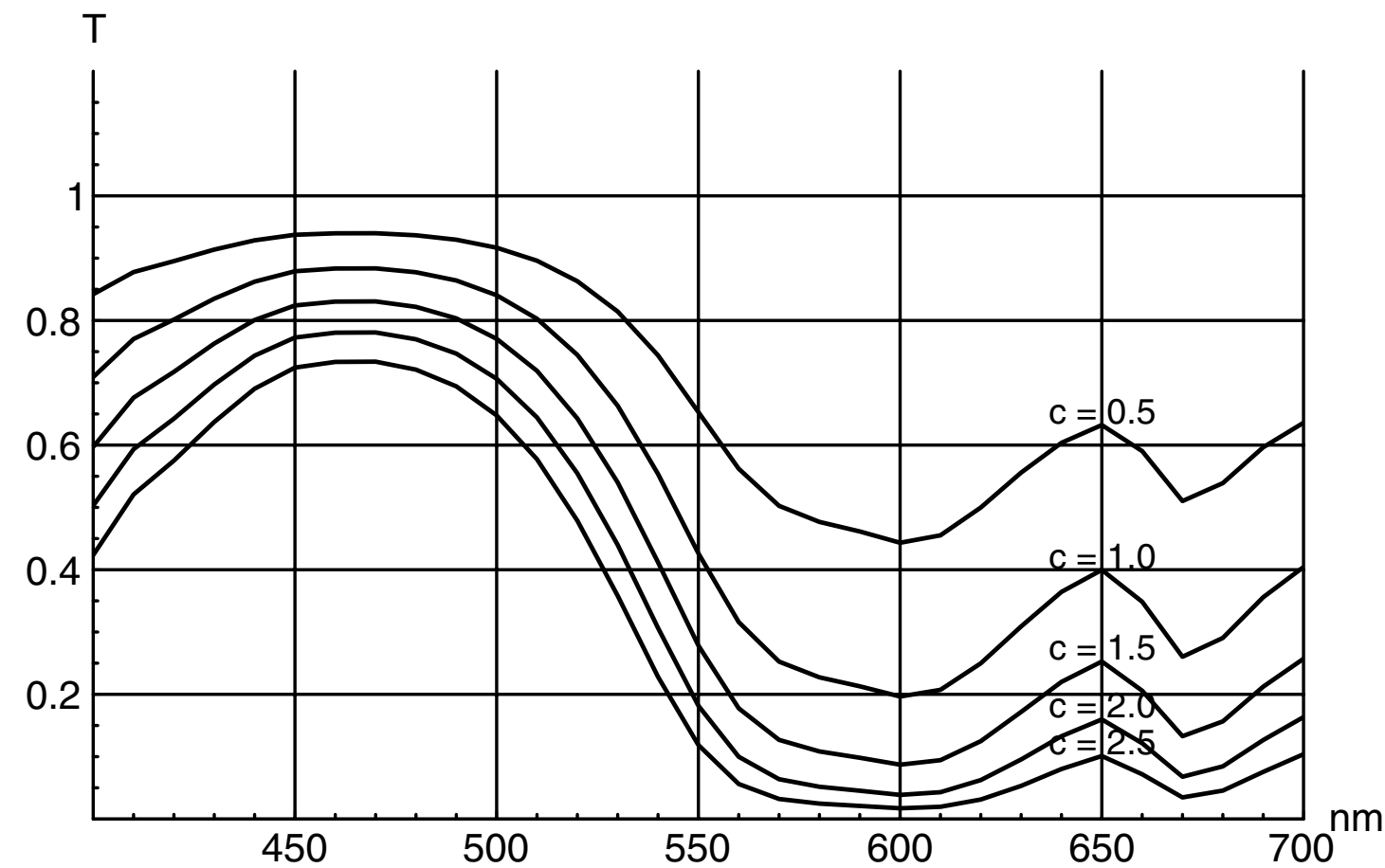
$$\Rightarrow e^{-0.03 \times 25} \approx 0.47$$



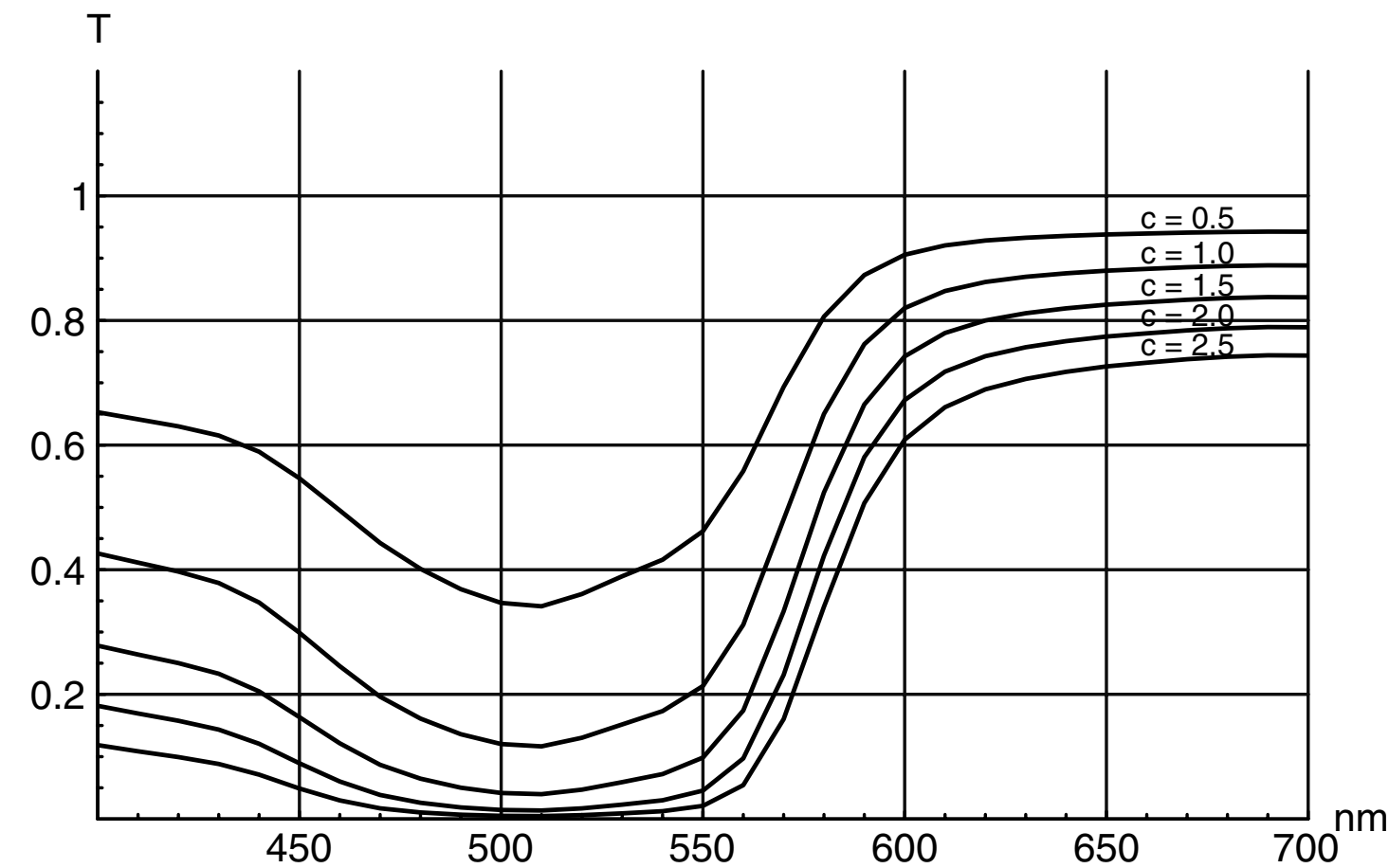
Absorption/Transmission vs. Concentration

$$I = I_0 e^{-\rho Q_a A s}$$

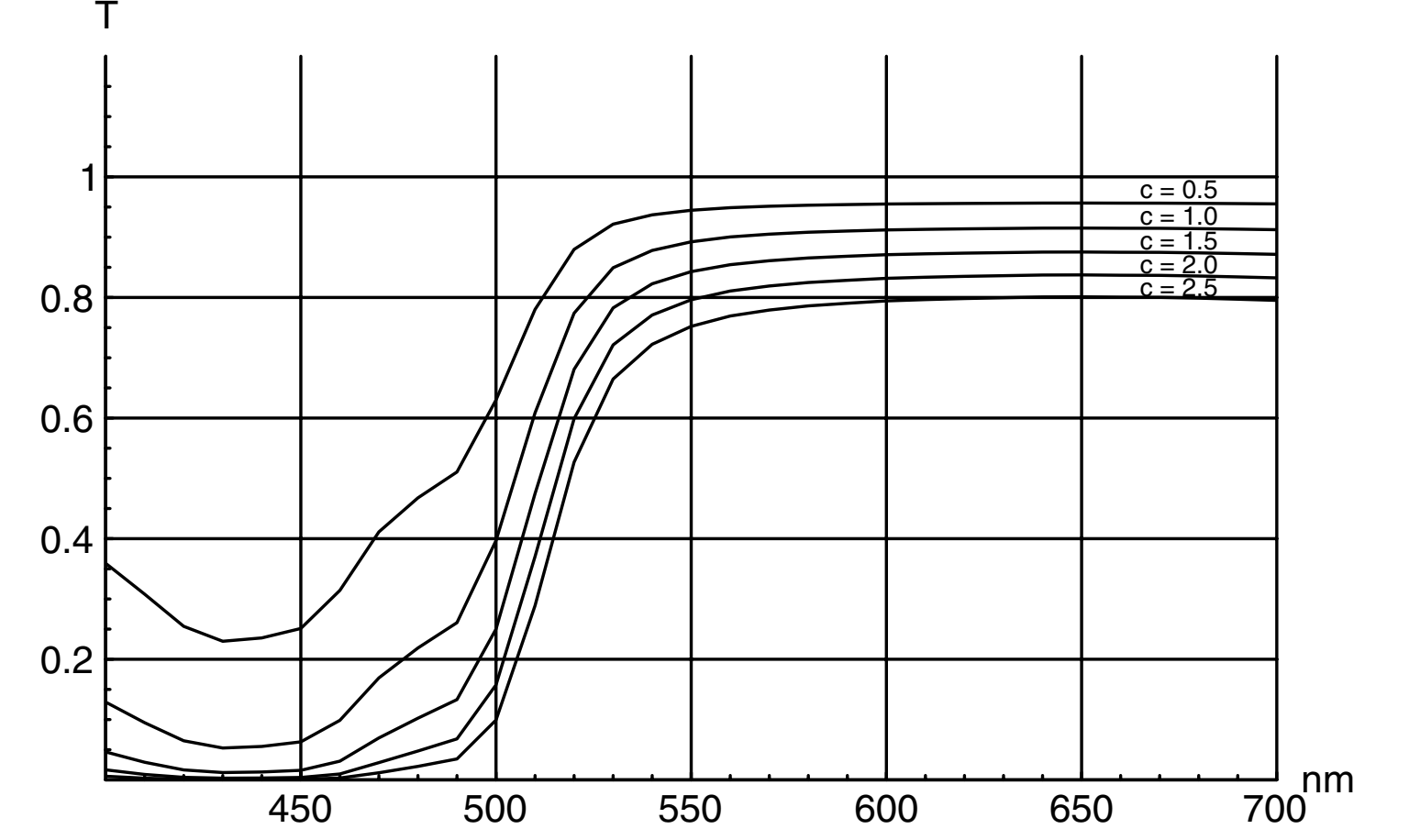
Cyan



Magenta



Yellow



Transmittance (1 - absorption) of three colored-ink at various concentrations (**c**).

Pure Scattering

(from a single particle)

Modeling Scattering

We need to model two aspects of scattering:

- How much is light scattered. This is captured by **scattering coefficient**.
- In which direction is a photon scattered. This is captured by the so-called “**phase function**”.

The general way to model scattering behavior amounts to solving Maxwell's equations for scattering. It's called the Mie solution.

- “Gustav Mie is the last person to discover the scattering behavior that bears his name.”

Importantly, people have worked out approximations under different conditions, using which we can readily explain many scattering behaviors.

Modeling Scattering

We will first consider cases where a photon is scattered by one particle.

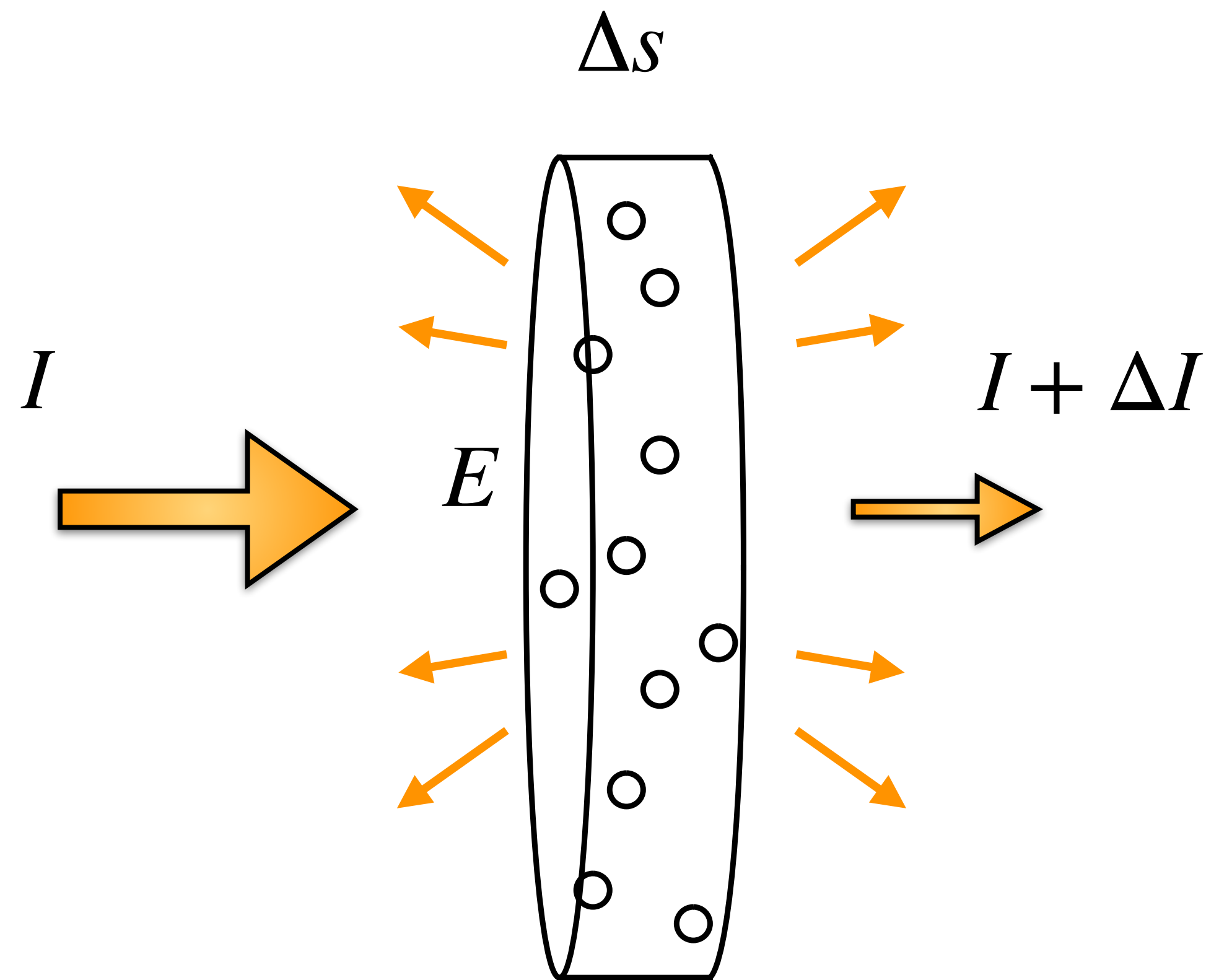
- This happens when the particles are reasonably apart of each other.
- A photon can be scattered by multiple particles if the particle concentration is high or when the material is thick.

We will then see general cases of multiple scattering + absorption.

We also assume elastic scattering, where scattered photon has the same wavelength as the incident photon.

- There are inelastic scattering, which does happen in nature but we won't consider here.

Modeling Scattering



* even though here we assume a photon is scattered by only a single particle, multiple particles can be scattering different photons.

Scattering efficiency (unitless): the probability that a photon is actually back-scattered when it hits a particle.

Concentration

Cross-section area

Distance traveled

$$I = I_0 e^{-\rho Q_s A s}$$

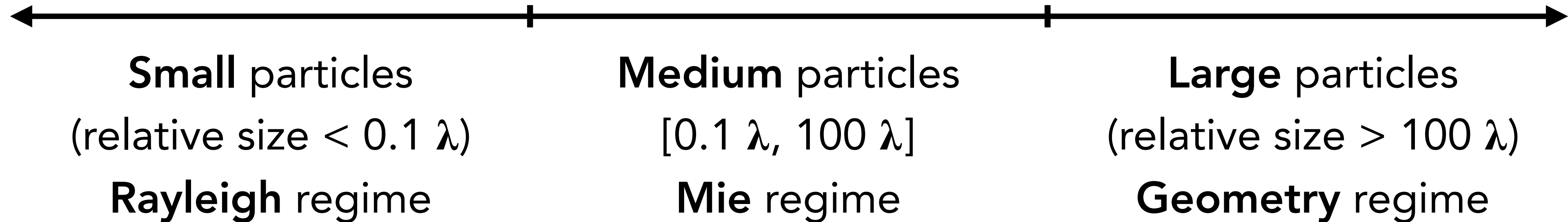
$$\mu = \rho Q_s A$$

is called **scattering coefficient**: Unit: 1/m

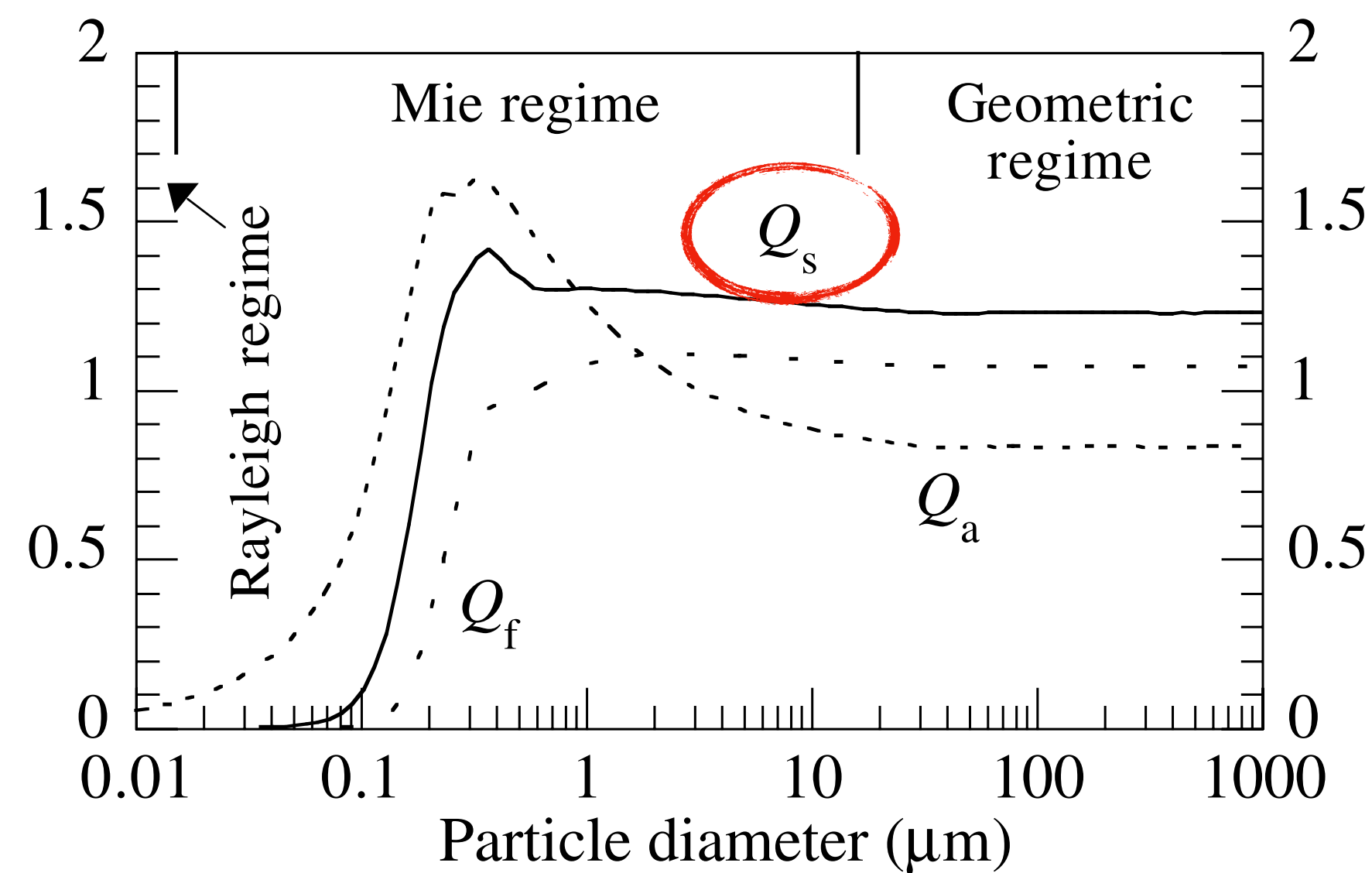
$$C_s = Q_s A$$

is called **effective scattering cross section**

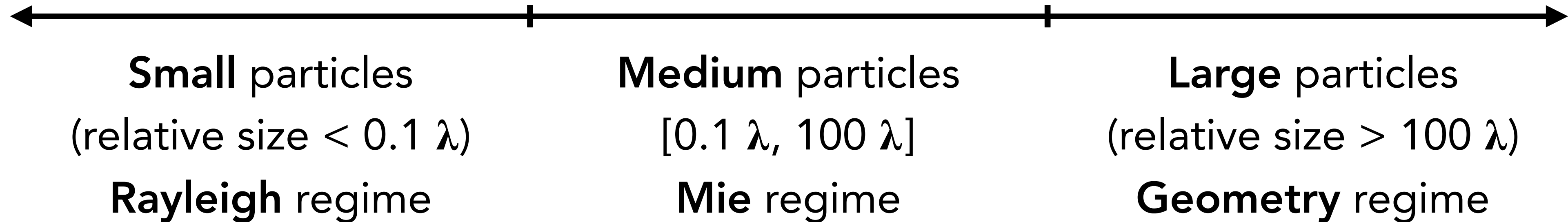
Three Scattering Regimes



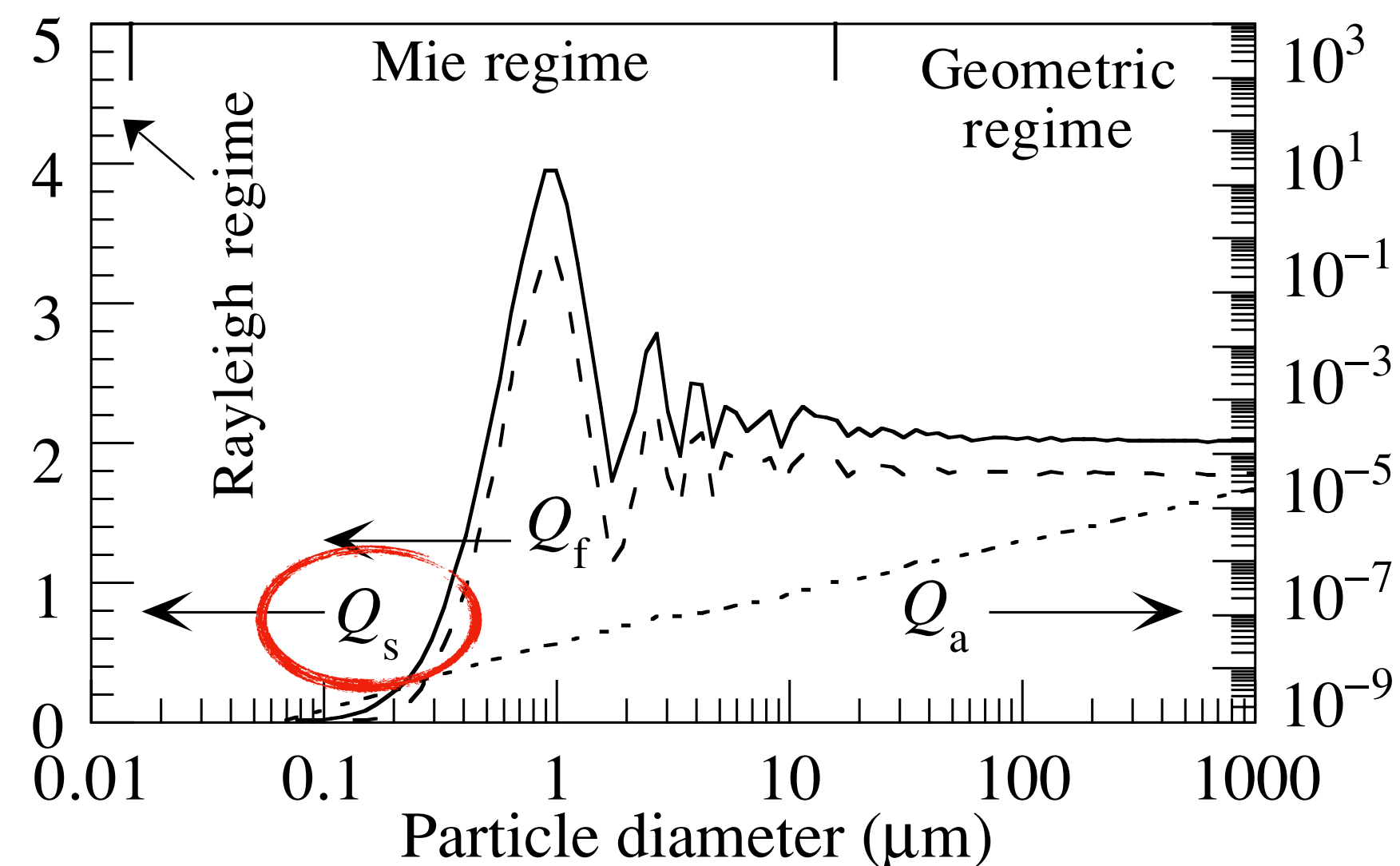
Soot particle
scattering 500
nm photons



Three Scattering Regimes



Water drop
scattering 500
nm photons



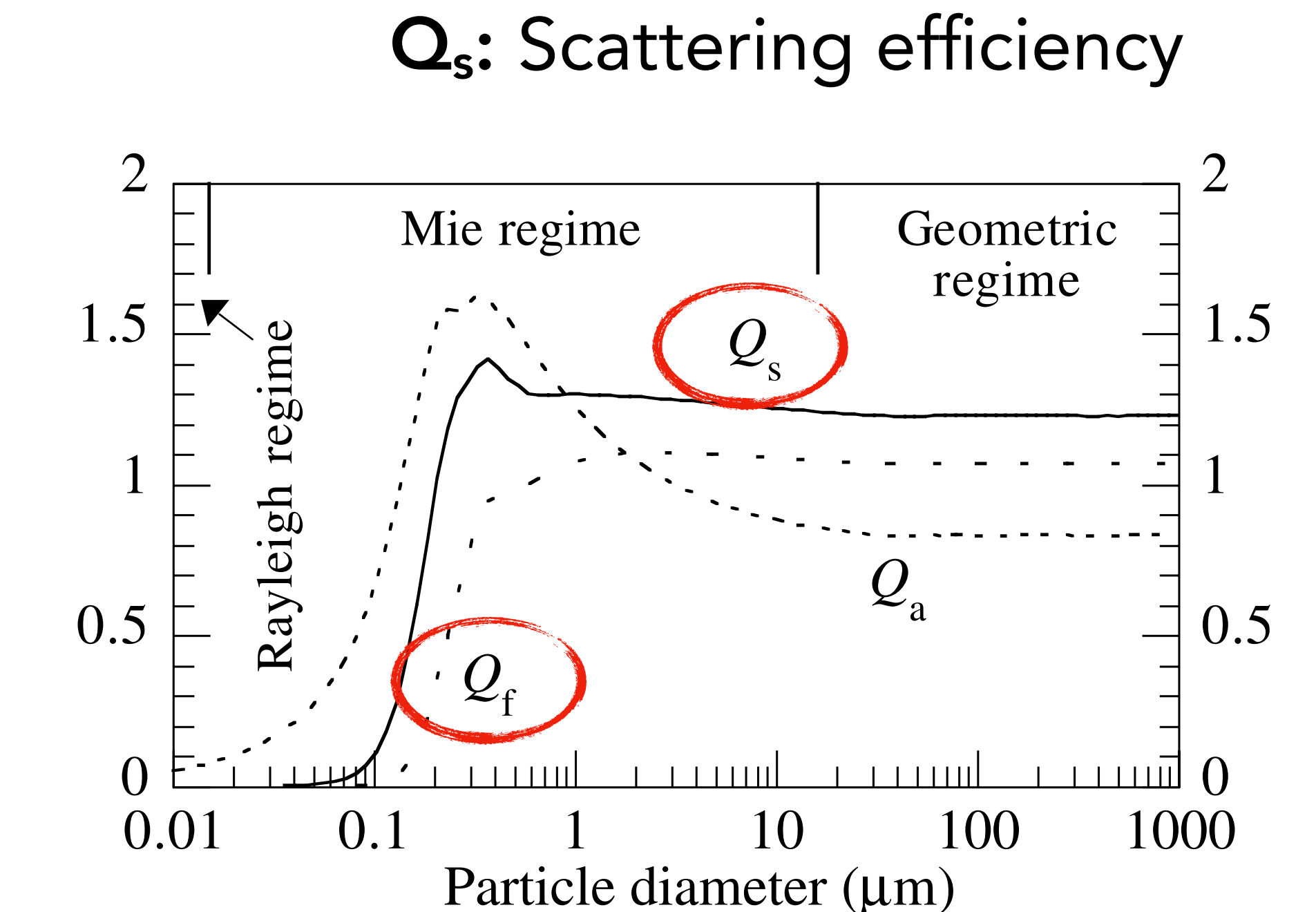
General Trends

Scattering efficiency

- Small particles scatter little; large particles scatter a lot.

Scattering direction

- Small particles scatter lights both forward and backward.
- More forward direction as particle size increases.

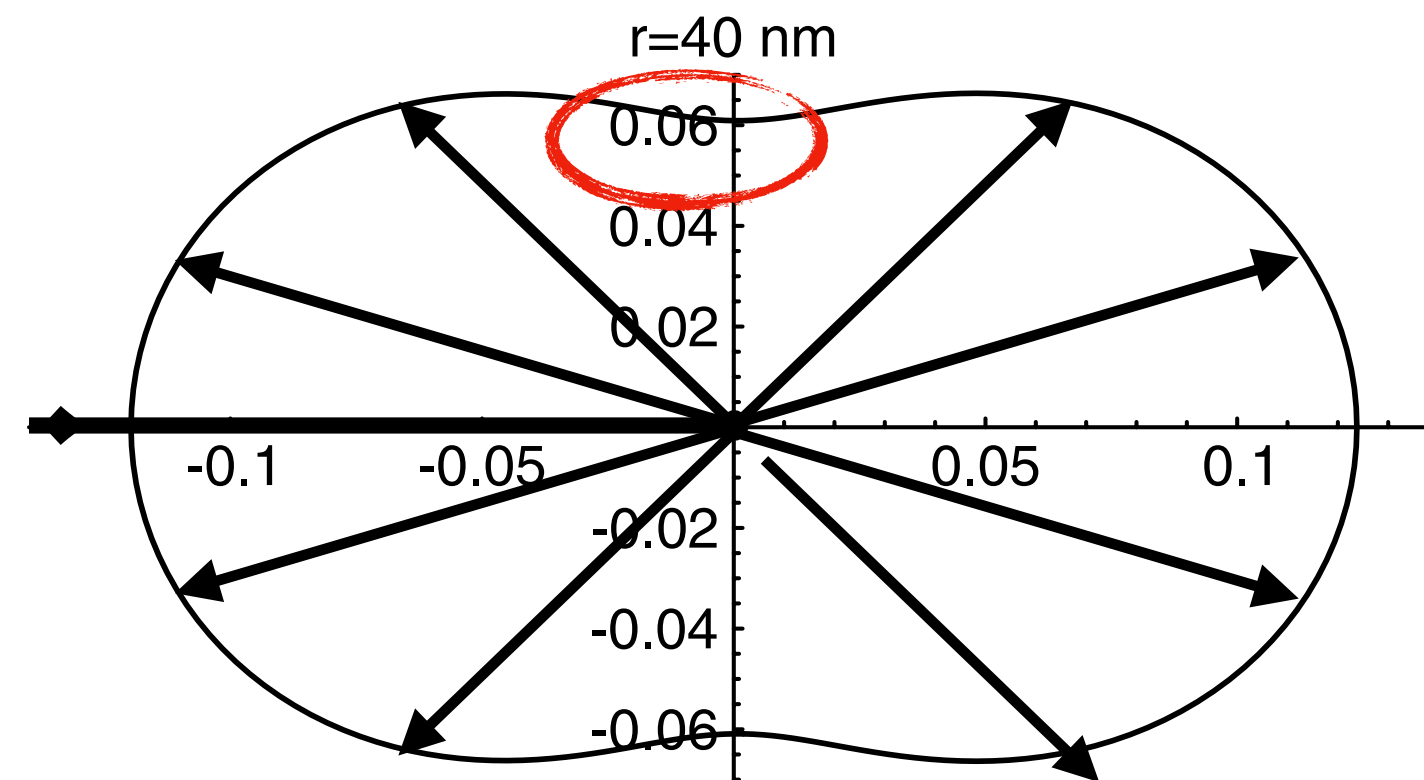


Q_f : Forward scattering efficiency. Q_f/Q_s shows how much light is scattered forward.

Scattering vs. Size

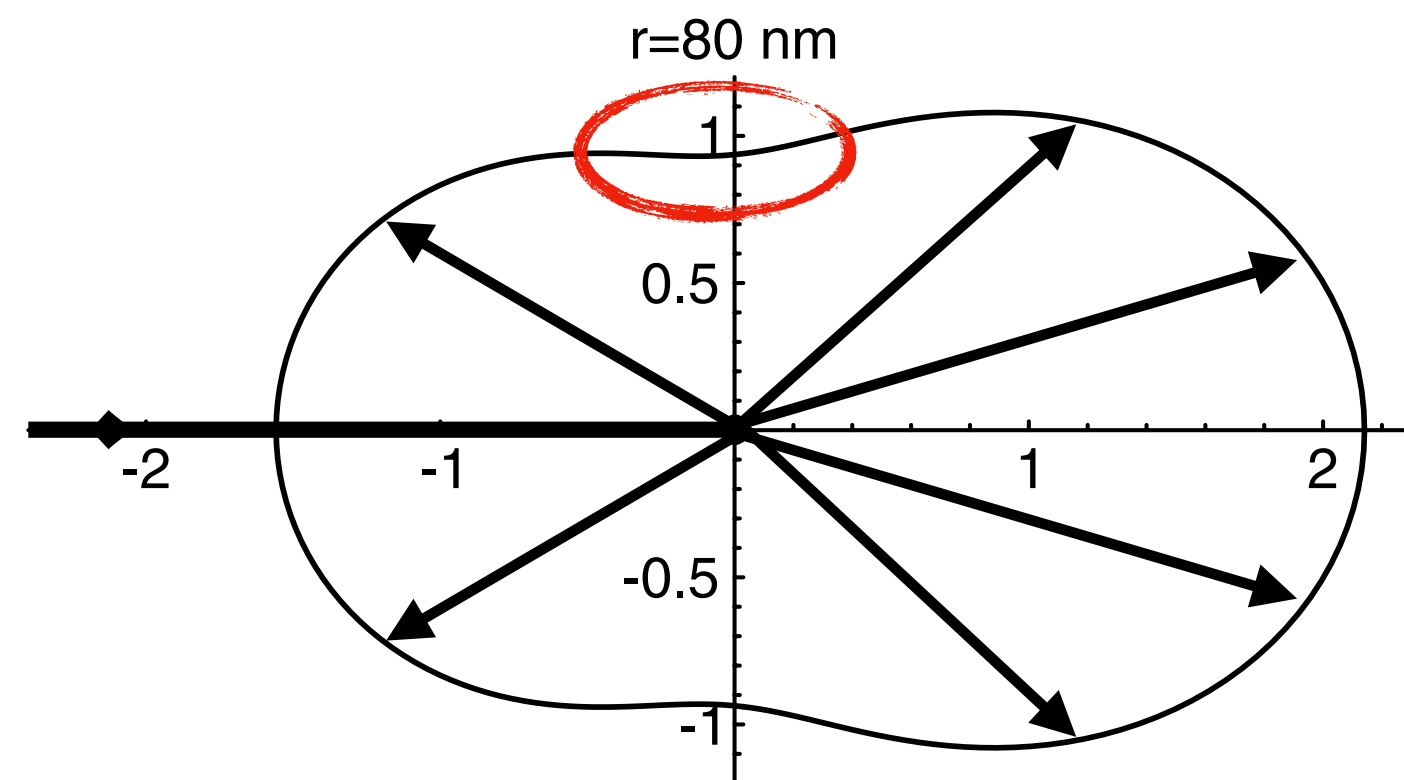
$r = 40 \text{ nm}$

$(< 0.1 \lambda)$



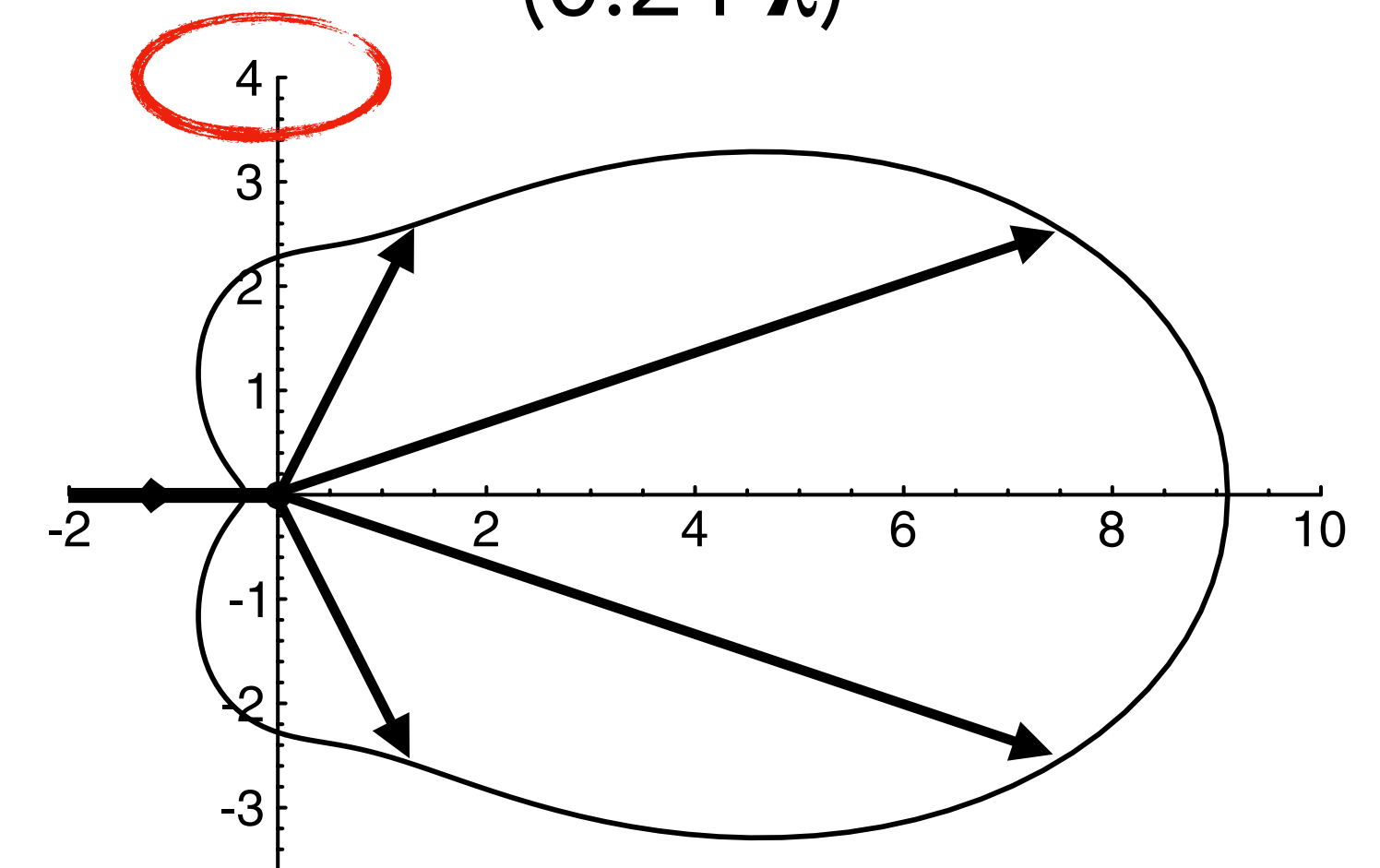
$r = 80 \text{ nm}$

(0.16λ)



$r = 120 \text{ nm}$

(0.24λ)

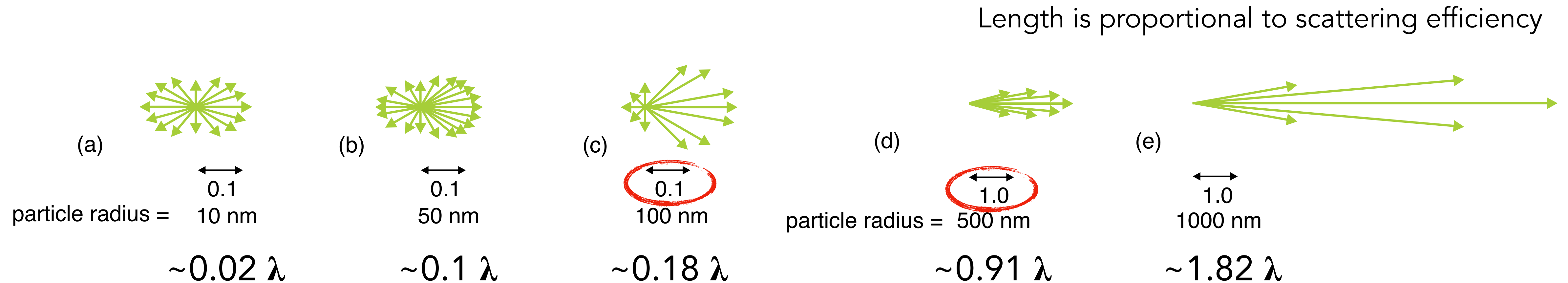


Spherical gold particle scattering **500 nm** photons.

As particle size increases

- significantly more forward scattering + higher scattering efficiency

Scattering vs. Size

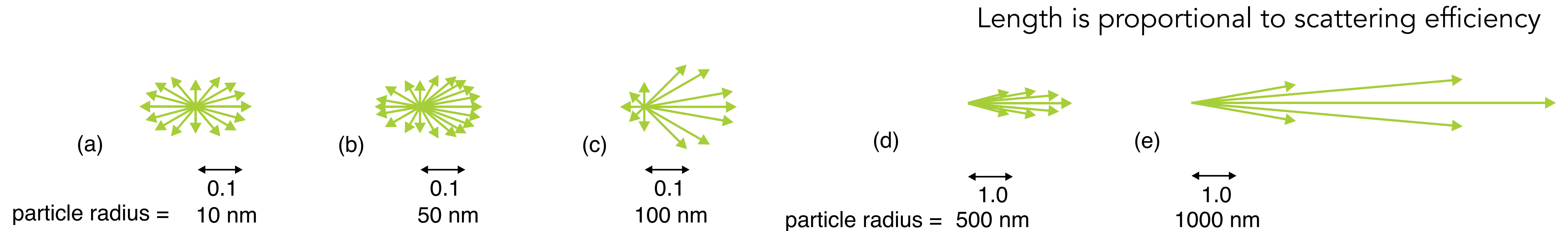


Scattering **550 nm** photons.

As particle size increases

- significantly more forward scattering + higher scattering efficiency

Scattering vs. Size



Small

Large

Small cross section area

Low scattering efficiency

Even backward and forward scattering

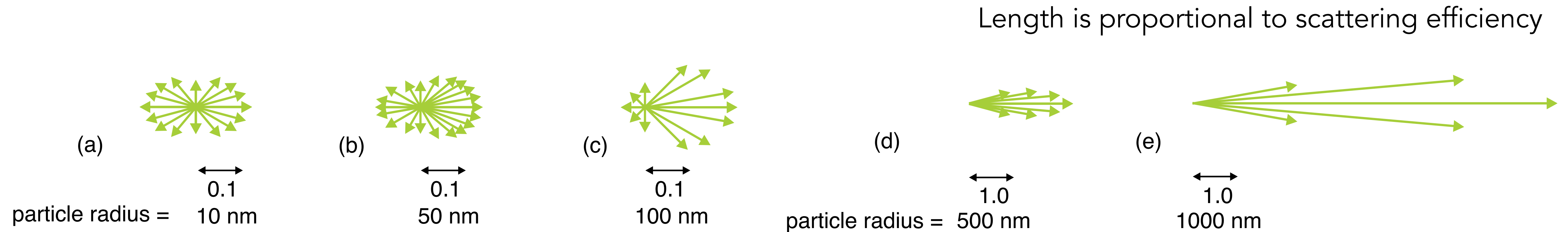
Large cross section area

High scattering efficiency

Mostly forward scattering

Increase
transparency

Scattering vs. Size



Small

Large

Small cross section area

Low scattering efficiency

Even backward and forward scattering

Increase
opacity

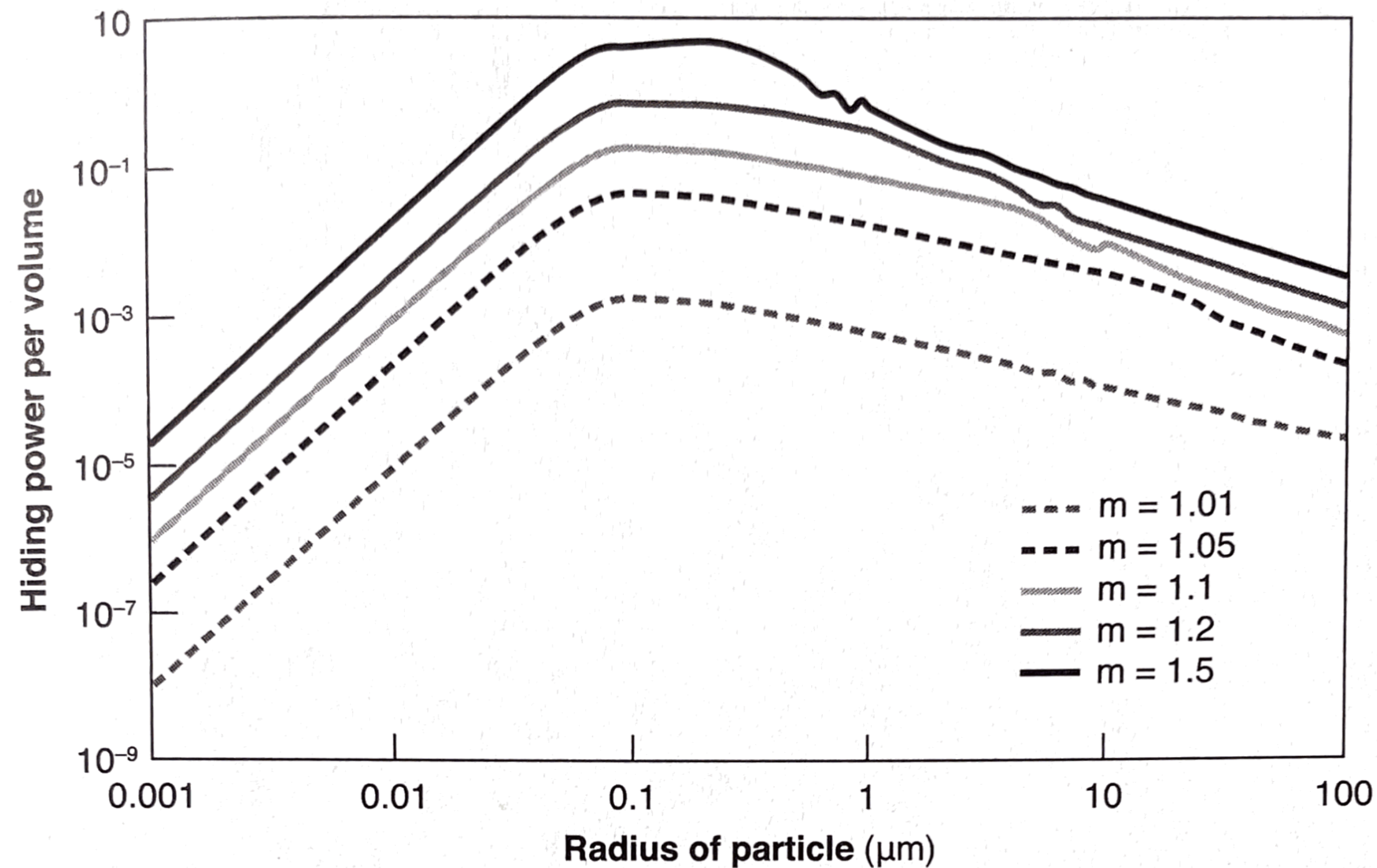
Large cross section area

High scattering efficiency

Mostly forward scattering

"Hiding Power" vs. Size

$$\frac{H}{V} = Q_s A(1 - g) \propto \frac{Q_s(1 - g)}{r}$$



There is a sweet spot in particle size that provides the highest opacity (hiding light behind it).

- Accidentally the sweet spot falls roughly within the visible range.

Hippopodius hippopus is normally transparent, but when disturbed its protein precipitate and form larger particles. It then becomes opaque.



Impact of Refractive Index

Part of scattering is caused by the refractive index mismatch between the particle and the surrounding medium.

- Water droplets surrounded by air in fog; pigments surrounded by oil in paint, etc.
- It's the relative index, ***m***, between the particle and the medium that matters.

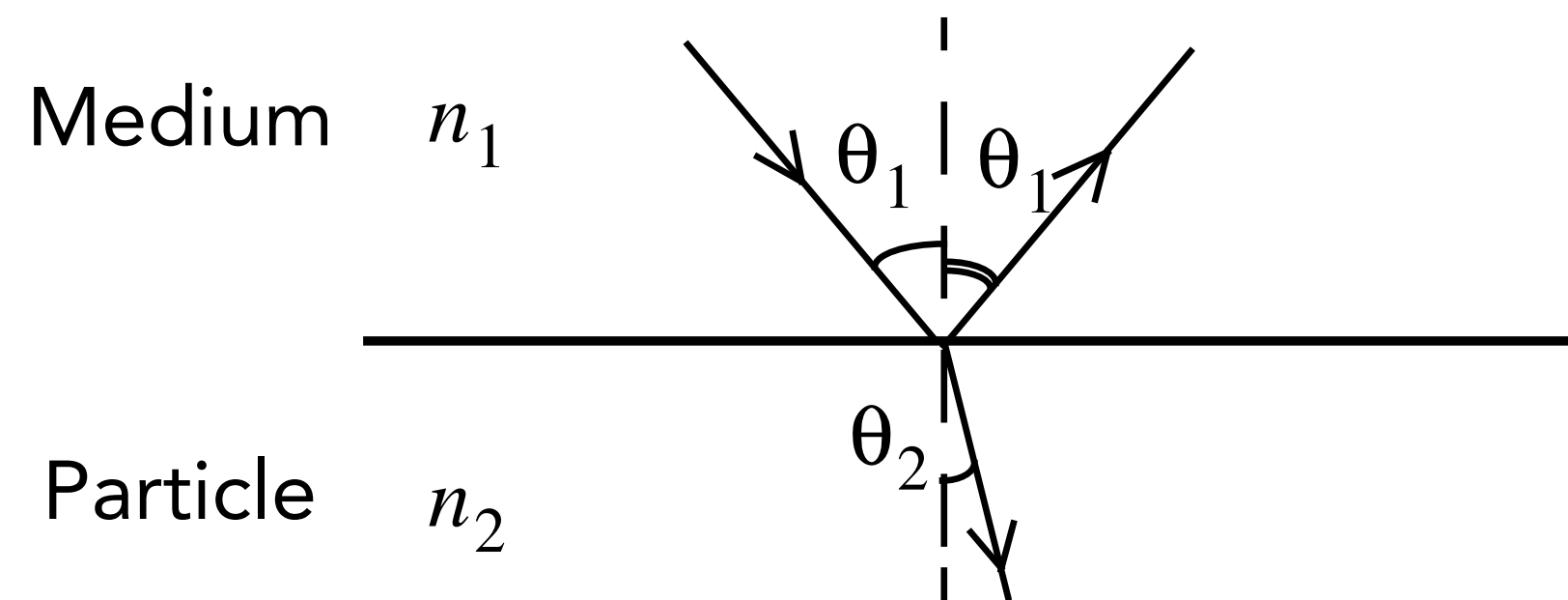
$$m = \frac{n_{particle}}{n_{medium}}$$

$$N_{air} \approx 1$$

$$N_{water} \approx 1.33$$

$$N_{glass} \approx 1.51$$

$$N_{SiO_2} \approx 1.46$$

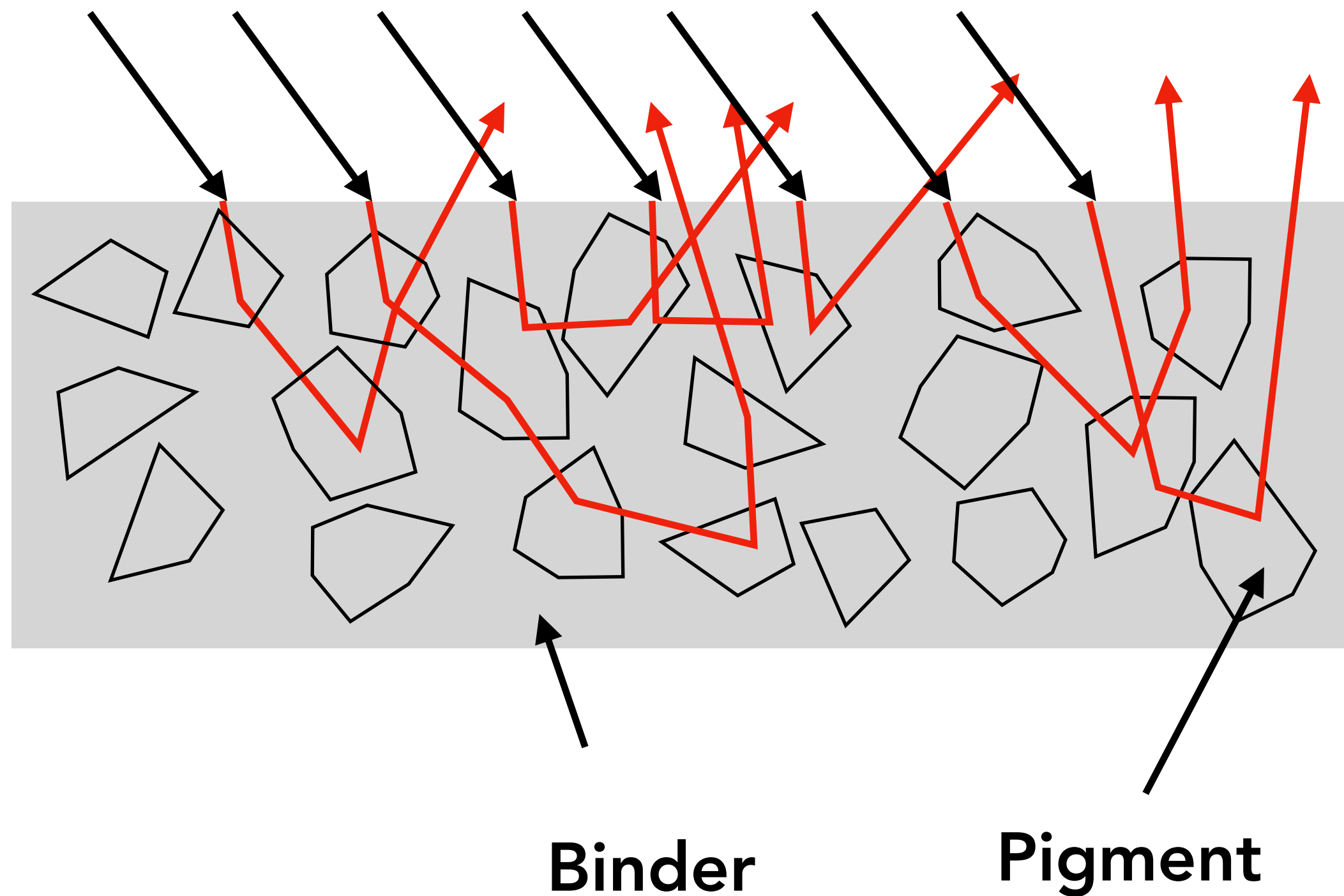


$$\frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2}$$

Snell's law

Aside: How Paints are Made

Paint $\sim$ Pigment + Binder

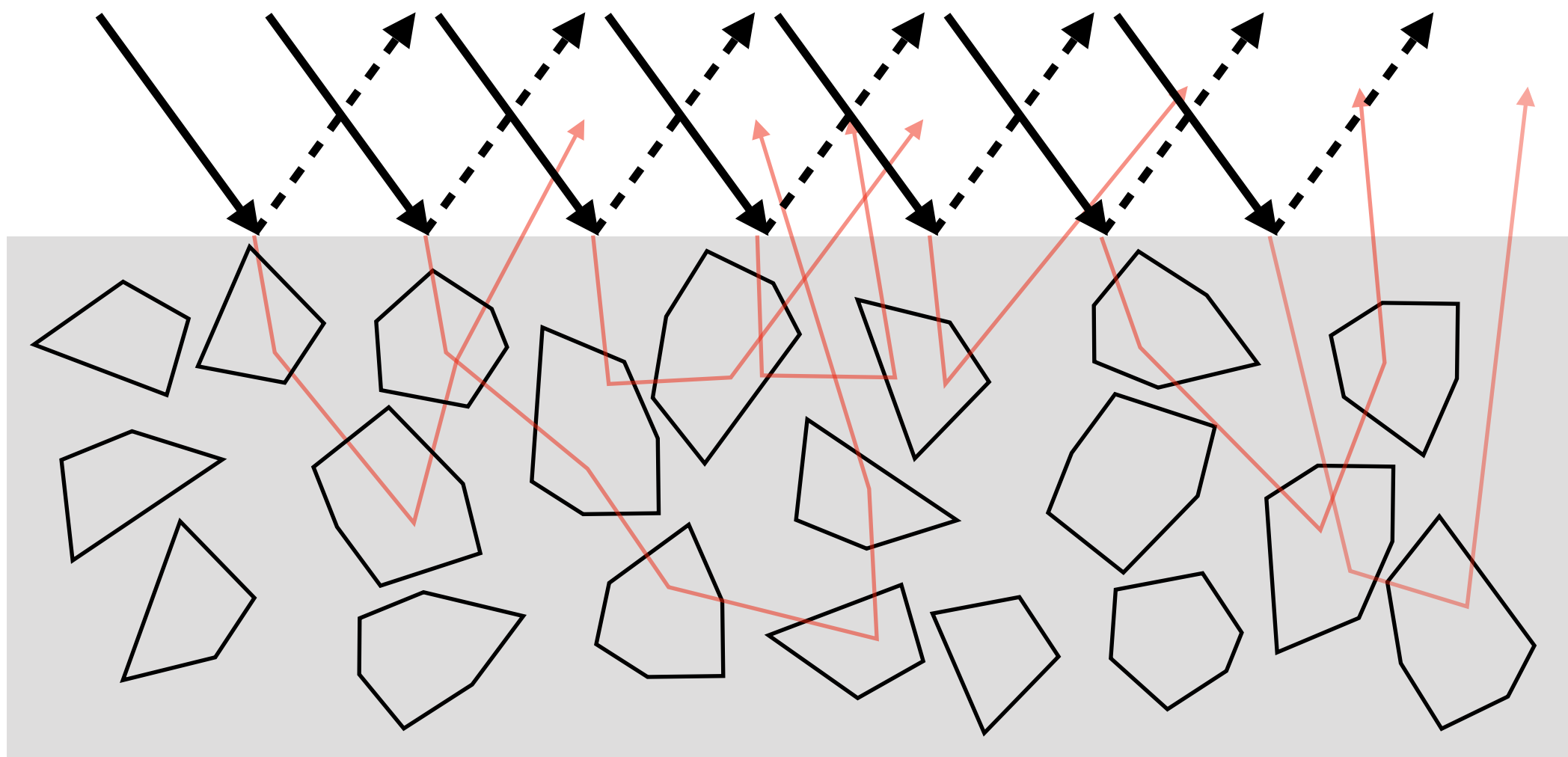


Binder (medium/vehicle) provides the adhesion needed to hold the pigments together.

Usually some additives are added:

- Glaze
- Extender to increase durability
- Glitter for shimmer

Paint $\sim$ Pigment + Binder

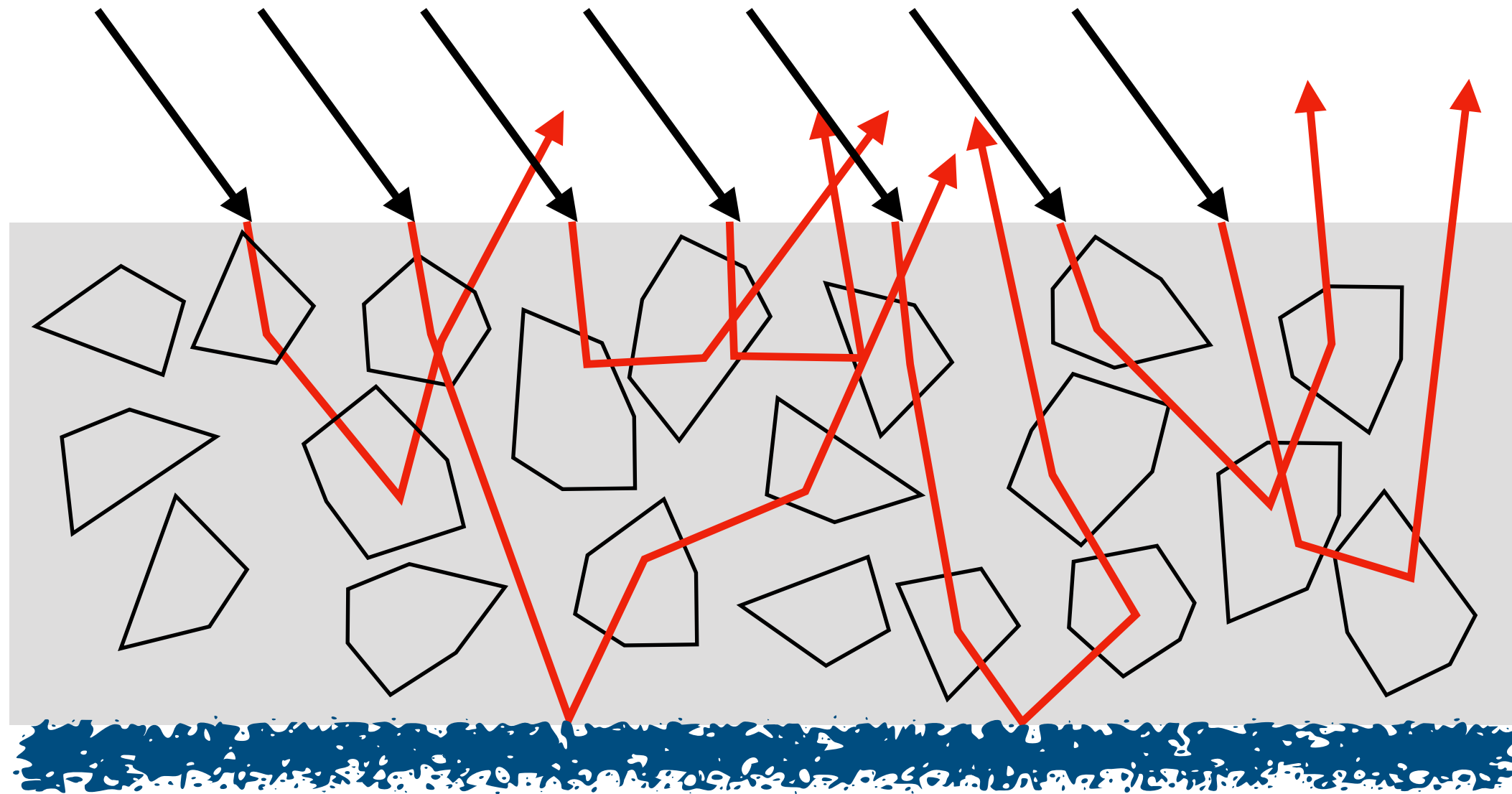


The incident lights are also reflected straight away at the air-medium surface (dashed lines)

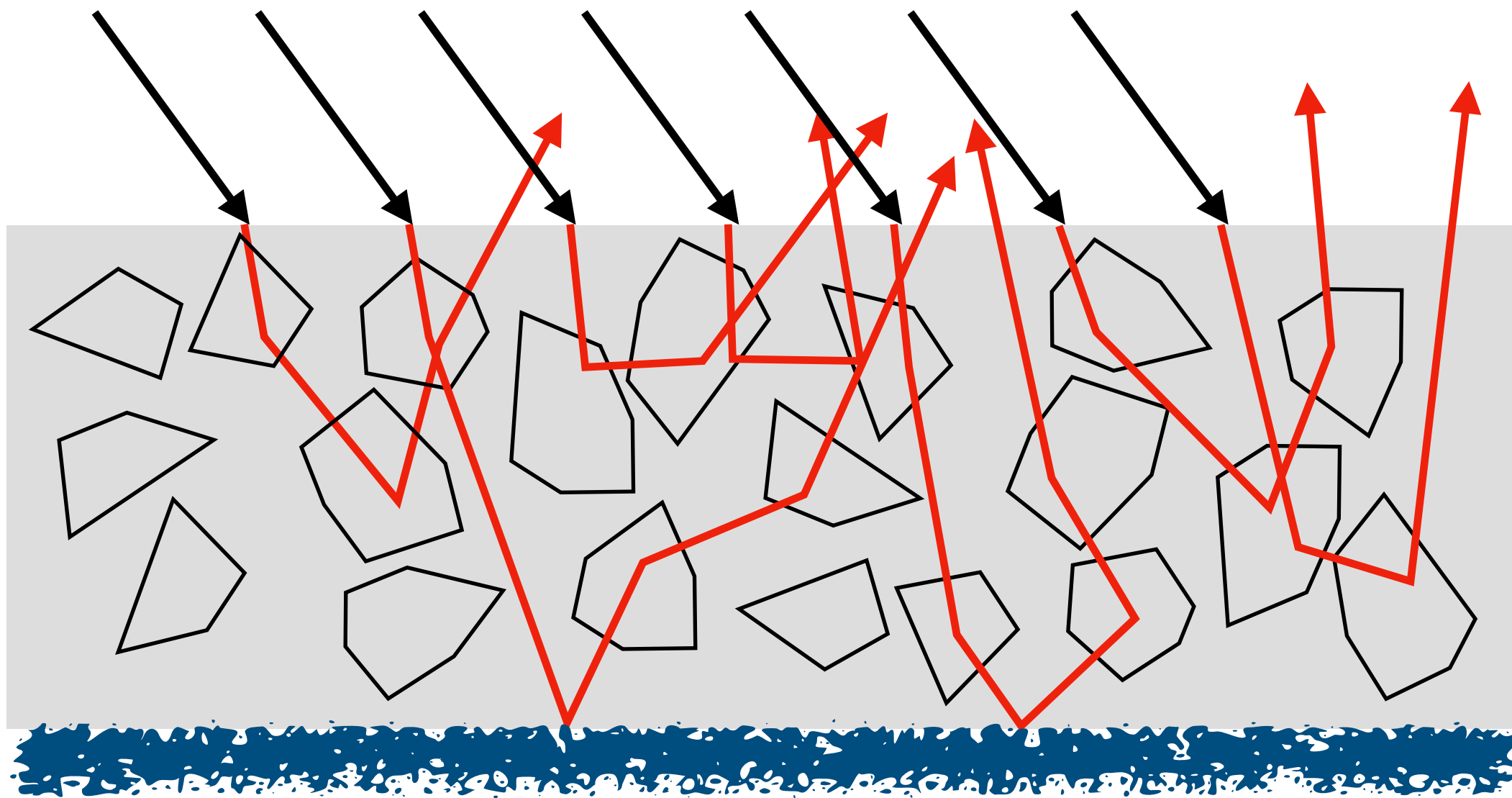
- We will ignore this for now and come back to it later

Note that surface reflection is just the macroscopic behavior of photon scattering off of the binder particles (scattering with interference).

Paint on Substrate



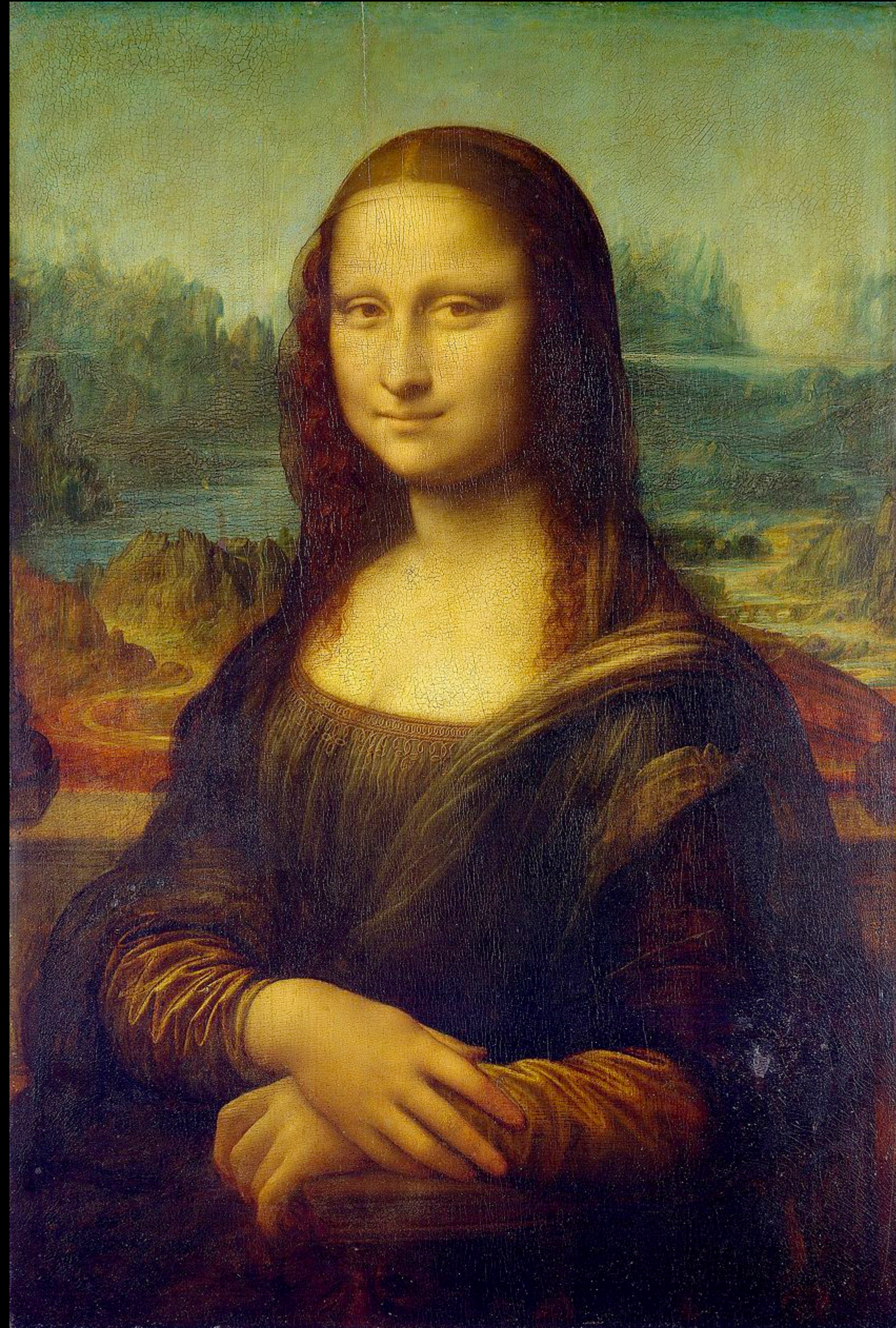
Paint on Substrate



The paint is applied to a substrate, e.g., a canvas or paper. The substrate is nothing more than another layer of scattering/absorbing material.

- We will show how layered material is modeled later.

Mona Lisa, Leonardo Da Vinci, 1506. The Louvre.

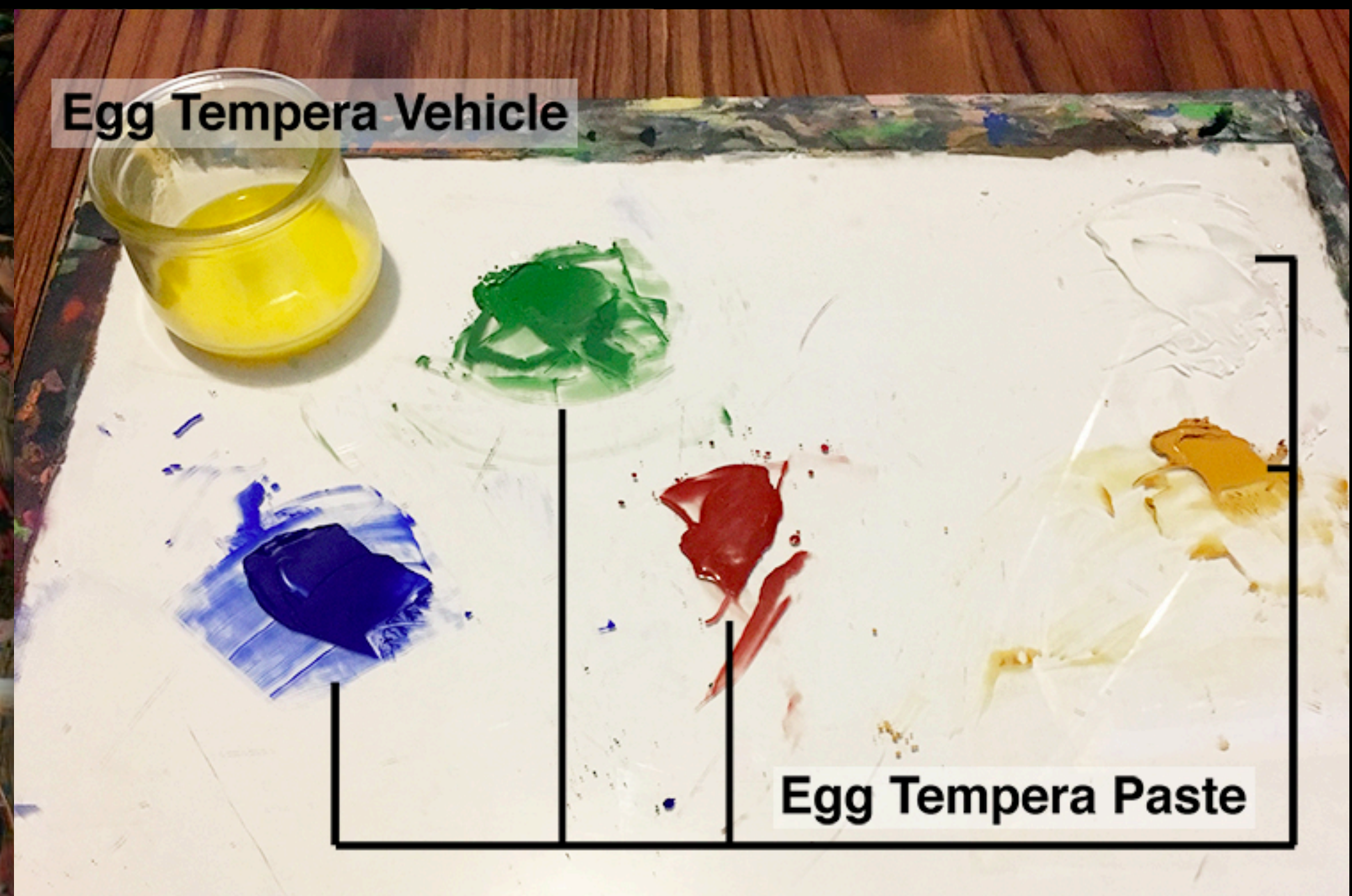
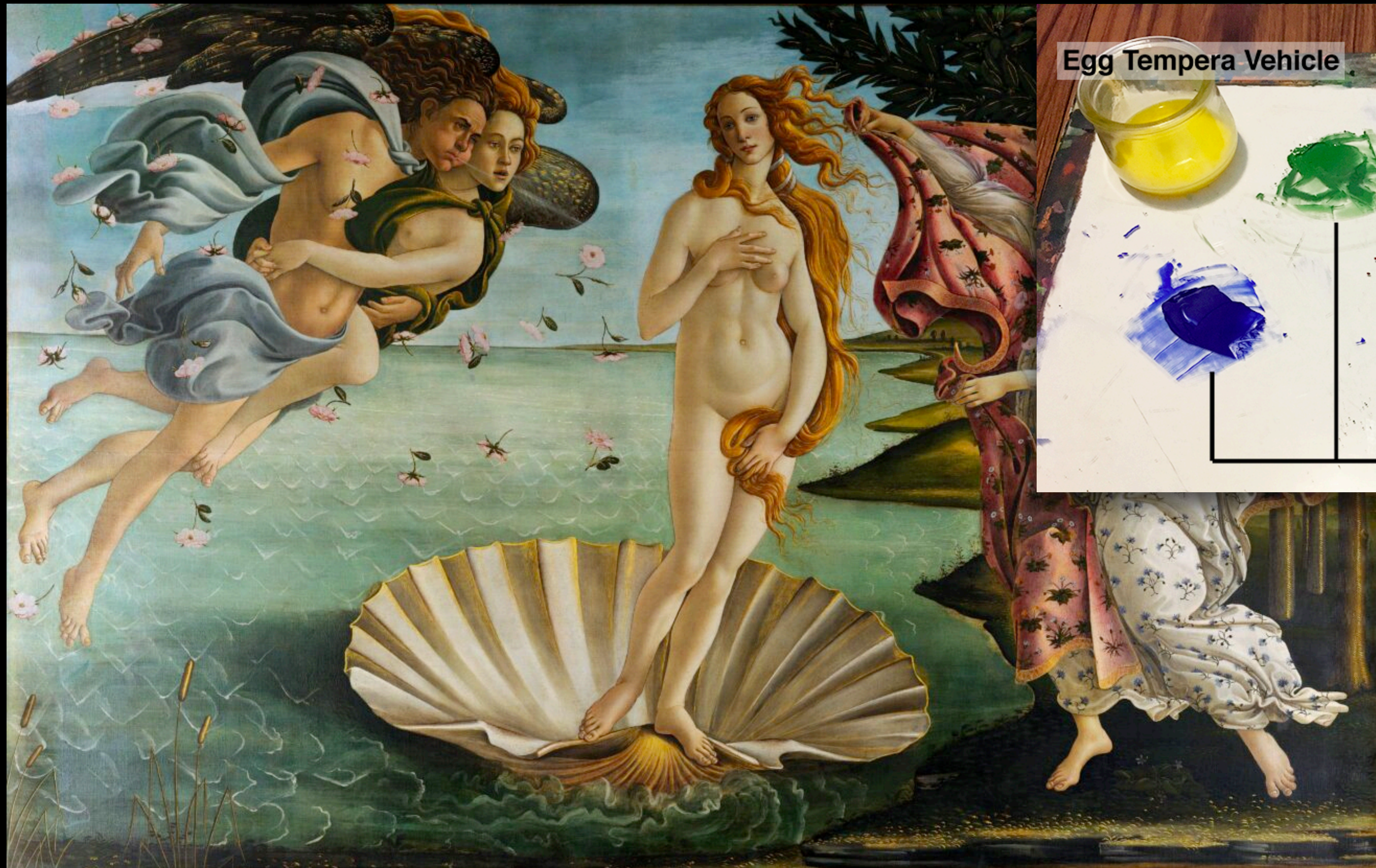


The Starry Night



Oil painting.
Binder is (linseed) oil.

The Birth of Venus, Sandro Botticelli, c 1485. Uffizi Galley, Florence.

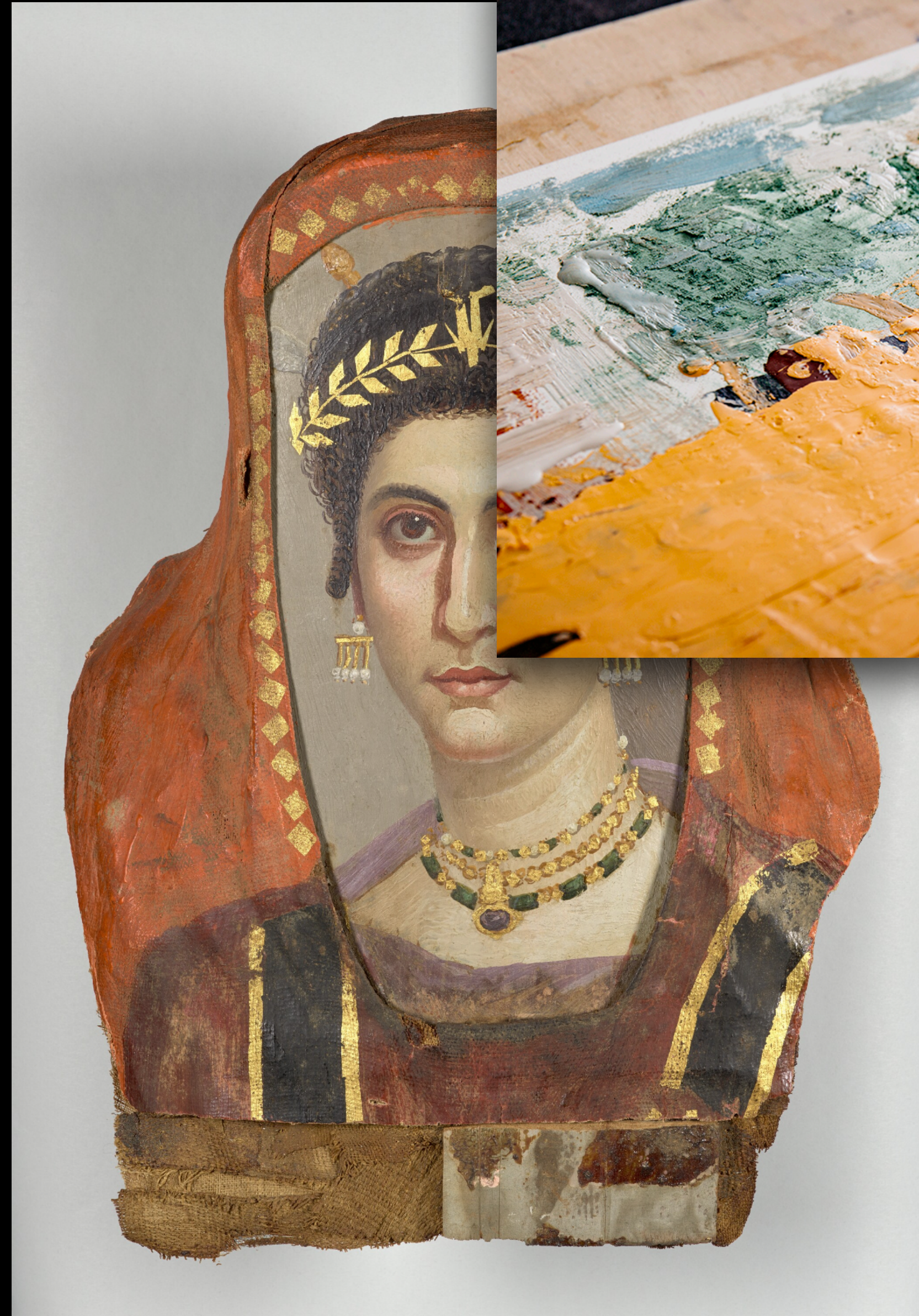


Tempera painting.
Binder is egg yolk.

Portrait of a Young Woman in Red,
A.D. 90–120. The Met.



Mummy Portrait of a Woman, A.D. 100. Getty.

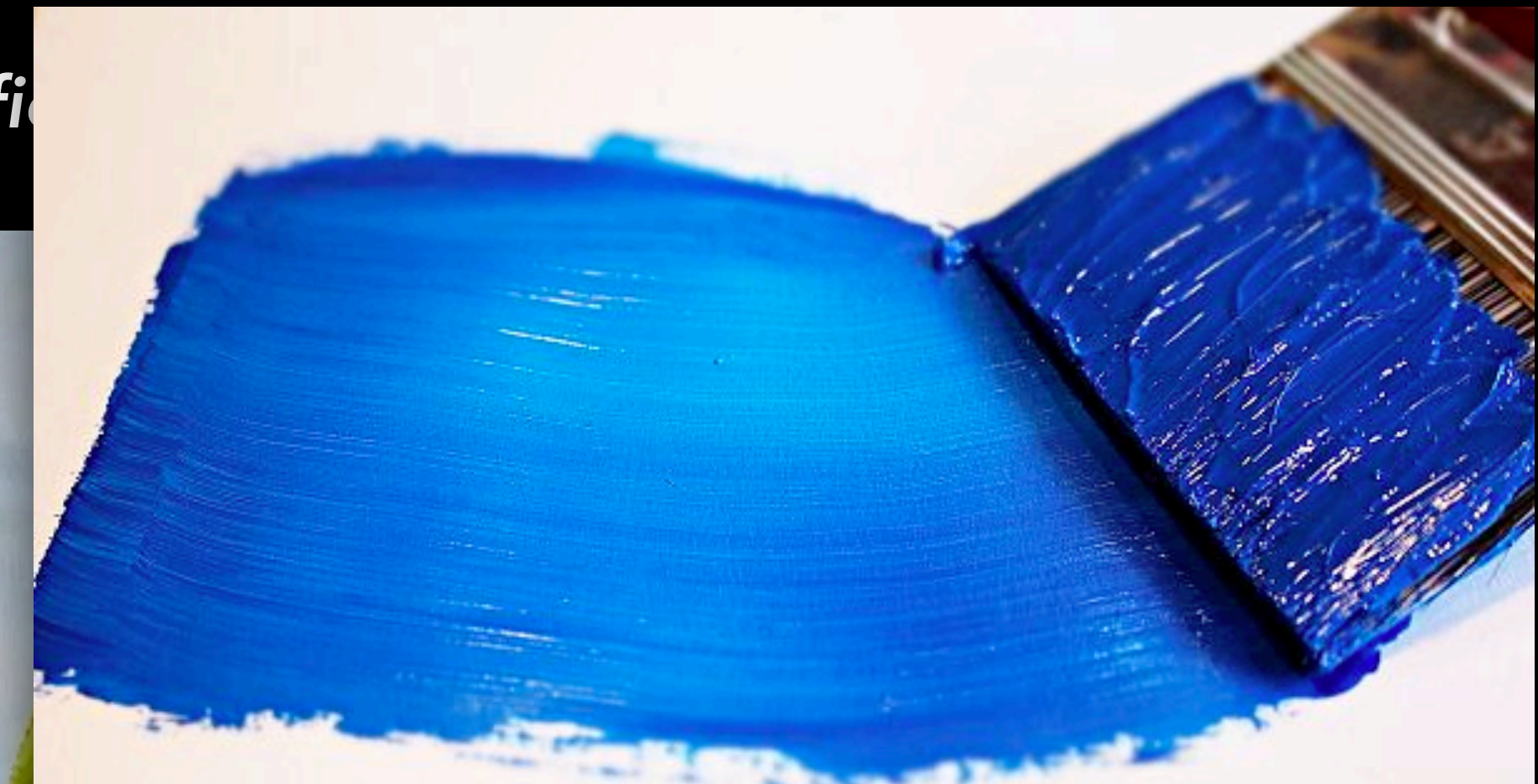


Encaustic painting.
Binder is (bees)wax.

Keep Moving Forward, Darien Bogart



Artificial



Acrylic painting.
Binder is "milk" + water.



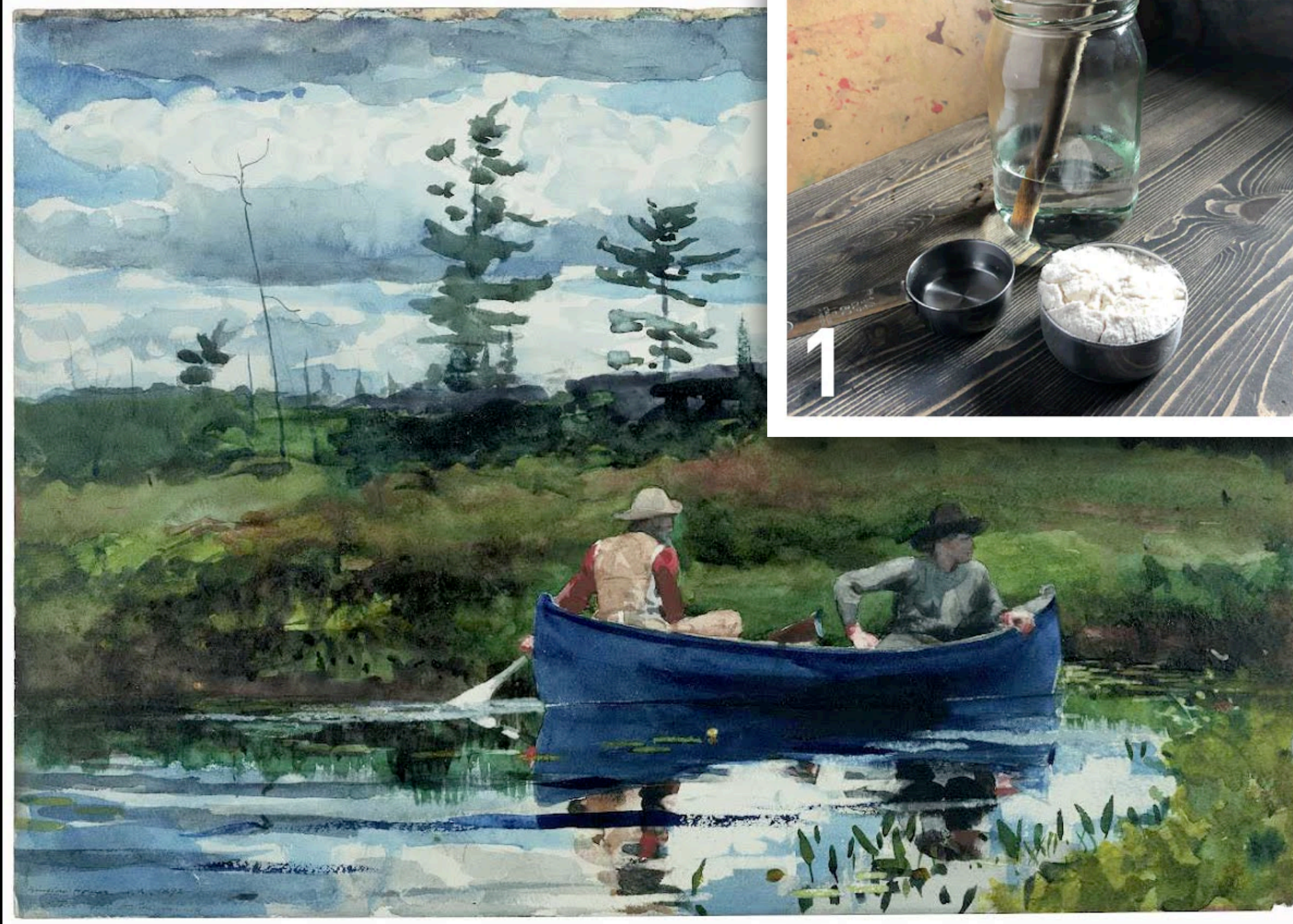
Stephen Quiller



Casein painting.
Binder is "milk" + water.



The Blue Boat, Winslow Homer, 1892. Museum



**"Watercolor" painting.
Binder is Gum Arabic + water.**

Pigment Size

average pigment particle size		
<i>microns</i>	<i>meters</i>	<i>representative pigments</i>
1000µm	10 ⁻³	= 1 millimeter
500µm		.
100µm	10 ⁻⁴	<i>coarse historical mineral pigments</i> modern luster & iridescent pigments
50µm		smallest particles visible without magnification cobalt violet manganese blue
10µm	10 ⁻⁵	cobalt green cobalt turquoise cerulean blue manganese violet black iron oxides
5µm		ultramarine blue (RS) viridian cobalt blue violet (brown) iron oxides yellow iron oxides

1µm	10 ⁻⁶	ultramarine blue (GS) red iron oxides cadmium red cadmium orange <i>semiopaque synthetic organics</i> diarylides pyrroles naphthols perinone orange
0.5µm		= 500 nanometers = wavelength of "blue green" light chromium oxide green cadmium yellow bismuth yellow titanium white transparent red iron oxides transparent yellow iron oxides <i>semitransparent synthetic organics</i> arylides benzimidazolones dioxazines perylenes
0.1µm	10 ⁻⁷	zinc white iron [prussian] blue <i>transparent synthetic organics</i> quinacridones phthalocyanines
0.05µm		carbon black
0.01µm	10 ⁻⁸	
0.0015µm	10 ⁻⁹	<i>glucose (corn syrup) molecule</i>
0.0003µm	10 ⁻¹⁰	<i>water molecule</i>

Disperse Pigments



Pigments are very tiny but usually agglomerate, which is the powders we buy.

We need to separate the pigment particles from the agglomerates and disperse them into the binder.





A Dutch Studio in the Sixteenth Century

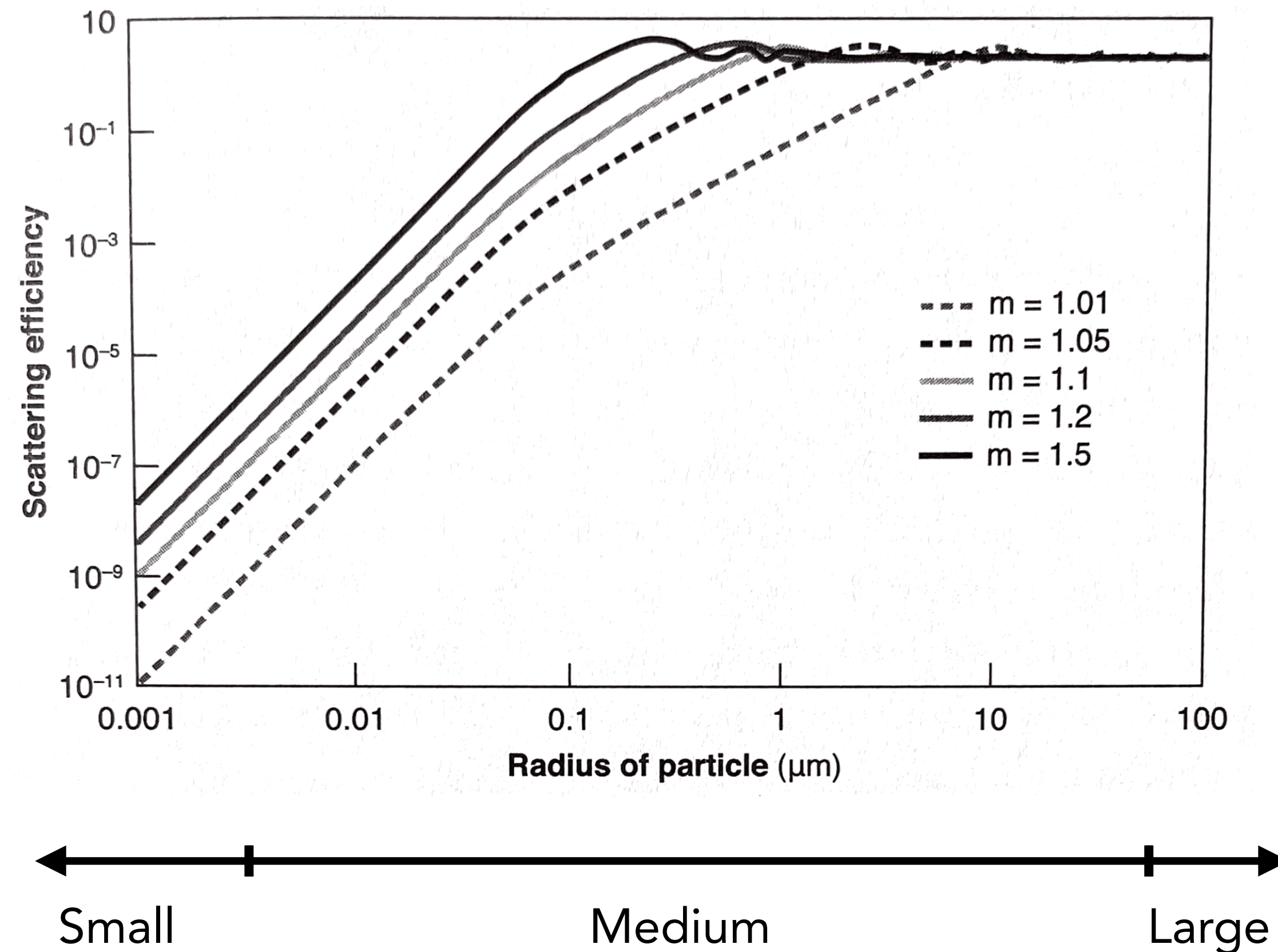


Back From Aside

Refractive Index vs. Scattering Efficiency

$$m = \frac{n_{particle}}{n_{medium}}$$

wavelength = 500 nm



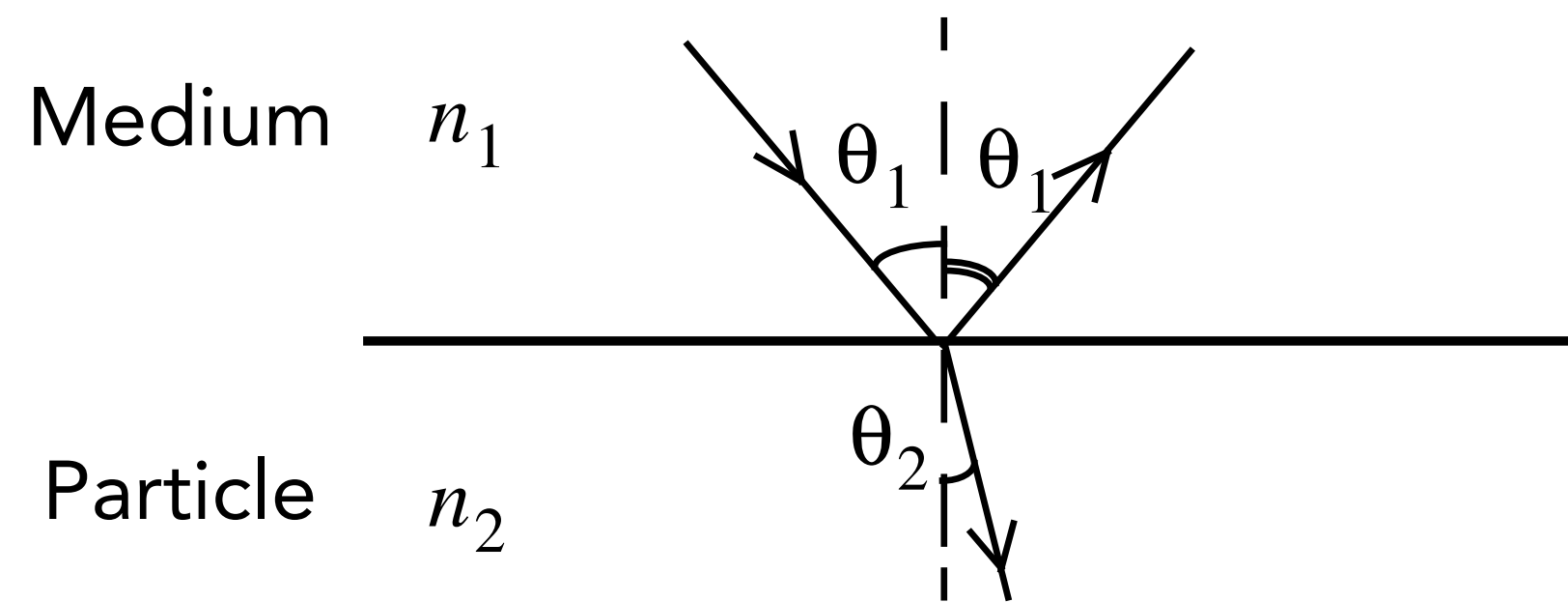
Observations:

- Large relative refractive index (m) increases scattering efficiency
- When m is close to 1, very little scattering until closer to the geometric (large) regime
- The scattering efficiency in the geometric regime is independent of refractive index m ,

Refractive Index vs. Scattering Direction

As m approaches 1 (i.e., the refractive indices of the medium and particle become more similar), forward scattering is more significant.

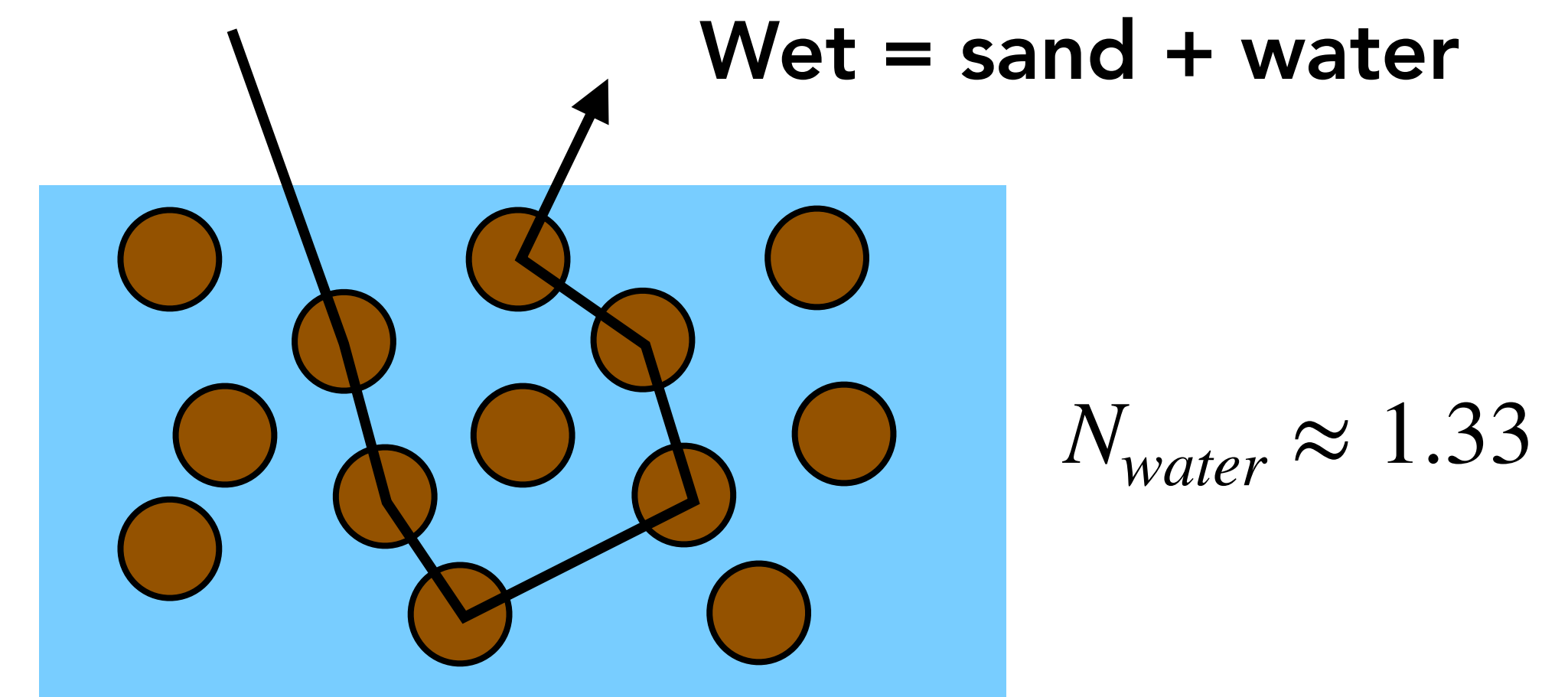
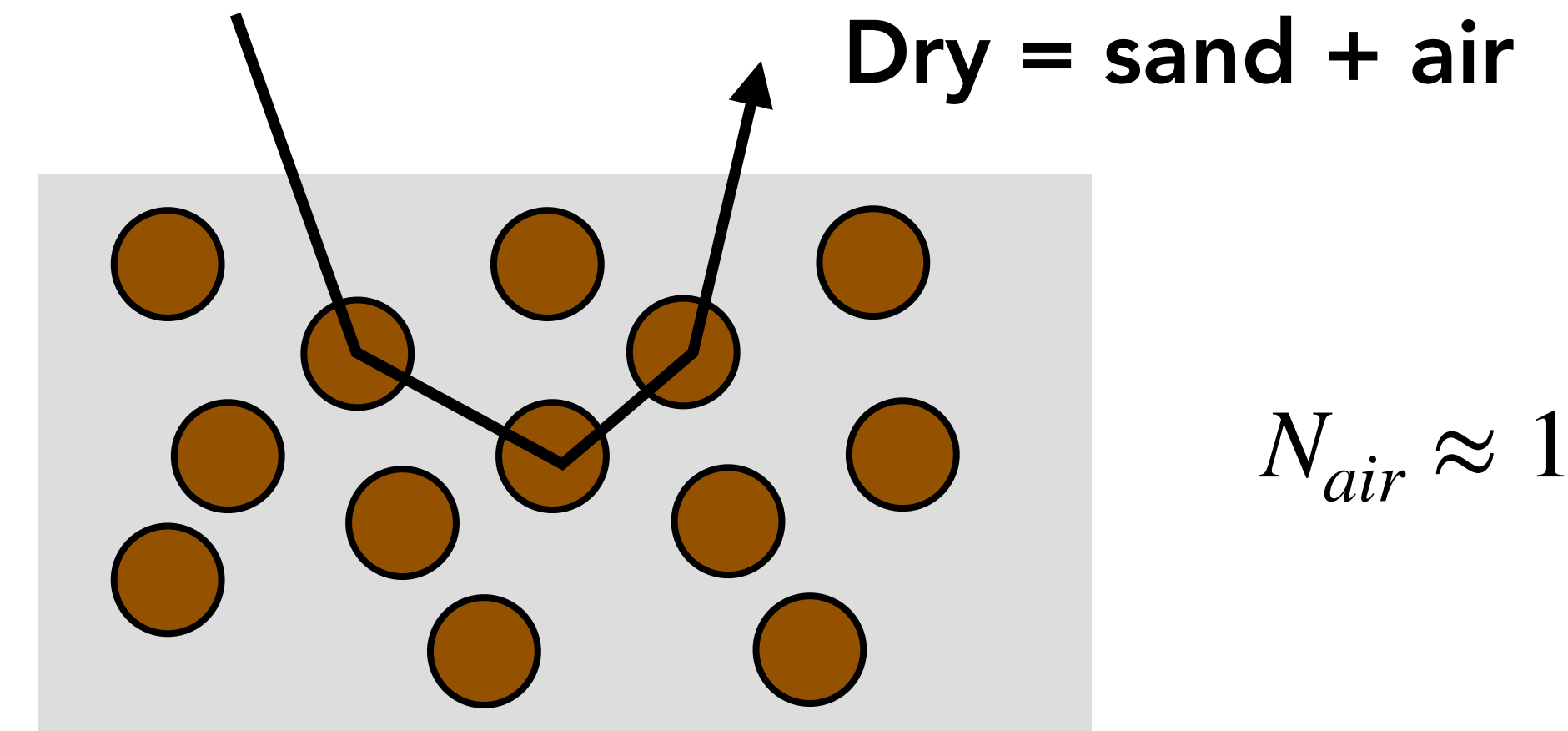
- Extreme case is when the medium and the particle are the same material, then scattering is completely forward ($\theta_1 = \theta_2$)!



$$\frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2}$$

Why Are Wet Things Darker?

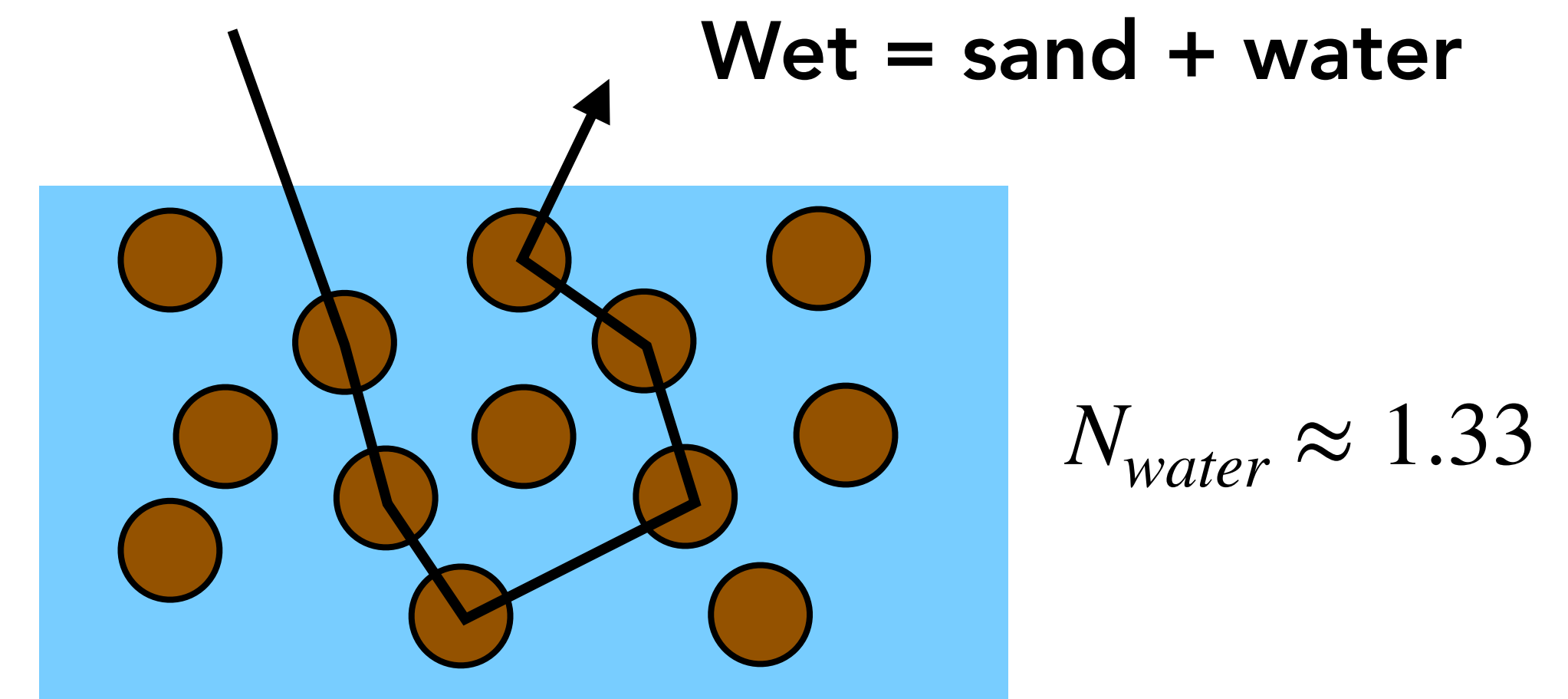
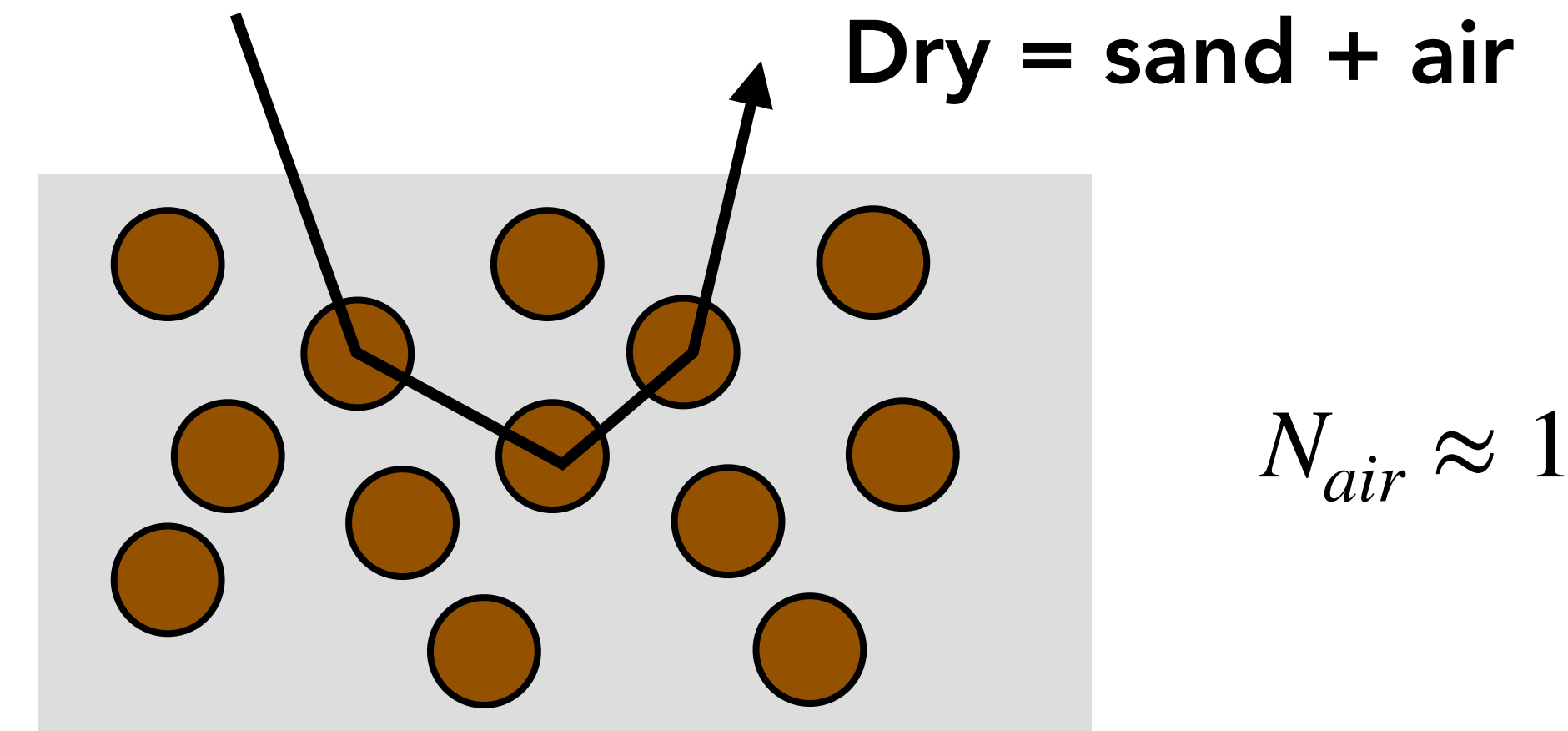
$$N_{SiO_2} \approx 1.46$$



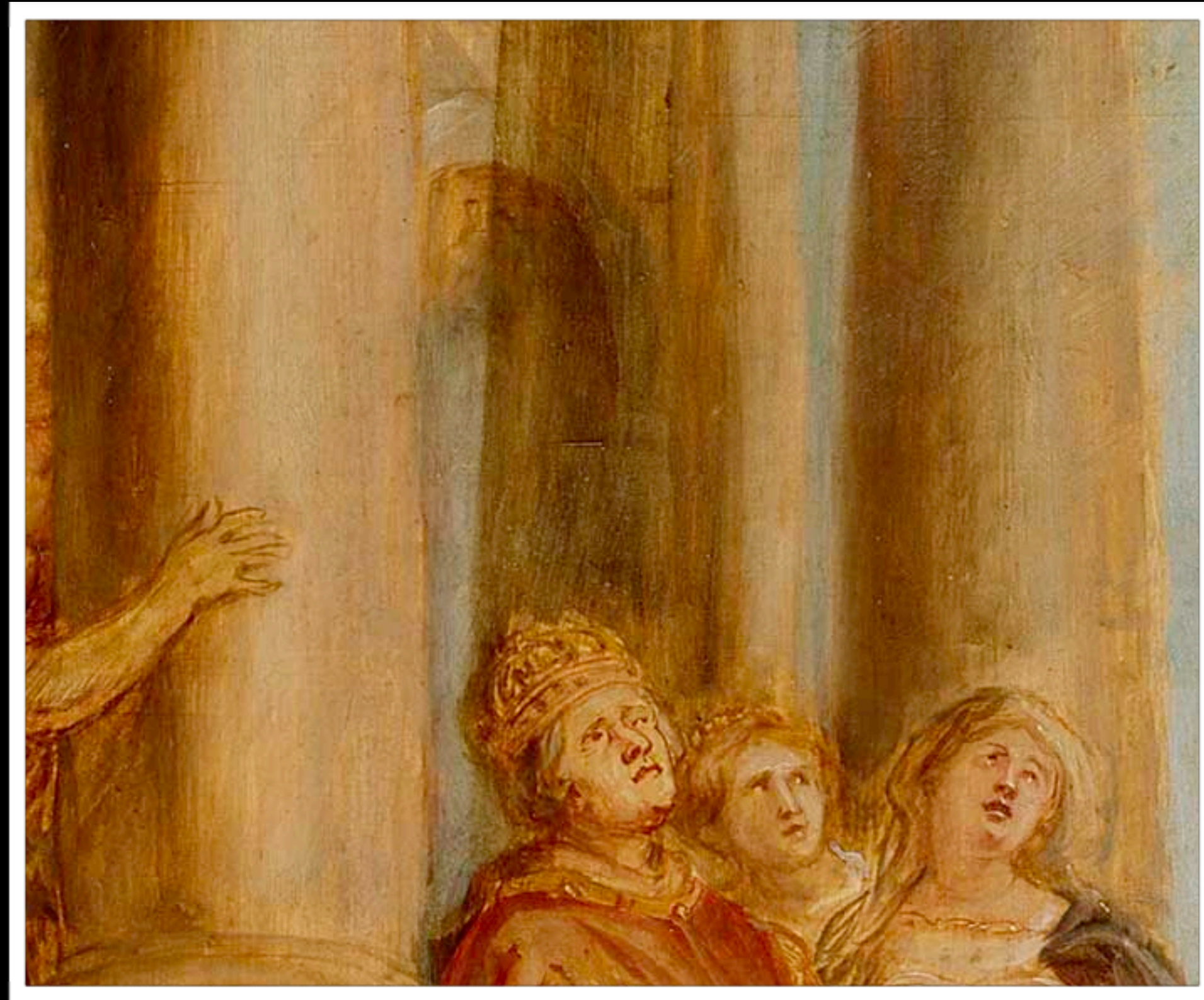
Why Are Wet Things Darker?

The RI of sand is much closer to water than to air. So light scattered forward more and travels longer distance and has more opportunities to be absorbed.

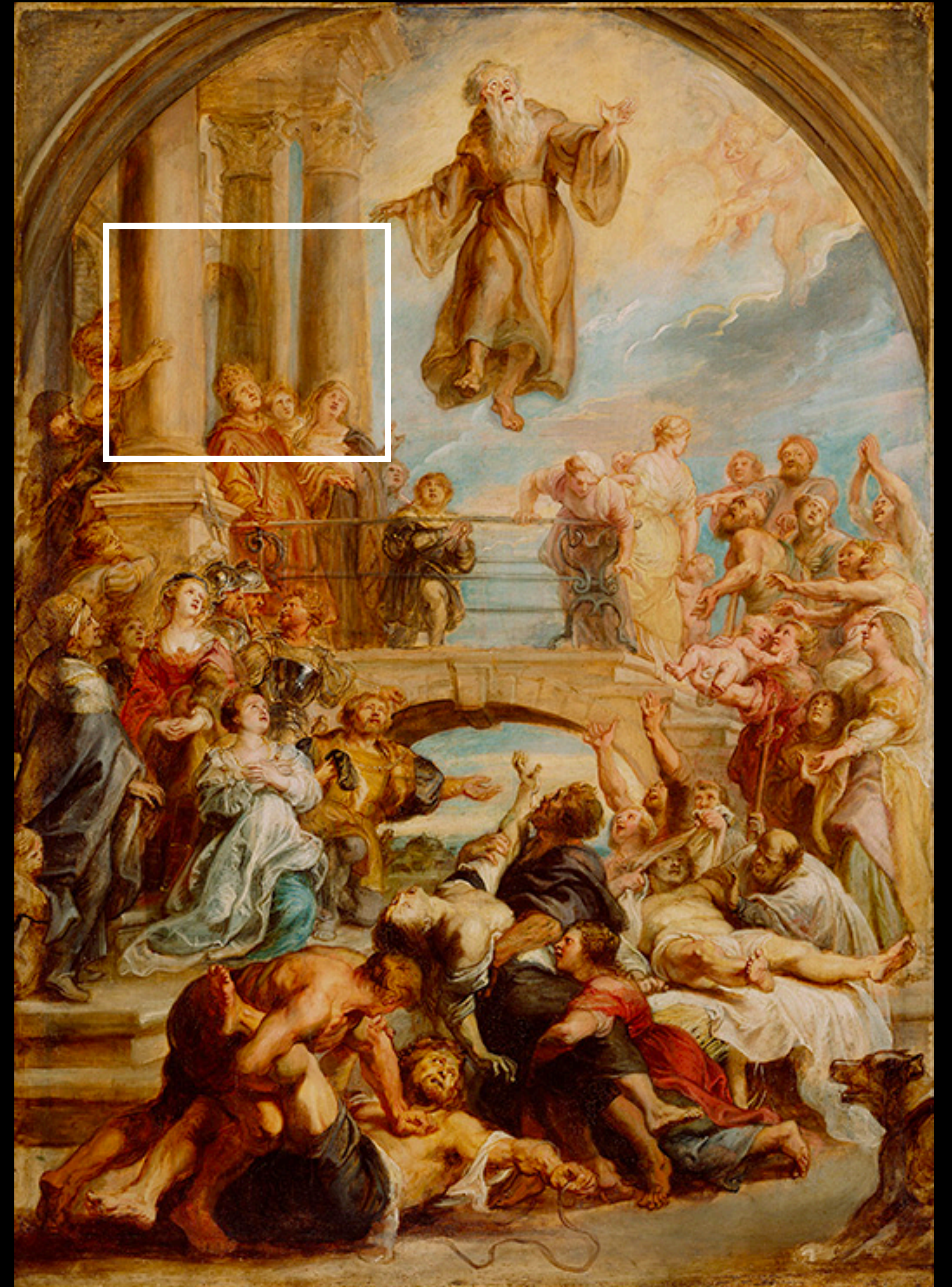
Note that scattering by sand is in the geometric regime (typical sand size is $[62.5 \mu\text{m}, 2 \text{ mm}]$), so scattering efficiency is high regardless of RI. The question is where photons scatter.



The Miracles of Saint Francis of Paola, P. P.
Rubens, 1627–8. J. Paul Getty Museum.



The refractive index of the linseed oil in the paint increases over time and matches that of the pigment. The paint layer becomes transparent, revealing the pentimento underneath.



Aside: Pentimento

The presence/emergence of earlier paints that have been painted over.

- Painter changing mind (pentimento is from “pentirsi” in Italian, which means repentance)
- Underdrawing

Can be identified using imaging techniques (e.g., X-ray), or from paint composition changes (previous slide).



The Old Guitarist, Pablo Picasso,
1903-1904. The Art Institute of Chicago.



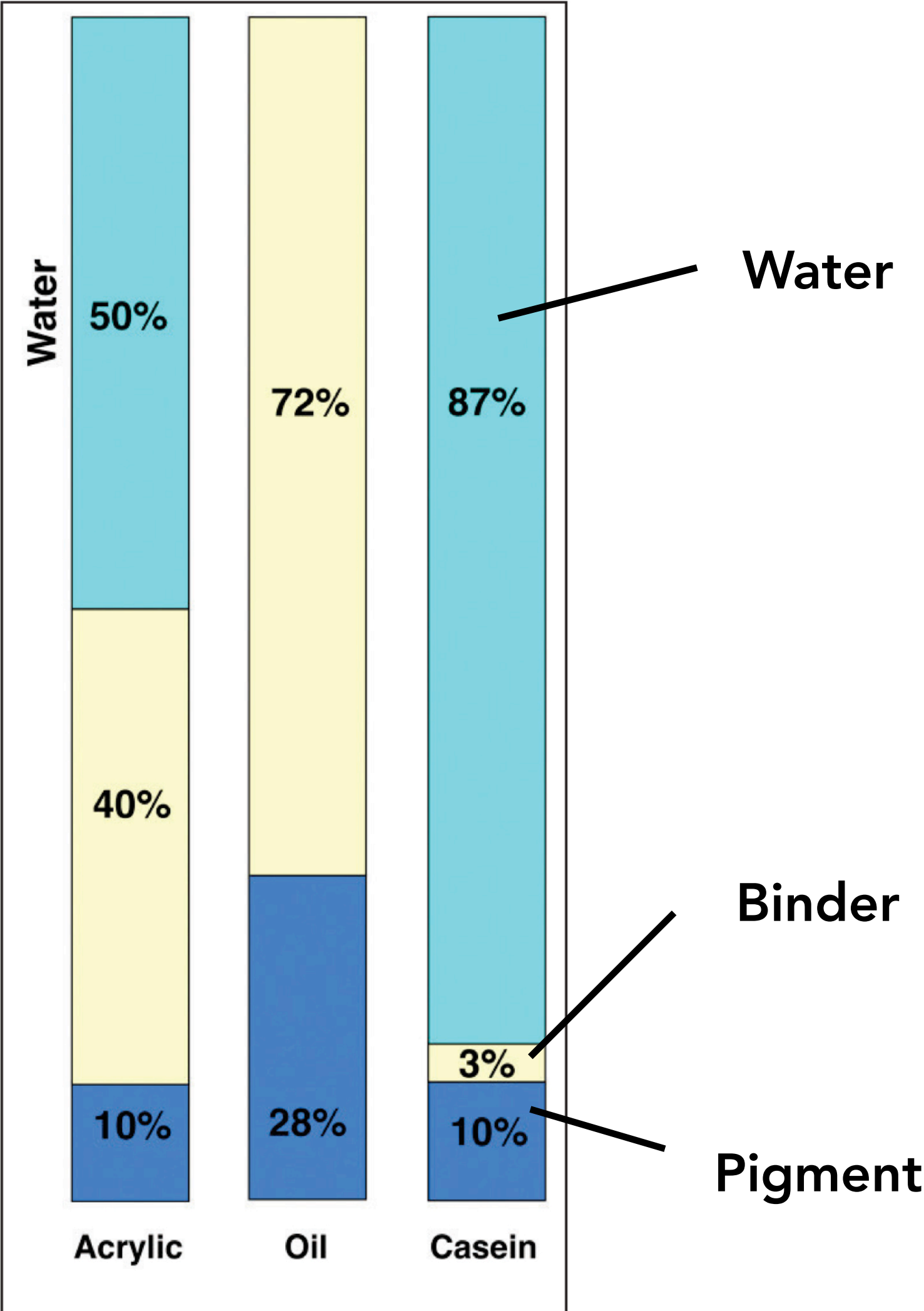
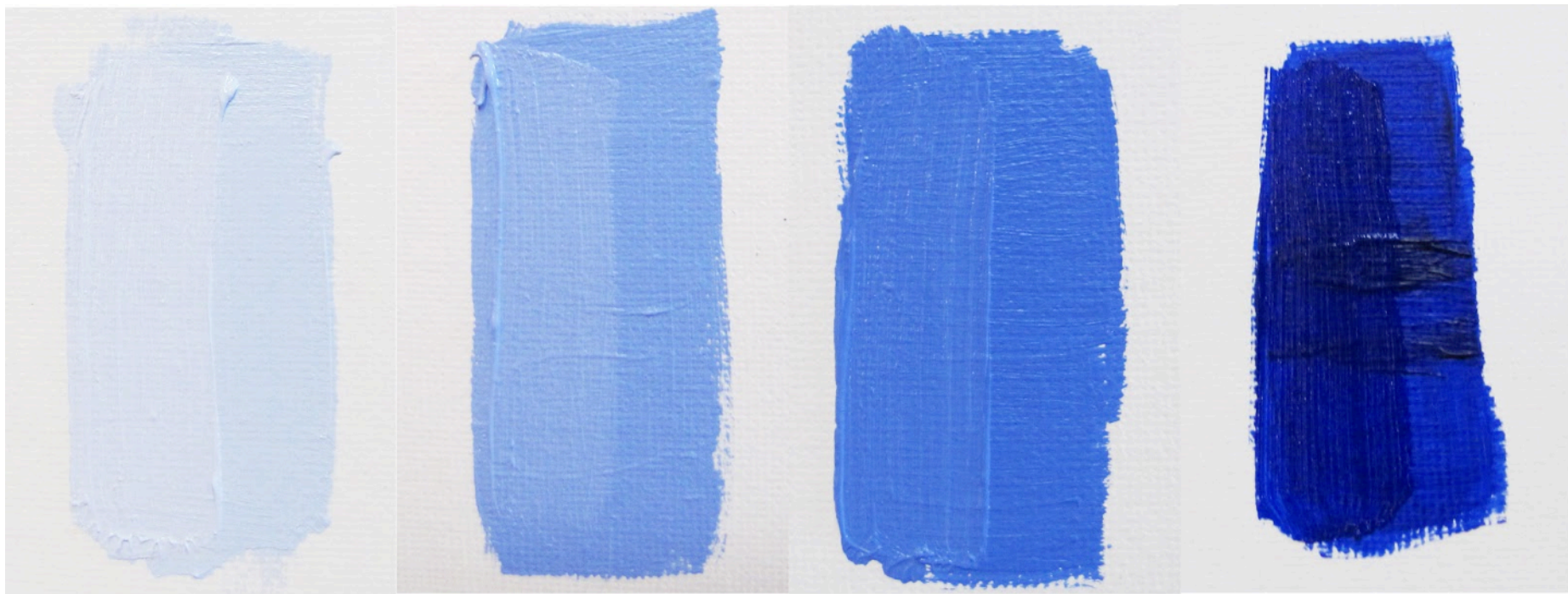
Acrylic Paints Dry Dark

Wet  Dry

Naphthol Red



Ultramarine Blue



Acrylic Paints Dry Dark

Wet $\longrightarrow$ Dry

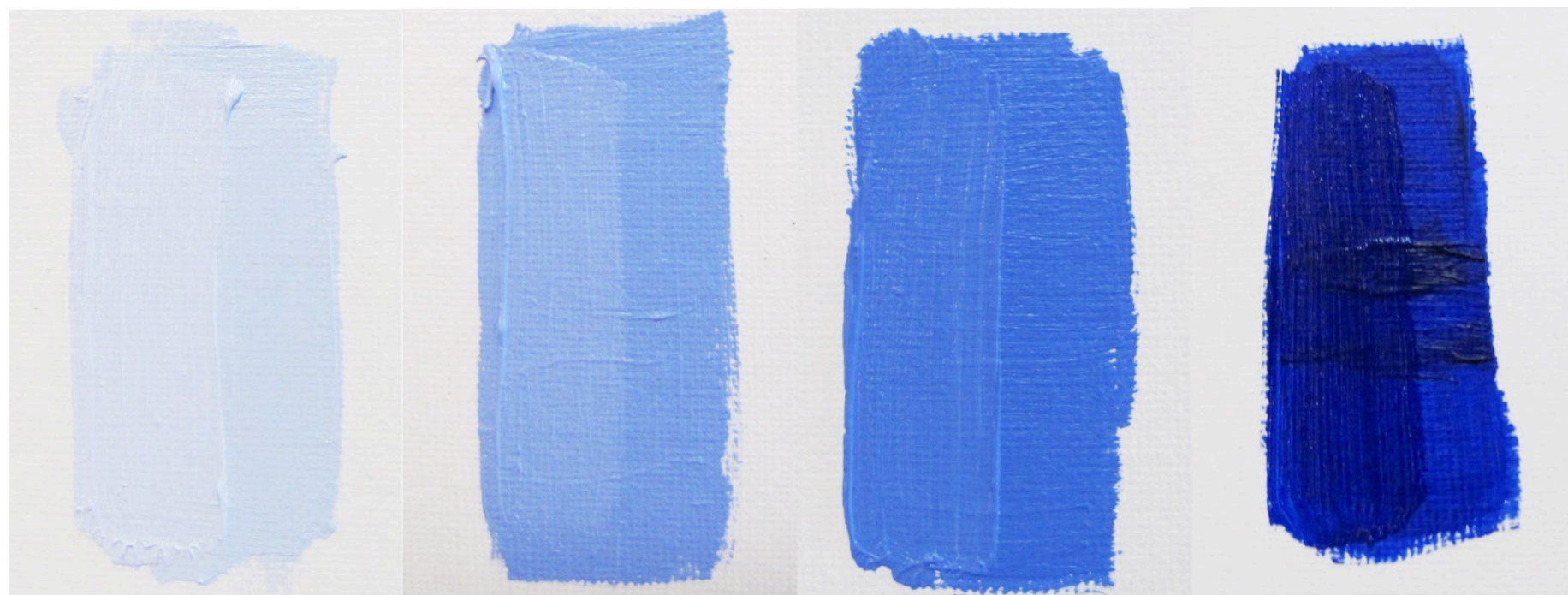
Naphthol Red

$$N_{\text{naphthol reds}} \approx 1.60$$



Ultramarine Blue

$$N_{\text{ultramarine-blue}} \approx 1.50$$



$$N_{\text{water}} \approx 1.33 \quad N_{\text{acrylic binder}} \approx 1.49$$

Reason: when water evaporates, the refractive indices of pigments and binder are closer, which encourages more forward scattering, which increases total absorption.

General Scenarios

(Absorption + Multiple Scattering)

Multiple Scattering

So far we assume that a photon is either absorbed or scattered by at most one particle before entering our eyes.

- Photon scattered by a particle will find its way out without encountering another scattering particle.

With high particle concentration or a thick colorant layer, photons can be scattered by multiple particles and might later be absorbed.

- A photon scattered might be later re-scattered.

Multiple scattering + absorption **is** the common case in nature, arts, and tech.

Solutions

Our goal is to estimate the (spectral) **reflectance** of a material (colorant) in the presence of multiple scattering and absorption.

There are two approaches in general:

- The general case amounts of solving the so-called **Radiative Transfer Equations**, for which analytical solutions are hard to obtained.
- Again, we make simplifications to allow modeling special, but common cases.

Radiative Transfer Equations

Change of radiance at $\mathbf{p}$ toward ω (directional derivative)

Extinction coefficient = absorption + scattering coefficients

Phase function: radiance distribution at $\mathbf{p}$ from ω_i to ω (the integration over ω_i is normalized to 1)

$$\omega \cdot \nabla L(\mathbf{p}, \omega) = -\sigma_t(\mathbf{p}, \omega)L(\mathbf{p}, \omega) + \sigma_a(\mathbf{p}, \omega)L_e(\mathbf{p}, \omega) + \sigma_s(\mathbf{p}, \omega) \int_{4\pi} p(\mathbf{p}, \omega, \omega_i)L(\mathbf{p}, \omega_i)d\omega_i$$

Incident radiance at $\mathbf{p}$ toward ω

Absorption coefficient

Emission

Scattering coefficient

Intuition: we account for the change of radiance at $\mathbf{p}$ along direction ω by

- Removing the absorbed and out-scattered (extinction) portion of incident ray
- Adding emission
- Adding in-scattering (integrated over all directions)

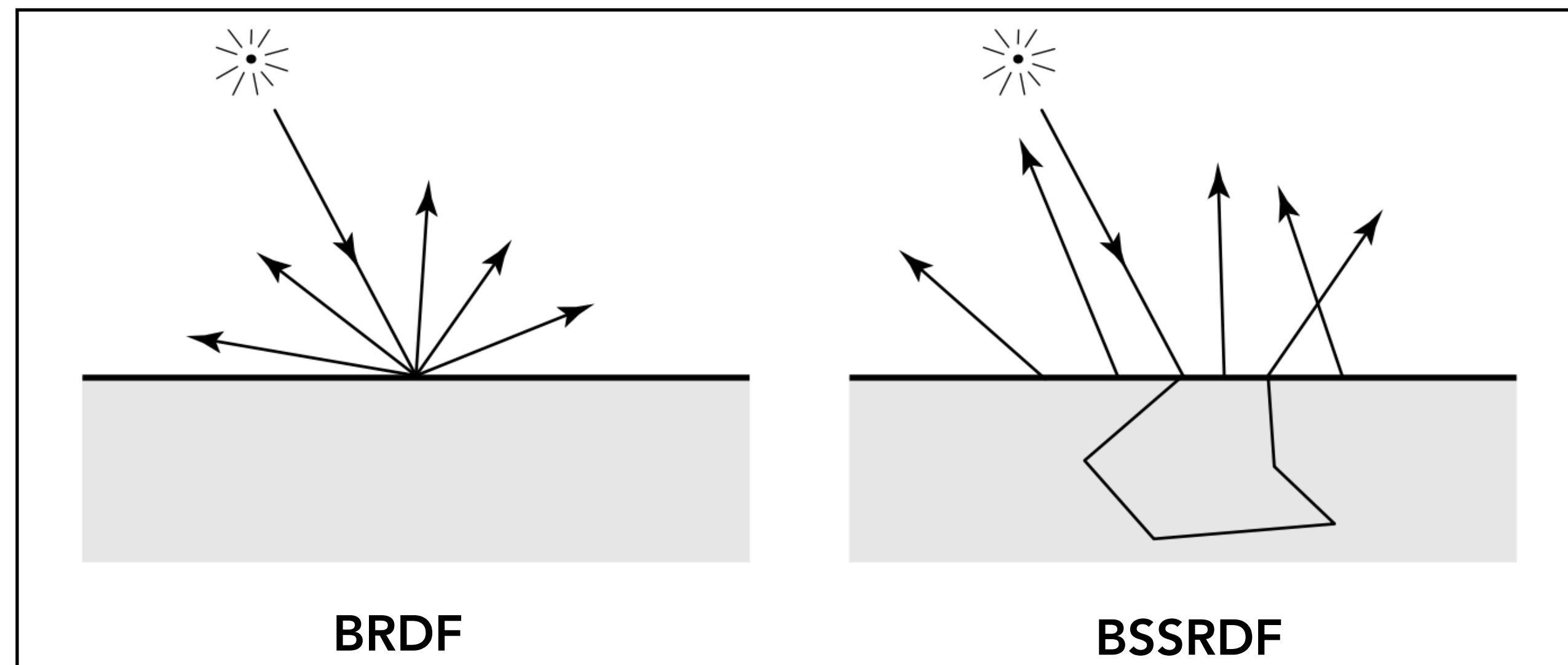
BSSDF: Combining Surface and Volume Scattering

Bidirectional Scattering Surface Reflectance Distribution Function (BSSRDF).

Models how the irradiance of incident rays affects radiance of other rays at **other spatial points**.

$$L_r(p_r, \omega_r) = \int_A \int_{\Omega} f_{bssdf}(p_i, \omega_i \rightarrow p_r, \omega_r) L_i(p_i, \omega_i) \cos \theta_i d\omega_i dA$$

Integrate over all points on the surface and from all directions



BRDF



[Jensen et al. 2001]

BSSRDF

Derive BSSRDF from radiative transfer equation based on diffusion theory and then use Monte Carlo integration to solve the integral equation.

[Jensen et al. 2001]

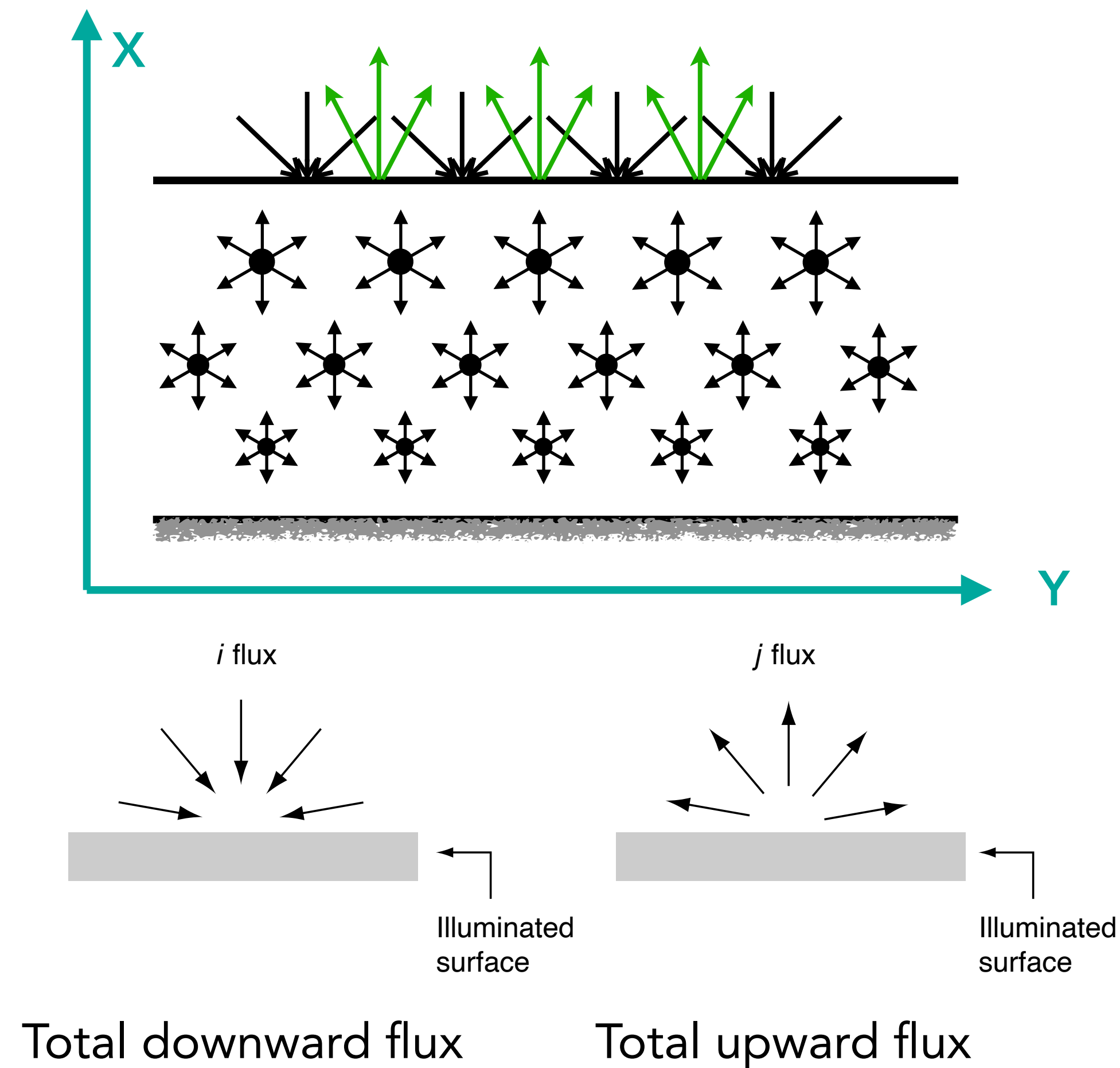


Perfect Diffuse Model

The assumptions we make are:

- Particles have the same scattering and absorption coefficients (homogeneity)
- Particles are randomly oriented (special case: they are all spheres)
- Incident light is uniform at all directions at all surface points (diffuse illumination)

Perfect Diffuse Model



Given the assumptions, there is no difference between different horizontal points (Y) at the same depth (X): the downward (and upward) flux for all Y s at the same X is the same.

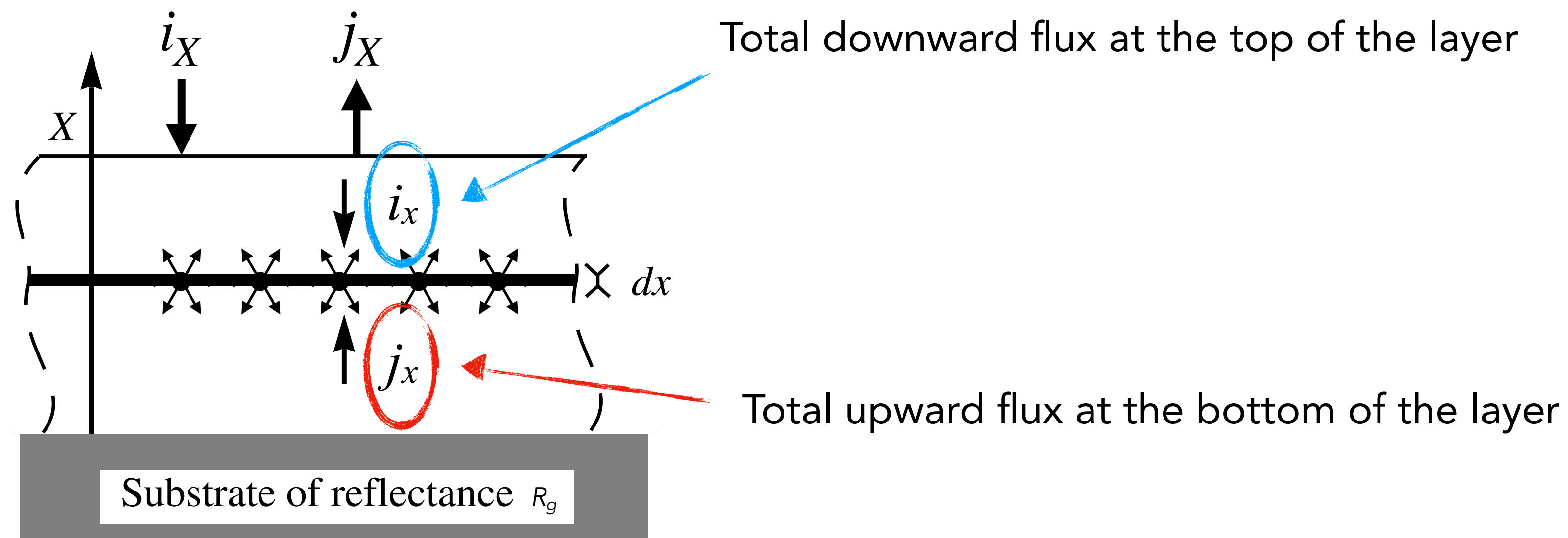
So the reflectance of any point is equivalent to the ratio of the total downward flux and the total upward flux at the depth where the point is.

Kubelka-Munk Equation

K : absorption coefficient

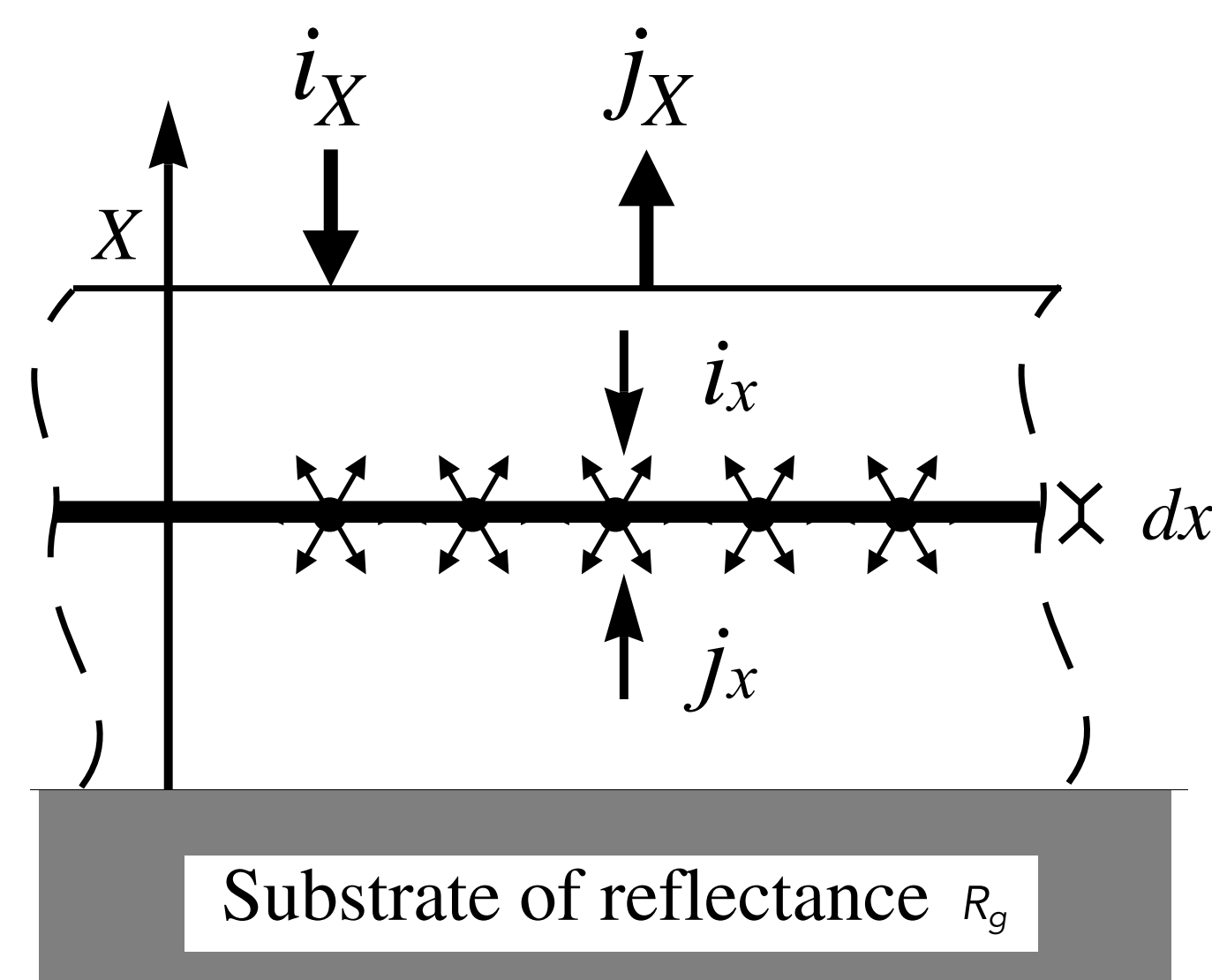
S : back-scattering coefficient

X : thickness



Kubelka-Munk Equation

K: absorption coefficient
S: **back-scattering** coefficient
X: thickness



Total downward flux gained Flux absorbed Flux scattered upward from *i* Flux scattered downward from *j*

$$\begin{aligned}
 di_x &= -Ki_x dx - Si_x dx + Sj_x dx \\
 -dj_x &= -Kj_x dx - Sj_x dx + Si_x dx
 \end{aligned}$$

Boundary conditions:

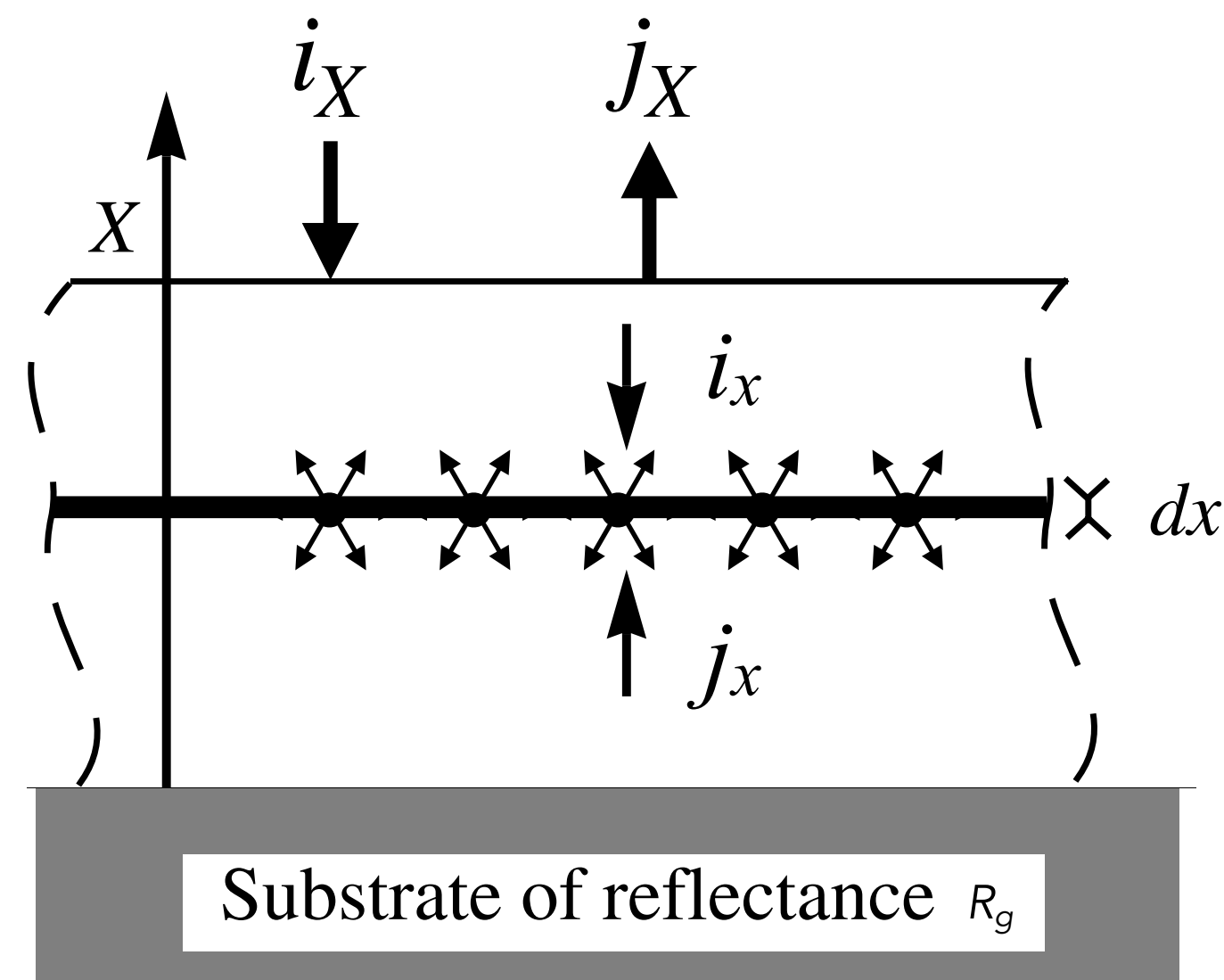
$$R_g = \frac{j_0}{i_0}$$

Reflectance of the layer:

$$R_X = \frac{j_X}{i_X} \quad \leftarrow \text{This is what we want to calculate!}$$

Kubelka-Munk Equation: General Solution

$$R_X = \frac{1 - R_g[a - b \coth(bSX)]}{a - R_g + b \coth(bSX)} \quad a = \frac{K + S}{S} \quad b = \sqrt{a^2 - 1} \quad \coth x = \frac{e^{2x} + 1}{e^{2x} - 1}$$



Reflectance depends on:

- Layer thickness **X**
- Absorption and scattering coefficients **K** and **S**
- Background reflectance: **R_g**

Kubelka-Munk Equation: No Scattering

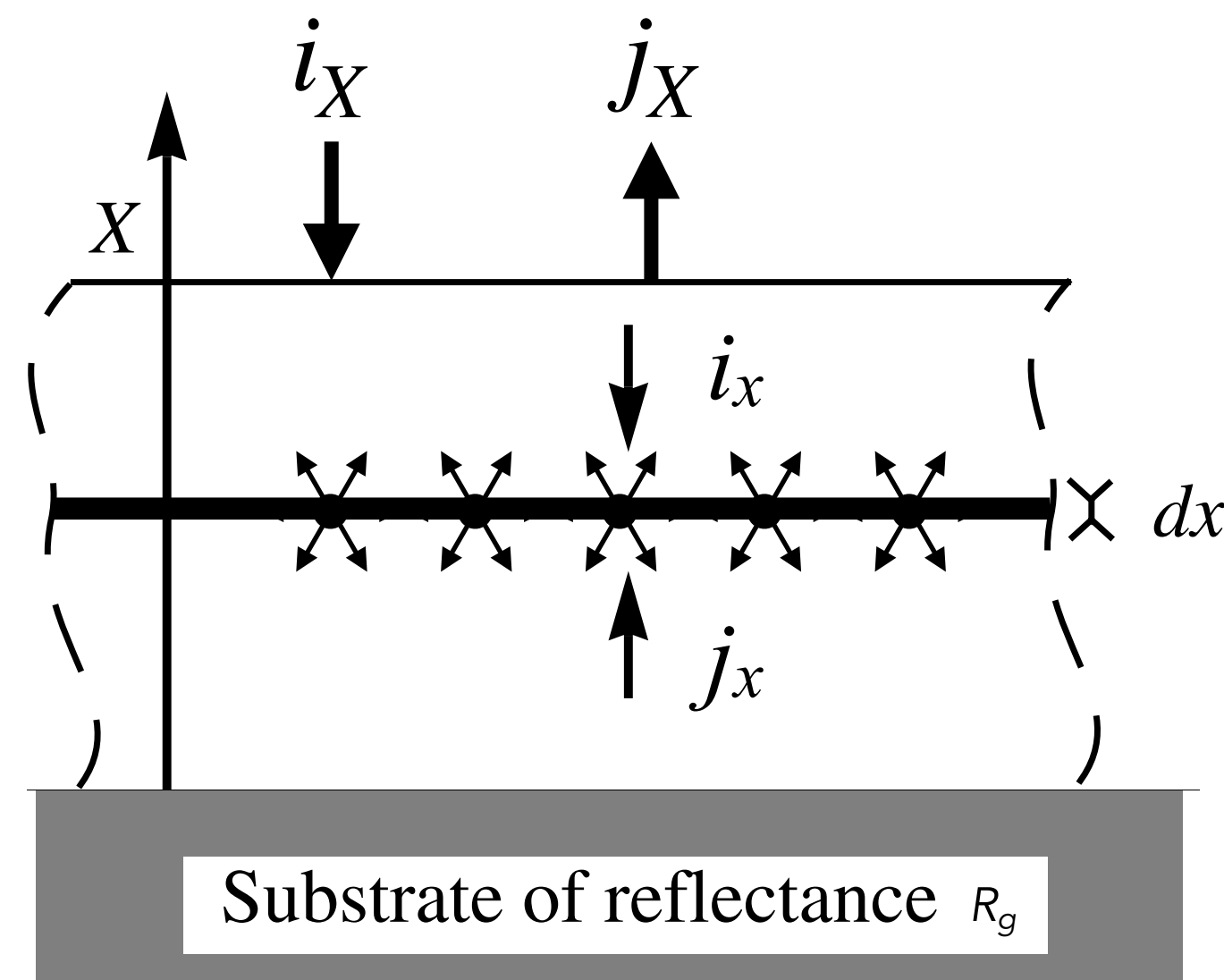
$$S = 0 \quad \Rightarrow$$

$$R_X = R_g e^{-2KX} = e^{-KX} R_g e^{-KX}$$

% of photons left after
they reach the bottom

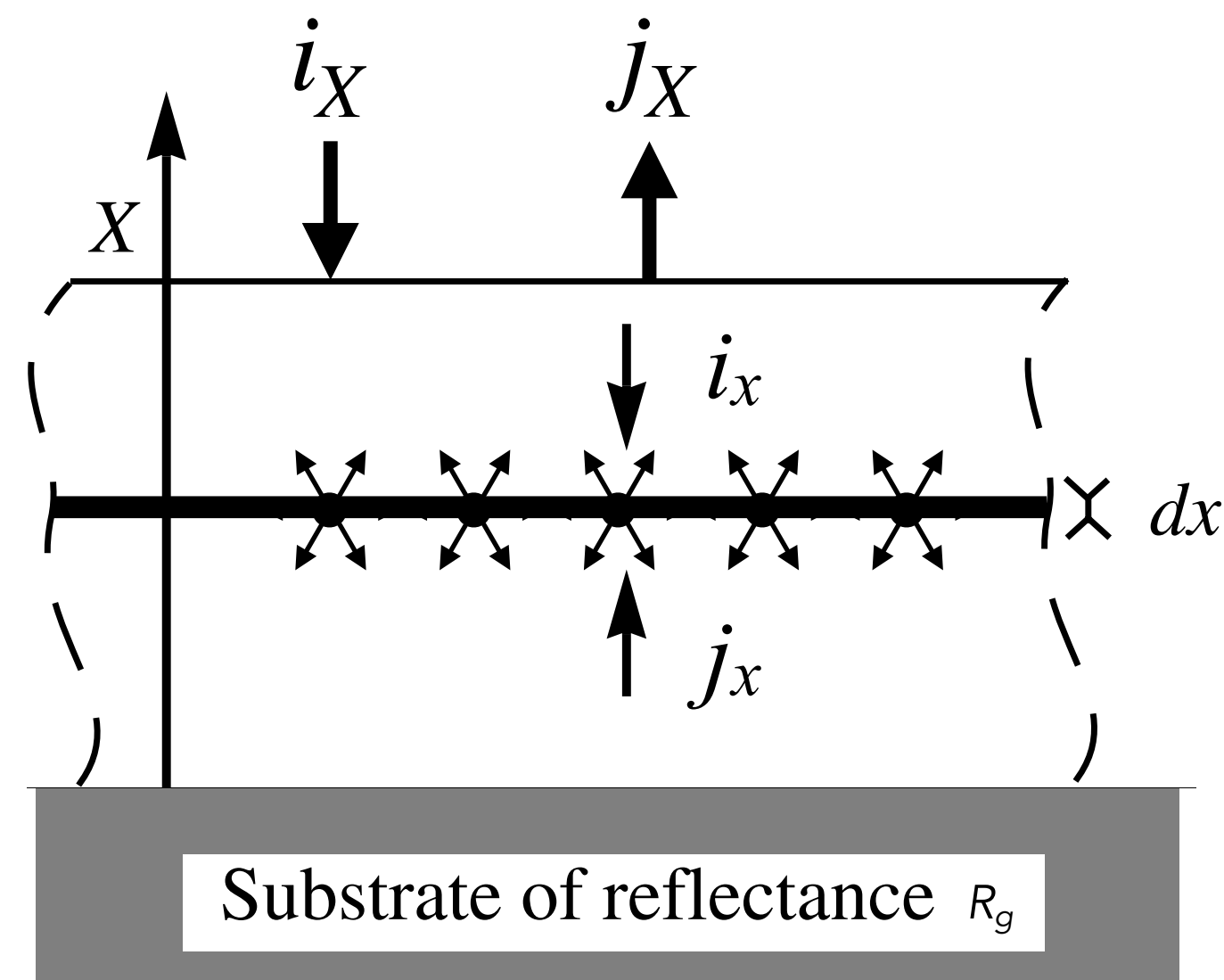
% of photons reflected
back by the substrate

% of photons left after
they reach the top again



Kubelka-Munk Equation: No Substrate

$$R_g = 0 \Rightarrow R_X = \frac{1}{a + b \coth(bSX)} \quad a = \frac{K + S}{S} \quad b = \sqrt{a^2 - 1}$$

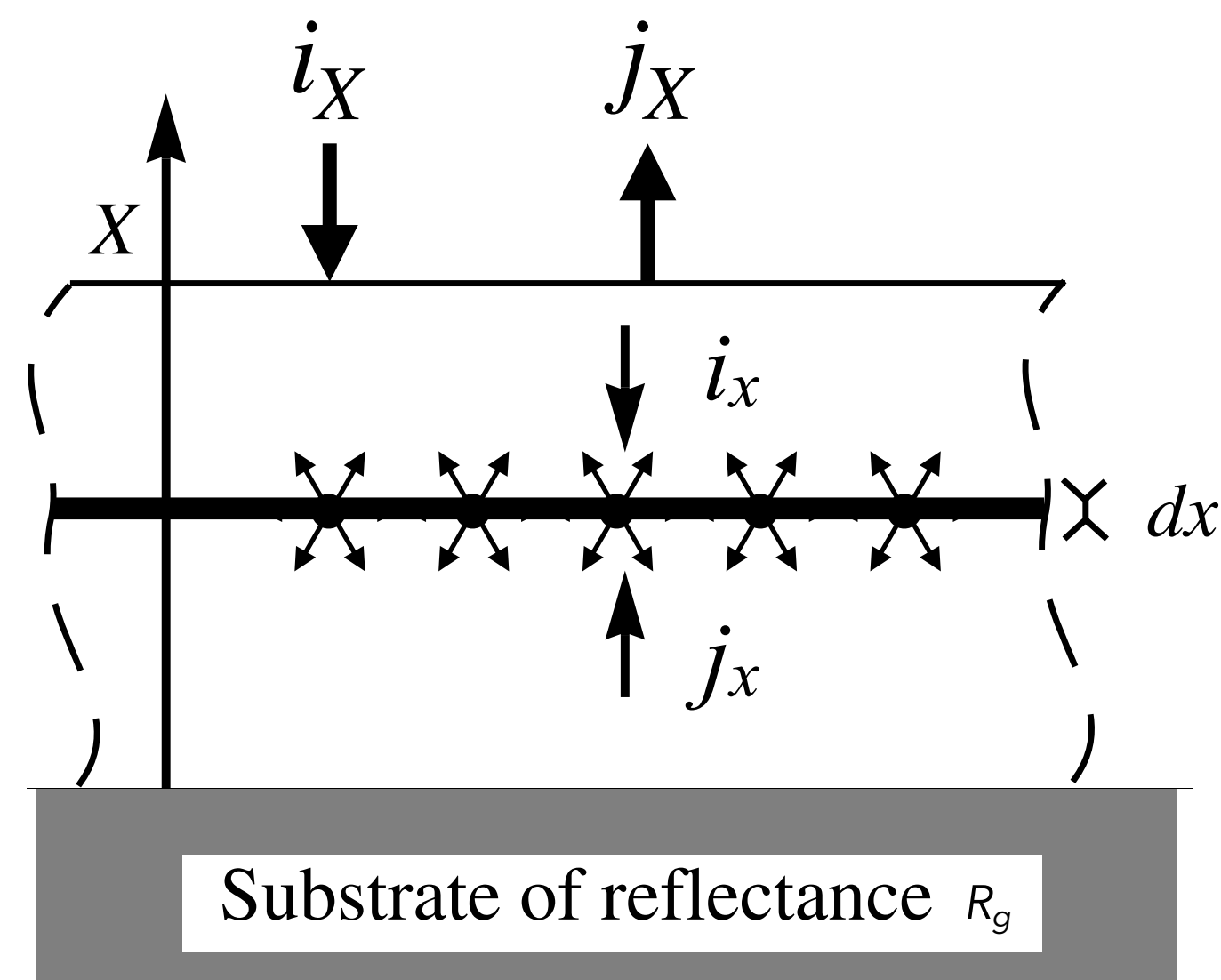


Substrate reflectance **R_g** is 0 now. Alternatively this can be thought of as having a **black substrate**. The effect is the same: whatever gets out of the layer never comes back.

Kubelka-Munk Equation: Complete Hiding

$$\lim_{X \rightarrow \infty} R_X = R_\infty = \frac{1}{a + b} = 1 + \frac{K}{S} - \sqrt{\left(\frac{K}{S}\right)^2 + 2\frac{K}{S}}$$

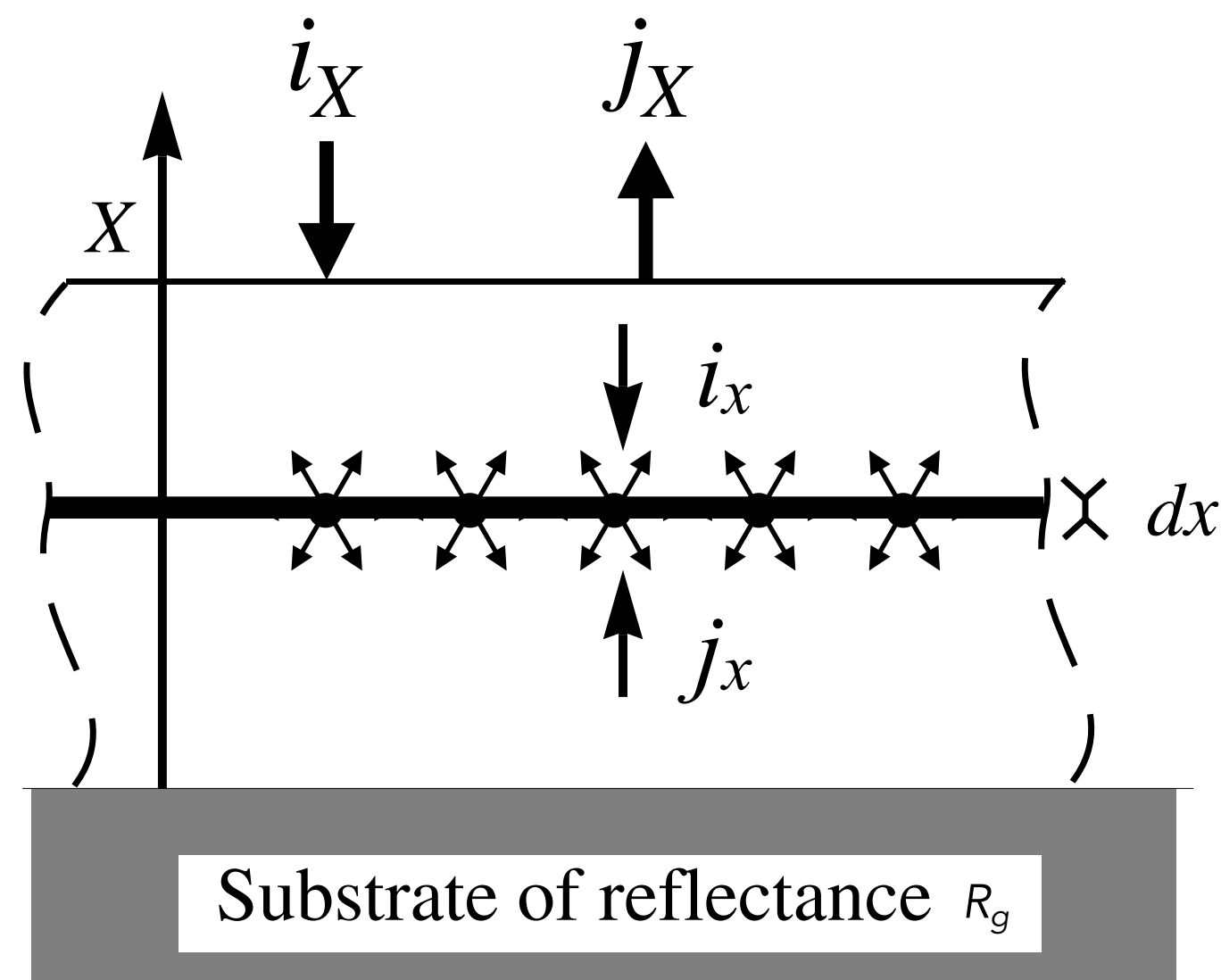
$$\frac{K}{S} = \frac{(1 - R_\infty)^2}{2R_\infty}$$



When the layer is so thick that the background reflectance is irrelevant, the resulting reflectance depends only on the absorption and scattering coefficients.

We say the "hiding is complete".

Hiding Power in Painting



$$HP = \frac{R_{black}}{R_{white}}$$

Reflectance when substrate is black.

Reflectance when substrate is white.

In paint industry, the way they quantify hiding power is to see how the surface reflectance changes when the background is white vs. black.

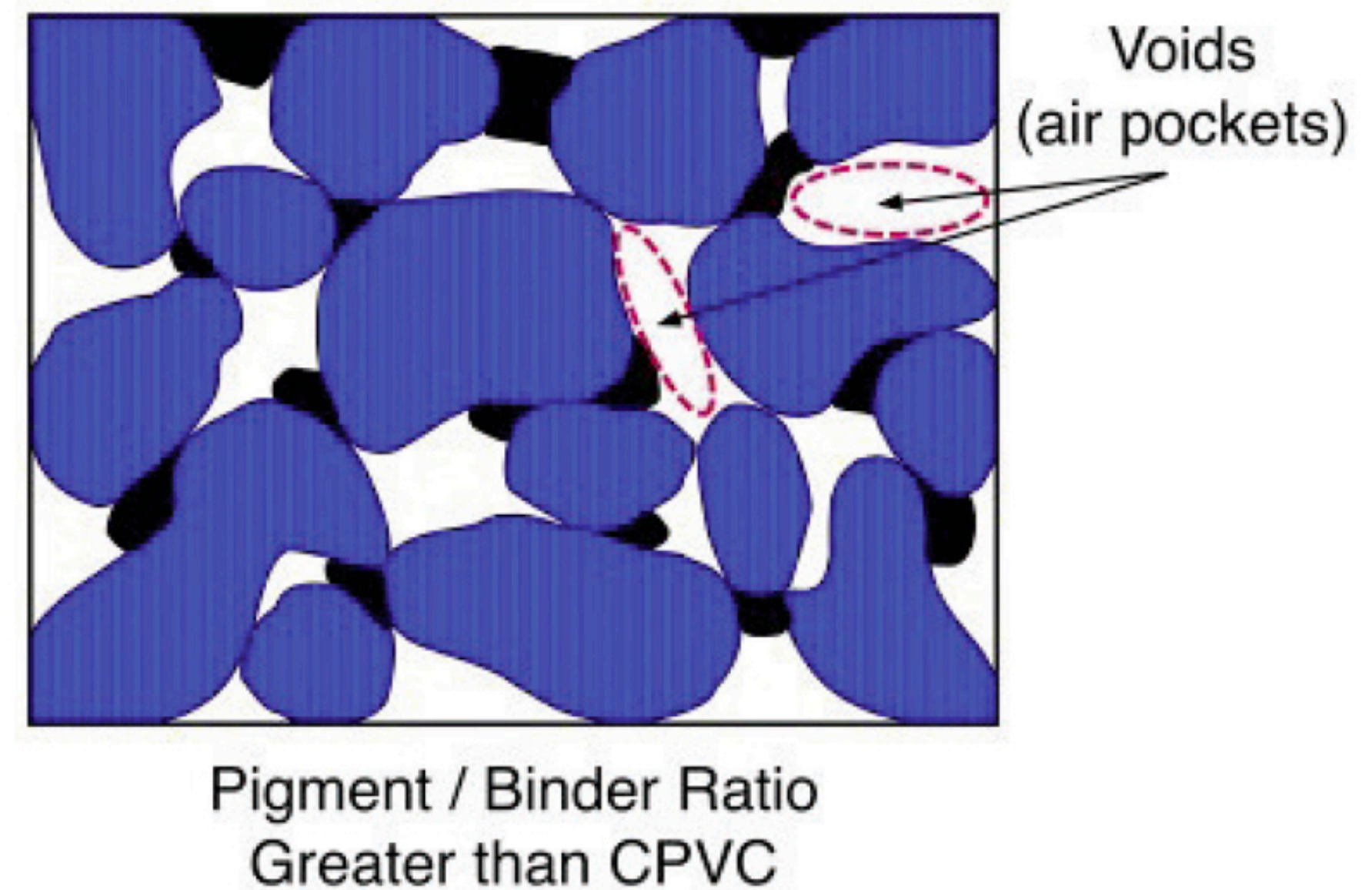
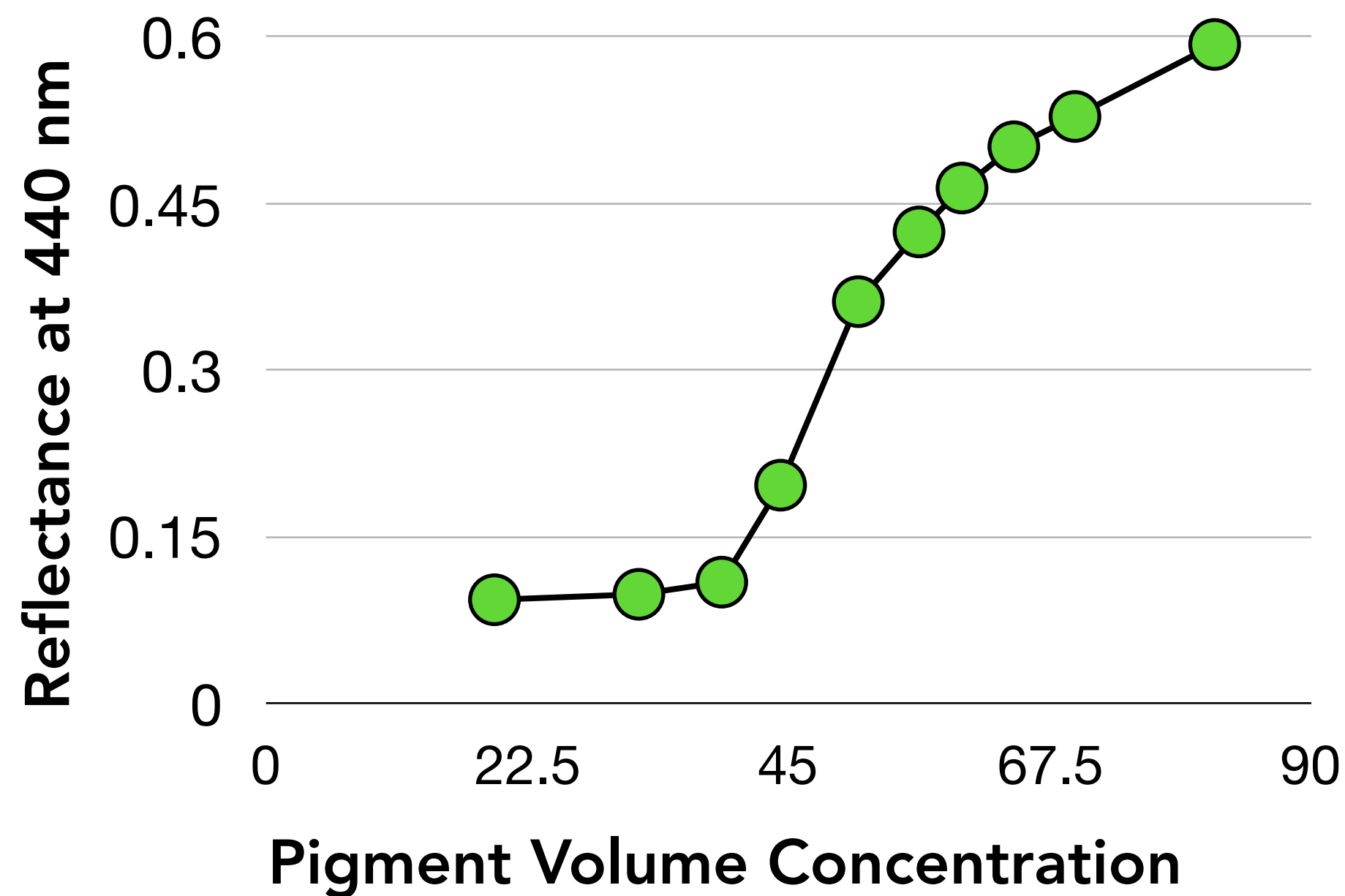
If the layer's hiding is complete, the reflectance ratio should 1, i.e., the background color doesn't matter and the colorant is **opaque**.

Reflectance vs. Concentration

$$\frac{K}{S} = \frac{(1 - R_{\infty})^2}{2R_{\infty}}$$

When hiding is complete, the reflectance of the colorant doesn't depend on the concentration.

- Until when critical concentration is reached, where the assumptions of K-M model break.

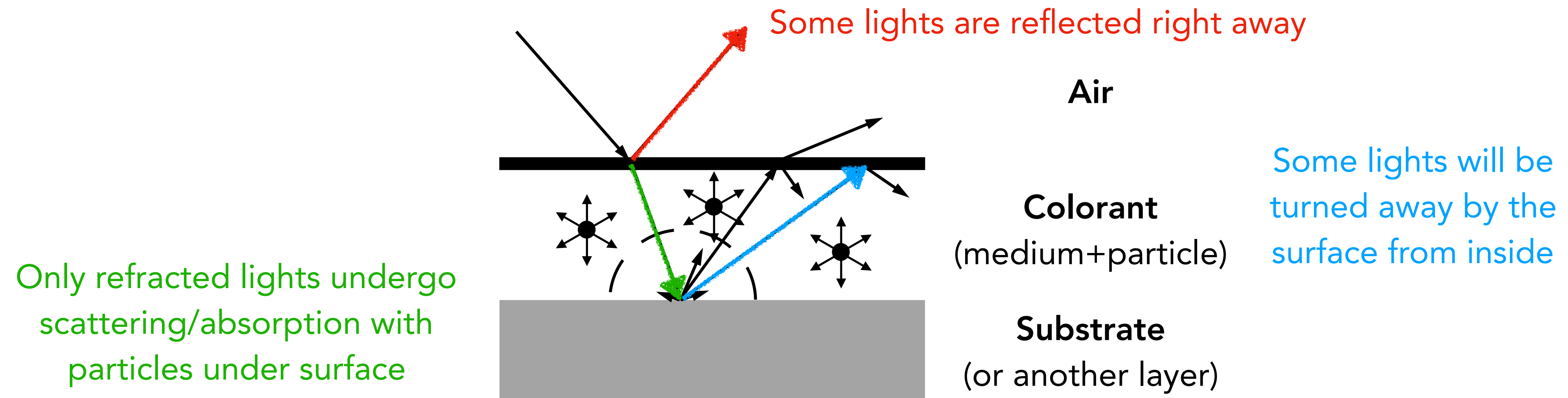


Surface Reflection

Considering Surface Reflection

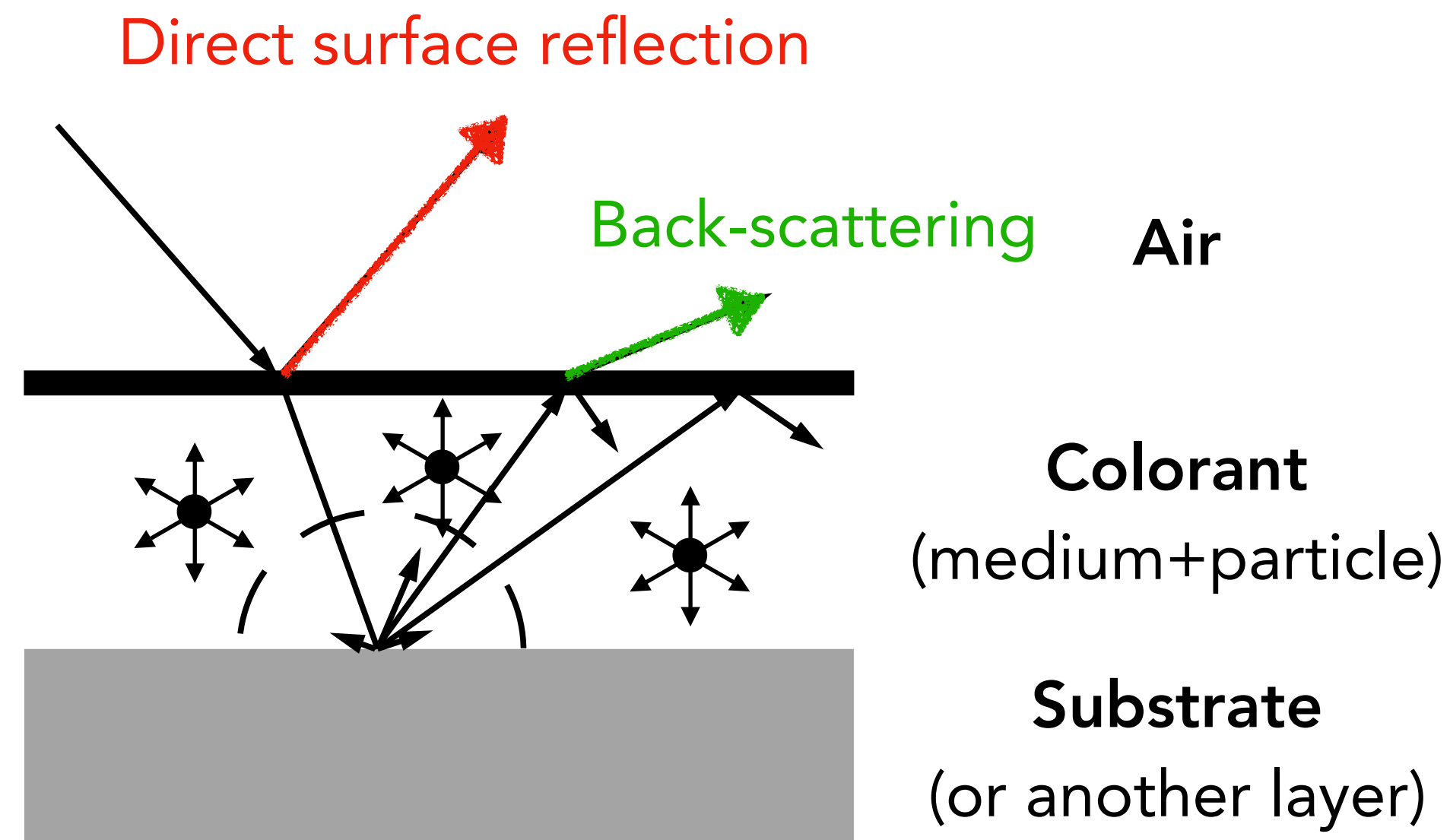
So far we have ignored reflection and refraction at the air/medium interface.

- Note that reflection and refraction are just macroscopic behaviors of light scattering (with interference) off of surface particles. We can reason about it using geometric optics, since a colorant is large enough for that.



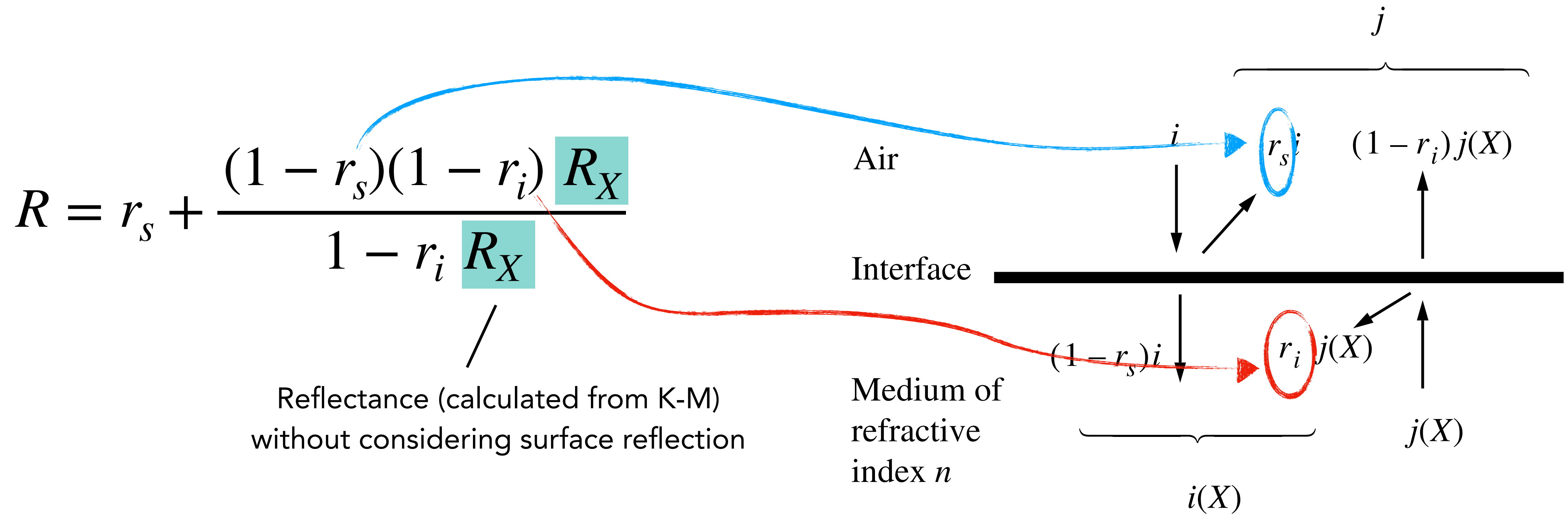
Reflectance = Surface Reflection + Back Scattering

The overall reflection is made up of both surface reflection (photons turned away right at the interface) and back-scattering from photon-particle interactions under the surface (usually called **sub-surface scattering**).

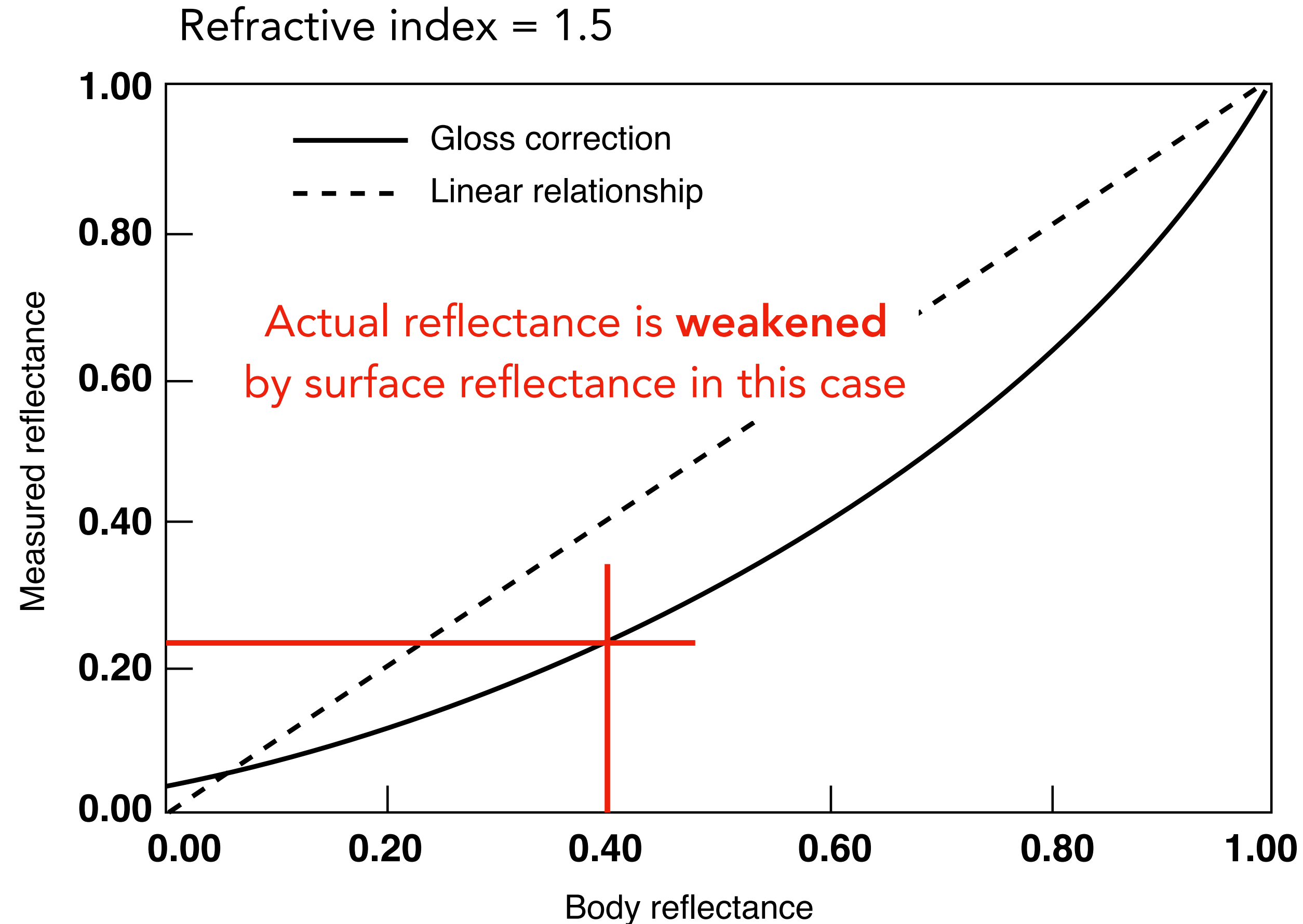


K-M Model With Surface Reflection Correction

This is called the **Saunderson correction**



K-M Model With Surface Reflection Correction



Using Fresnel equation

- r_s is 0.092 for diffuse light and 0.04 for when incident lights are perpendicular to the surface.
- r_i is 0.06 for when the internal lights crossing the interface are diffuse.

In practice, parameters are fit with observations, just like other parameters in K-M model.

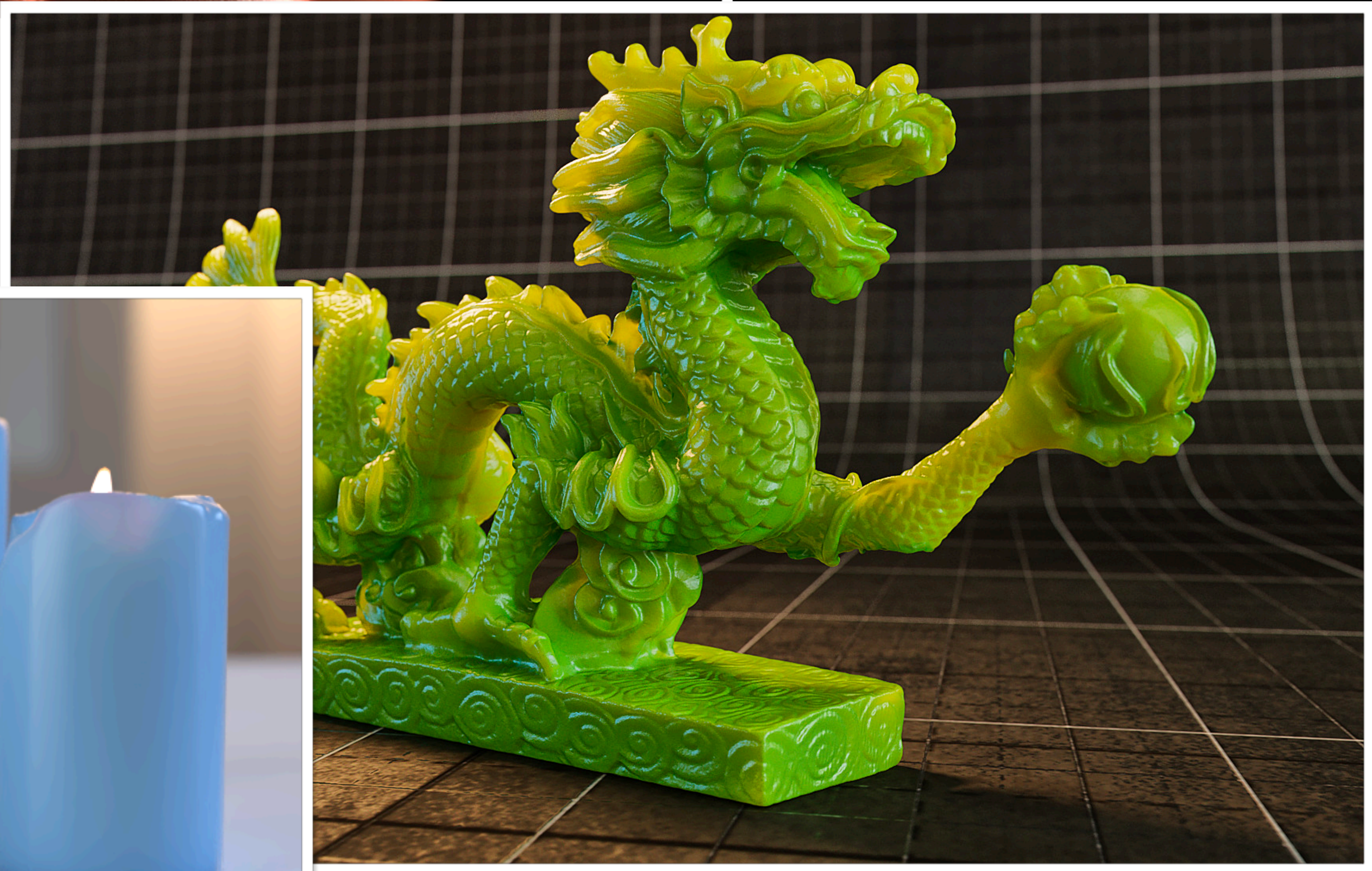
Material Classification

In graphics, material is mainly classified into two categories.

- Those that can modeled by considering only surface reflection: absorption efficiency of sub-surface particles is so high that back-scattering contributes little to reflectance.
- Those where sub-surface scattering must be modeled. In graphics, people say:
 - “modeling **translucent** material must consider sub-surface scattering”
 - “sub-surface scattering is caused by photons exiting at a point different from the incident point”

A big part of photorealistic rendering is material modeling. It’s always a trade-off between rendering time vs. realism.

- But note that you don’t have to be physically correct, just need to be photorealistic.



Modeling Surface

What's this type?

Surface reflection is modeled by the “roughness” of the surface.

- Rough surfaces can be modeled as a collection of small mirrors or “*microfacets*”.

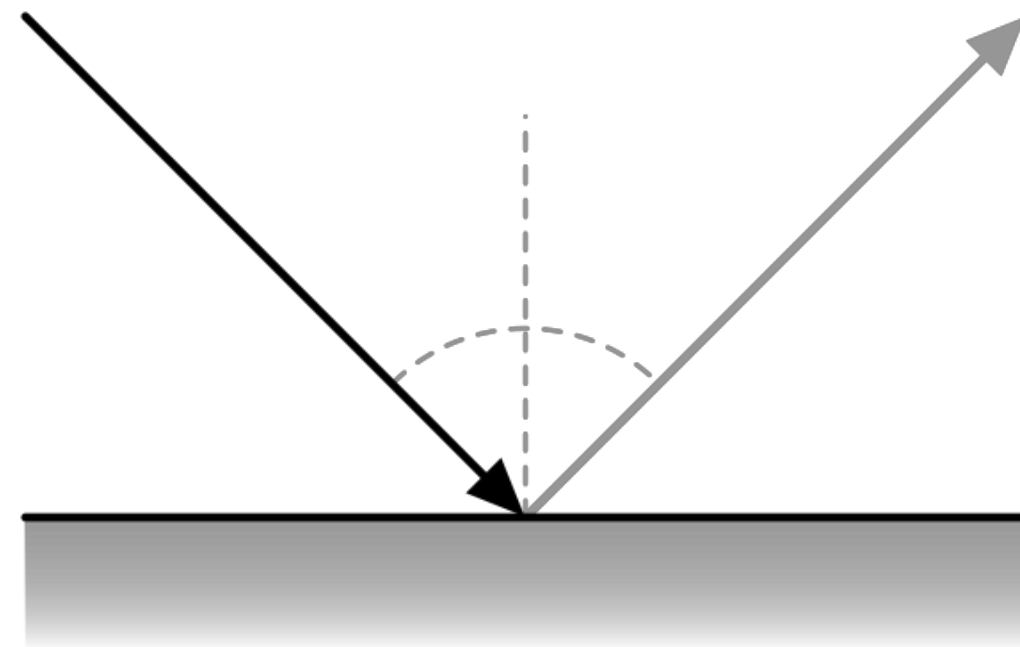
Surface types:

- Purely reflective. No roughness; perfect mirror.
- Glossy (shiny). Microfacet orientations are similar but not the same.
- Diffuse (matte). Lights reflected multiple times between “micro-mirrors”. Result: reflecting evenly to all angles regardless incident angle.

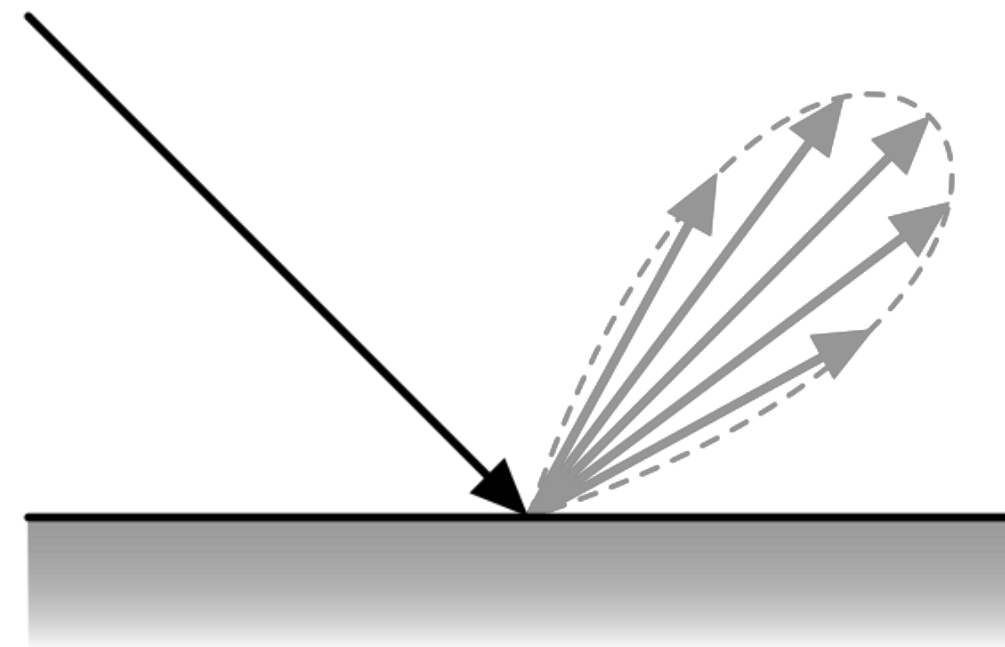


Surface Types

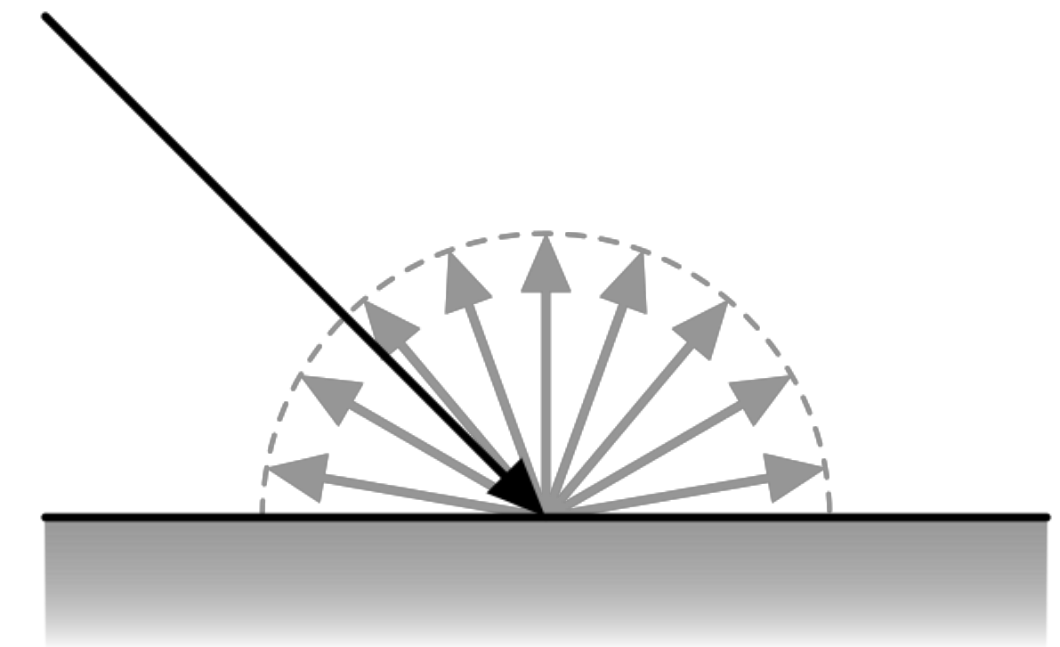
Mirror/Glass (Specular)



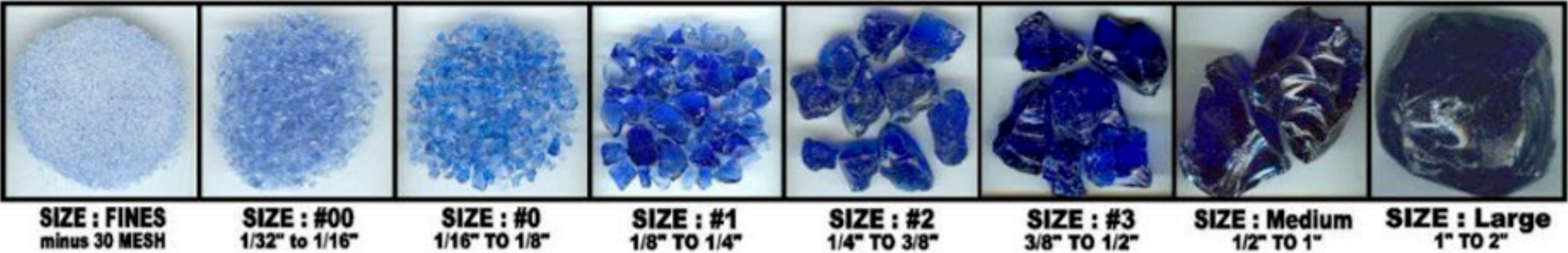
Glossy (Specular)

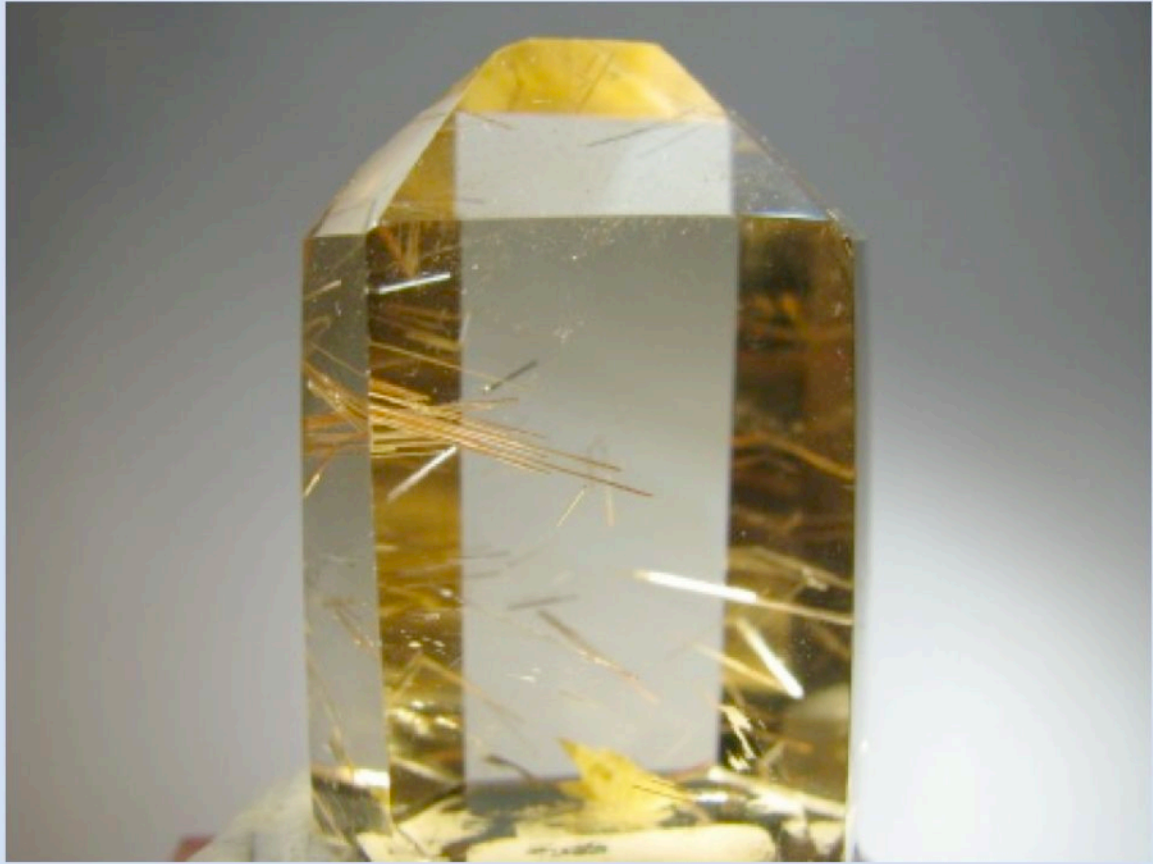


Diffuse (Not Specular)



Puzzle





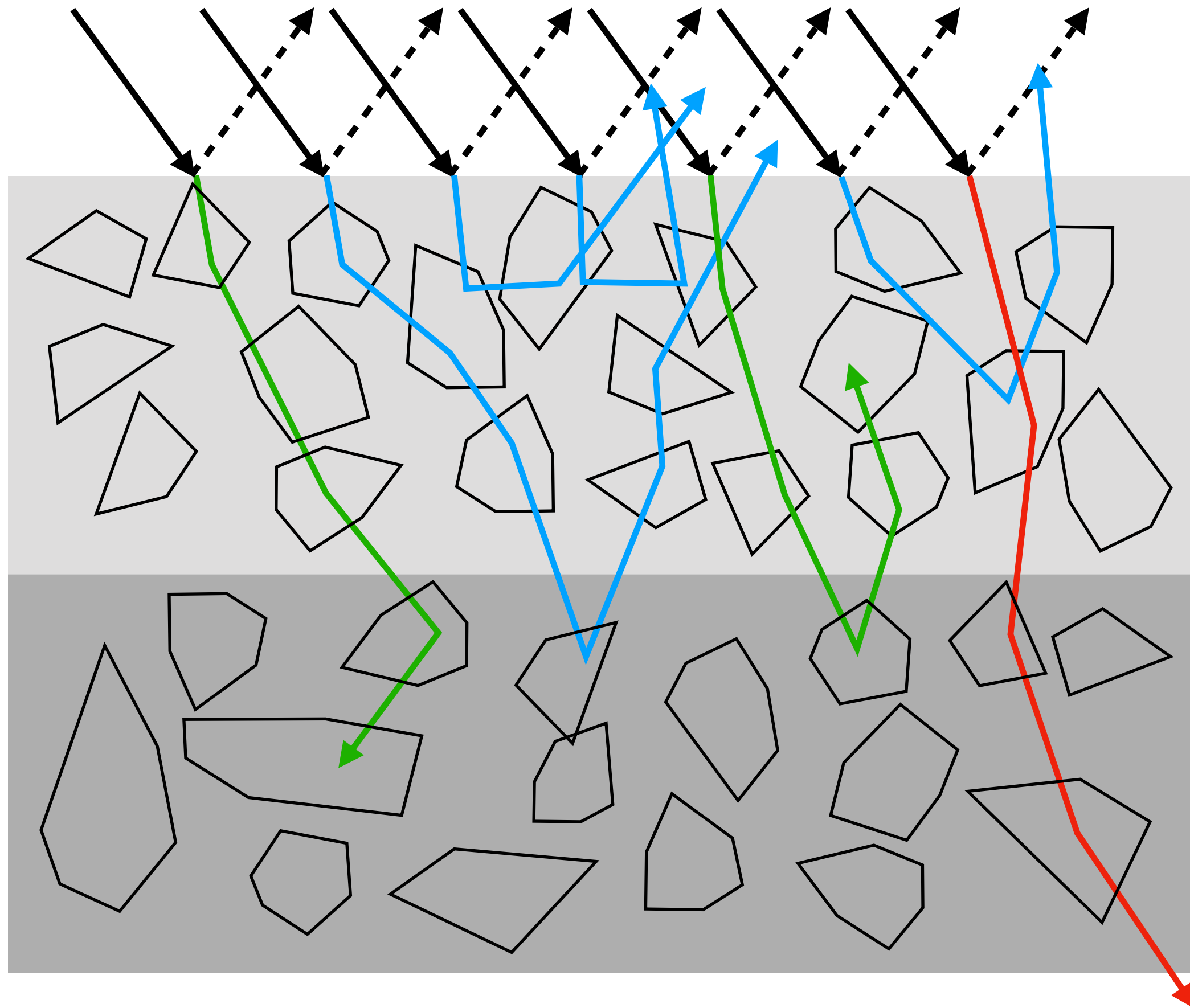


Modernist
Encaustic

Titanium-rutile crystal & Rutile pigment

Layering vs. Mixing

Layering



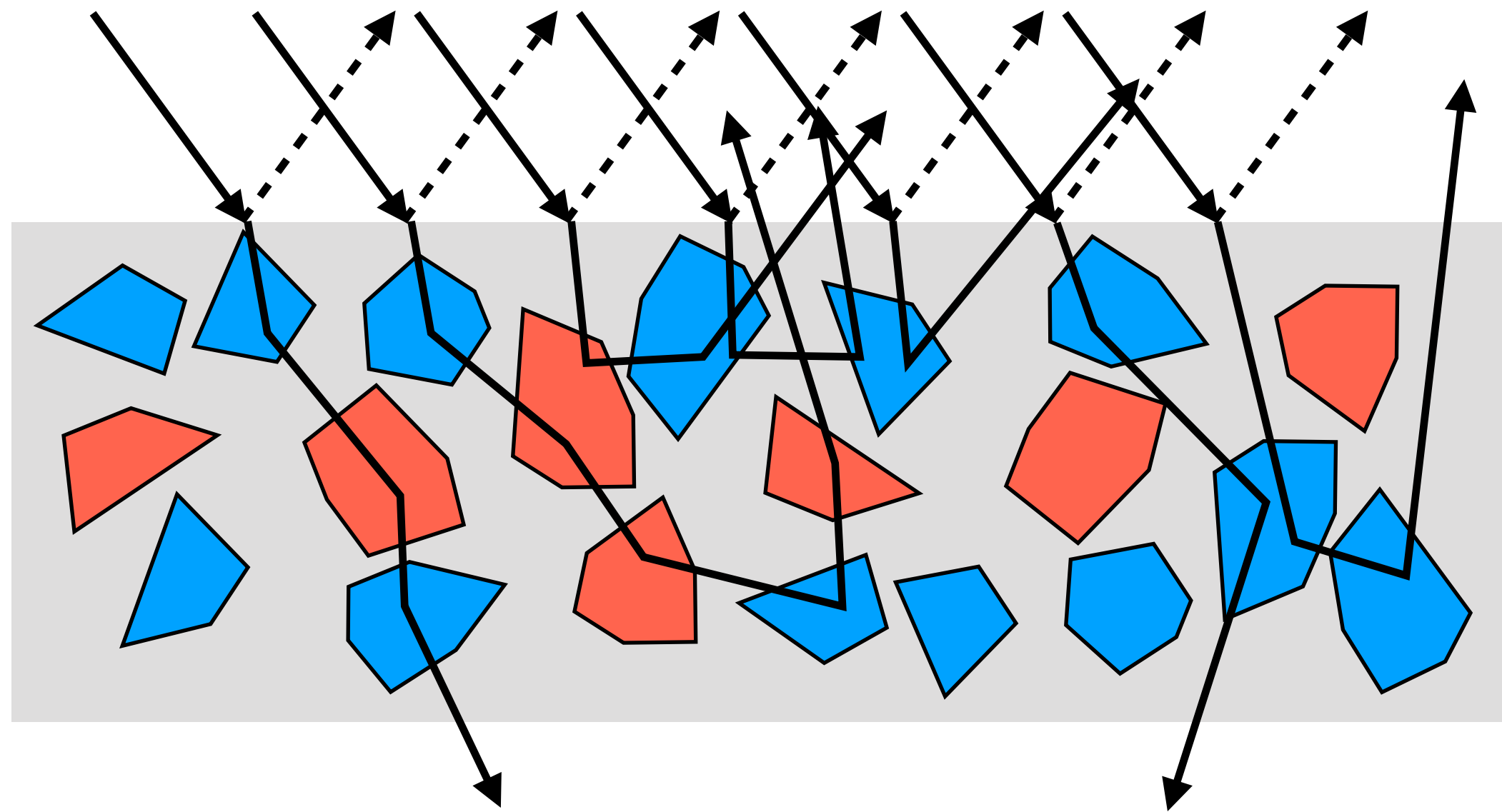
$$R_1 = \frac{1 - R_g[a_1 - b_1 \coth(b_1 S_1 X_1)]}{a_1 - R_g + b_1 \coth(b_1 S_1 X_1)}$$

$$R_2 = \frac{1}{a_2 + b_2 \coth(b_2 S_2 X_2)}$$

* Ignoring **Saunderson** correction here

Mixing

$$R_X = \frac{1 - R_g[a - b \coth(bSX)]}{a - R_g + b \coth(bSX)} \quad a = \frac{K + S}{S} \quad b = \sqrt{a^2 - 1} \quad \coth x = \frac{e^{2x} + 1}{e^{2x} - 1}$$



$$K = \sum_i K_i c_i$$

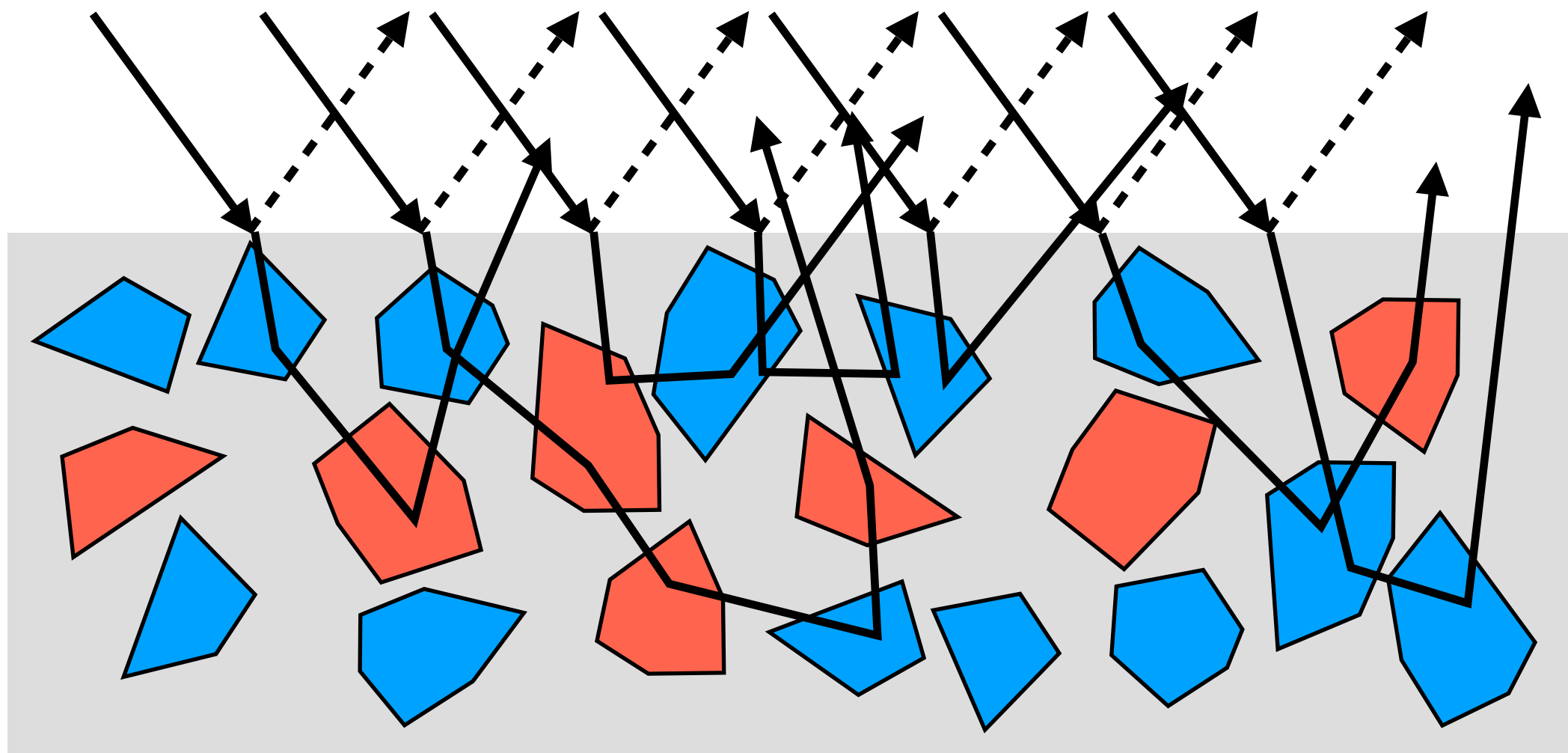
Relative concentration of each pigment

$$S = \sum_i S_i c_i$$

Absorption and scattering coefficients of each pigment

Mixing When Hiding is Complete

$$\frac{K}{S} = \frac{\sum_i K_i c_i}{\sum_i S_i c_i} = \frac{(1 - R_\infty)^2}{2R_\infty}$$



This set-up is commonly used to determine the K/S coefficients of different pigment.

- Based on reflectance measurement.
- See [this](#) paper.

K/S can also be theoretically derived, but since K-M is an empirical approximation, fitting observed data is better.

Layering == Mixing For Transparent Materials

Transparent materials do not scatter light and only absorb lights. They only take photons out.

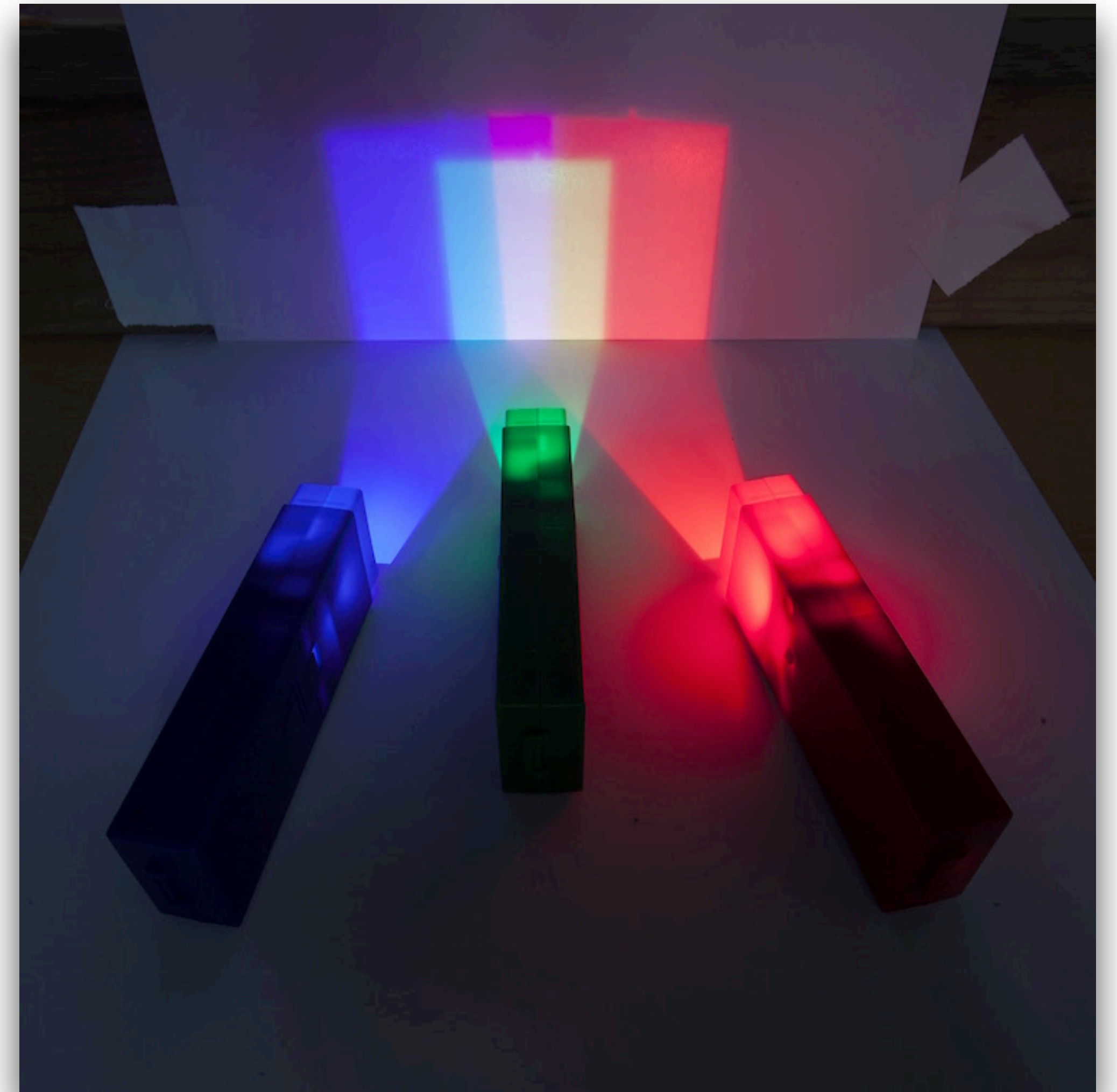
It's not important whether the pigments are mixed or layered, i.e., where the pigment particles are. What matters is just the total amount of particles.

- assuming that the absorption behaviors of individual particles are independent.

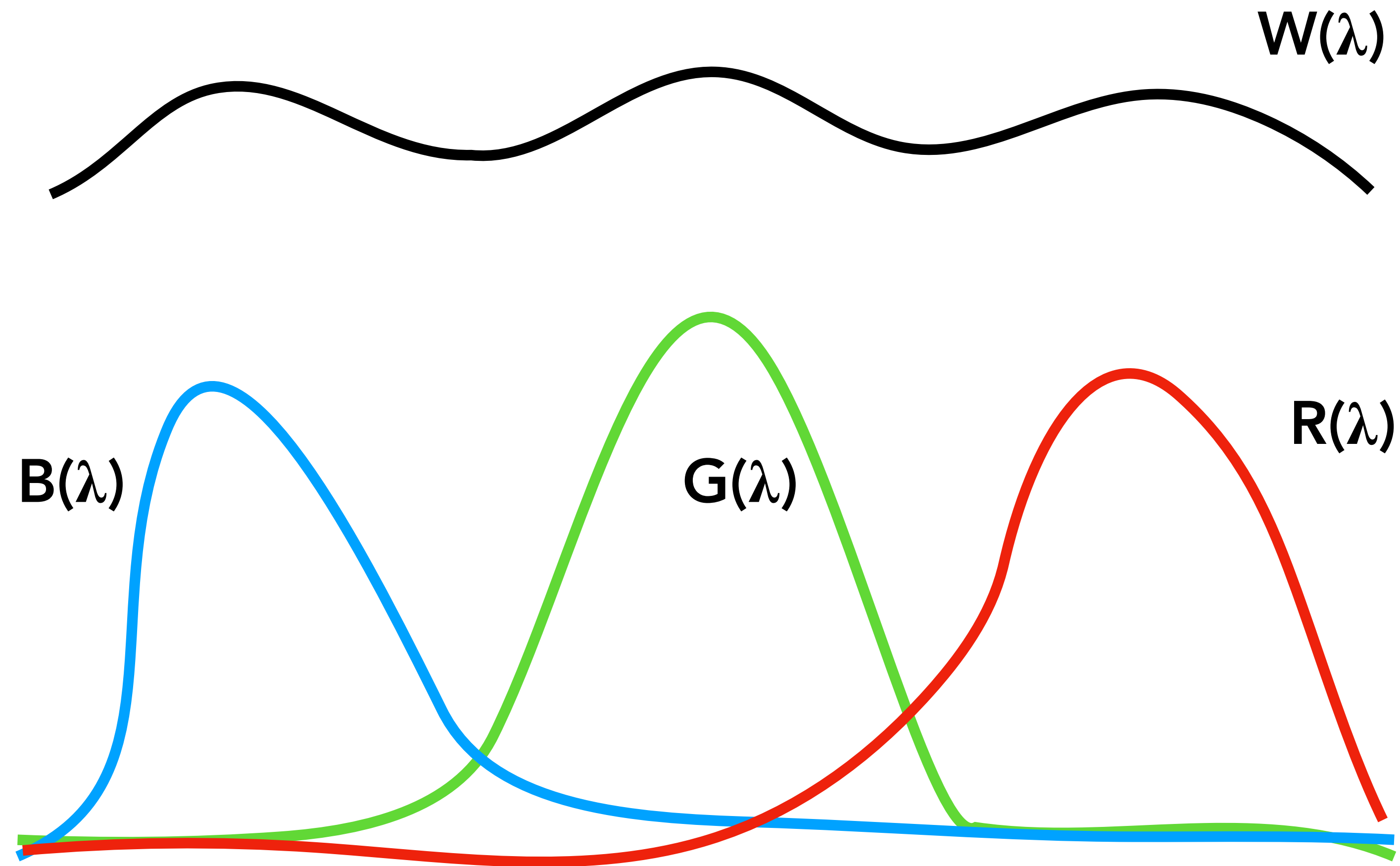
Light Colors are Additive

Mixing lights:

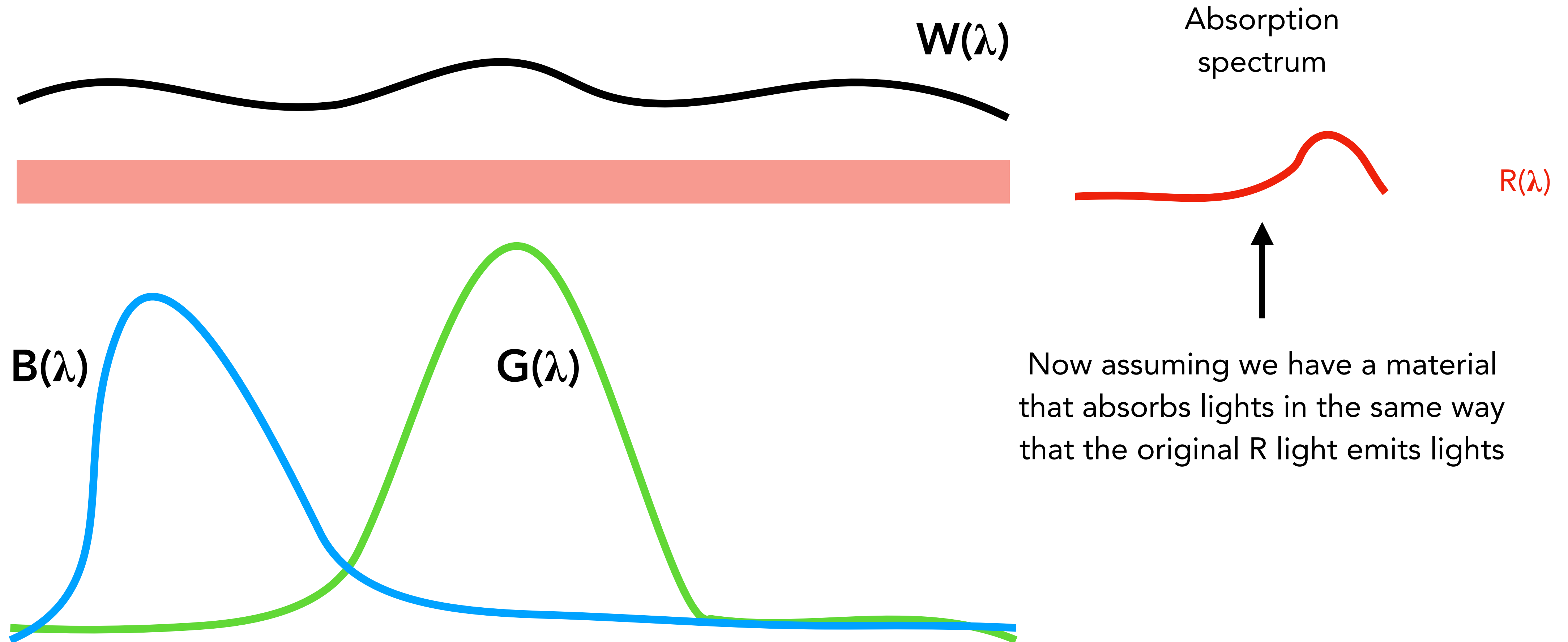
- If we have three lights whose *linear* RGB values are $[R_1, G_1, B_1]$, $[R_2, G_2, B_2]$, and $[R_3, G_3, B_3]$, when we mix them the mixed light's color is $[R_1 + R_2 + R_3, G_1 + G_2 + G_3, B_1 + B_2 + B_3]$.
- Can you prove it?
- The key is that the spectral power distribution of the mixed light is the sum of that of each light. From SPD to RGB color is linear.



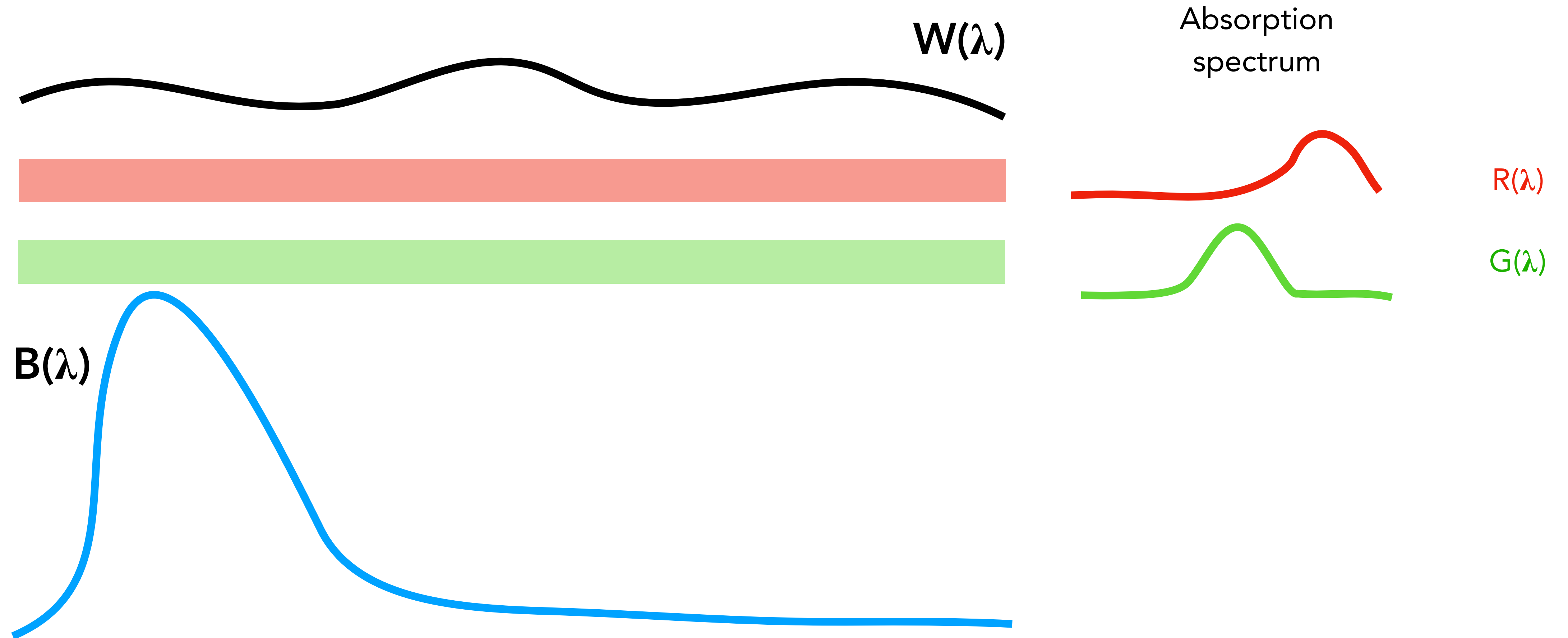
What is Subtractive Color Space Really?



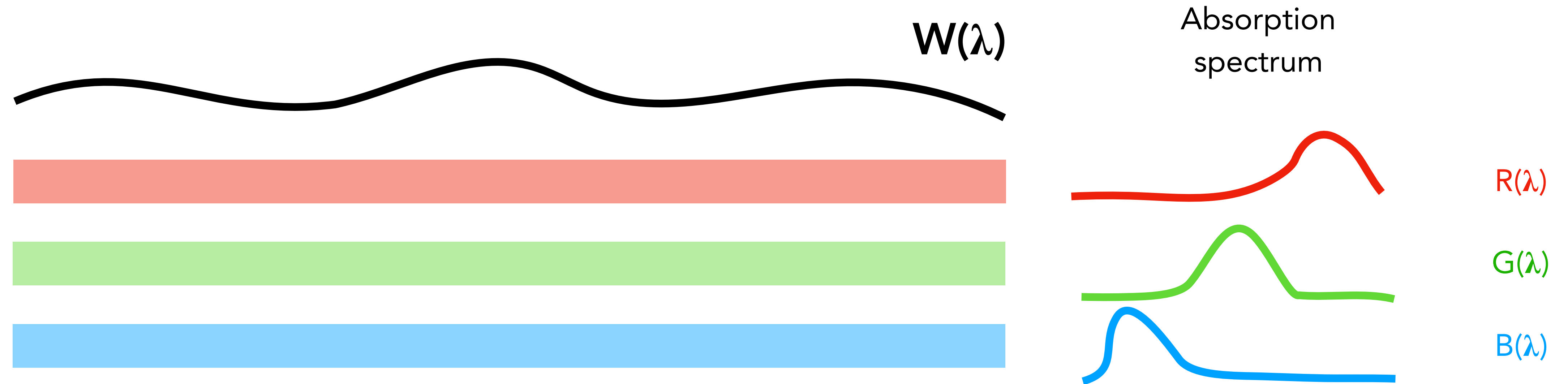
What is Subtractive Color Space Really?



What is Subtractive Color Space Really?

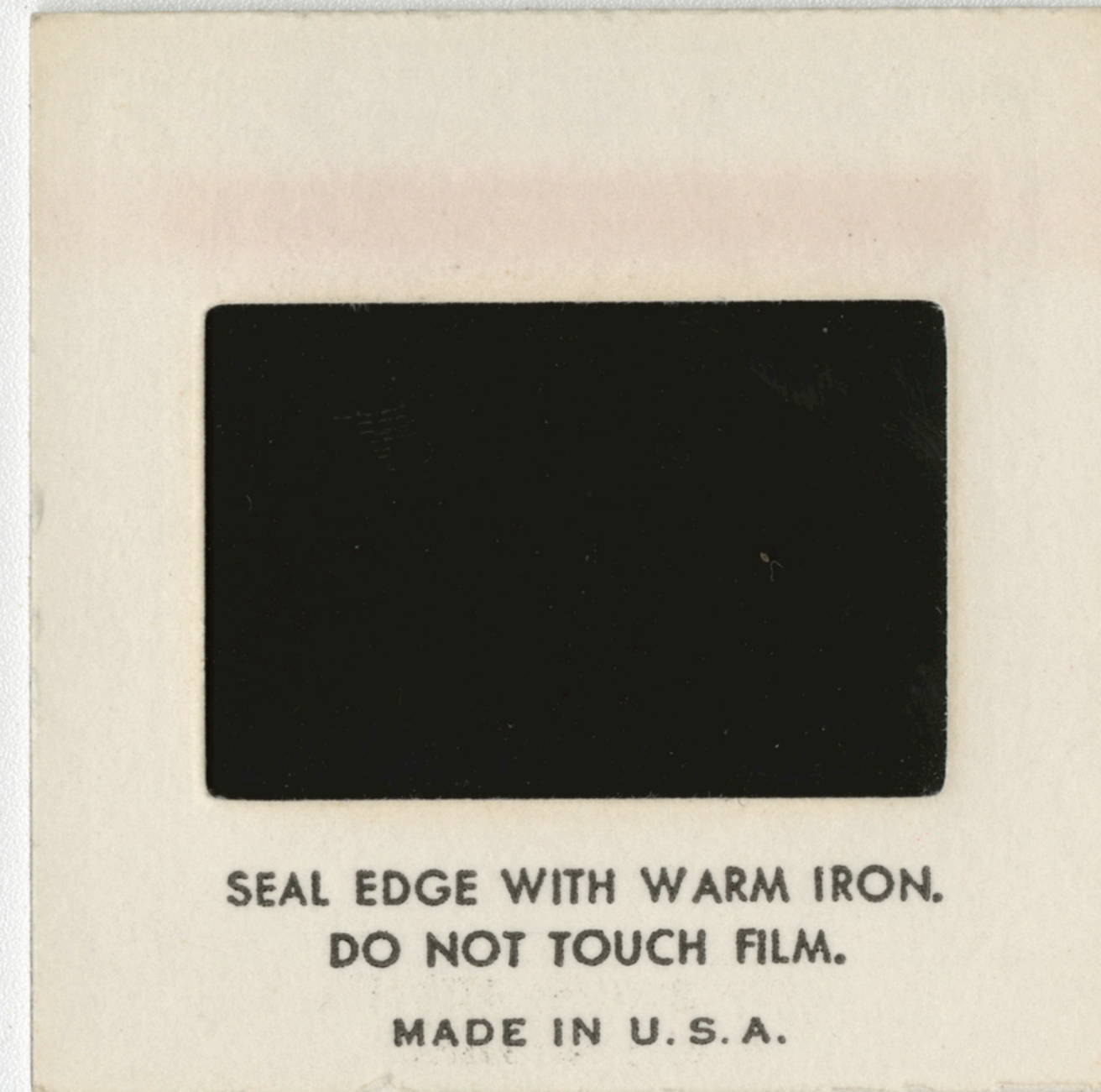
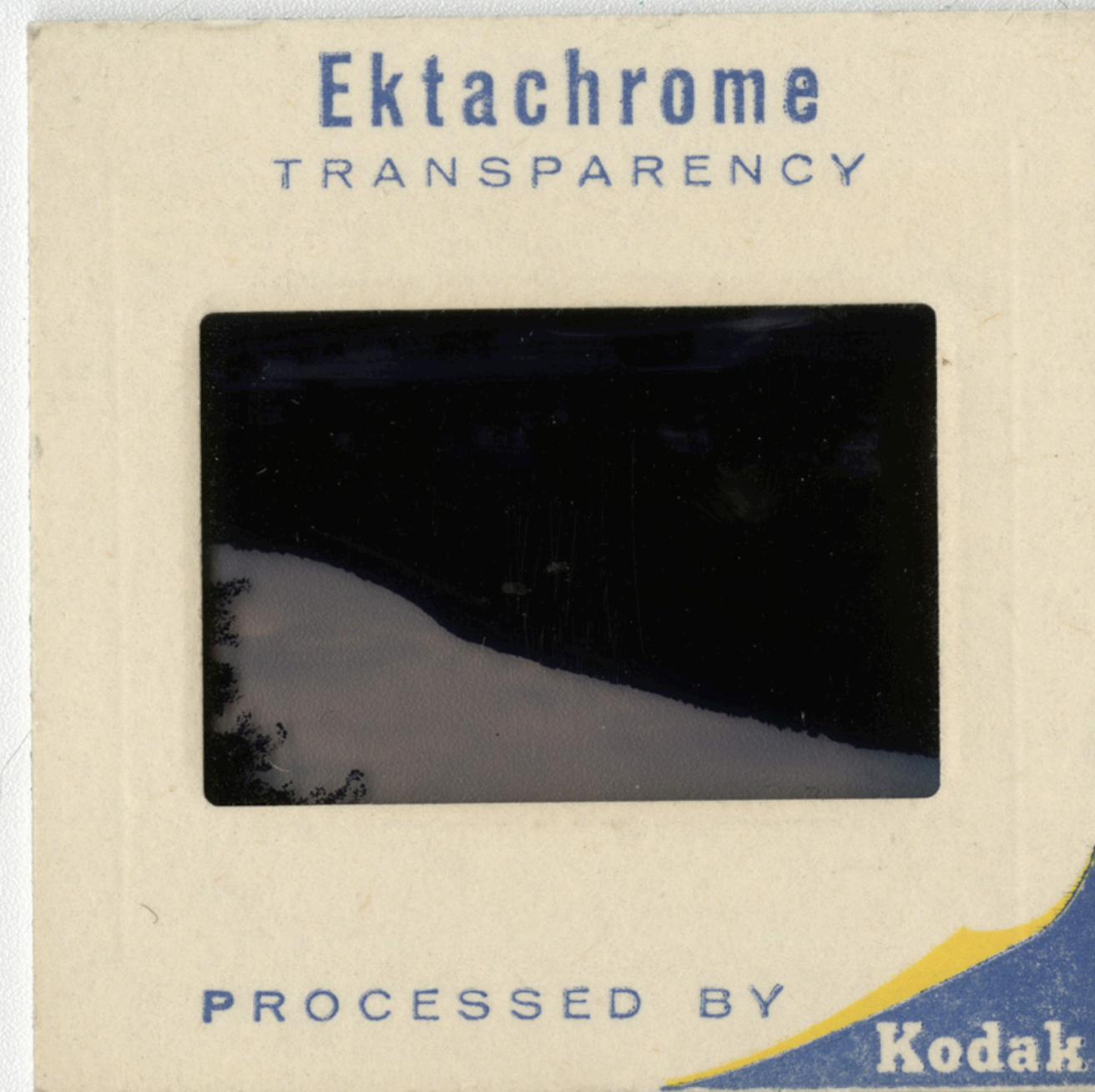
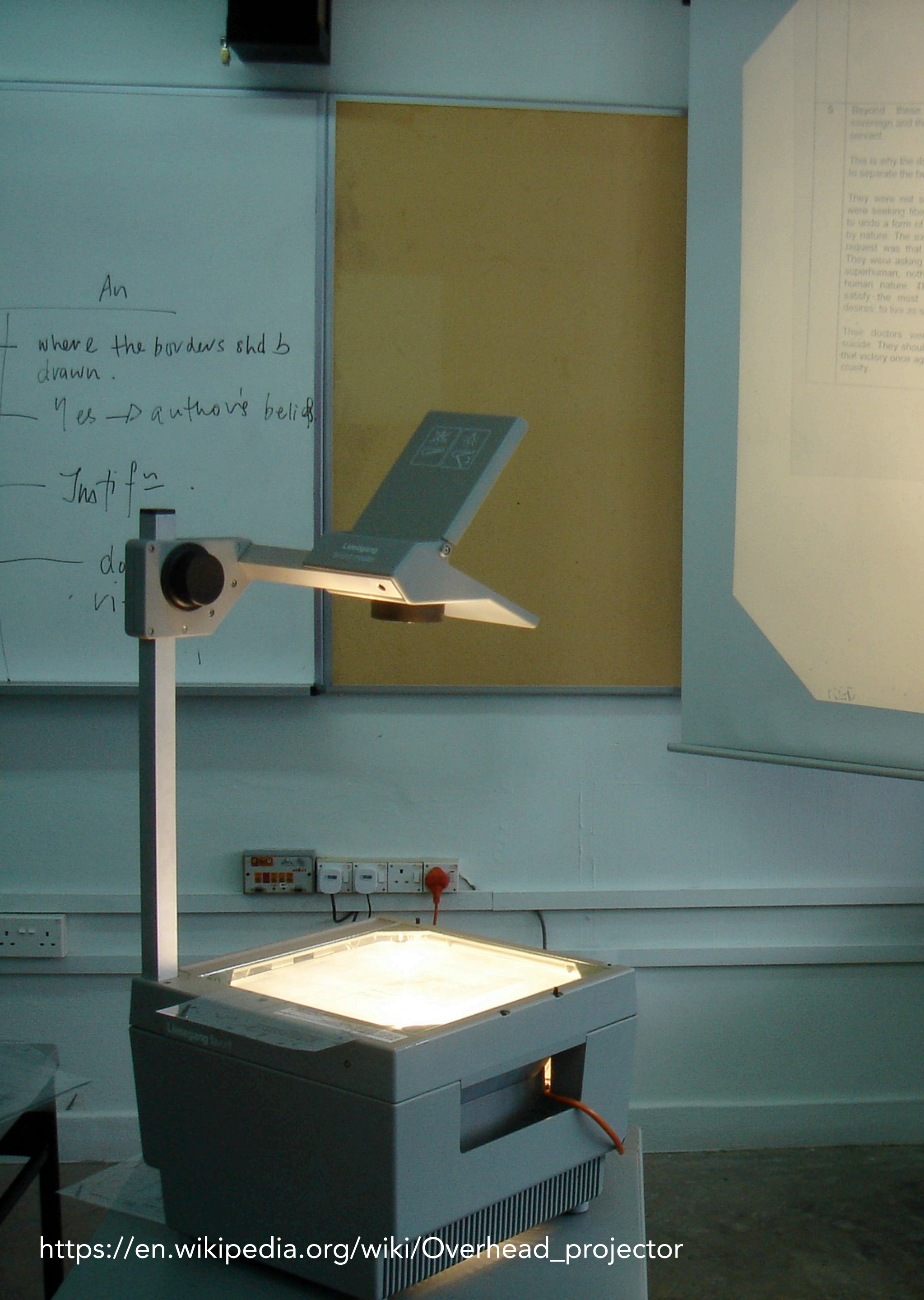


What is Subtractive Color Space Really?



Subtractive color space applies **ONLY** to strictly transparent material.

- Nearly perfect for slides/transparencies printed with dyes, pretty good for traditional photographic prints on photographic media.
- Rather poor for today's digital color printers, because they also **scatter** lights.



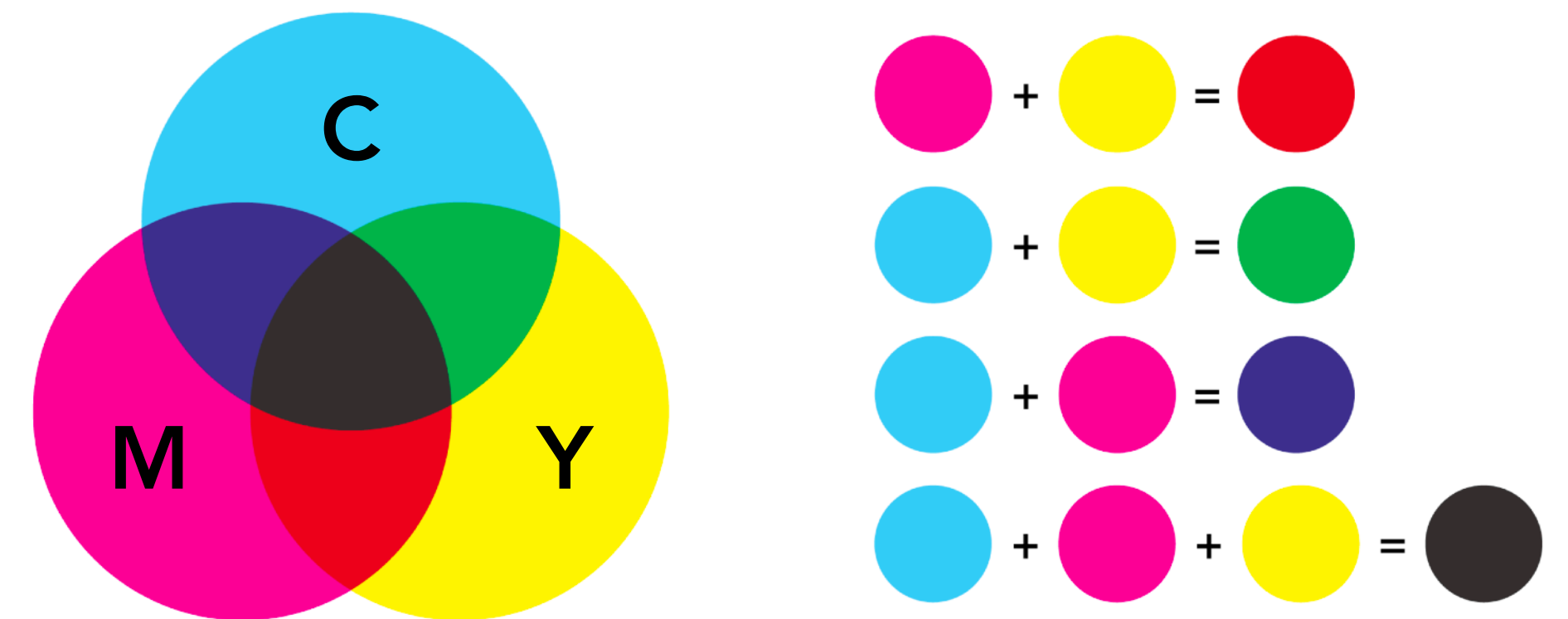
Is Mixing Materials Subtractive?

Conventional wisdom is that subtractive color model should be used when discussing paint/material mixing. Really?

Cyan + Yellow = Green because:

- Cyan material absorbs Red light; Yellow material absorbs Blue light
- Cyan and Yellow mixture would then absorb both Red and Blue, leaving only Green

By this logic, Blue + Yellow = Black/Gray





Mixing Materials is (Generally) NOT Subtractive!

Mixing paints: does subtractive color space apply? A few issues.

- First of all, a paint's color depends on illumination. So it's meaningless to discuss the color of a paint (or any non-emitting material) without illumination information.
- But more importantly paints also **scatter** lights.

To calculate the color of a paint mixture, we should:

- Get the spectral scattering and absorption coefficient for each pigment
- Calculate the reflectance spectrum of the mixture using the K-M model
- Use the spectrum of the illumination to calculate the R, G, B values of the mixture

True or False?

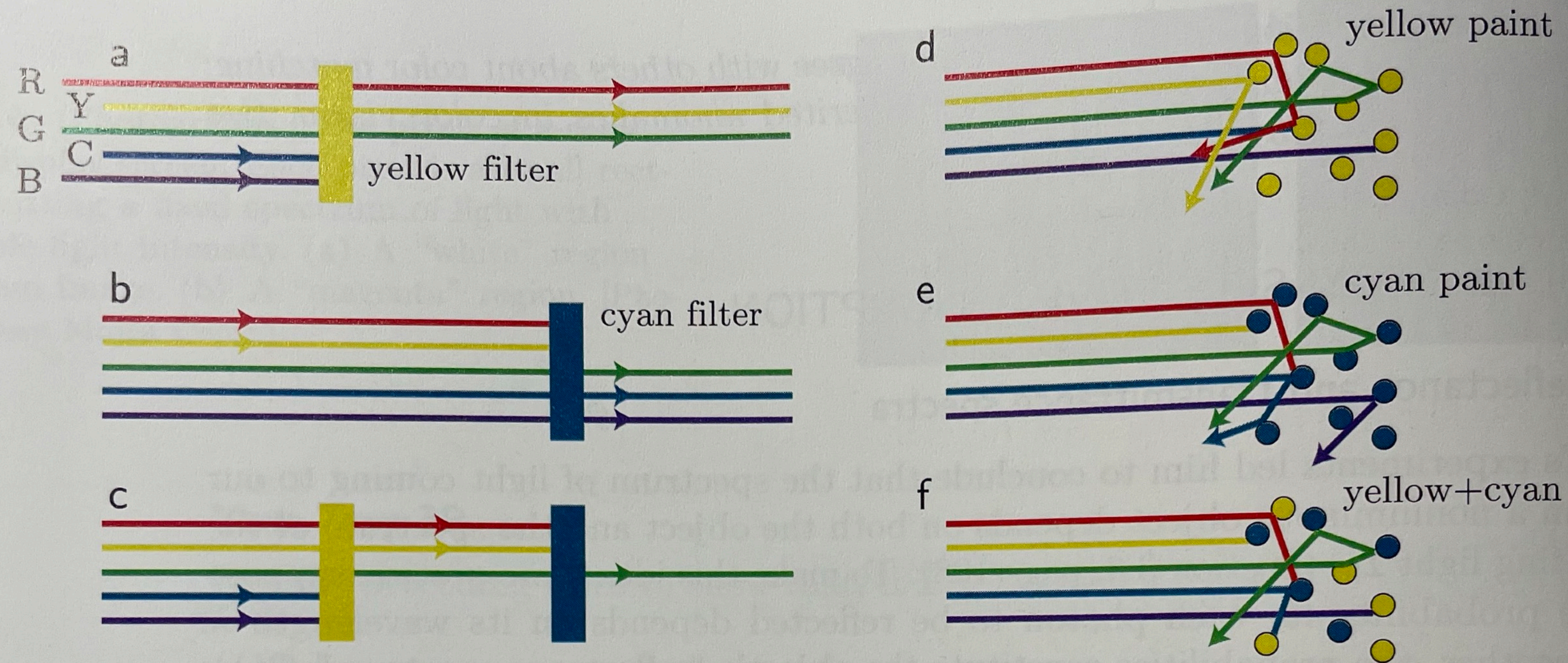
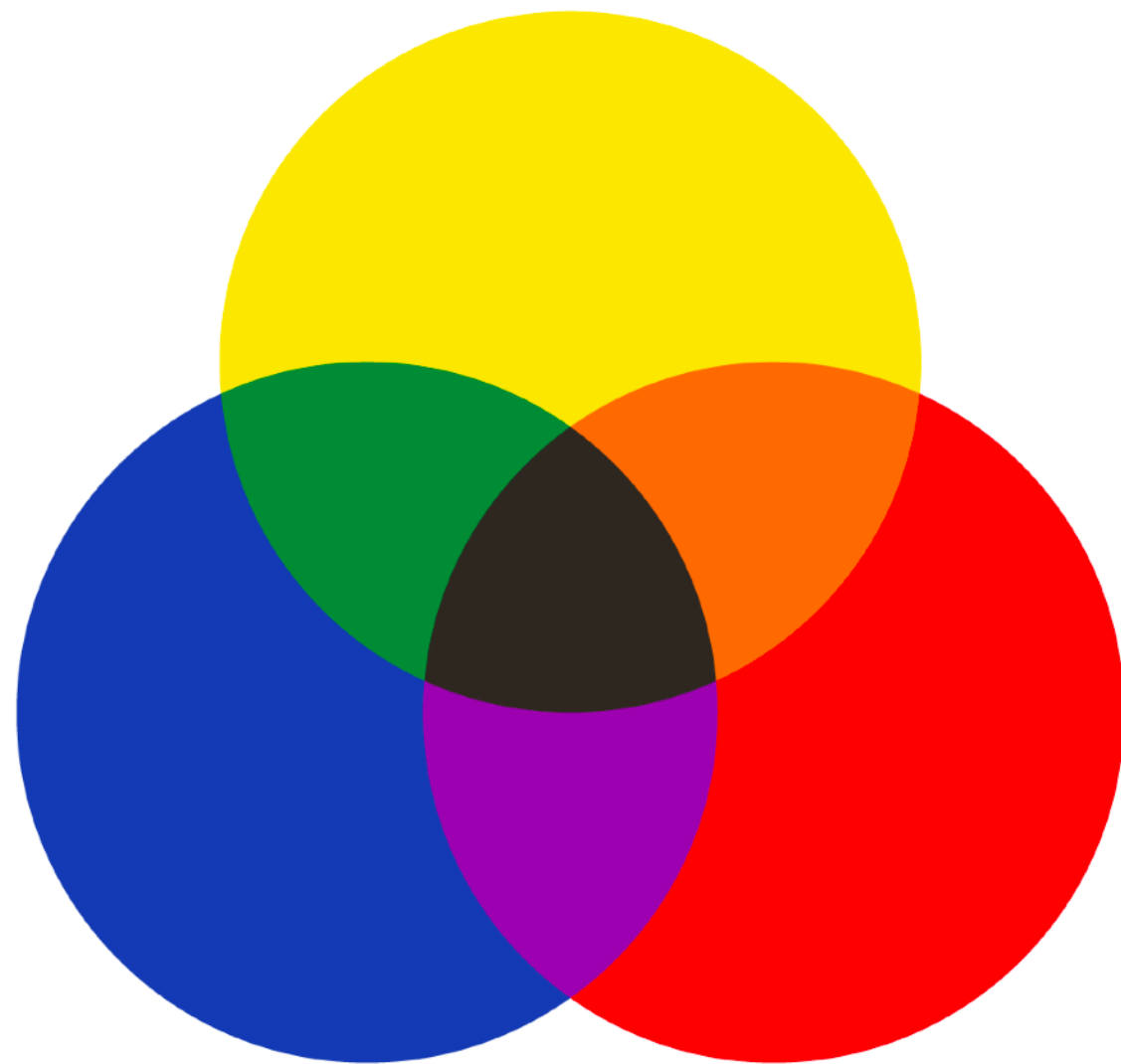


Figure 3.7: [Schematics.] **Subtractive color combination.** (a) A “yellow” filter transmits a band of wavelengths including spectral green, yellow, and red. It blocks other wavelengths, including those corresponding to blue and cyan. (b) Similarly, a “cyan” filter instead blocks red and yellow light. (c) Placing these two filters in succession blocks everything except green light. (d,e,f) Yellow and cyan chromophores are distributed within the fibers of a piece of paper. Multiple scattering of light combines their effects, similarly to (c).

Artists Know Better



The **RYB** model used by artists

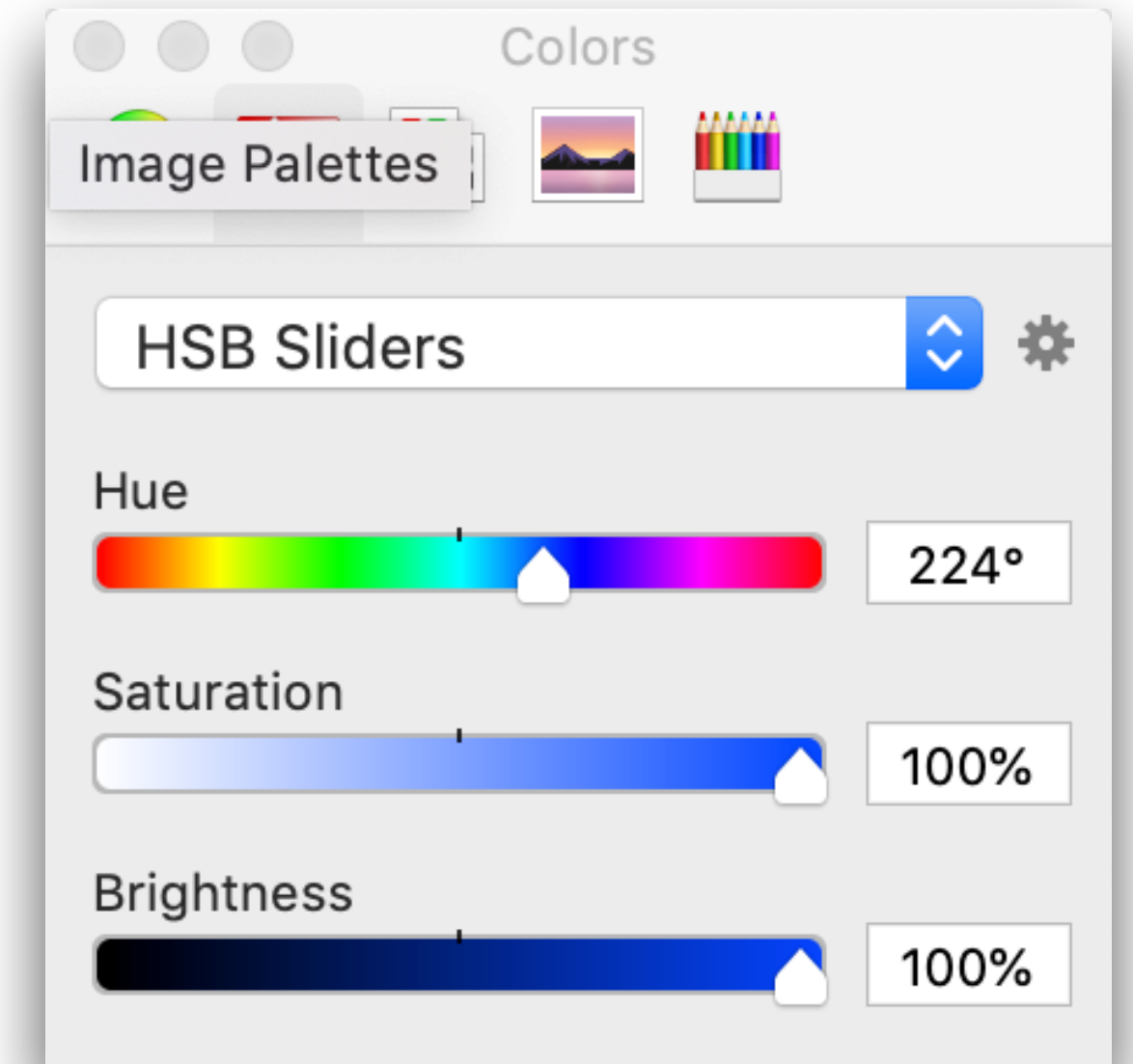
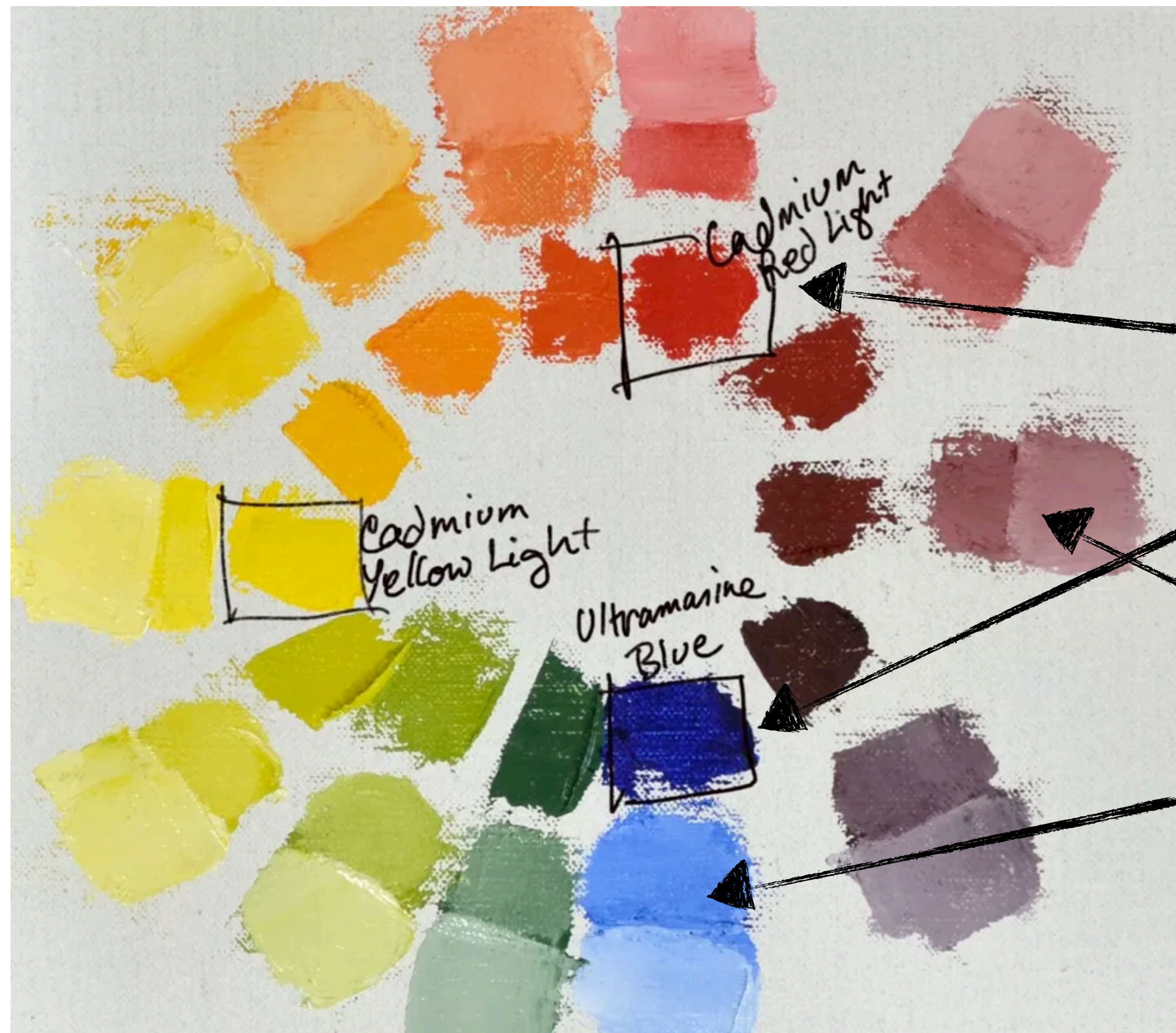


Add White to Adjust Saturation Levels

Skillful painters usually use just three primary paints plus white to create different saturation levels



Masstone vs. Tint



Masstone: pure color

Tint: mixing pure color with white (desaturation)

Spectral reflectance vs. cobalt blue tint

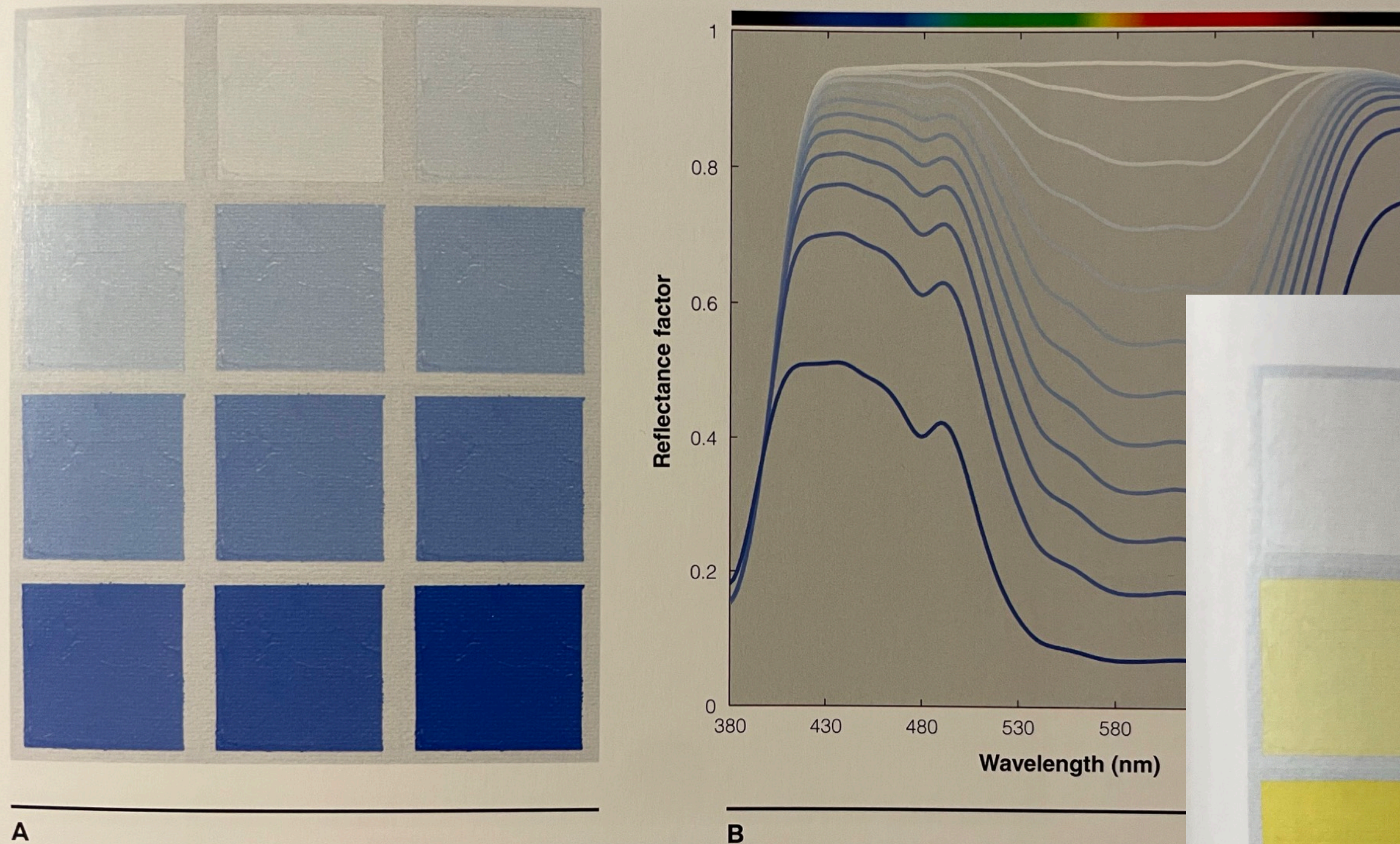


FIGURE 1.6. (A) Tint ladder of titanium white and cobalt blue paint. (B) Reflectance spectra. The high reflectance at very long wavelengths and the dip-like shape at short wavelengths are characteristic for cobalt blue, in a binding medium.

Spectral reflectance vs. arylide yellow tint

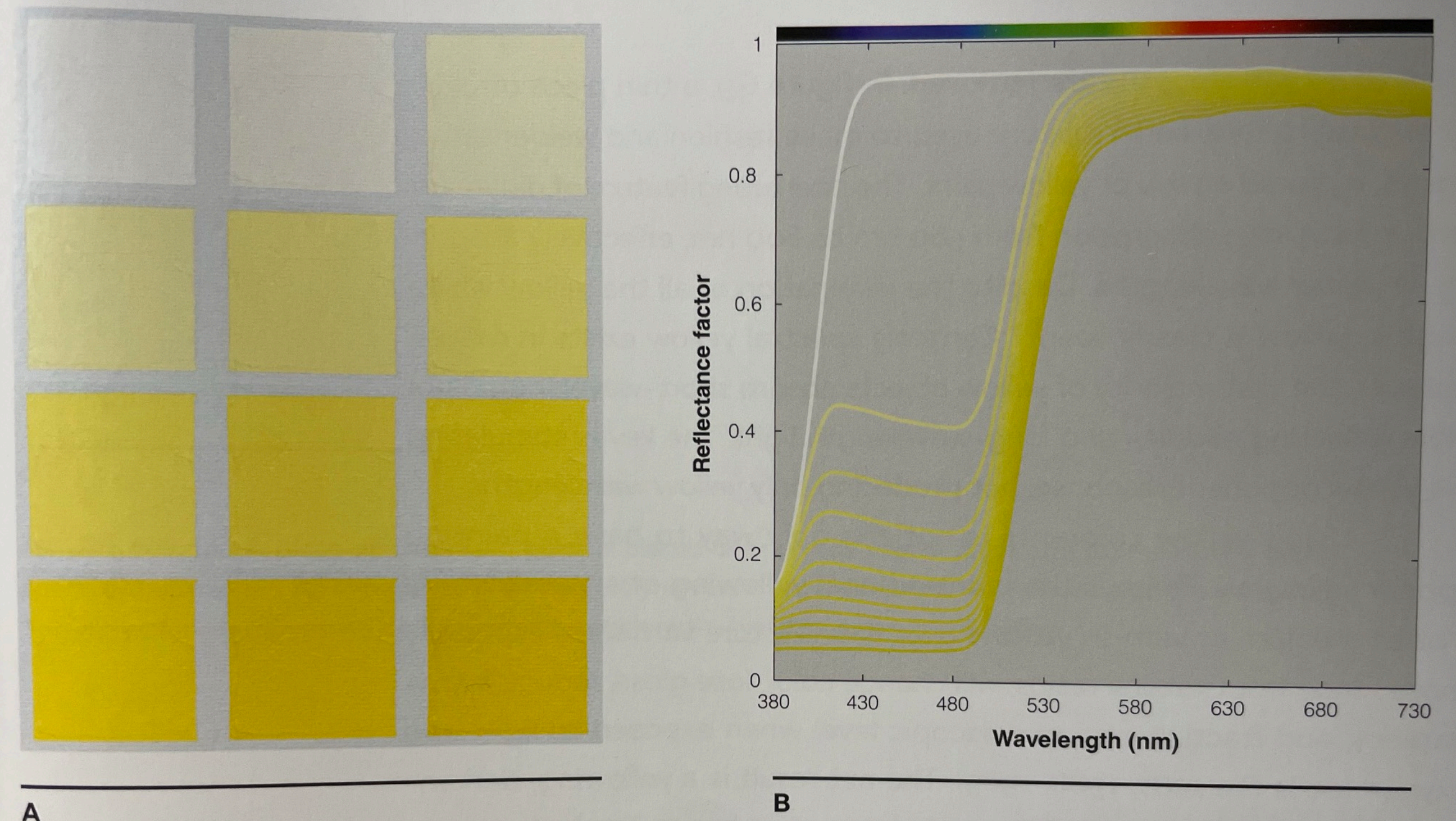


FIGURE 6.7. (A) Tint ladder of arylide yellow and titanium white, with (B) its reflectance spectra. With an increase in the concentration of yellow, absorption of short-wavelength light increases, reducing reflectance.

Applicability of K-M Model

A precise theoretical material model would require modeling of both the 3D microstructure and the non-isotropic nature of the flux. However, we will almost never have the data required for such modeling and therefore have to rely on simplifying assumptions and often empirical modifications, which is what K-M does.

The accuracy is determined by the validity of the simplifications and how well the empirical modifications are able to fit the data. At some point, the empirical modifications are more like parametric functions to **fit observations** rather than rigorously justified physical models.

Painting vs. Dyeing vs. Printing

Painting vs. Dyeing vs. Printing

Similarities:

- Mixing colorants to produce color
- All have a substrate (e.g., canvas vs. textile vs. paper) on which colorants are applied

The Kubelka-Munk model has been applied to all three areas for predicting colors, reverse-engineering colorants given a sample, etc.





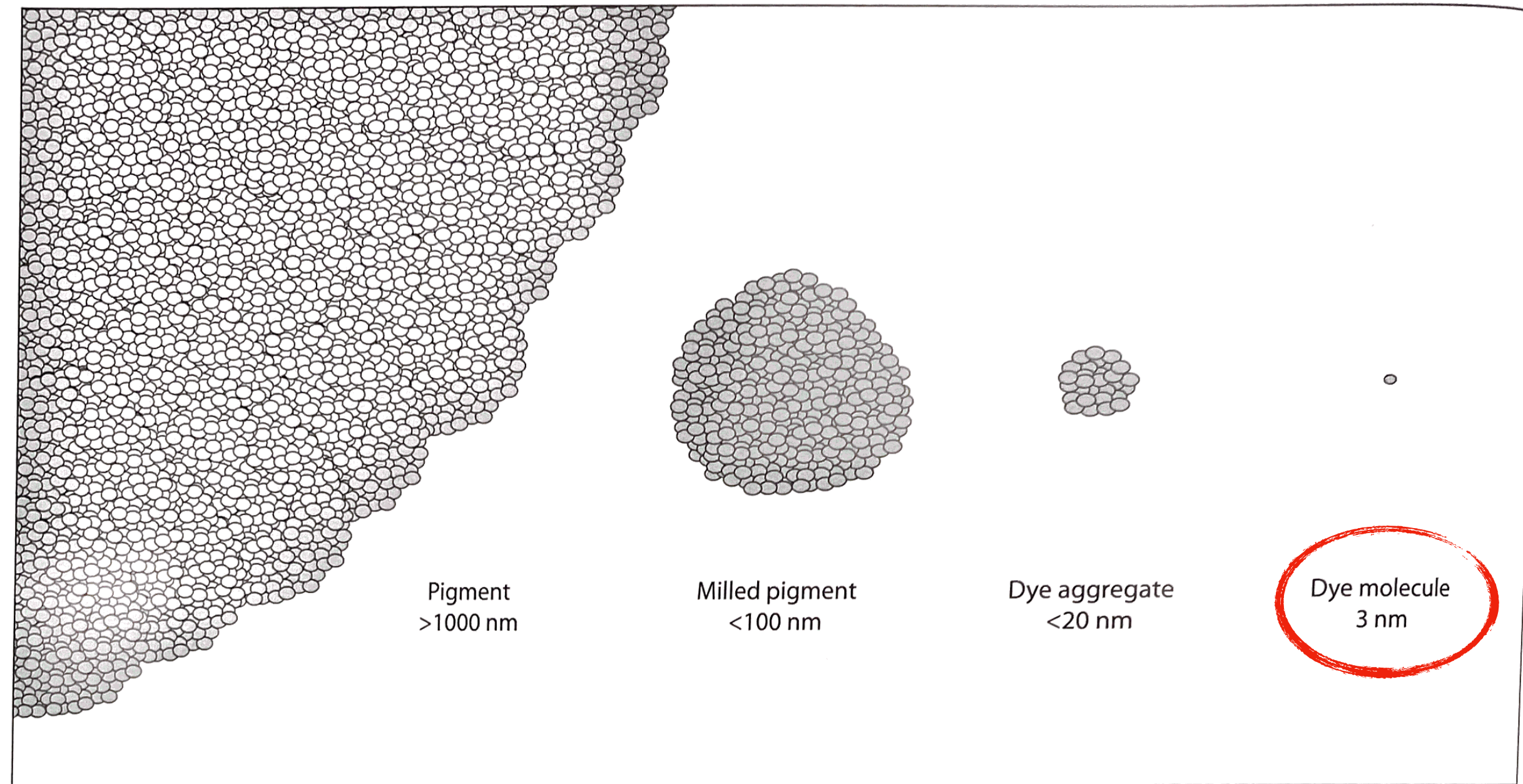
Colorant Differences

Painting uses **pigments**, and dyeing uses **dyes**.

- Dye molecules are orders of magnitude smaller than pigment particles.
- Dyes are water soluble (chemically bond with the water molecules).
- So dyes scattering efficiency is quite low, mostly transparent/translucent.

Printing inks can either be pigments or dyes.

Colorant Differences



Layering vs. Mixing

Painting can be layering or mixing depending on what artists decide to do.

- Glazing/vanishing are translucent layers.
- Pentimento is intentionally painting over/hiding under-layers.

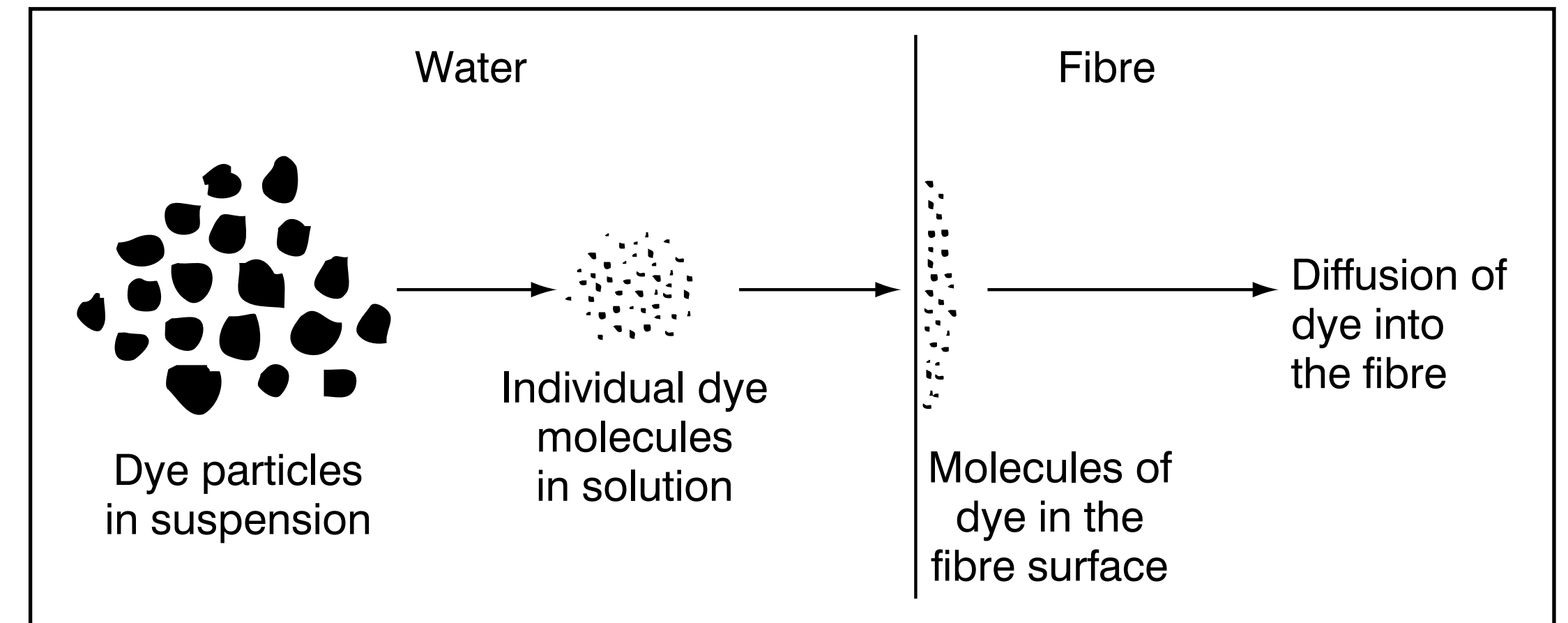
Dyeing colorants are usually (pre-)mixed.

Printing inks are complicated; they are neither completely layered nor completely mixed.

Substrate Interaction

Painting: paint usually completely hides the substrate so the color usually doesn't depend on the substrate.

Dyeing: dyes are molecularly dispersed into the substrate (fibers). Dyes, while absorb lights like crazy, do not scatter nearly as much as the fibers, which contribute to most of the scattering and participate in color formation.



An impressionistic painting of a mountain landscape. The foreground shows a path or stream winding through green foliage. The middle ground features a dark, dense forest. The background is dominated by a large, rugged mountain peak with patches of white, possibly snow or light-colored rock. The sky is a mix of light blue and white, suggesting a bright, hazy day. The brushstrokes are thick and visible, giving the painting a textured, expressive feel.

Dyes usually penetrate directly into the fibers.
The scattering efficiency of dyes is low.



Substrate Interaction

Printing: somewhat in-between.

- The ink-mixture penetrates into the substrate (paper), but doesn't completely absorb lights (you can still see light from the other side!)
- So the paper contributes to color formation and is effectively just another colorant in the K-M model.

Cross sections of Epson Ultrachrome 7 pigment-based ink printed on paper. Pigment particles are larger. They are not fully penetrated.

Cross sections of Epson R340 dye-based ink printed on paper. Dye particles are smaller; they penetrate into paper well.

