CSC 576 - 2015 Fall: Homework 3

Submission before the class on Oct. 22

Requirements

LaTeX generated Homework is preferred. One additional point can be obtained if your homework is created from LaTeX. Due to the request from some students, the homework is posted online right now, but would be updated probably every week until it is formally released. (*), (**), or (***) indicates the difficulty of each question. Please submit your homework before our class on the date above. Any late submission would not be accepted no matter what reasons.

Please indicate the names of your classmates if you discuss any question with them or ask for help from them.

(**) Question 1: 2 points

Define $\|X\|_{\infty,1}$ to be $\sum_i \|X_i\|_{\infty}$. Reformulate the following problem into linear programing problem:

$$
\min_{X \in \mathbb{R}^{n \times p}} \|X\|_{\infty,1}
$$

s.t. $AX = B$.

(**) Question 2: 2 points

For any nonnegative random variable $x$, prove the following inequality: for any $t > 0$

$$
P(x \geq t) \leq \frac{E(x)}{t}.
$$

(For simplicity, you can assume that $x$ is a continuous random variable and has the density function.)

(**) Question 3: 2 points

Provide a solution to the following optimization problem and prove it

$$
\min_{x \in \mathbb{R}^p} \frac{1}{2}\|x - t\|^2 + \lambda\|x\|_1
$$

where $\lambda > 0$.  

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(**) Question 4: 2 points

Provide a solution to the following optimization problem and prove it

\[
\min_{x \in \mathbb{R}^p} \frac{1}{2} \| x - t \|^2 + \lambda \| x \|
\]

where \( \lambda > 0 \).

(***) Question 5: 3 points

Provide a solution to the following optimization problem and prove it

\[
\min_{x \in \mathbb{R}^p} \frac{1}{2} \| x - t \|^2 + \lambda \| x \|_{\infty}
\]

where \( \lambda > 0 \).

(**) Question 6: 3 points

Read the class note [http://www.cs.rochester.edu/u/jliu/CSC-576/class-note-4.pdf](http://www.cs.rochester.edu/u/jliu/CSC-576/class-note-4.pdf) and prove why the solution to (4) solves (3). (Hint: prove by contradiction.)