CSC 252: Computer Organization Spring 2019: Lecture 3

Instructor: Yuhao Zhu

Department of Computer Science University of Rochester

Action Items:

- Trivia 1 is due tomorrow, midnight
- Main assignment due Feb. 1, midnight

- Programming Assignment 1 is out
 - Details: <u>http://cs.rochester.edu/courses/252/spring2019/</u> <u>labs/assignment1.html</u>
 - Due on Feb 1, 11:59 PM
 - Trivia due Friday, 1/25, 11:59 PM
 - You have 3 slip days (not for trivia)

20	21	22	23	24	25	26
				Today	Trivia	
27	28	29	30	31	Feb 1	2
					Due	

• TA review sessions schedule is posted

Review Session Schedule

Two 1-hour review sessions are offered each week. Review sessions are not mandatory. During review sessions, TAs might review course materials from the past week, go over problem sets and past exams, provide an overview of programming assignments, etc. They will be interactive and you are encouraged to ask questions. It is up to the TAs to decide how to run it.

Tuesday 6 PM - 7 PM, in WH 2506 (Rotating across Olivia, Jessica, Max, Sam, Yawo)

Thursday 7:30 PM - 8:30 PM, in WH 2506 (Rotating across Amir, Minh, Yiyang, Yu)

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 - If all don't know, ask me.

Previously in 252...

- Computers are built to understand bits: 0 and 1
 - 0: low (no) voltage; 1: high voltage
- Integer representations (Fixed-point really)





• Two's Complement

• Two's Complement



Unsigned	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

• Two's Complement

-4 -3 -2 -1 0 1 2 3

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• Two's Complement

-4 -3 -2 -1 0 1 2 3

 $b_2b_1b_0$

 2^{2} 2^{1} 2^{0}

Weights in Unsigned

Signed	Unsigned	Binary
0	0	000
1	1	001
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• Two's Complement

Weights in **-2² 2¹ 2⁰** Signed

Signed	Unsigned	Binary
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-4 -3 -2 -1 0 1 2 3 $b_2b_1b_0$ 2^{2} 2^{1} 2^{0} Weights in Unsigned Weights in -2² 2¹ 2⁰ Signed

 $101_2 = 1^*2^0 + 0^*2^1 + (-1^*2^2) = -3_{10}$

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- Only 1 zero
- There is (still) a bit that represents sign!
- Unsigned arithmetic still works

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+) 101 111

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-2	110
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•	Only	1	zero
---	------	---	------

- There is (still) a bit that represents sign!
- Unsigned arithmetic still works

$$\begin{array}{c} 010 \\ +) 101 \\ 111 \end{array} \begin{array}{c} 2 \\ +) -3 \\ -1 \end{array}$$

Signed	Binary
0	000
1	001
2	010
3	011
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-1	111

Only 1 z	zero		Signed	Binary
There is	(~+:11)		0	000
i nere is (still) a bit that represents sign!			1	001
Unsigned arithmetic still works			2	010
5				011
			-4	100
	010	2	-3	101
+)	101	+) -3	-2	110
	111	-1	-1	111

• 3 + 1 becomes -4 (called overflow. More on it later.)

 Suppose you want to define a variable that represents a person's age. What assumptions can you make about this variable's numerical value?

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 - Integer
 - Non-negative
 - Between 0 and 255 (8 bits)
- Define a data type that captures all these attributes: unsigned char in C
 - Internally, an **unsigned char** variable is represented as a 8-bit, non-negative, binary number

• What if you want to define a variable that could take negative values?

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 - Theoretically could be either signed-magnitude or two's complement
 - The C language designers chose two's complement
 - This is where math meets computer science

C Data Type	32-bit	64-bit
(unsigned) char	1	1
(unsigned) short	2	2
(unsigned) int	4	4
(unsigned) long	4	8

	W			
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
ТМах	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

C Data Type		32-bit	64-bit
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Data Types (in C)

			W	
	8	16	32	64
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- C Language
 - •#include <limits.h>
 - •Declares constants, e.g.,

•ULONG_MAX

•LONG_MAX

•LONG_MIN

• Values platform specific

Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
 - Summary
- Representations in memory, pointers, strings

One Bit Sequence, Two Interpretations

• A sequence of bits can be interpreted as either a signed integer or an unsigned integer

Signed	Unsigned	Binary
0	0	000
1	1	001
2	2	010
3	3	011
-4	4	100
-3	5	101
-2	6	110
-1	7	111

Signed vs. Unsigned Conversion in C

- What happens when we convert between signed and unsigned numbers?
- Casting (In C terminology)
 - Explicit casting between signed & unsigned

```
int tx, ty = -4;
unsigned ux = 7, uy;
tx = (int) ux; // U2T
uy = (unsigned) ty; // T2U
```

- Implicit casting
 - e.g., assignments, function calls

tx = ux; uy = ty;

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- Implicit casting
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tx = ux;

$$uy = ty;$$

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Mapping Between Signed & Unsigned

• Mappings between unsigned and two's complement numbers: Keep bit representations and reinterpret



Bits
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

Signed
0
1
2
3
4
5
6
7
-8
-7
-6
-5
-4
-3
-2
-1

Unsigned
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Bits
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

Signed
0
1
2
3
4
5
6
7
-8
-7
-6
-5
-4
-3
-2
-1

T2U



Bits
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

Signed	
0	
1	
2	
3	
4	
5	
6	
7	
-8	
-7	
-6	
-5	
-4	
-3	
-2	
-1	

T2U

U2T



Bits	Signed	
0000	0	
0001	1	
0010	2	
0011	3	
0100	4	
0101	5	
0110	6	
0111	7	
1000	-8	
1001	-7	
1010	-6	
1011	-5	
1100	-4	
1101	-3	
1110	-2	
1111	-1	



Bits	
0000	
0001	
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1111	





Conversion Visualized

Signed → Unsigned

- Ordering Inversion
- Negative → Big Positive

2's Complement Range





Conversion Visualized



Conversion Visualized



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The Problem

short int x = 15213; int ix = (int) x; short int y = -15213; int iy = (int) y;

C Data Type	64-bit
char	1
short	2
int	4
long	8

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- Should always be able to preserve the value, but how?

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- Converting from smaller to larger integer data type
- Should always be able to preserve the value, but how?

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	0000000 0000000 00111011 01101101
У	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 1111111 11000100 10010011

Signed Extension

- Task:
 - Given *w*-bit signed integer *x*
 - Convert it to (w+k)-bit integer with same value

Signed Extension

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- Rule:
 - Make *k* copies of sign bit:

•
$$X' = X_{W-1}, \dots, X_{W-1}, X_{W-1}, X_{W-2}, \dots, X_0$$

k copies of MSB

Signed Extension

- Task:
 - Given *w*-bit signed integer *x*
 - Convert it to (w+k)-bit integer with same value
- Rule:
 - Make k copies of sign bit:

•
$$X' = x_{w-1}, \dots, x_{w-1}, x_{w-1}, x_{w-2}, \dots, x_0$$



Another Problem

unsigned short x = 47981; unsigned int ux = x;

	Decimal	Hex	Binary
x	47981	BB 6D	10111011 01101101
ux	47981	00 00 BB 6D	0000000 0000000 10111011 01101101

Unsigned (Zero) Extension

- Task:
 - Given *w*-bit unsigned integer *x*
 - Convert it to (w+k)-bit integer with same value
- Rule:
 - Simply pad zeros:

•
$$X' = 0, ..., 0, x_{w-1}, x_{w-2}, ..., x_0$$



Yet Another Problem

int x = 53191; short sx = (short) x;

	Decimal	Hex	Binary
x	53191	00 00 CF C7	0000000 0000000 11001111 11000111
sx	-12345	CF C7	11001111 11000111

Yet Another Problem

int	х	=	53191;	
short	sx	=	(short)	x;

	Decimal	Hex	Binary
x	53191	00 00 CF C7	0000000 0000000 11001111 11000111
sx	-12345	CF C7	11001111 11000111

• Truncating (e.g., int to short)

- Can't always preserve the numerical value
- C's implementation: leading bits are truncated, results reinterpreted

Yet Another Problem

int	х	=	53191;	
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• Truncating (e.g., int to short)

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Questions?

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• Similar to Decimal Addition

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- Similar to Decimal Addition
- Suppose we have a new data type that is 3-bit wide (c.f., **short** has 16 bits)

Normal	010		2	
Cooo	+) 101	+)	5	
Lase	111		7	

Unsigned	Binary
0	000
1	001
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- Similar to Decimal Addition
- Suppose we have a new data type that is 3-bit wide (c.f., **short** has 16 bits)
- Might overflow: result can't be represented within the size of the data type

Normal	010 +) 101	2 +) 5
Case	111	7
Overflow	110	6
Case	+) 101	+) 5

Unsigned	Binary
0	000
1	001
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- Similar to Decimal Addition
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Normal	010 +) 101	2 +) 5	6 7
Case	111	7	
Overflow	110 +) 101	6 +) 5	
Case	1011	11	True Sum

Unsigned Binary

З

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- Might overflow: result can't be represented within the size of the data type

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Case	111	7	
Overflow	110 +) 101	6 +) 5	
Case	1011	11	True Sum
	011	3	Sum with

Unsigned	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

same bits

Unsigned Addition in C

Operands: w bitsuTrue Sum: w+1 bits+ vDiscard Carry: w bitsUAdd_w(u, v)
Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

 Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)

Signed	Binary
0	000
1	001
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-4	100
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 Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)

Normal	010	2
	+) 101	+) -3
Case	111	-1

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

- Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

Normal Case	010 +) 101	2 +) -3
	111	-1
Overflow	110 +) 101	-2 +) -3
Lase	1011	-5

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
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-2	110
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	111	-1
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Case	1011	-5
	011	3

Signed	Binary
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	010	2
Normal	+) 101	+) -3
Case	111	-1
Overflow	110	-2
Case	+) 101	+) -3
	1011	-5
	011	3

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111
	Signed 0 1 2 3 -4 -3 -2 -1

Min

Negative Overflow

- Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

Normal	010 +) 101	2 +) -3	Min	100 101 110
Case	111	-1	-1	111
Overflow	110 +) 101	-2 +) -3	011 +) 001	3 +) 1
Case	1011 011	-5	0100	4

Signed

0

2

З

Binary

000

001

010

011

Negative Overflow

- Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

			Min> -4	100
	010	2	-3	101
Normal	+) 101	+) -3	-2	110
Case	111	-1	-1	111
•	110	-2	011	3
Overflow	+) 101	+) -3	+) 001	+) 1
Case	1011	-5	0100	4
	011	3	100	-4

Signed

0

2

3

Binary

000

001

010

011

Negative Overflow

- Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

100
101
110
111
3
1
4
-4

Negative Overflow

Positive Overflow

Signed

0

2

3

Max

Binary

000

001

010

011

Operands: w bits

True Sum: w+1 bits

Discard Carry: w bits



Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

+) 111 +) 110 1101

+)	111 110				
1101					

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111







• This is not an overflow by definition



- This is not an overflow by definition
- Because the actual result can be represented by the bit width of the datatype (3 bits here)

















In	Out
0	1
1	0







Note: Serial structure on top, parallel on bottom.

Basic Logic Gates





















NAND

Basic Logic Gates











AND



NAND

A	В	\mathbf{C}_{in}	S	Cou
				t
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



Α	В	\mathbf{C}_{in}	S	Cou
				t
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = (~A \& ~B \& C_{in})$$



	Α	В	C _{in}	S	C _{ou}
-					t
ست	0	0	0	0	0
	0	0	1	1	0
	0	1	0	1	0
	0	1	1	0	1
	1	0	0	1	0
	1	0	1	0	1
	1	1	0	0	1
	1	1	1	1	1



Α	В	\mathbf{C}_{in}	S	C _{ou}
				t
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1
	A 0 0 1 1 1 1	 A B 0 0 0 1 0 1 0 1 1 1 1 1 1 1 1 	 A B C_{in} 0 0 0 0 1 0 1 0 1 0 1 0 1 0 1 1 1 0 1 1 1 1 	A B C _{in} S 0 0 0 0 0 0 0 1 1 1 0 1 0 1 1 0 1 0 1 1 1 0 0 1 1 1 0 1 1 0 1 1 0 1 1 1 1 1 1 1

Truth Ta	able
----------	------

	Α	В	\mathbf{C}_{in}	S	C _{ou}
					t
	0	0	0	0	0
	0	0	1	1	0
	0	1	0	1	0
	0	1	1	0	1
	1	0	0	1	0
	1	0	1	0	1
	1	1	0	0	1
	1	1	1	1	1



	Α	В	C _{in}	S	Cou
_					t
	0	0	0	0	0
	0	0	1	1	0
	0	1	0	1	0
	0	1	1	0	1
	1	0	0	1	0
	1	0	1	0	1
	1	1	0	0	1
	1	1	1	1	1

$$S = (~A \& ~B \& C_{in})$$

$$| (~A \& B \& ~C_{in})$$

$$| (A \& ~B \& ~C_{in})$$

$$| (A \& B \& C_{in})$$

$$C_{ou} = (~A \& B \& C_{in})$$

$$| (A \& ~B \& C_{in})$$



A	В	\mathbf{C}_{in}	S	\mathbf{C}_{ou}
				t
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Add two bits and carry-in, produce one-bit sum and carry-out.

C_{ou} = (~A & B & C_{in}) | (A & ~B & C_{in}) | (A & B & ~C_{in}) | (A & B & C_{in})



Add two bits and carry-in, produce one-bit sum and carry-out.

C_{ou} = (~A & B & C_{in}) | (A & ~B & C_{in}) | (A & B & ~C_{in}) | (A & B & C_{in})


Add two bits and carry-in, produce one-bit sum and carry-out.

Cou = (~A & B & Cin) | (A & ~B & Cin) | (A & B & ~Cin) | (A & B & Cin)



Add two bits and carry-in, produce one-bit sum and carry-out.

Cou = (~A & B & Cin) | (A & ~B & Cin) | (A & B & ~Cin) | (A & B & Cin)



Add two bits and carry-in, produce one-bit sum and carry-out.





Add two bits and carry-in, produce one-bit sum and carry-out.















- Ripple-carry Adder
 - Simple, but performance linear to bit width



- Ripple-carry Adder
 - Simple, but performance linear to bit width
- Carry look-ahead adder (CLA)
 - Generate all carriers simultaneously



• Design digital components from basic logic gates

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- Key idea: use the truth table!

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• Goal: Computing Product of *w*-bit numbers *x*, *y*

Original Number (w bits)















- Goal: Computing Product of *w*-bit numbers *x*, *y*
- Exact results can be bigger than w bits
 - Up to 2w bits (both signed and unsigned)

Original Number (w bits)

Product (2w bits)



Unsigned Multiplication in C



- Standard Multiplication Function
 - Ignores high order w bits
- Implements Modular Arithmetic $UMult_w(u, v) = u \cdot v \mod 2^w$

Signed Multiplication in C

Operands: <i>w</i> bits			И	• • •	
		*	V	• • •	
True Product: 2*w bits	<i>u</i> · <i>v</i>	••		• • •	
Discard w bits: w bits				• • •	

• Standard Multiplication Function

- Ignores high order w bits
- Some of which are different for signed vs. unsigned multiplication
- Lower bits are the same

Power-of-2 Multiply with Shift

- Operation
 - u << k gives u * 2^k
 - 001₂ << 2 = 100₂ (1 * 2² = 4)
 - Both signed and unsigned

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- Most machines shift and add faster than multiply
 - Compiler generates this code automatically
 - u << 3 == u * 8
 - (u << 5) (u << 3) == u * 24

- Implement power-of-2 divide with shift
 - u >> k gives $\lfloor u / 2^k \rfloor$ ($\lfloor 2.34 \rfloor = 2$)
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- $234_{10} >> 2 = 2.34_{10}$, truncated result is 2 ($\lfloor 2.34 \rfloor = 2$)
- $1101_2 >> 2 = 0011_2$ (true result: 11.01_2 . $\lfloor 13 / 4 \rfloor = 3$)

Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
 - Summary
- Representations in memory, pointers, strings

Byte-Oriented Memory Organization

- Programs refer to data by address
 - Conceptually, envision it as a very large array of bytes: byte-addressable
 - In reality, it's not, but can think of it that way
 - An address is like an index into that array
 - and, a pointer variable stores an address

Machine Words



- Any given computer has a "Word Size"
 - Nominal size of a memory address
 - Until recently, most machines used 32 bits (4 bytes) as word size
 - Limits addresses to 4GB (2³² bytes)
 - Increasingly, machines have 64-bit word size
 - Potentially, could have 18 EB (exabytes) of addressable memory
 - That's 18.4 X 1018

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Example Data Representations (in Bytes)

Word Size	4	8
C Data Type	32-bit	64-bit
char	1	1
short	2	2
int	4	4
long	4	8
float	4	4
double	8	8
pointer	4	8

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double	8	8
pointer	(4)	(8)

Word-Oriented Memory Organization

- Addresses Specify Byte Locations
 - Address of first byte in word
 - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



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0x100	0x101	0x102	0x103	

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Representing Integers



Hex: FFFFC493	
---------------	--

int B = -15213;



Representing Integers



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Announcement

- Check the course website before asking
 - http://www.cs.rochester.edu/courses/252/spring2019/
- Direct ALL questions regarding assignments to the TAs
 - They have done them. They have debugged them. They know them inside out.
 - If one doesn't know, ask another.
 - If all don't know, ask me.