CSC 252: Computer Organization
Spring 2021: Lecture 2

Instructor: Yuhao Zhu

Department of Computer Science
University of Rochester

Action Items:

- **Get CSUG account**
- **Make sure you have VPN setup!!!!**
- **Sign up for Blackboard**

Announcement

- Make sure you can access CSUG machines!!!
- Programming assignment 1 will be posted today.
  - It is in C language. Seek help from TAs.
  - TAs are best positioned to answer your questions about programming assignments!!!
- Programming assignments do NOT repeat the lecture materials. They ask you to synthesize what you have learned from the lectures and work out something new.

Previously in 252…

Problem

Algorithm

Program

Instruction Set Architecture (ISA)

Microarchitecture

Circuit

Previously in 252…

Problem

Algorithm

Program

Instruction Set Architecture (ISA)

ISA is the contract between software and hardware.

Microarchitecture

Circuit

Previously in 252…

Problem

Algorithm

	Renting
Service provider	Landlord
Service receiver	YOU
Contract	Lease
Contract's language	Natural language (e.g., English)

Circuit

Previously in 252…

Problem

Algorithm

	Renting	Computing
Service provider	Landlord	Hardware
Service receiver	YOU	Software
Contract	Lease	ISA
Contract's language	Natural language (e.g., English)	Assembly programming language

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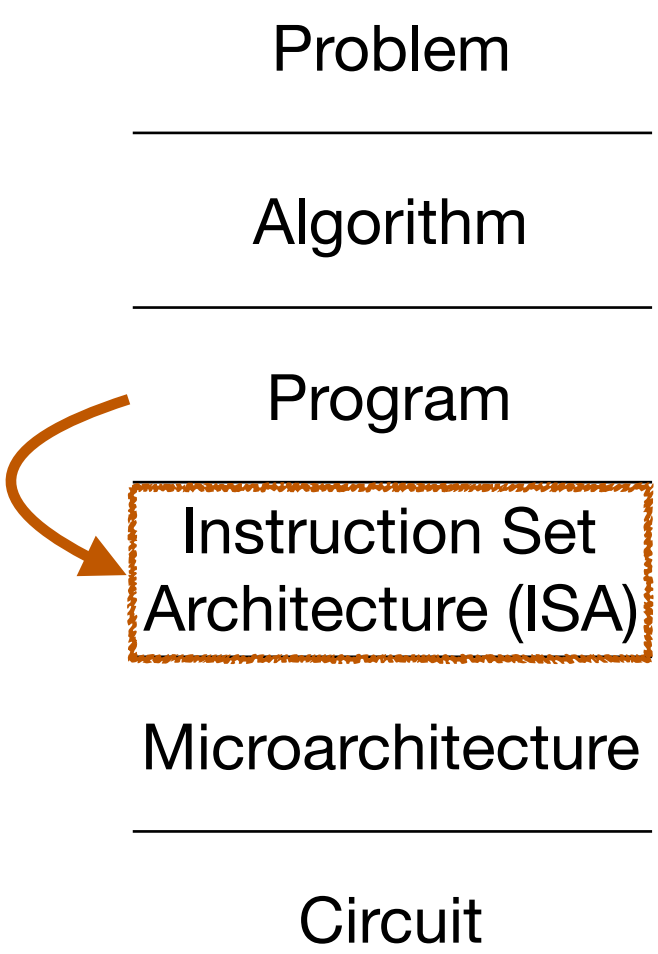
Circuit

Previously in 252…

- How is a human-readable program translated to a representation that computers can understand?



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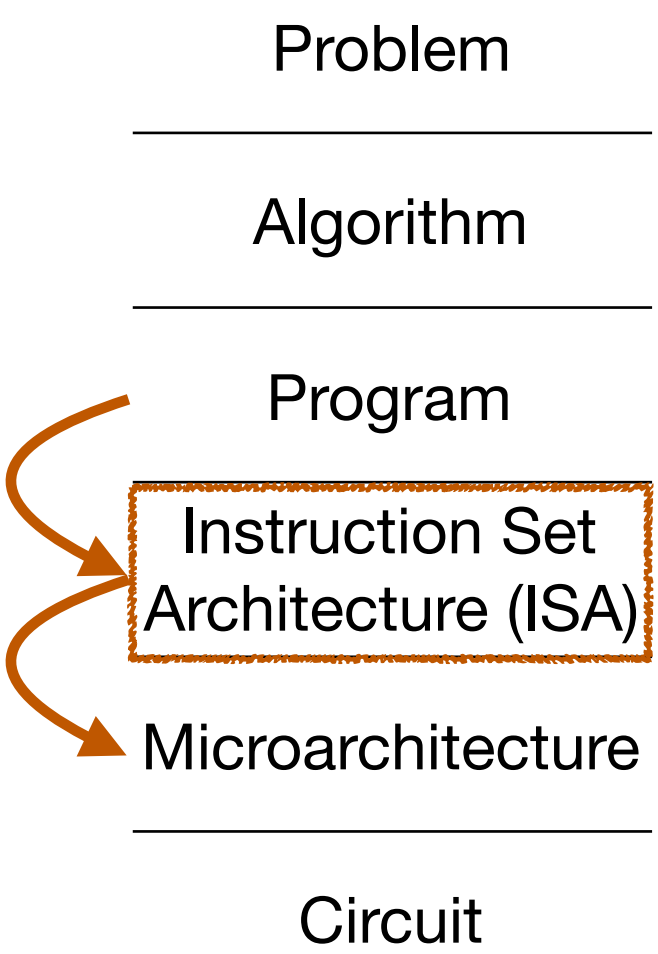
Previously in 252…

- How is a human-readable program translated to a representation that computers can understand?
- How does a modern computer execute that program?



ISA is the contract between software and hardware.

Previously in 252…

C Program

```
void add() {
  int a = 1;
  int b = 2;
  int c = a + b;
}
```

Assembly program

```
movl    $1, -4(%rbp)
movl    $2, -8(%rbp)
movl    -4(%rbp), %eax
addl    -8(%rbp), %eax
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Previously in 252…

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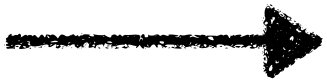

Executable Binary

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01101010   …
11010101   …
01110001   …
```

Previously in 252…

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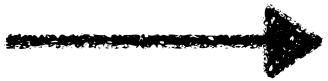

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- What's the difference between an assembly program and an executable binary?
  - They refer to the same thing — a list of instructions that the software asks the hardware to perform
  - They are just different representations
- Instruction = Operator + Operand(s)

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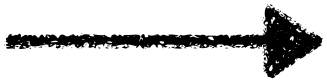

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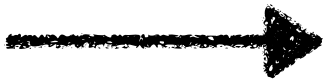

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Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

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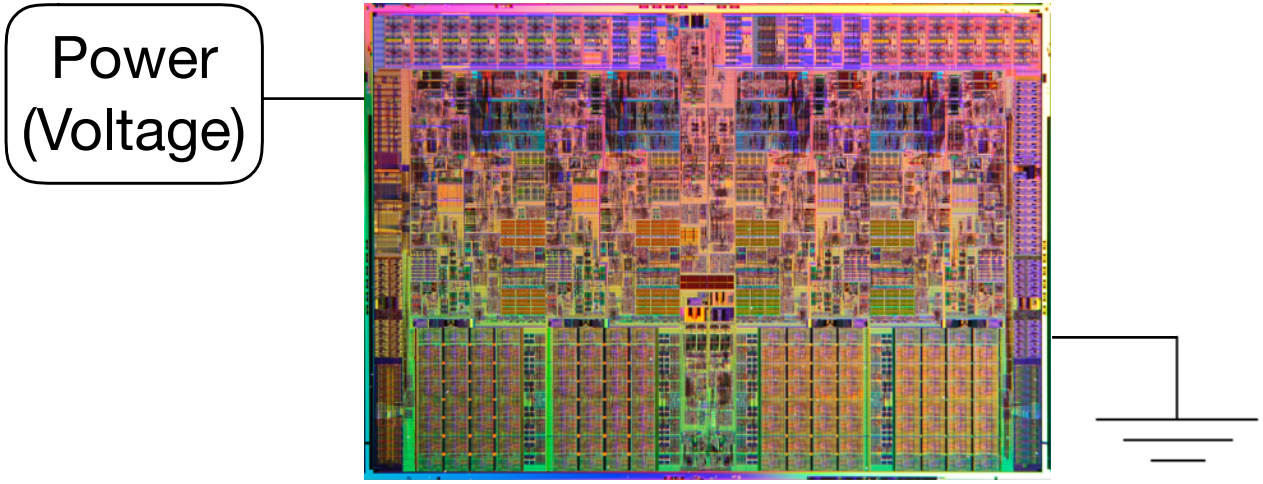

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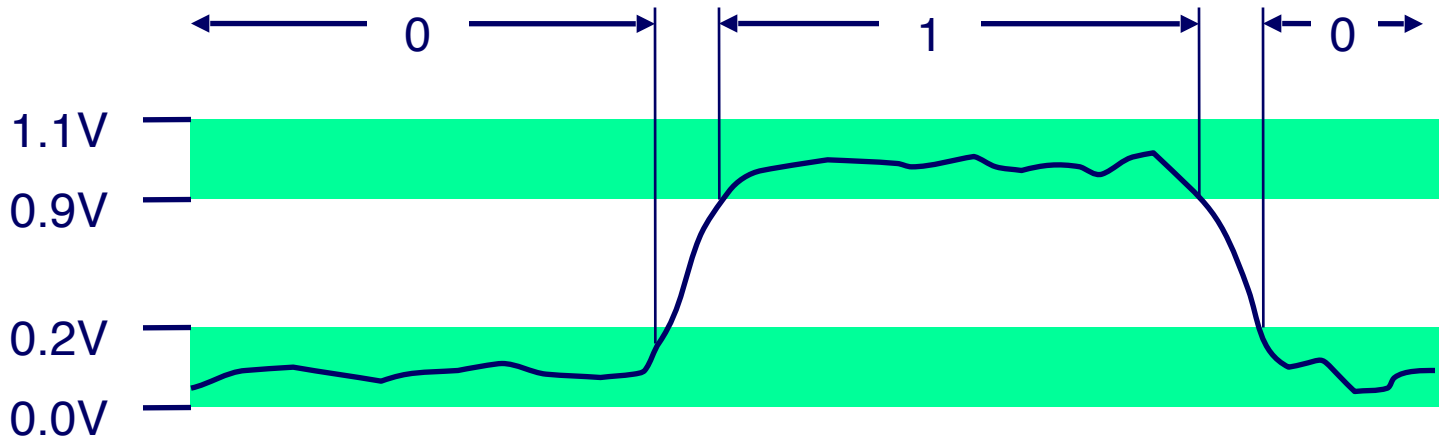

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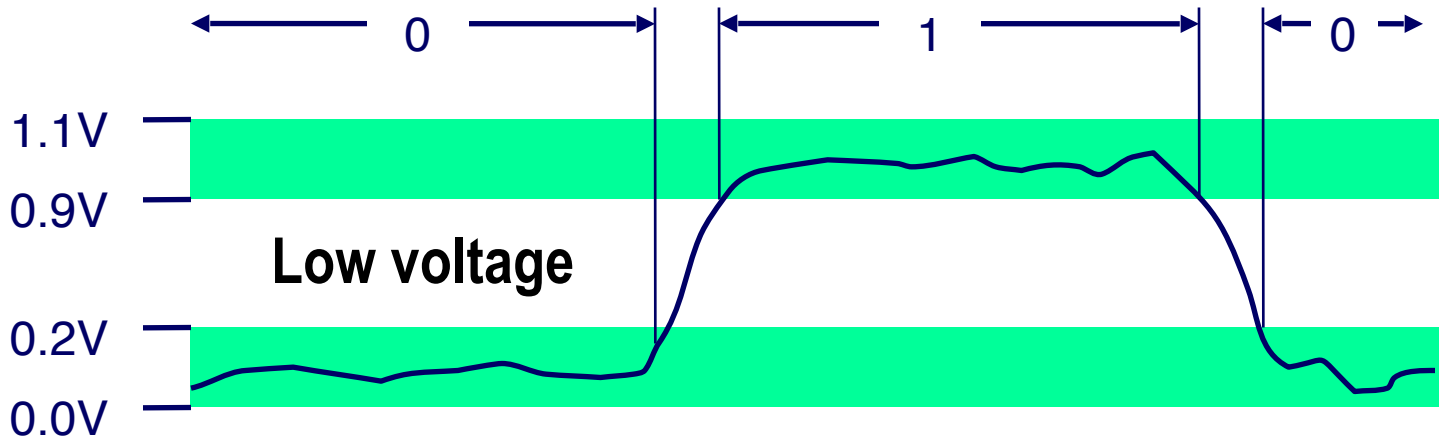

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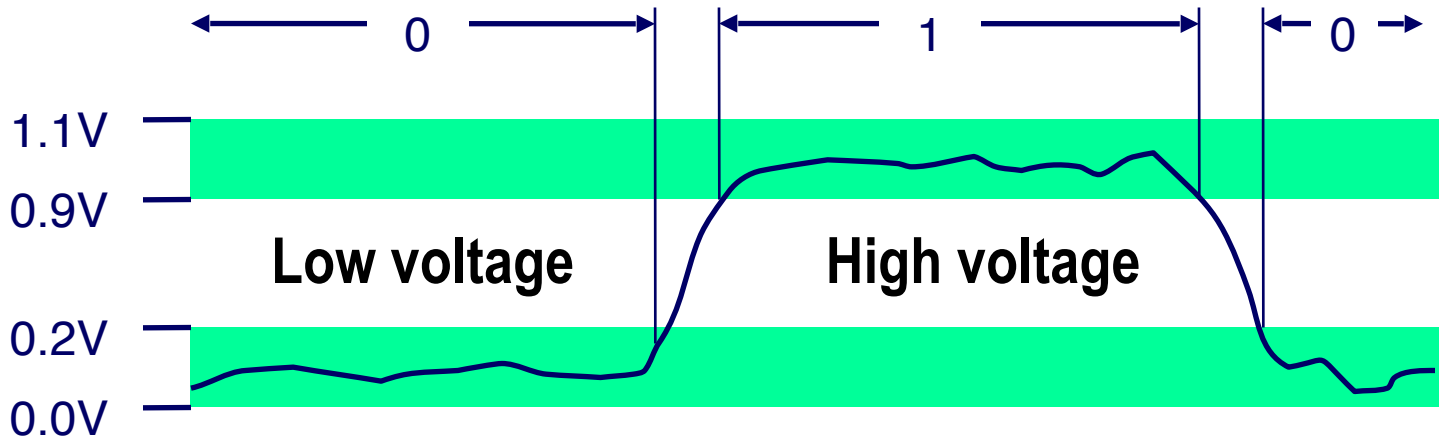

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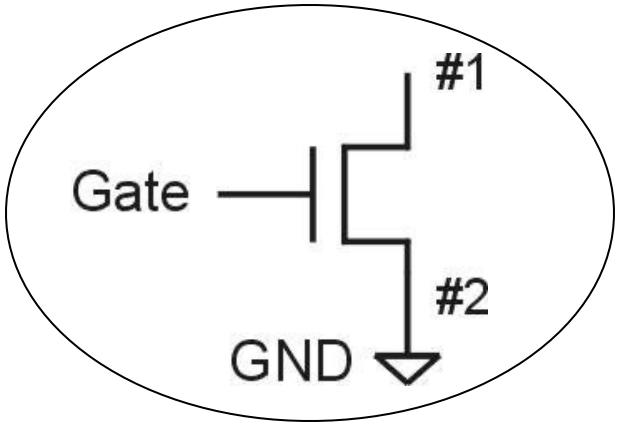

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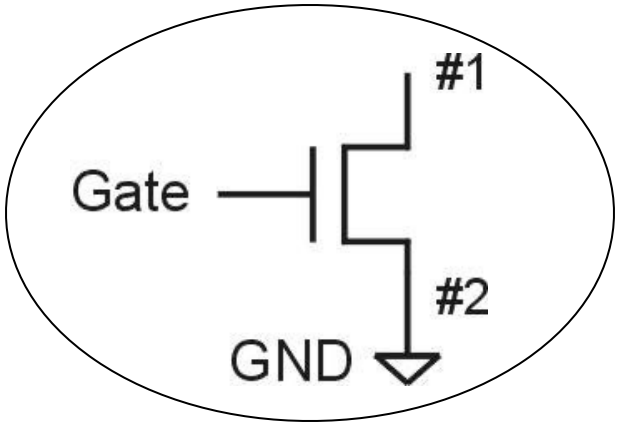

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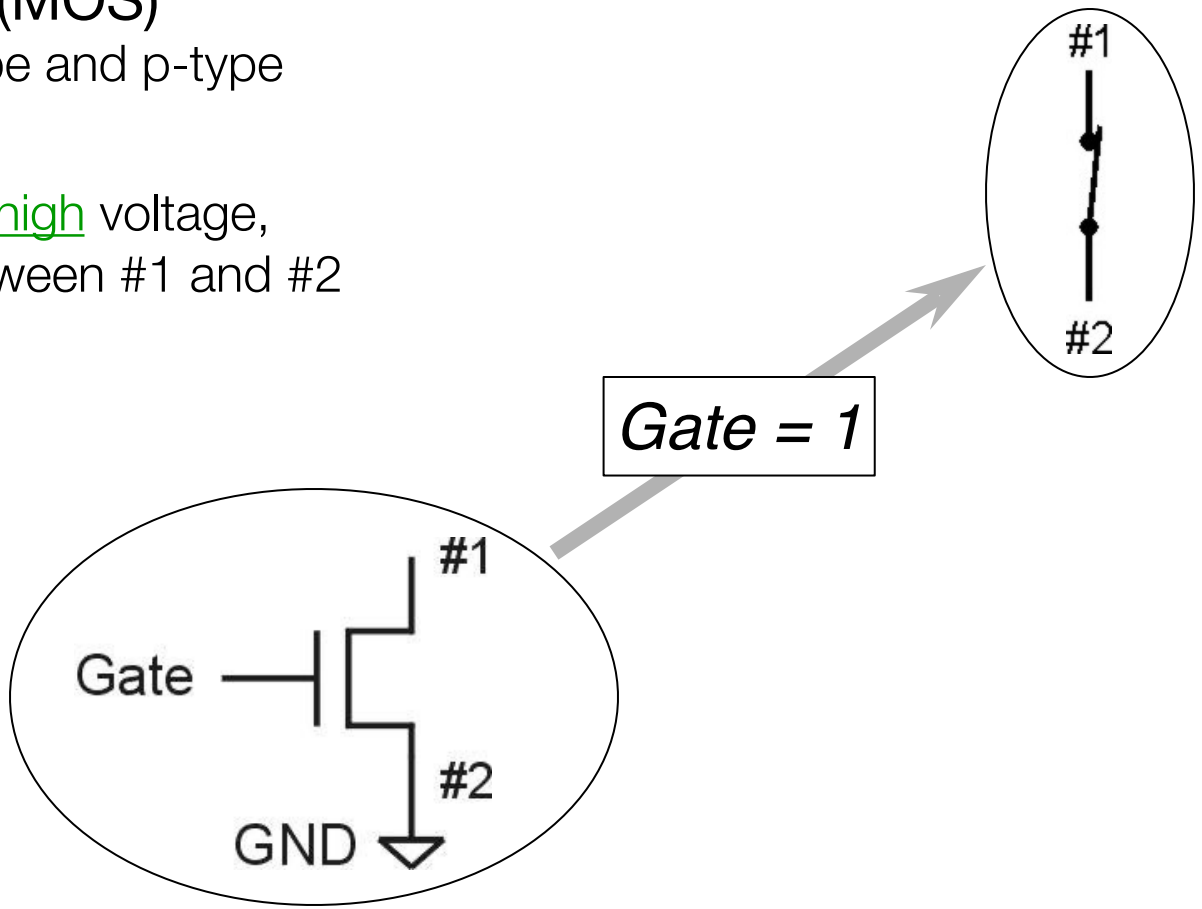

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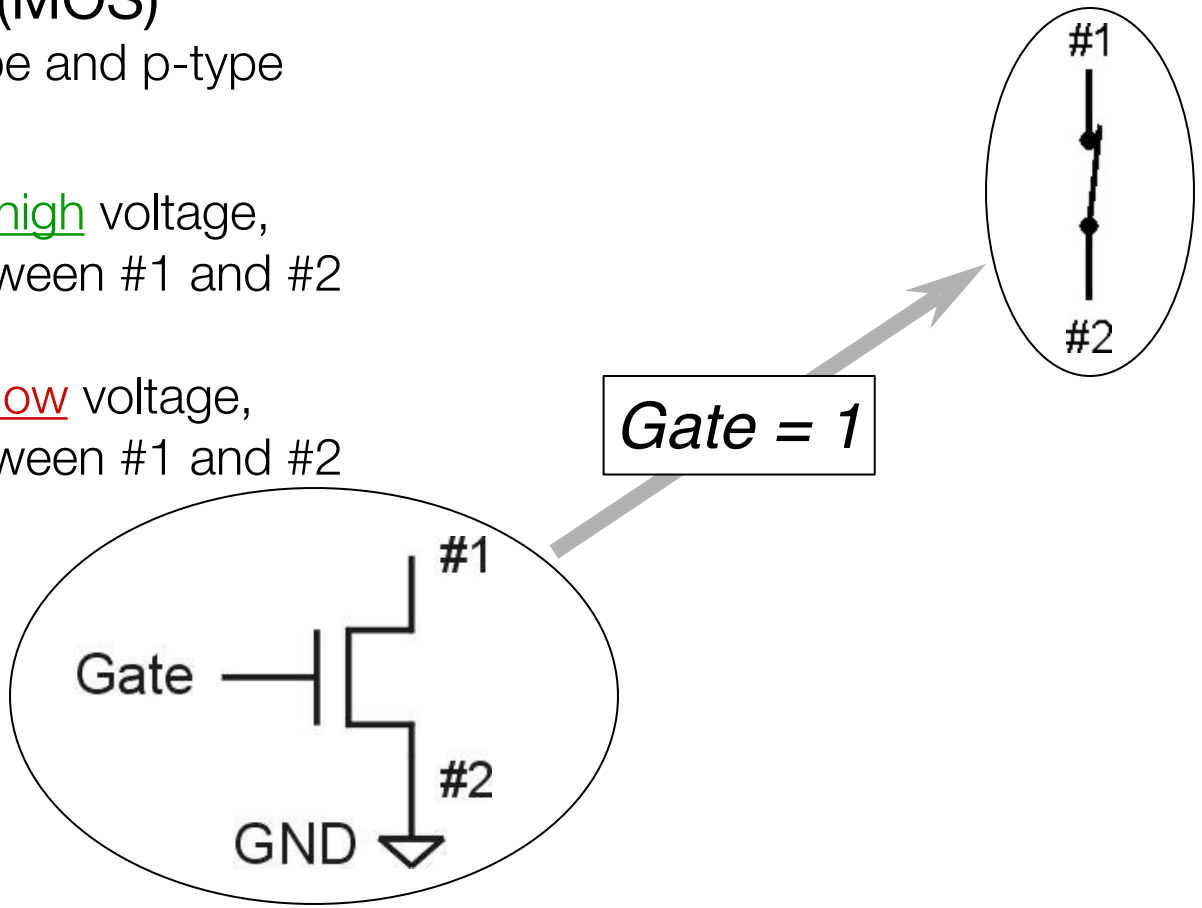

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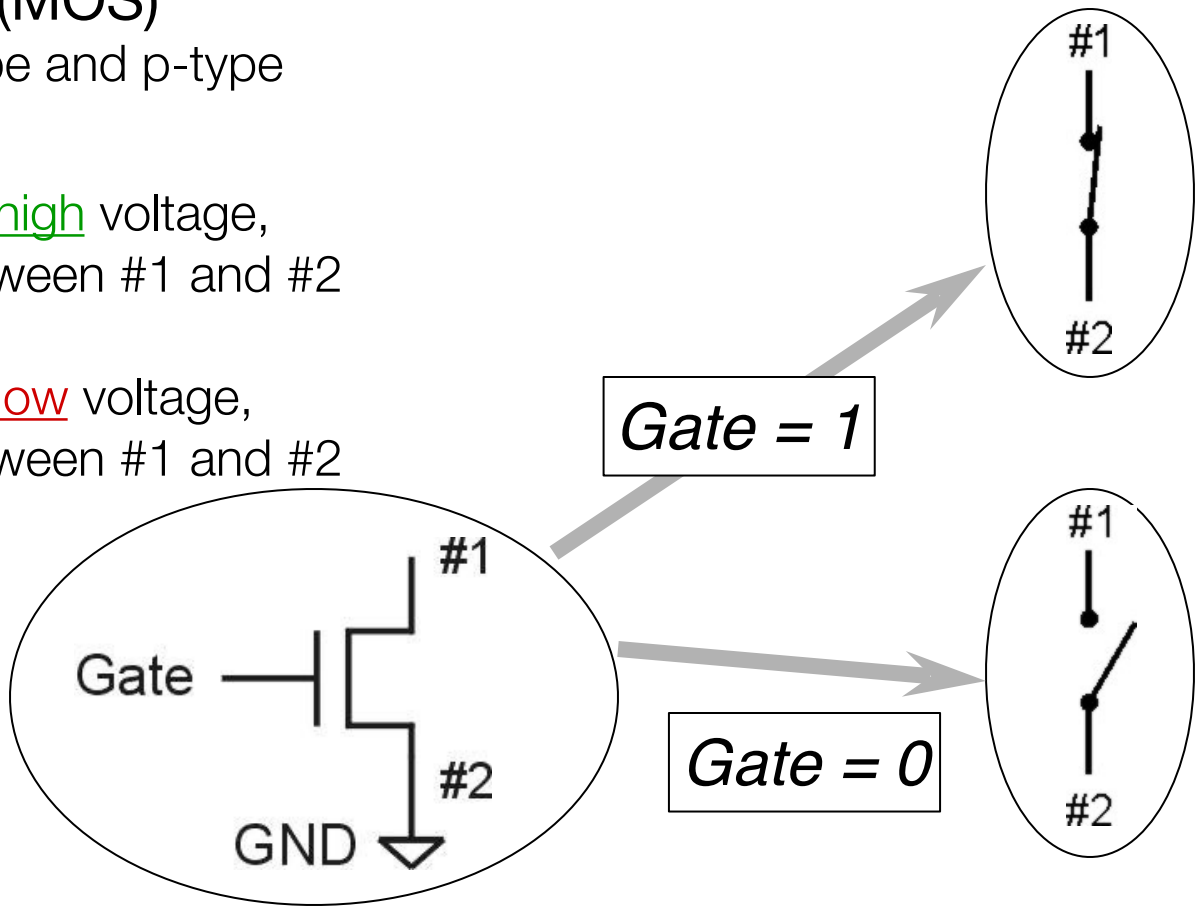

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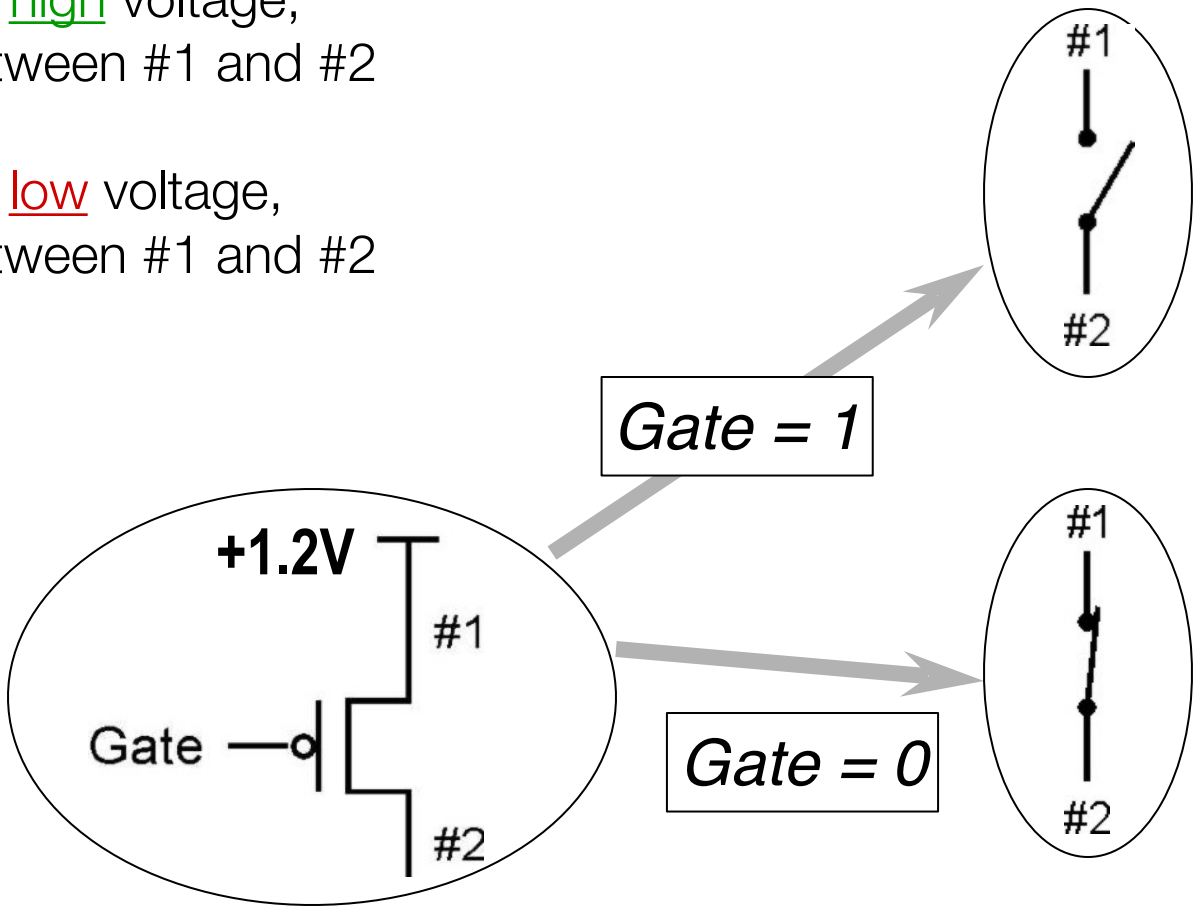
Why Bits?

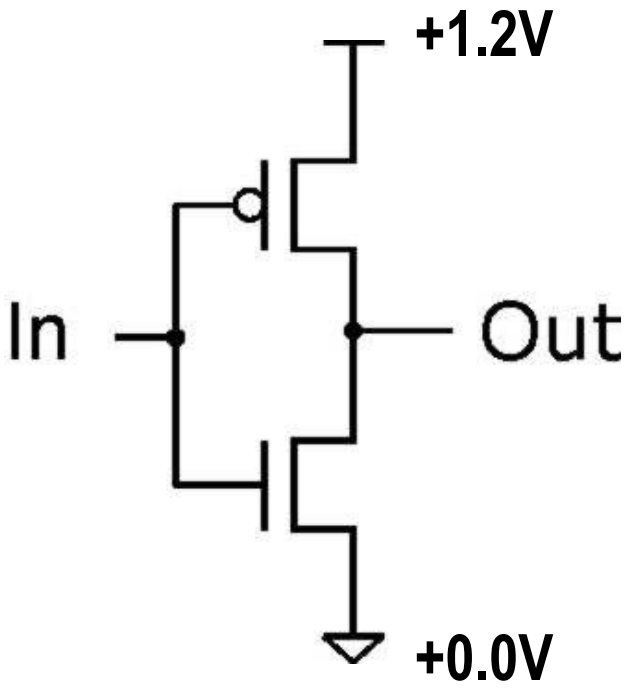
p-type is *complementary* to n-type (PMOS)

- when Gate has high voltage,
  open circuit between #1 and #2
  (switch open)
- when Gate has low voltage,
  short circuit between #1 and #2
  (switch closed)



Terminal #1 must be
connected to +1.2V

Inverter



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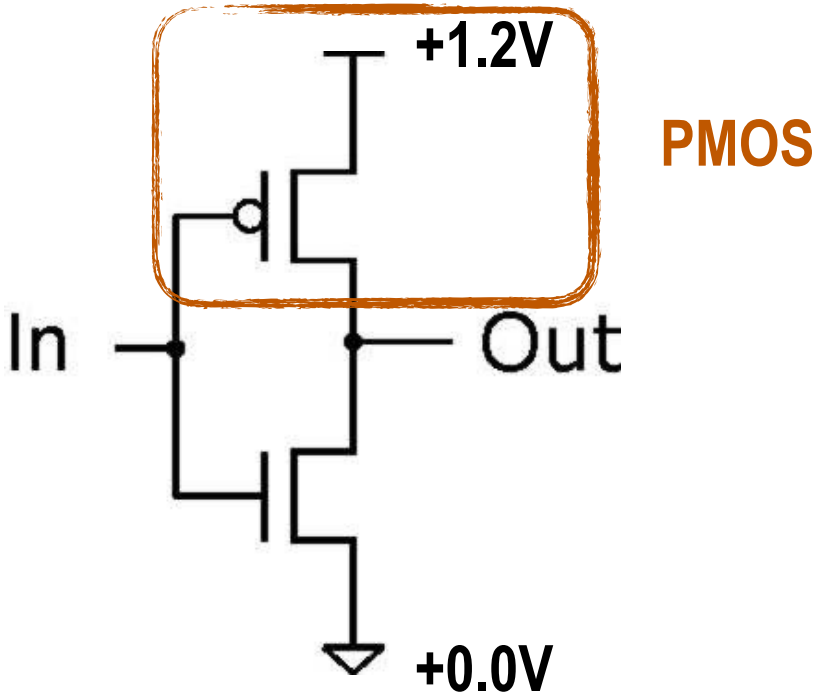

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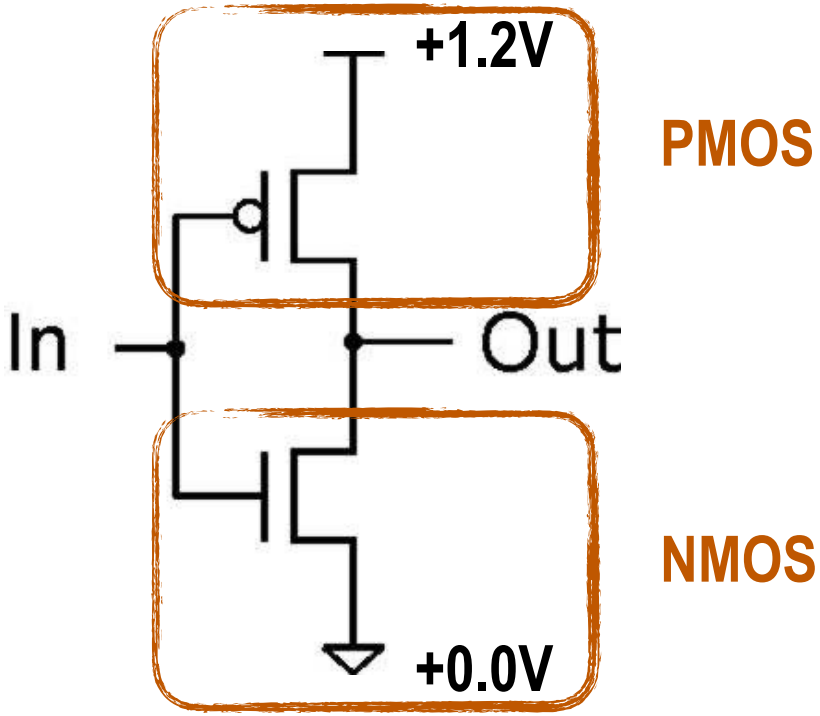

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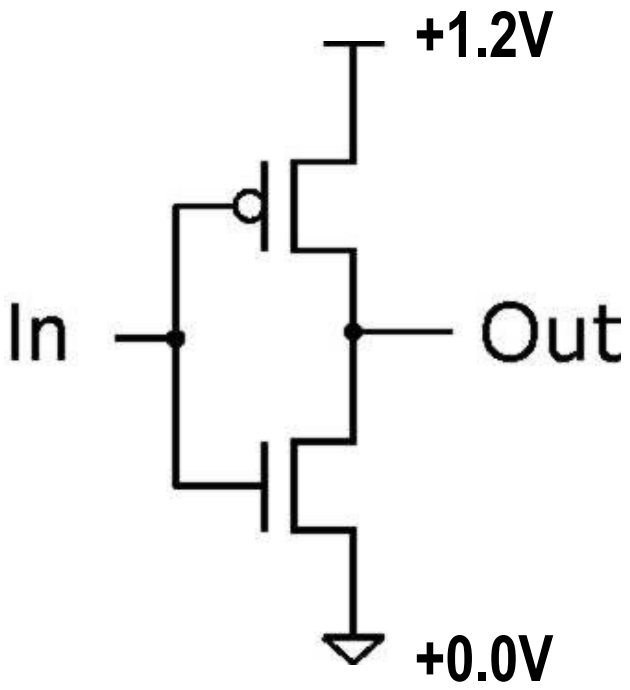

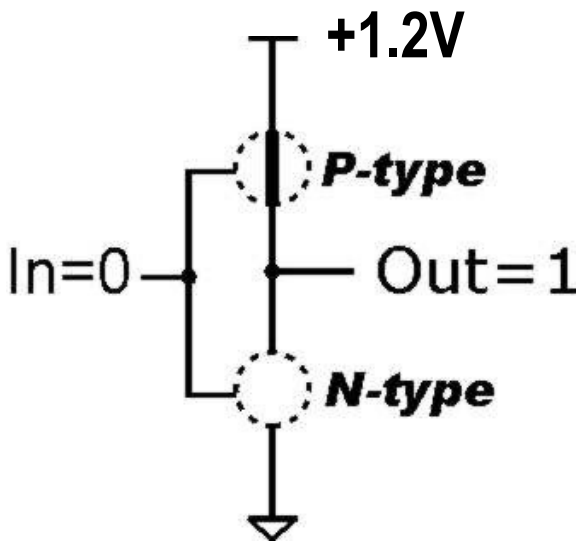

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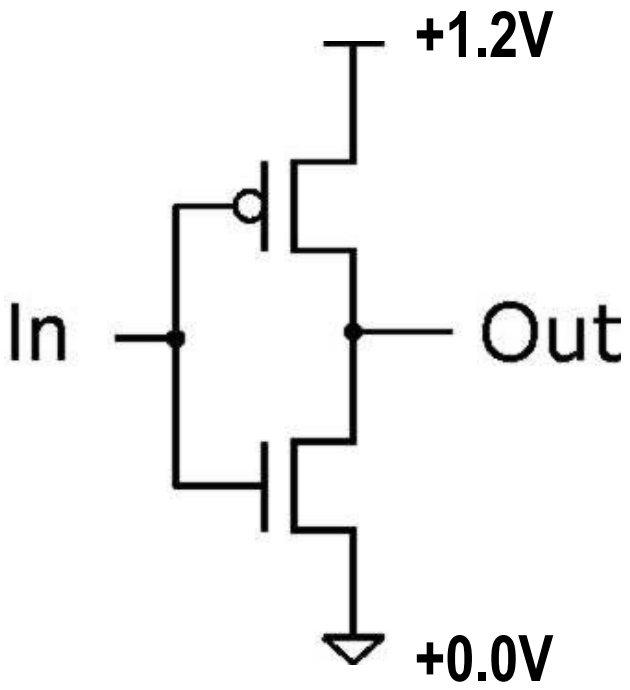
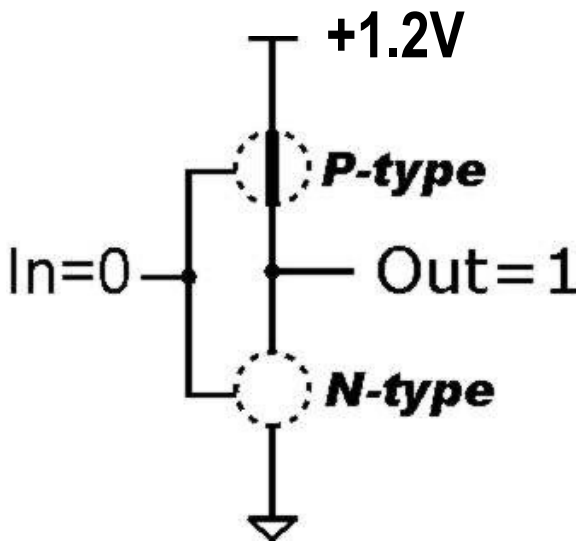
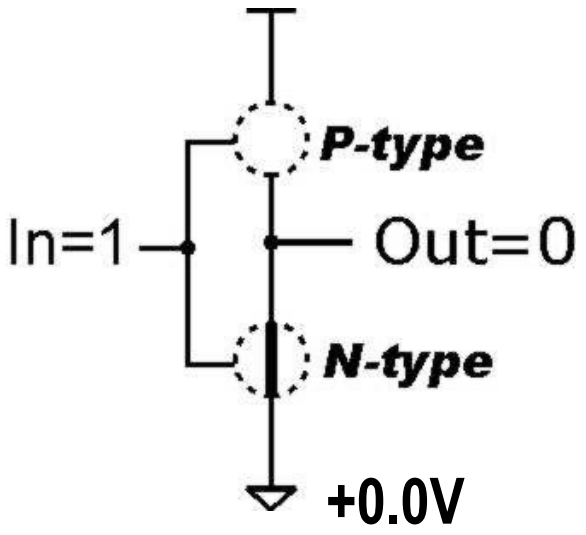
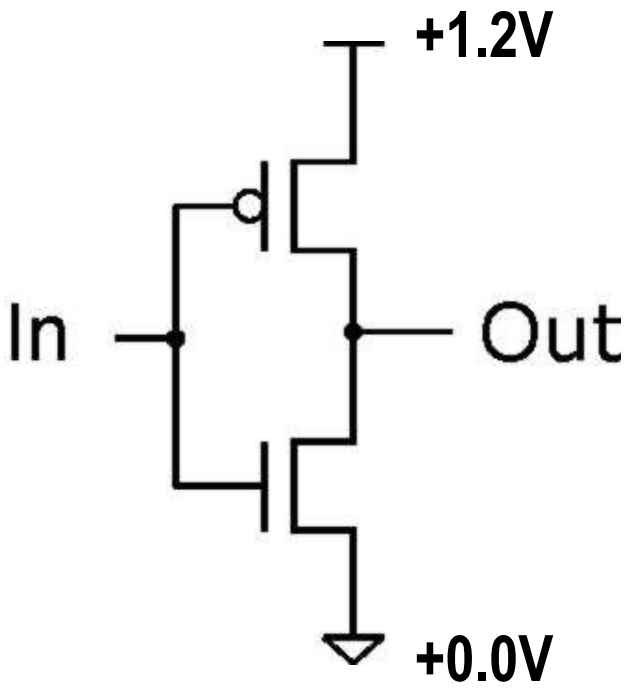

+1.2V
In
Out
+0.0V

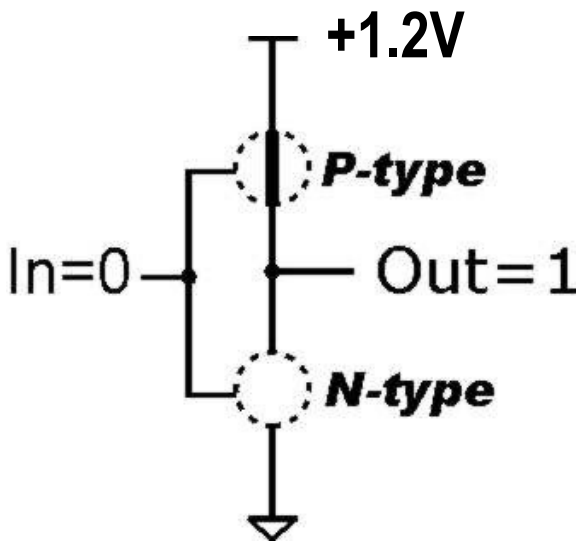

+1.2V
P-type
In=0
Out=1
N-type

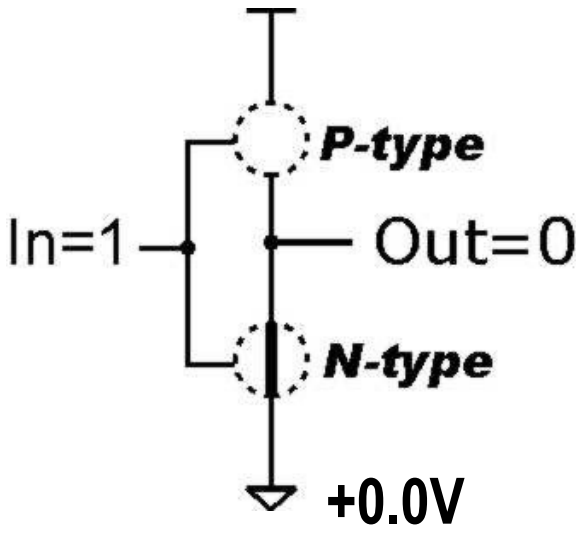

P-type
In=1
Out=0
N-type
+0.0V

Inverter



In	Out
0	1
1	0





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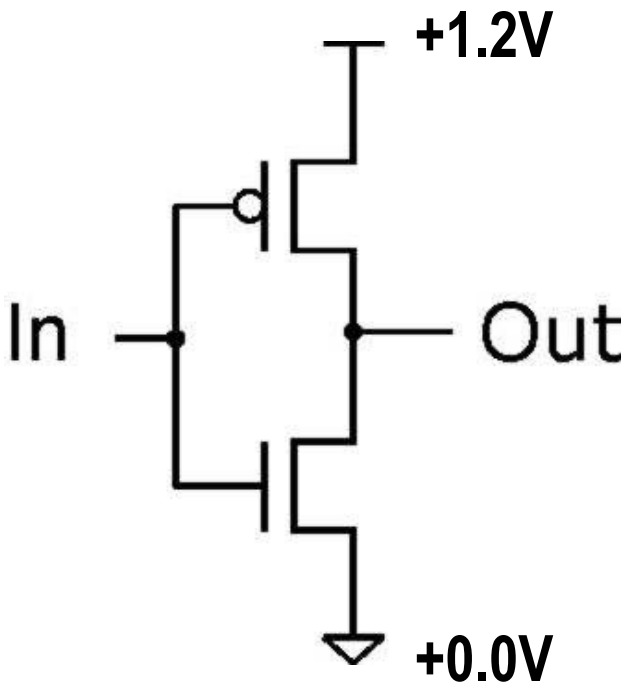

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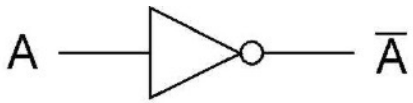
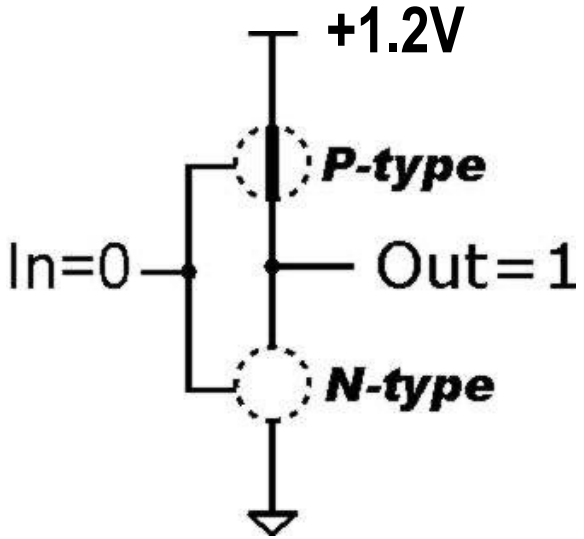
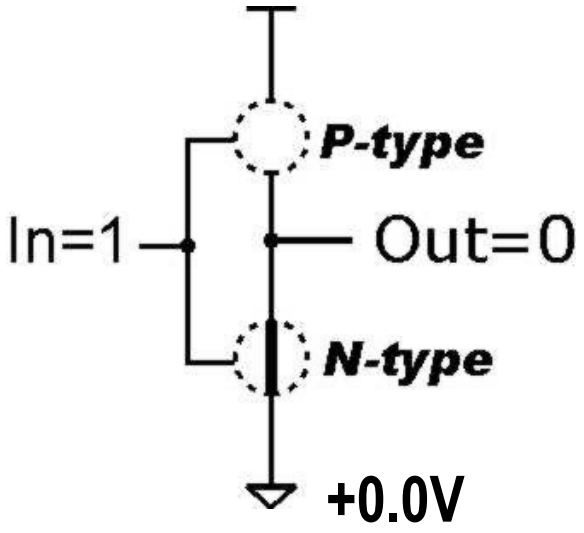
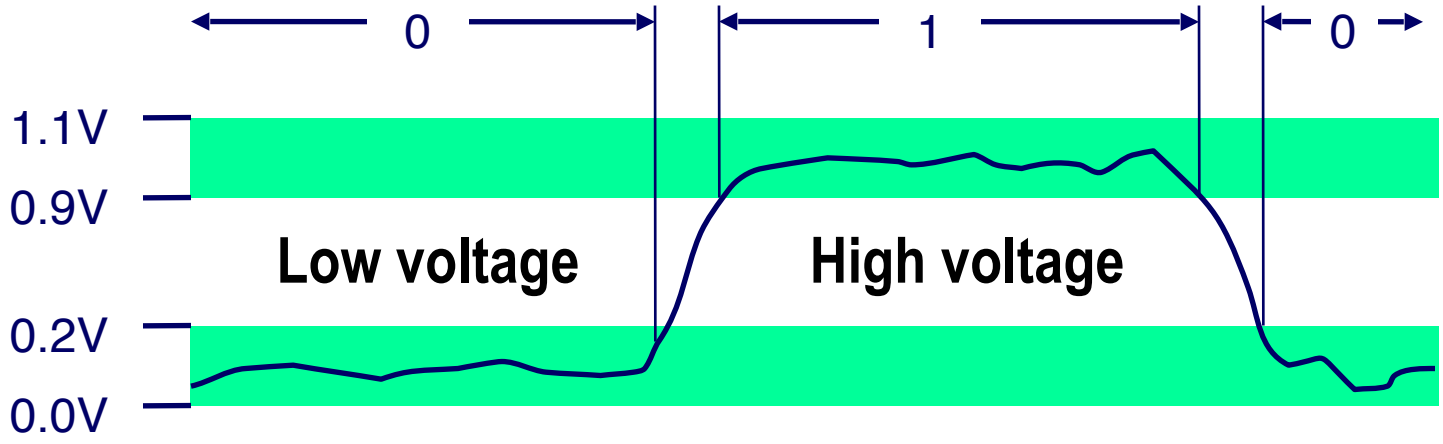
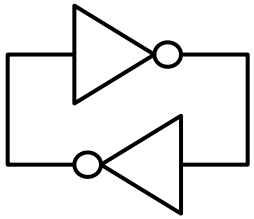






Store/Access Data





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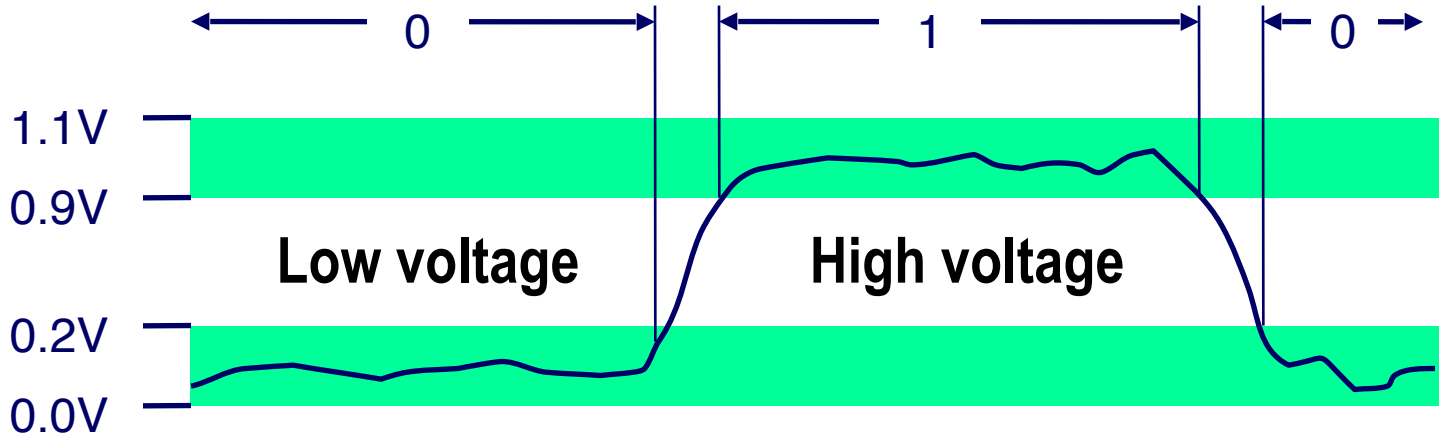

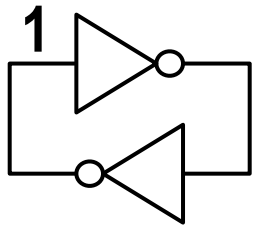

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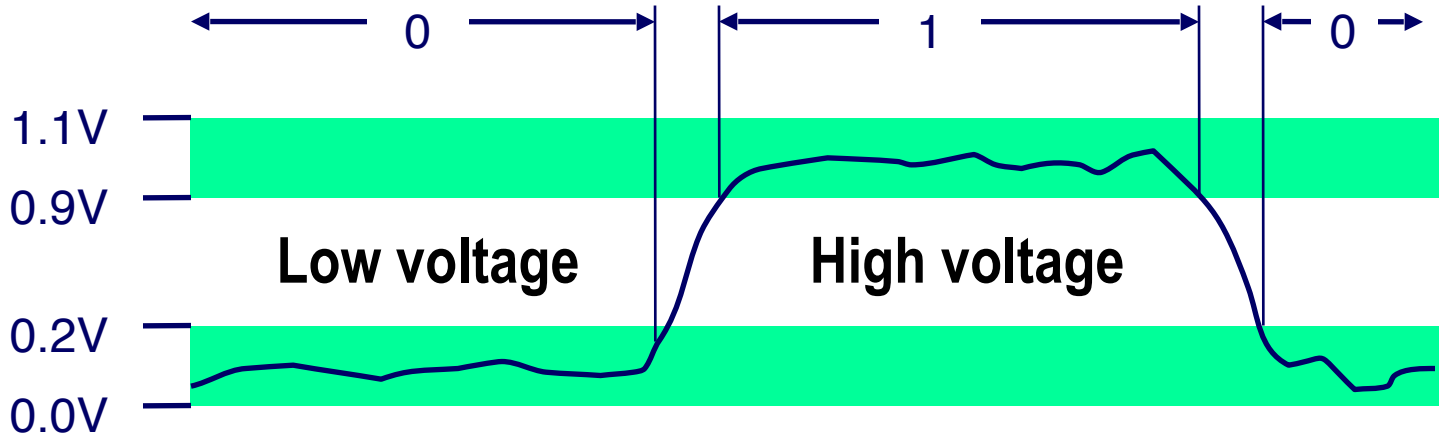

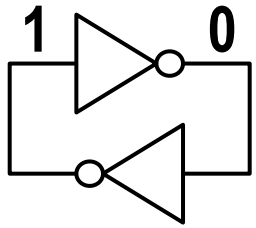

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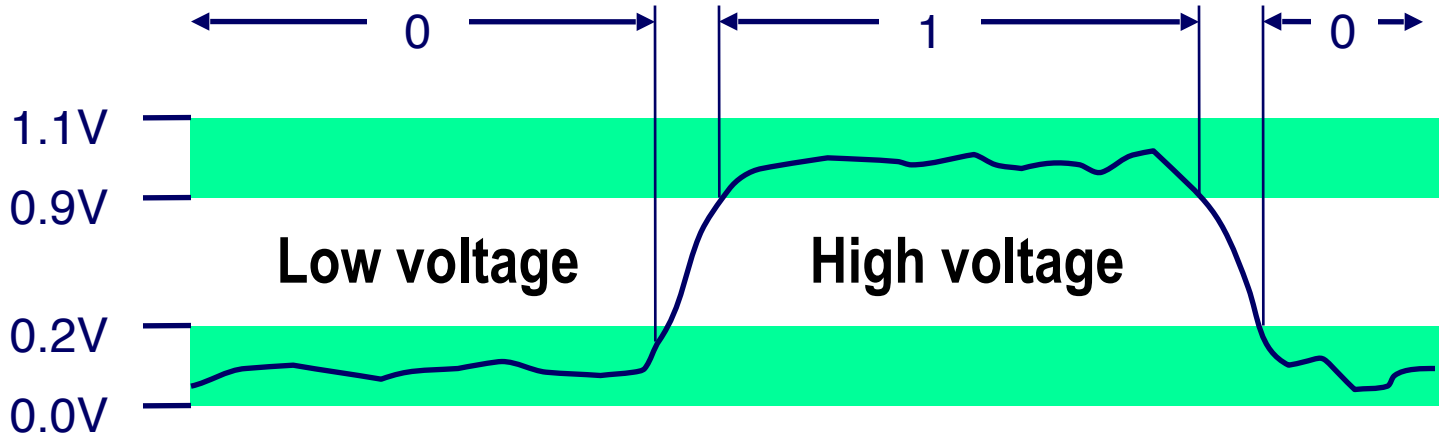

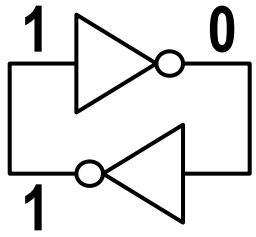

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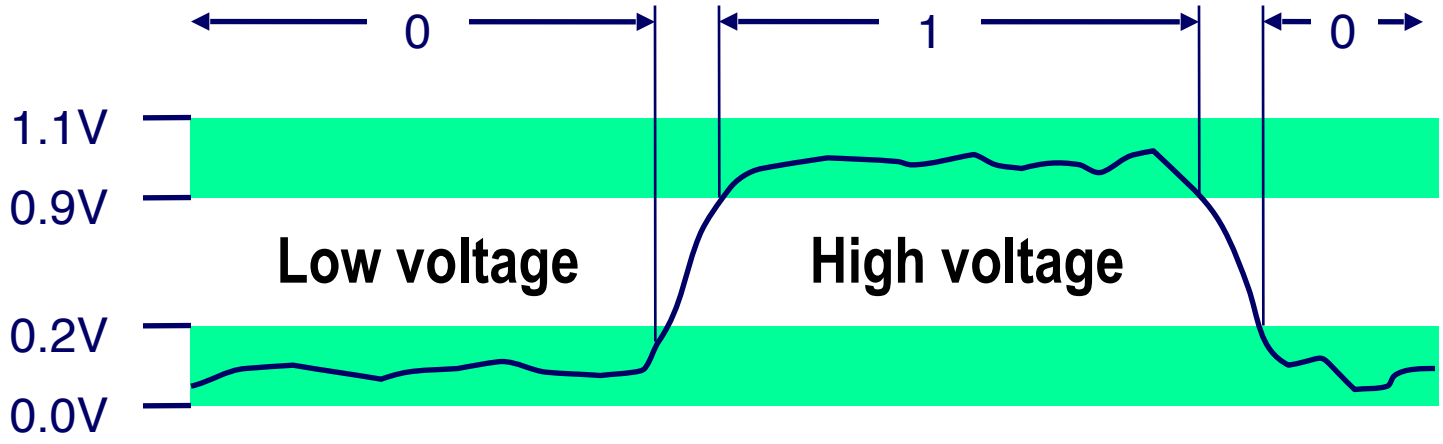
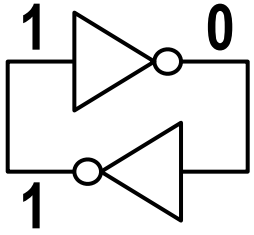


- Two cross coupled inverters store a single bit
  - Feedback path persists the value in the “cell”



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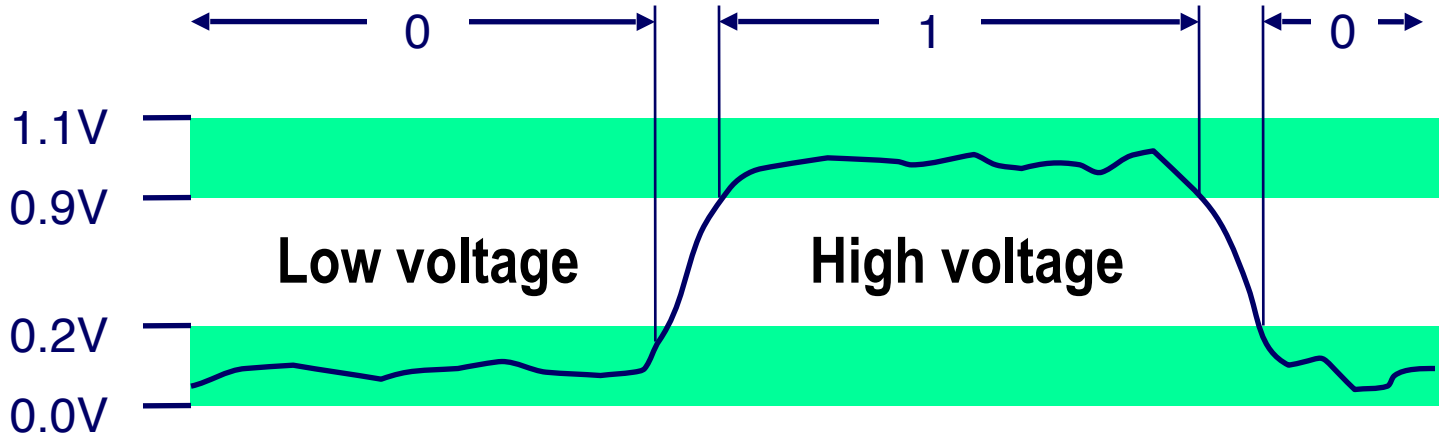

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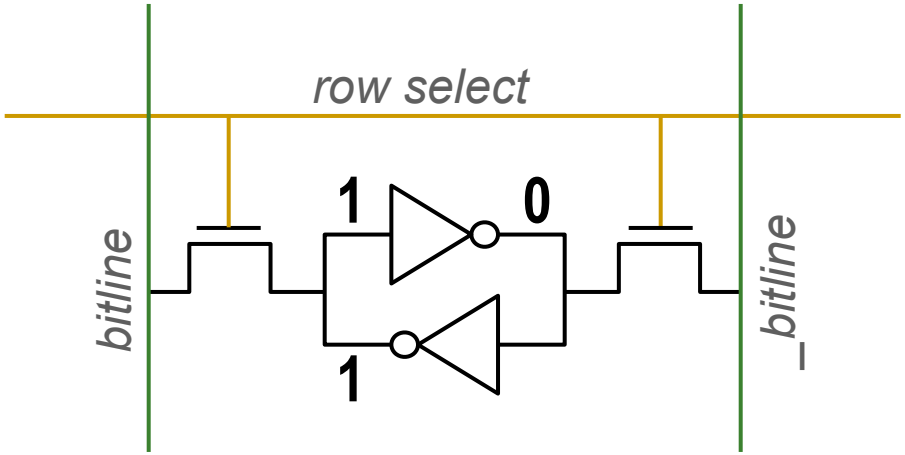

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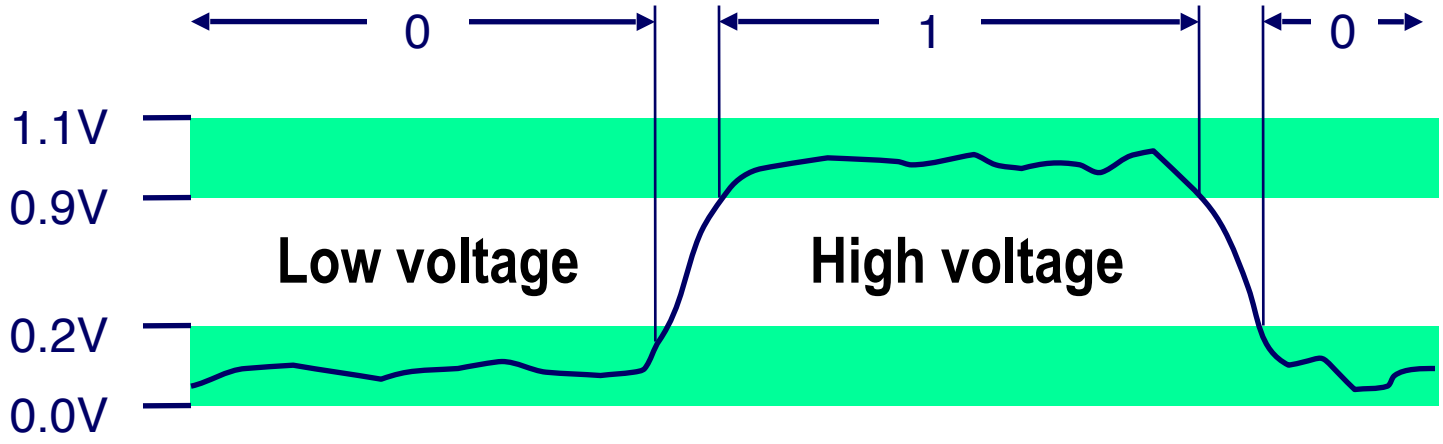

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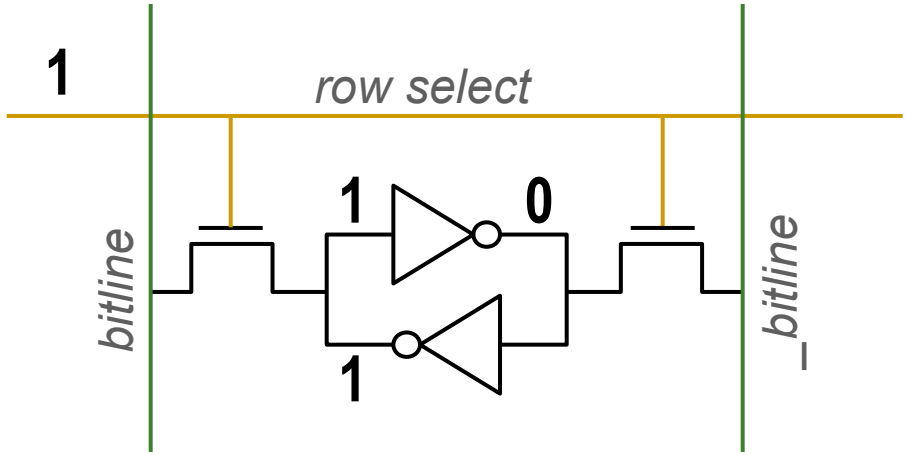

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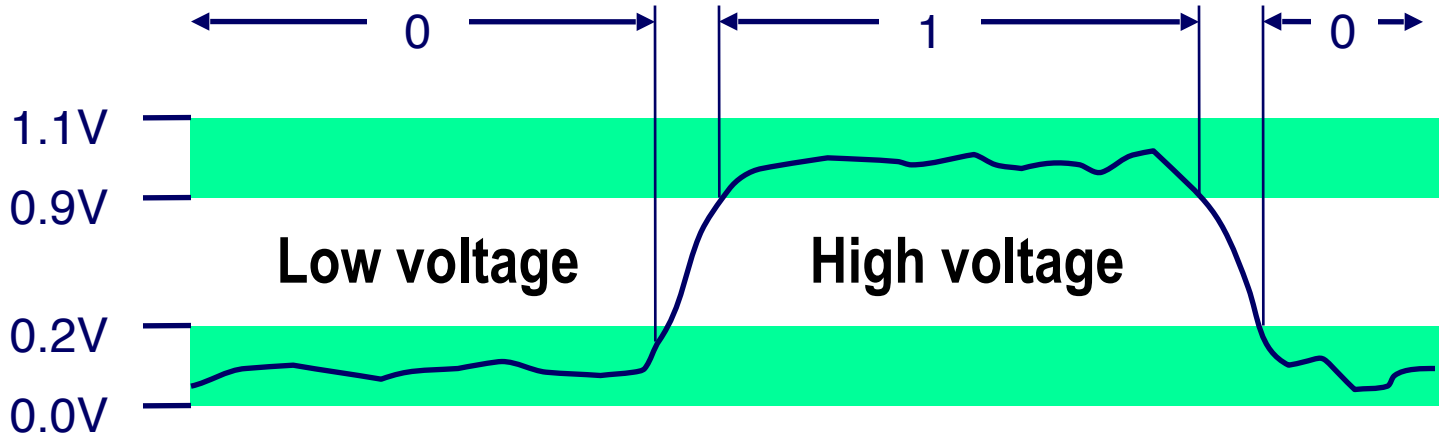

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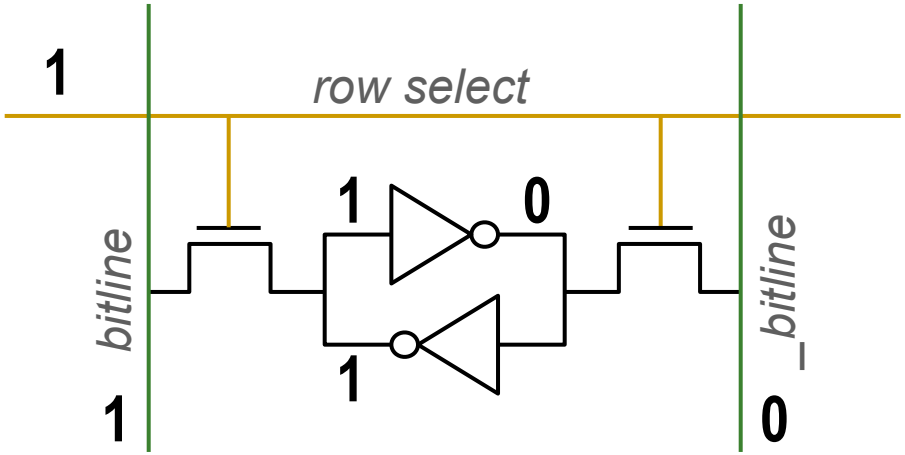

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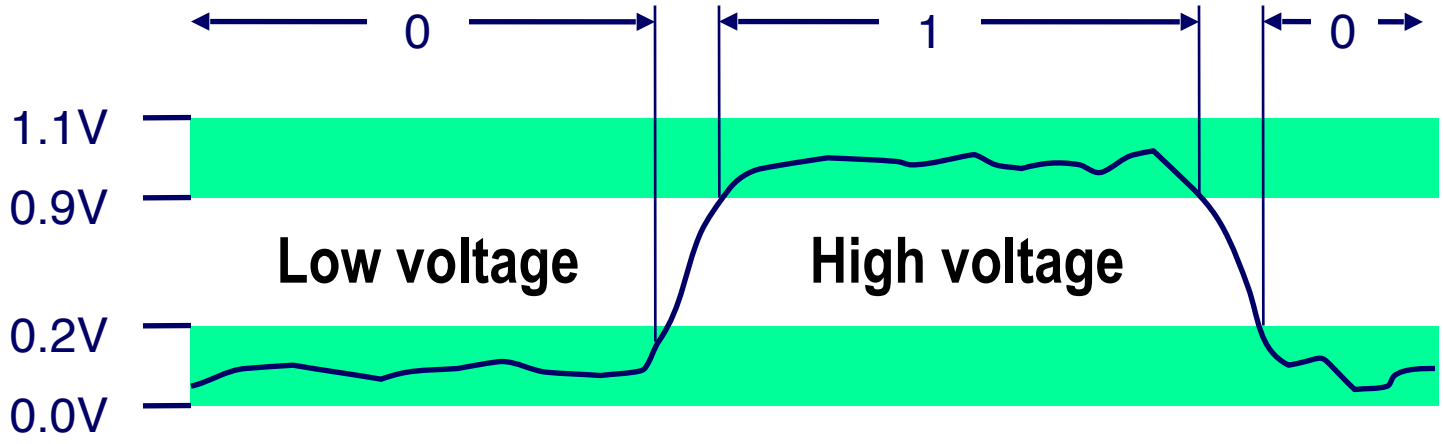

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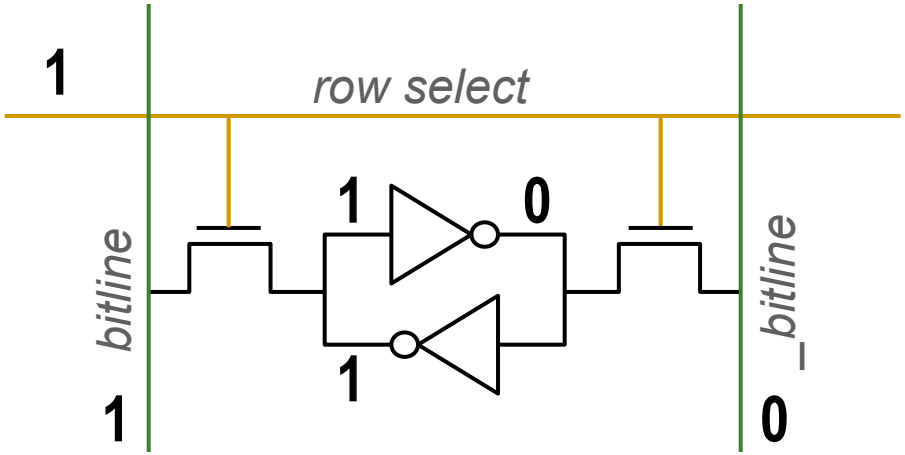

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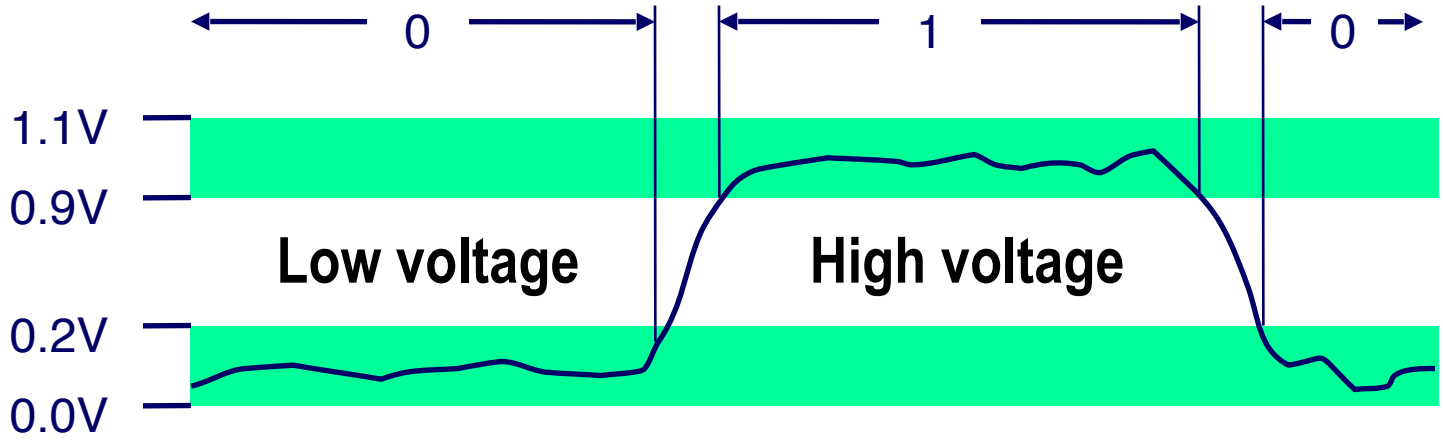

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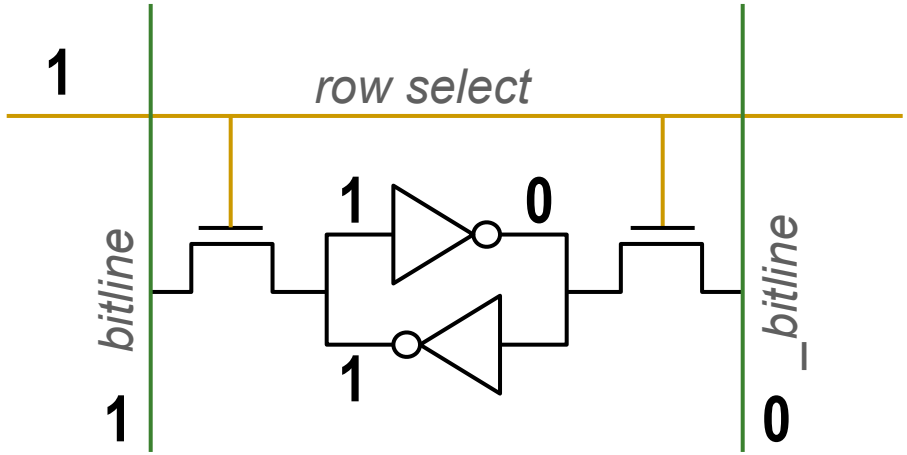

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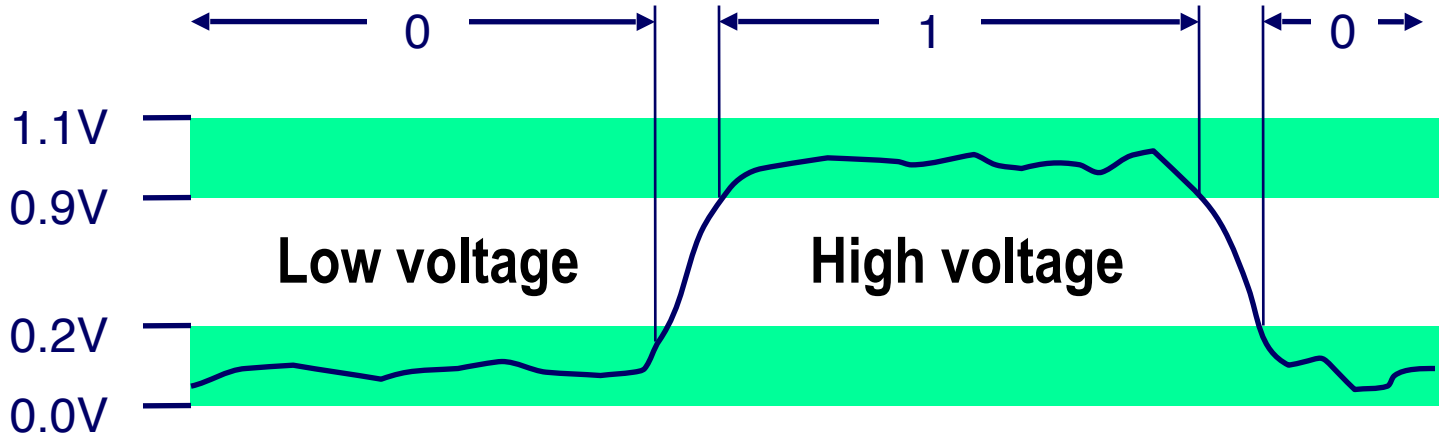

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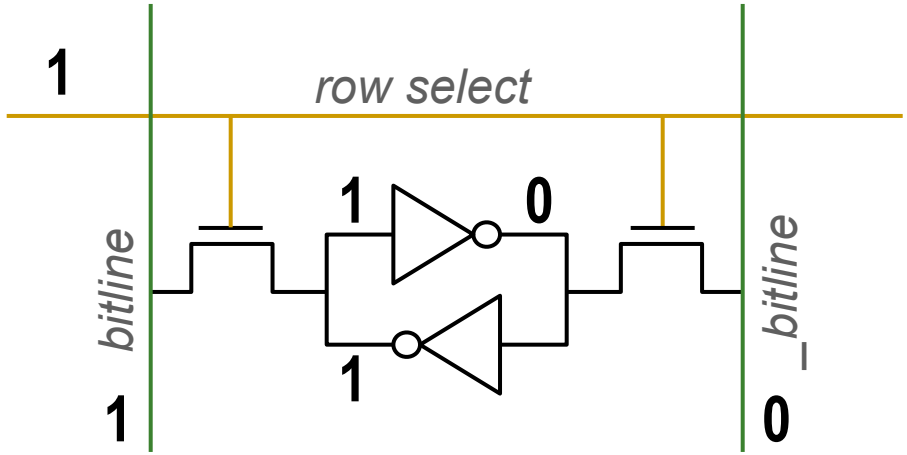

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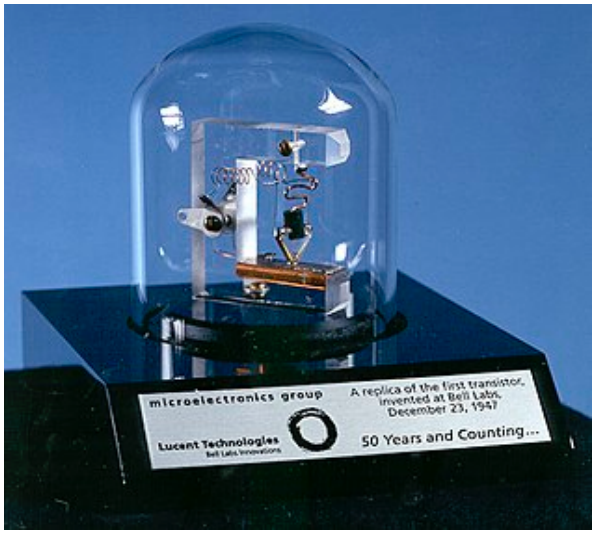

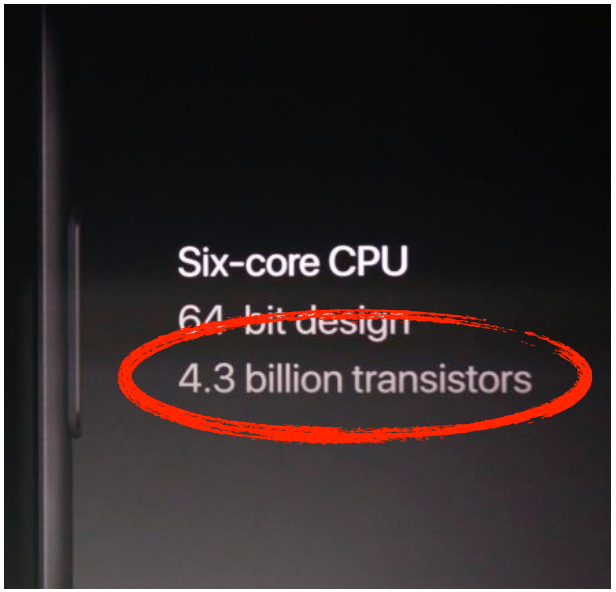

Why Limit Ourselves Only to Binary?

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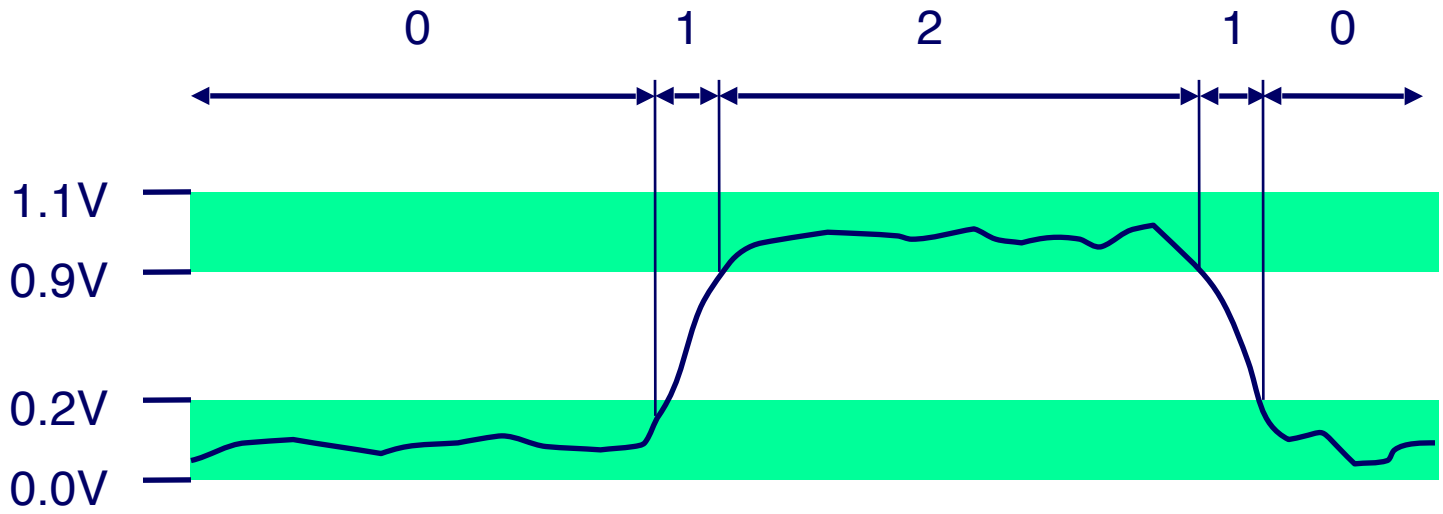

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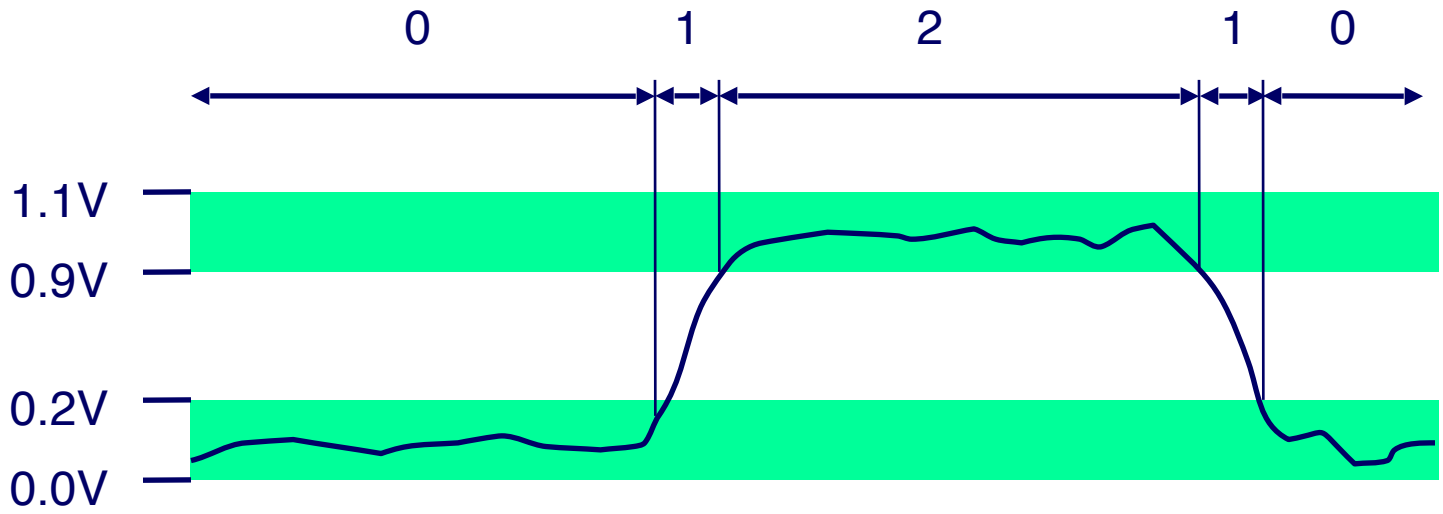
- Answer: Noise



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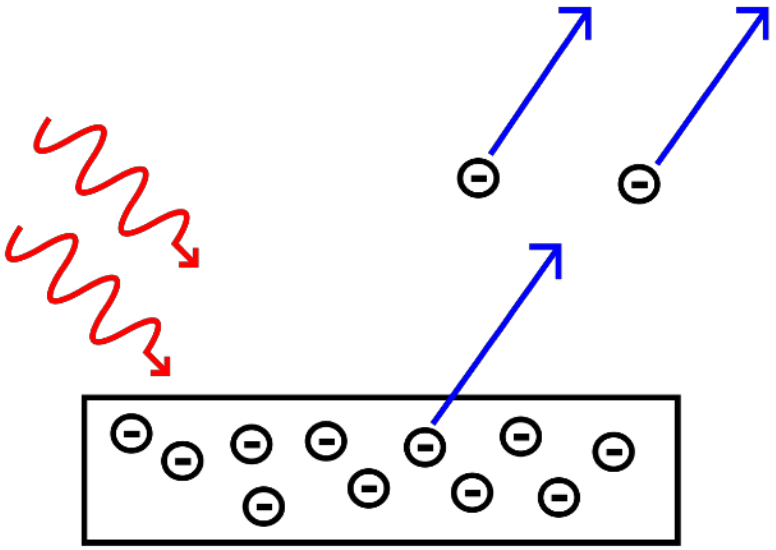

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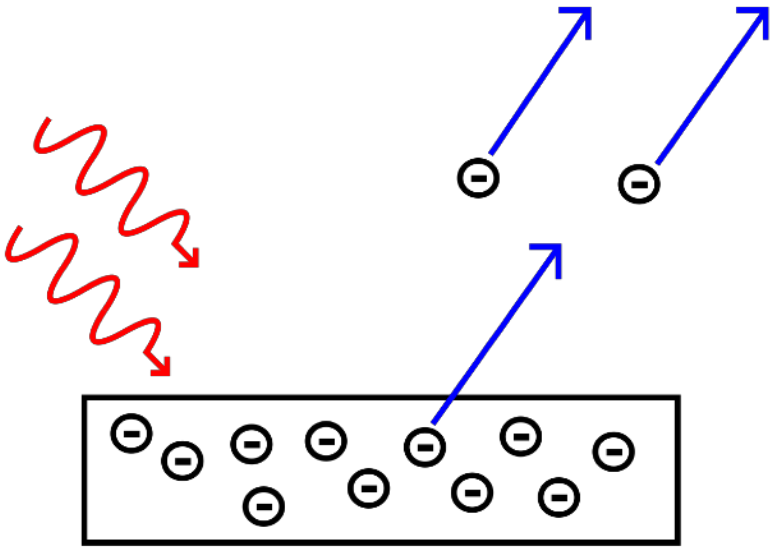

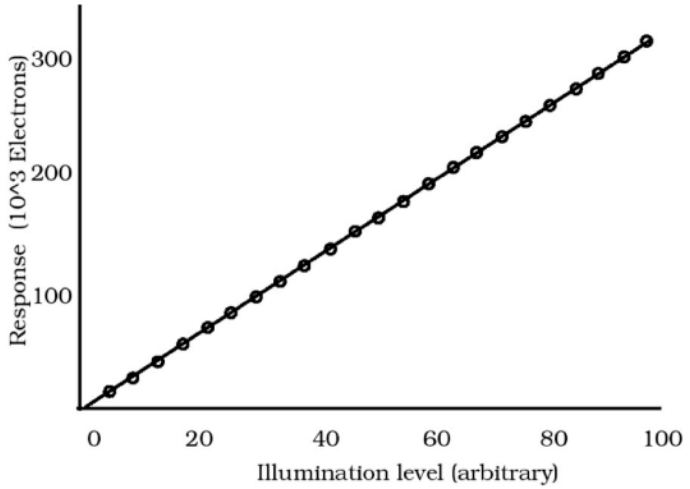

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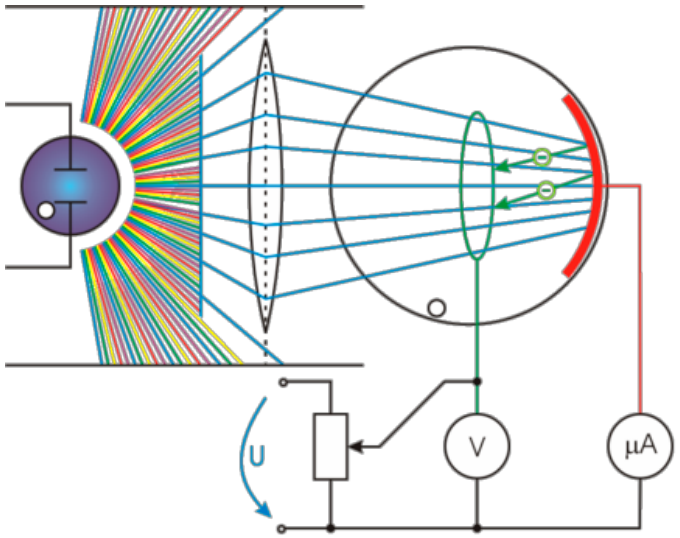

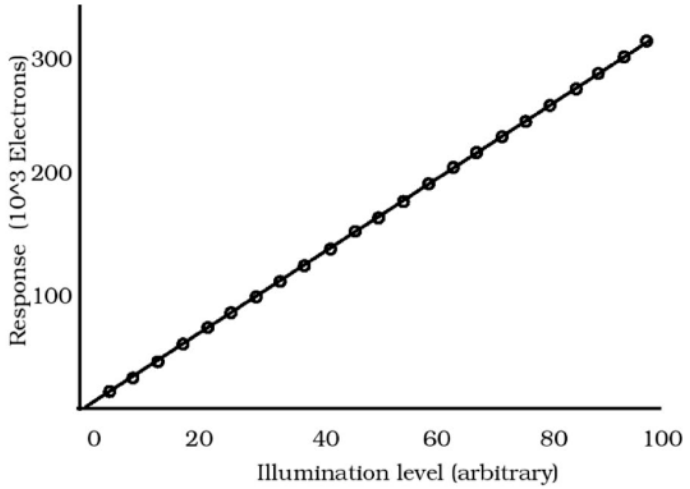

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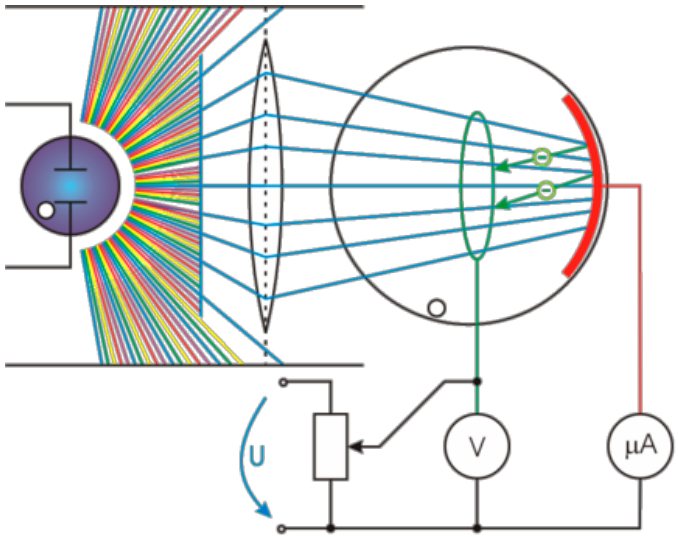

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Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

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```
  0110
+ 0101
------
  1011
```

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```
  0110
+ 0101
------
  1011
```

```
  6
+ 5
---
 11
```

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

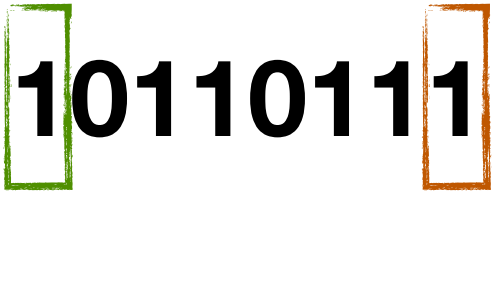
Hexdecimal (Hex) Notation

- Base 16 Number Representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - Four bits per Hex digit
  - $11111110_2 = FE_{16}$
- Write $FA1D37B_{16}$ in C as
  - 0xFA1D37B
  - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Bit, Byte, Word

- Byte = 8 bits
  - Binary 00000000_2 to 11111111_2; Decimal: 0_{10} to 255_{10}; Hex: 00_{16} to FF_{16}
  - Least Significant Bit (LSb) vs. Most Significant Bit (MSb)



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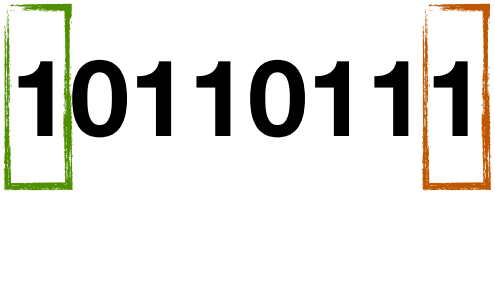


- Word = 4 Bytes (32-bit machine) / 8 Bytes (64-bit machine)
  - Least Significant Byte (LSB) vs. Most Significant Byte (MSB)

Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

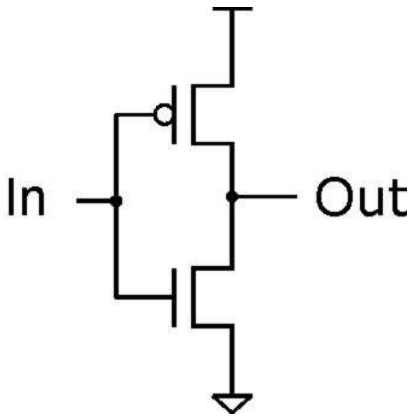
Bit-level manipulations

Not

- **~A = 1 when**

A=0

~	
0	1
1	0



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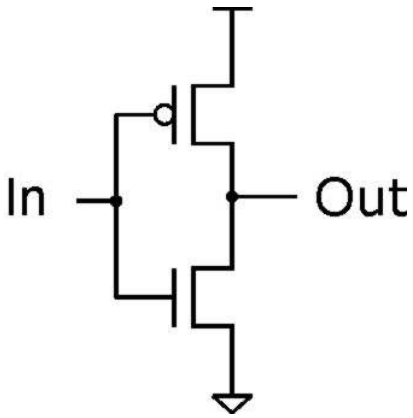

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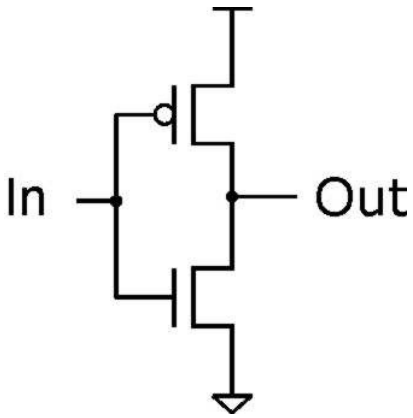

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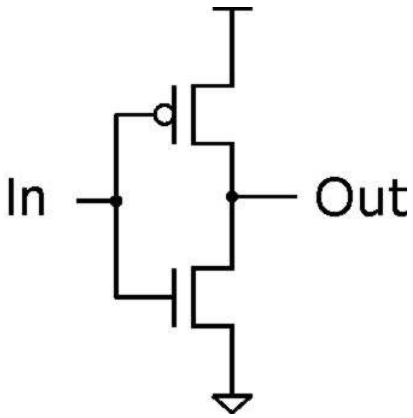

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Exclusive-Or (Xor)

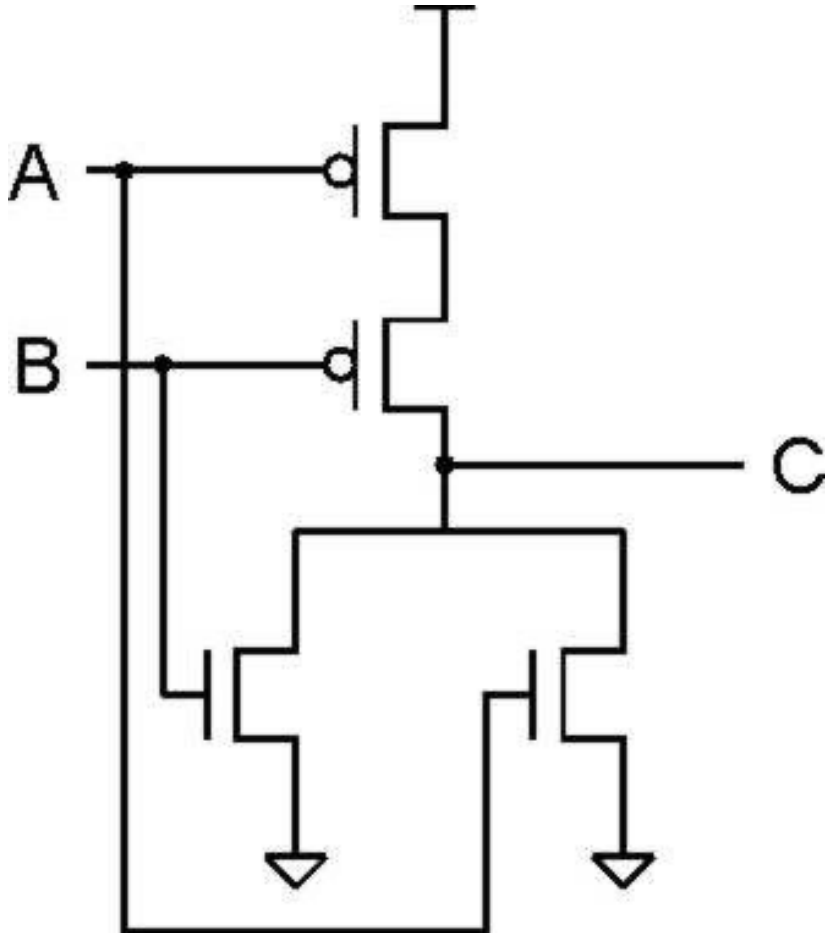
- A^B = 1 when either A=1 or B=1, but not both

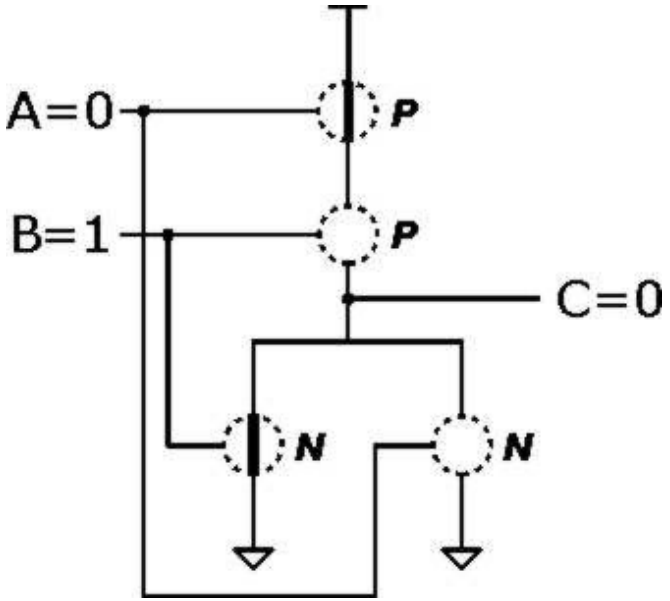
^	0	1
0	0	1
1	1	0

NOR (OR + NOT)

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

NOR (OR + NOT)





A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Bit Vector Operations

- Operate on Bit Vectors
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
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```

Bit-Level Operations in C

- Operations &, |, ~, ^ Available in C
  - Apply to any “integral” data type
    - `long, int, short, char, unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- Examples (Char data type)
  - `~0x41 → 0xBE`
    - $\sim 01000001_2 \rightarrow 10111110_2$
  - `~0x00 → 0xFF`
    - $\sim 00000000_2 \rightarrow 11111111_2$
  - `0x69 & 0x55 → 0x41`
    - $01101001_2 \ \& \ 01010101_2 \rightarrow 01000001_2$
  - `0x69 | 0x55 → 0x7D`
    - $01101001_2 \ | \ 01010101_2 \rightarrow 01111101_2$

Contrast: Logic Operations in C

- Contrast to Logical Operators
  - `&&, ||, !`
    - View 0 as “False”
    - Anything nonzero as “True”
    - Always return 0 or 1
    - Early termination (e.g., 0 && 1 && 1)
- Examples (char data type)
  - `!0x41  → 0x00`
  - `!0x00  → 0x01`
  - `!!0x41  → 0x01`

  - `0x69 && 0x55  → 0x01`
  - `0x69 || 0x55  → 0x01`
  - `p && *p` (avoids null pointer access)

Shift Operations

- Left Shift: `x << y`
  - Shift bit-vector **x** left **y** positions
    - Throw away extra bits on left
  - Fill with 0's on right
- Right Shift: `x >> y`
  - Shift bit-vector **x** right **y** positions
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  - Logical shift
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  - Arithmetic shift
    - Replicate most significant bit on left
- Undefined Behavior
  - Shift amount < 0 or ≥ total amount of bits

Argument x	01100010
<< 3	
Log. >> 2	
Arith. >> 2	

Argument x	10100010
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<< 3	00010*000*
Log. >> 2	
Arith. >> 2	

Argument x	10100010
<< 3	
Log. >> 2	
Arith. >> 2	

Shift Operations

- Left Shift: `x << y`
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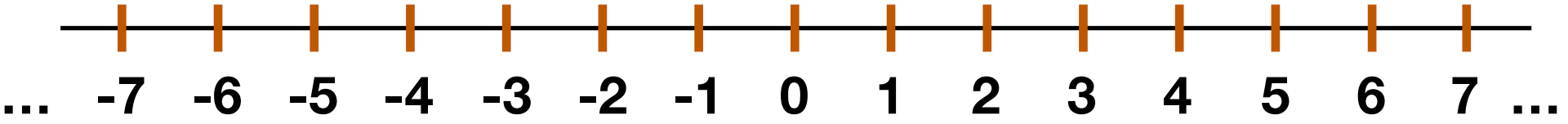
Arith. >> 2	*11*101000

Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

Representing Numbers in Binary

- Different types of number
  - Integer (Negative and Non-negative)
  - Fractions
  - Irrationals



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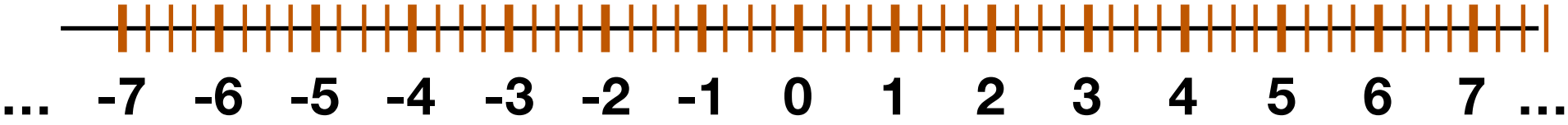

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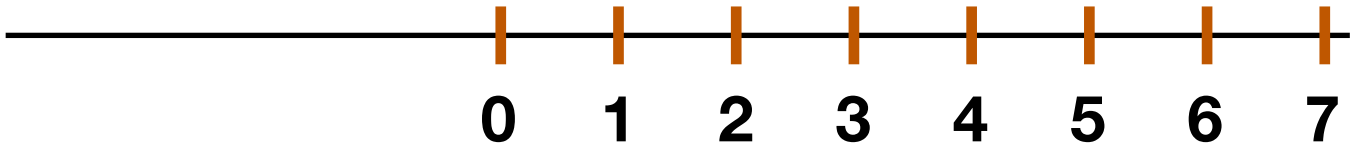

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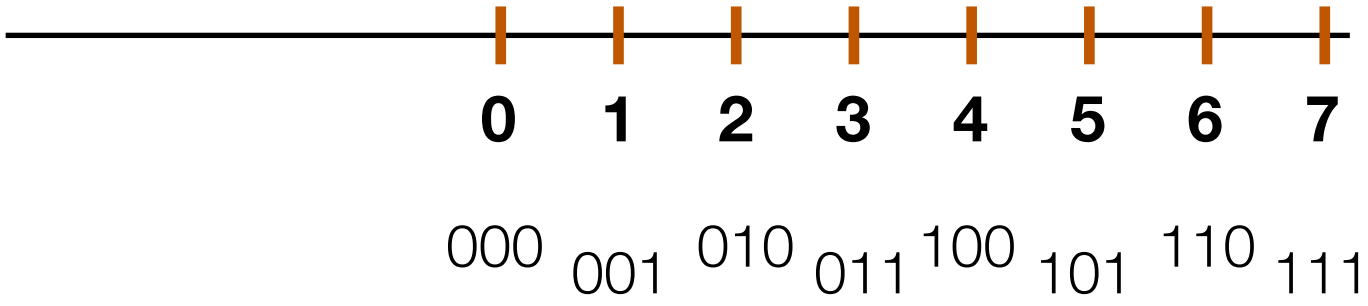

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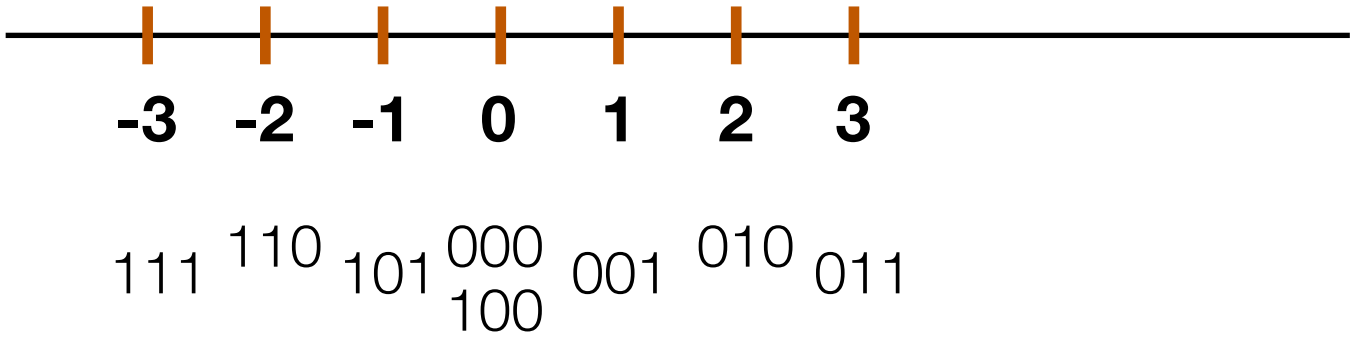

Sign-Magnitude Implications

- Bits have different semantics
  - Two zeros…
  - Normal arithmetic doesn't work
  - Make hardware design harder

Signed Value	Binary
0	000
1	001
2	010
3	011
-0	100
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-2	110
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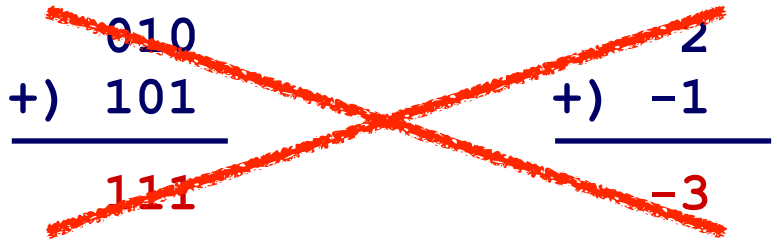

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- Solution 2: Two’s Complement

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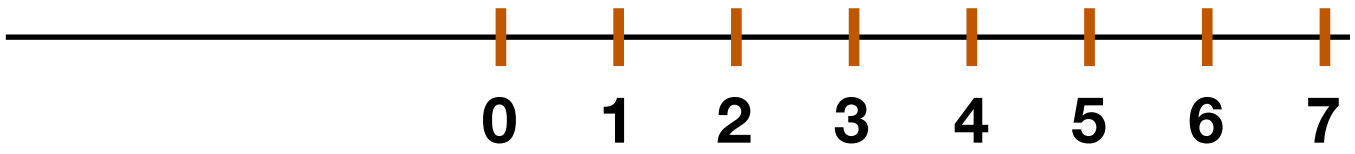

Unsigned	Binary
0	000
1	001
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4	100
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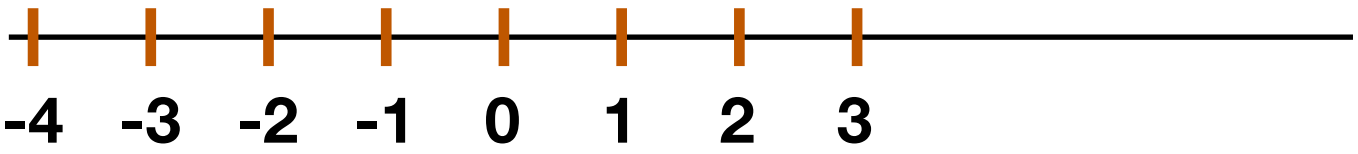
7	111

Encoding Negative Numbers

- Solution 2: Two's Complement



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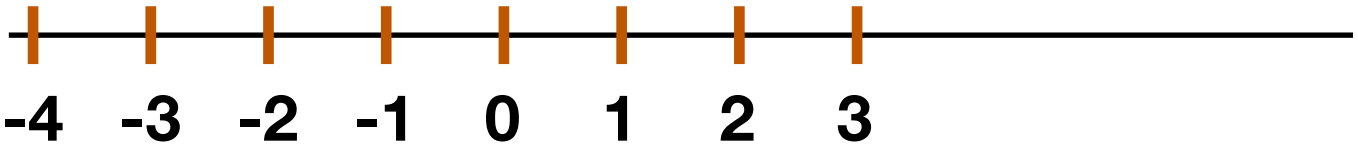

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Encoding Negative Numbers

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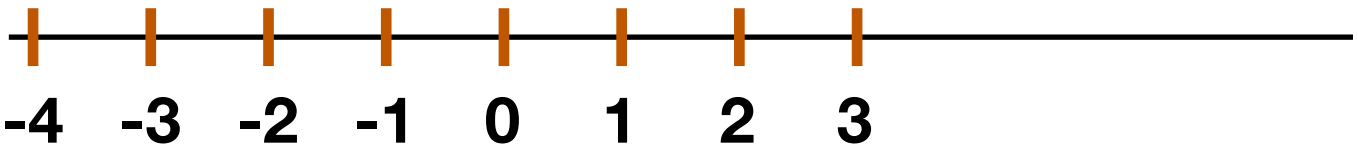

Signed Weight	Unsigned Weight	Bit Position
2^0	2^0	0
2^1	2^1	1
-2^2	2^2	2

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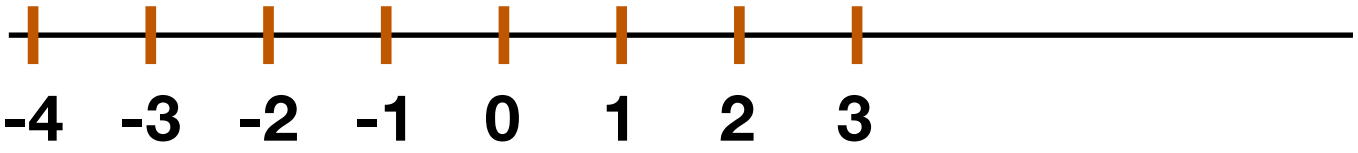

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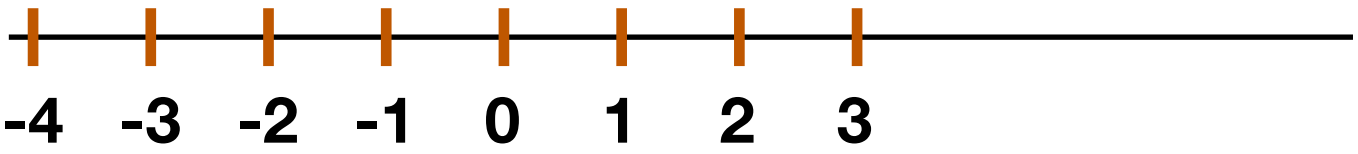

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$101_2 = 1*2^0 + 0*2^1 - 1*2^2 = -3_{10}$

Two-Complement Encoding Example

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Two-Complement Implications

- Only 1 zero
- Usual arithmetic still works
- There is a bit that represents sign!
- Most widely used in today's machines

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```
   010
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```

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- Unsigned Values
  - *UMin* = 0

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Values for *W* = 16

	Decimal	Hex	Binary
UMax	**65535**	**FF FF**	**11111111 11111111**
TMax	**32767**	**7F FF**	**01111111 11111111**
TMin	**-32768**	**80 00**	**10000000 00000000**
-1	**-1**	**FF FF**	**11111111 11111111**
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  - *UMin* = 0
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    111…1
- Two's Complement Values
  - *TMin* = -2^{w-1}
    100…0
  - *TMax* = $2^{w-1} - 1$
    011…1
- Other Values
  - Minus 1
    111…1

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0	**0**	**00 00**	**00000000 00000000**

Data Representations in C (in Bytes)

- By default variables are signed
- Unless explicitly declared as unsigned (e.g., `unsigned int`)
- Signed variables use two-complement encoding

C Data Type	32-bit	64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8

Data Representations in C (in Bytes)

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
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- C Language
  - `#include <limits.h>`
  - Declares constants, e.g.,
    - `ULONG_MAX`
    - `LONG_MAX`
    - `LONG_MIN`
  - Values platform specific