CSC 576 - 2015 Fall: Homework 4

Submission before the class on Nov. 5

Requirements

\LaTeX generated Homework is preferred. One additional point can be obtained if your homework is created from \LaTeX. Due to the request from some students, the homework is posted online right now, but would be updated probably every week until it is formally released. (*) (**) or (***) indicates the difficulty of each question. Please submit your homework before our class on the date above. Any late submission would not be accepted no matter what reasons.

Please indicate the names of your classmates if you discuss any question with them or ask for help from them.

(*) Question 1: 1 point

Prove \( \|X\|_F = \|\text{diag}(\Sigma)\|_2 \) where the SVD of \( X \) is \( X = U\Sigma V^\top \).

(**) Question 2: 2 point

Given an arbitrary matrix \( M \), provide the solution to the following problem:

\[
\min_X \frac{1}{2} \|X - M\|_F^2 + \lambda \|X\|_*.
\]

(Hint: think about homework 3: Question 3).

(**) Question 3: 2 point

Given matrices \( A \) and \( B \) (both have full column rank), provide the solution to the following problem:

\[
\max_x \frac{\|Ax\|_\infty}{\|Bx\|}.
\]

(**) Question 4: 6 points

You are provided a data matrix \( A \in \mathbb{R}^{100 \times 300} \) and two observations: \( b_1 = Ax^*_1 \) and \( b_2 = Ax^*_2 \). Please recover

- \( x^*_1 \) (it is known to be a sparse vector)
- \( x^*_2 \) (it is known to be a stepwise vector)
print out your recovered $x_1^*$ and $x_2^*$, and briefly describe your optimization formulations. (Hint: you can formulate both into linear programing and call any “linear programing” solvers to solve it, or solve it by using CVX [http://cvxr.com/cvx/] if you are using Matlab.)