# CSC2/455 Software Analysis and Improvement
# Model Checking

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URCS

# Outline

A Tour of CBMC

Some (unneeded?) History

Model Checking

Liveness Properties

Postscript

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# Wrong Min

```
#include <assert.h>

int nondet_int();

int min(int a, int b) {
  if(a < b)
    return a;
  else
    return a;
}

void check_min() {
  int x, y, r;

  x = nondet_int();
  y = nondet_int();

  r = min(x, y);

  assert(r == x || r == y);
  assert(r <= x && r <= y);
}
```

# CBMC

```
$ cbmc --function check_min wrong_min.c
CBMC version 5.10 (cbmc-5.10) 64-bit x86_64 linux
Parsing wrong_min.c
Converting
Type-checking wrong_min
Generating GOTO Program
Adding CPROVER library (x86_64)
Removal of function pointers and virtual functions
Generic Property Instrumentation
Running with 8 object bits, 56 offset bits (default)
Starting Bounded Model Checking
size of program expression: 48 steps
simple slicing removed 2 assignments
Generated 2 VCC(s), 2 remaining after simplification
Passing problem to propositional reduction
converting SSA
Running propositional reduction
Post-processing
Solving with MiniSAT 2.2.1 with simplifier
403 variables, 1026 clauses
SAT checker: instance is SATISFIABLE
Solving with MiniSAT 2.2.1 with simplifier
403 variables, 0 clauses
SAT checker inconsistent: instance is UNSATISFIABLE
Runtime decision procedure: 0.00148068s

** Results:
[check_min.assertion.1] assertion r == x || r == y: SUCCESS
[check_min.assertion.2] assertion r <= x && r <= y: FAILURE

** 1 of 2 failed (2 iterations)
VERIFICATION FAILED
```

# Tracing CBMC

```
$ cbmc --trace --function check_min wrong_min.c
 ...

[check_min.assertion.2] assertion r <= x && r <= y: FAILURE

Trace for check_min.assertion.2:

 ...

State 20 file wrong_min.c line 15 function check_min thread 0
----------------------------------------------------
  x=1073741824 (01000000 00000000 00000000 00000000)

State 21 file wrong_min.c line 16 function check_min thread 0
----------------------------------------------------
  y=1 (00000000 00000000 00000000 00000001)

State 24 file wrong_min.c line 18 function check_min thread 0
----------------------------------------------------
  a=1073741824 (01000000 00000000 00000000 00000000)

State 25 file wrong_min.c line 18 function check_min thread 0
----------------------------------------------------
  b=1 (00000000 00000000 00000000 00000001)

State 30 file wrong_min.c line 18 function check_min thread 0
----------------------------------------------------
  r=1073741824 (01000000 00000000 00000000 00000000)

Violated property:
  file wrong_min.c line 21 function check_min
  assertion r <= x && r <= y
  r <= x && r <= y
```

# Generating test cases

```
$ cbmc --cover branch --function min wrong_min_2.c
CBMC version 5.10 (cbmc-5.10) 64-bit x86_64 linux
 ...
converting SSA
Aiming to cover 3 goal(s)
Running propositional reduction
Post-processing
Solving with MiniSAT 2.2.1 with simplifier
334 variables, 569 clauses
SAT checker: instance is SATISFIABLE
Covered function min entry point
Covered function min block 1 branch false
Solving with MiniSAT 2.2.1 with simplifier
334 variables, 0 clauses
SAT checker: instance is SATISFIABLE
Covered function min block 1 branch true
Runtime decision procedure: 0.00146064s

** coverage results:
[min.coverage.1] file wrong_min_2.c line 6 function min entry point: SATISFIED
[min.coverage.2] file wrong_min_2.c line 6 function min block 1 branch false: SATISFIED
[min.coverage.3] file wrong_min_2.c line 6 function min block 1 branch true: SATISFIED

** 3 of 3 covered (100.0%)
** Used 2 iterations
Test suite:
a=0, b=1
a=1, b=0
```

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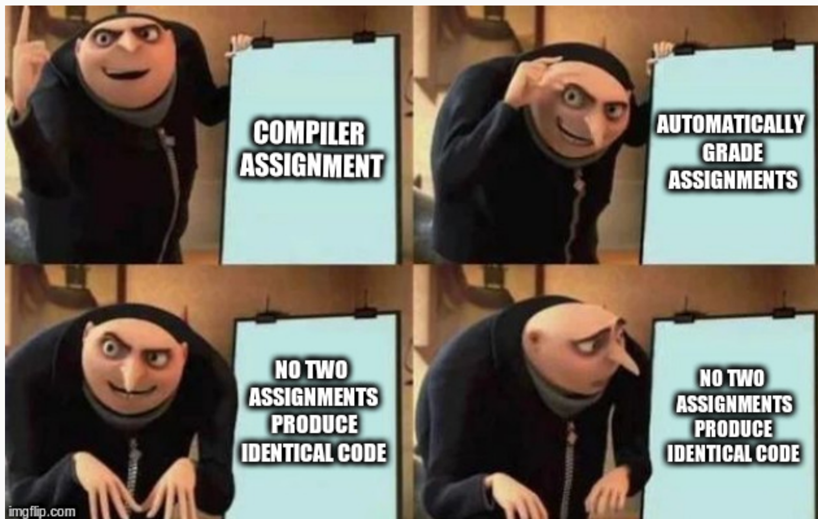
# The Plan



## Check for Equivalence

- $A$: Original source program
- $B$: Compiler-generated program (e.g. your 3-address code)
- Is $A = B$?
    - Program equivalence problem
    - Undecidable in general

## Test?

- Develop test cases
- Run $B$ with these test cases
  - Works
  - Tests may miss bugs
- Also, many programs harder to test
  - Don't have `main`
  - Accept input interactively
  - Buggy compilers may introduce infinite loops

# Solution

- Ended up using bounded model checking for C
  - CBMC
- Allows me to check that certain properties hold across all executions
- Can still require manual inspection

# Computing the minimum of three numbers

```
int min_of_3_before(int x, int y, int z) {
   int min3;

   if(x > y) {
      if(y > z) {
         min3 = z;
      } else {
         min3 = y;
      }
   } else {
     if(x > z) {
         min3 = z;
     } else {
         min3 = x;
     }
   }

   return min3;
}
```

# Another implementation

```
int min_of_3_after(int x, int y, int z) {
   int min3;

   if(x > y && y > z) {
     min3 = z;
   } else {
     if(x > y)
       min3 = y;
     else
       min3 = x;
   }

   return min3;
}
```

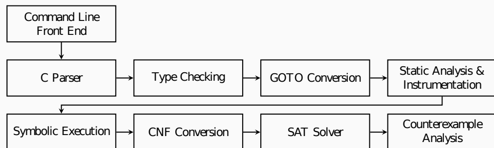
## Checking equivalence: before and after

```
int nondet_int();
void check_eqv() {
  int a, b, c;

  a = nondet_int();
  b = nondet_int();
  c = nondet_int();

  assert(min_of_3_before(a, b, c) == min_of_3_after(a, b, c));
}
```

# Results

```
$ cbmc --trace --function check_eqv min3.c
CBMC version 5.10 (cbmc-5.10) 64-bit x86_64 linux
 ...
[check_eqv.assertion.1] assertion return_value_min_of_3_before == return_value_min_of_3_after: FAIL

State 20 file min3.c line 45 function check_eqv thread 0
----------------------------------------------------
  a=1 (00000000 00000000 00000000 00000001)

State 21 file min3.c line 46 function check_eqv thread 0
----------------------------------------------------
  b=1073741824 (01000000 00000000 00000000 00000000)

State 22 file min3.c line 47 function check_eqv thread 0
----------------------------------------------------
  c=0 (00000000 00000000 00000000 00000000)

 ...

State 32 file min3.c line 13 function min_of_3_before thread 0
----------------------------------------------------
  min3=0 (00000000 00000000 00000000 00000000)

 ...
State 46 file min3.c line 33 function min_of_3_after thread 0
----------------------------------------------------
  min3=1 (00000000 00000000 00000000 00000001)

Violated property:
  file min3.c line 49 function check_eqv
  assertion return_value_min_of_3_before == return_value_min_of_3_after
  return_value_min_of_3_before == return_value_min_of_3_after



** 1 of 1 failed (1 iteration)
VERIFICATION FAILED
```

# CBMC Architecture




- CBMC translates entire program into a "GOTO" program in SSA form
- It then "executes" every statement in the program
  - Values it does not know about are turned into "symbols" (Symbolic Execution)
- Program is then converted into boolean formulae (CNF)
- The formula is handed off to a SAT solver

Source: Kroening and Tautschnig, CBMC – C Bounded Model Checker (Competition Contribution), TACAS 2014

# Loops: Definite Bounds

```
for(i = 0; i < 10; i++) {
  ...
}
```

CBMC will unroll loop.

# Loops: Symbolic Bounds

```
for(i = 0; i < N; i++)
   B;
```

gets unrolled by a fixed number (B is body), with unroll assert:

```
i = 0;
if(i < N) {
  B;
  i++;

  if(i < N) {
     B;
     i++;

     assert(N == 2);
  }
}
```

- If `assert` fails, unrolling was insufficient.
  - Not sound!
  - Otherwise, conclusion is sound

# Other complications

- Pointers, arrays, dynamic memory allocation, etc.
- See CPROVER manual for more details

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# Basic Ideas

- Formula $\varphi$
  - Correctness (Safety) property
  - Propositional logic
  - Example: the argument to the `assert` statements
- Intepretation $\mathcal{K}$
  - More on this later
- We ask: $\mathcal{K} \models \varphi$?
  - Is $\varphi$ true in $\mathcal{K}$?

# Transition System

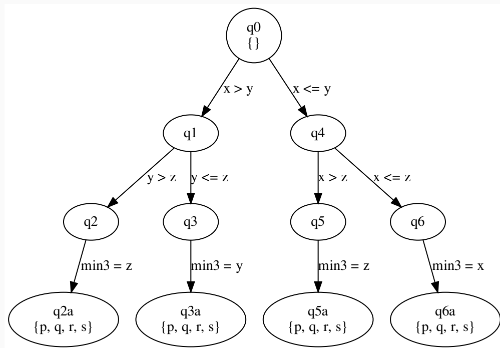
- $\mathcal{T} = (Q, I, E, \delta)$
  - set of states $Q$ (e.g. values of all variables)
  - initial states $I \in Q$
  - action labels $E$ (e.g. program statements)
  - (total) transition relation $\delta \subset Q \times E \times Q$
- A run of $\mathcal{T}$ is the same as a trace of states
  - $s0e0s1...$ where $(s0, e0, s1) \in \delta$, and $s0 \in I$
- A reachable state is a state that exists in some run.

# Kripke Structures

```
min3 = min_of_3(x, y, z);
assert(min3 == x || min3 == y || min3 == z);
assert(min3 <= x);
assert(min3 <= y)
assert(min3 <= z)
```

- Let $\mathcal{V}$ be a set of propositions
  - e.g. `min3 <= x`
  - e.g. `min3 <= y`
- A Kripke structure $\mathcal{K} = (Q, I, E, \delta, \lambda)$ is a transition system where:
  - $\lambda : Q \to 2^{\mathcal{V}}$
- $\lambda$ is a function that maps a state $q$ to the (subset) of propositions from $\mathcal{V}$ that are true in that state
  - $q \models P$ where $P \in \mathcal{V}$

# Kripke structure for our min-of-3 example




- Let $p$ be the "must be one of inputs" proposition
- Let $q, r, s$ be the $<= x, <= y, <= z$ proposition
- (Note: True propositions in internal states not shown)

# Invariants

- An invariant is a *safety property* for the system that holds in every reachable state
- An inductive invariant holds in the initial state, and is preserved by all transitions
  - including transitions from unreachable states
  - more on this when we discuss Hoare Logic

# Invariant Checking Algorithm: High level details

- Assume finite Kripke structure
- Given an invariant to check,
    - Enumerate all reachable states
    - Check that invariant holds in all of them

# Invariant Checking Algorithm: Pseudocode

```
def verify_inv(ks, inv):
    done = set()
    todo = set()

    for s in ks.initial_states():
        if s in done: continue

        todo.add(s)

        while len(todo) > 0:
             ss = todo.pop()
             done.add(ss)

             if not ss.satisfies(inv): return False

             for succ in ss.successors():
                 if succ not in done: todo.add(succ)

    return True
```

based on Figure 3.3 in S. Merz, An introduction to Model Checking.

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# Progress

- Does something "good" eventually happen?
- Does the system ever deadlock?
- Does the system livelock?
  - An action $e$ is no longer possible after a particular state $q_i$
- These require reasoning over *sequences* of states
  - These can be infinite even in a finite Kripke structure

These properties need a *temporal* logic, that incorporates notions of (logical) "time points" into formulae we want to check.

# Specifying temporal properties in PTL

- Let $\sigma = q_0 q_1 ...$ be a sequence of states
  - $\sigma_i$ is the state $i$
  - $\sigma|_i$ is the suffix $q_i q_{i+1} \ldots$ of $\sigma$
- Let $\varphi$ be a formula
- $\sigma \models \varphi$ if $\varphi \in \lambda(\sigma_0)$
- $\mathrm{X}\varphi$ (also a formula), read as "next $\varphi$",
  - $\sigma \models \mathrm{X}\varphi$ if $\sigma|_1 \models \varphi$
- $\varphi \mathrm{U} \psi$ (also a formula), read as "$\varphi$ until $\psi$"
  - $\sigma \models \varphi \mathrm{U} \psi$ if and only if there exists $k \in N$
  - $\sigma|_k \models \psi$
  - for all $1 \leq i < k$, $\sigma|_i \models \varphi$
  - Note: $\varphi$ can continue to hold after $k$

# More temporal properties

- $\mathrm{F}\varphi$, “eventually $\varphi$”
    - $\mathrm{true}\mathrm{U}\varphi$
- $\mathrm{G}\varphi$, “always $\varphi$”
    - $\neg\mathrm{F}\neg\varphi$
- $\varphi\mathrm{W}\psi$, “$\varphi$ unless $\psi$”
    - $(\varphi\mathrm{U}\psi) \vee G\psi$
- $\mathrm{GF}\varphi$
- $\mathrm{FG}\varphi$

# Some examples of invariants

- $\mathrm{G}\neg(\mathit{own}_1 \wedge \mathit{own}_2)$
  - where $\mathit{own}_1$ and $\mathit{own}_2$ are propositions representing states in which locks for resource are obtained by process 1 and 2
- Other properties (see the reading)
  - weak and strong fairness
  - precedence
  - etc.

# Existential and Universal Properties: CTL

- Branching time logic for properties of systems
  - Computation Tree Logic (CTL)
- $\mathrm{EX}\varphi$, there exists a transition where $\varphi$ holds from current state
- $\mathrm{EG}\varphi$, exists a path from current state where $\varphi$ holds on all states
- $\mathrm{EU}$, exists a path until...
- Also $\mathrm{AX}$ properties, properties that hold on all possible paths from current state

# Verifying PTL and CTL invariants?

- State sequences of infinite length possible
- How do we check invariants?

# Büchi Automata

- $\omega$-automaton
  - run on infinite strings
- strings represent state sequences (actually $\lambda(q_0)\lambda(q_1)...$)
- non-deterministic as well as deterministic
  - but non-deterministic Büchi automata more powerful

# But, but SAT, Logic?

Büchi-automata have a very close relation to logic.

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# Further Reading and Links

- Stephan Merz, An Introduction to Model Checking
  - Accessible and good introduction, with links to other material
- Javier Esparza, Automata Theory: An algorithmic approach
  - See Chapters 8, 9 and 14
- Spin Model Checker
- Selected industrial applications
  - CACM, "How Amazon Web Services Uses Formal Methods"
  - CACM, "A Decade of Software Model Checking with SLAM"
- A segue into compiler verification
  - Ken Thompson, Reflections on Trusting Trust, Turing Award Lecture 1984
  - The COMPCERT project